$$Z \in \{0,1\}^{T \times N \times N}$$

$$A \in \mathbb{R}^{T \times N \times N}$$

$$p(Z) = \prod_t \prod_{i<j} p(z_{ij}^t)$$

$$= \prod_t \prod_{i<j} \mu_i^T \mu_j^{\,z_{ij}^t} \, (1 - \mu_i^T \mu_j)^{1 - z_{ij}^t}$$

$$p(A \mid Z, \theta) = \prod_t \prod_{i<j} \mathrm{Pois}(A_{ij}^t ; \lambda_{ij})^{z_{ij}^t} \, \delta(A_{ij}^t)^{1 - z_{ij}^t}$$

Loss function to optimize:

$$\mathcal{L}(q, \theta, \mu) = \sum_{z} q(z) \log \frac{p(z, \theta \mid A)}{q(z)}$$

$$q(z) = \frac{p(z, \theta \mid A)}{\sum_{z} p(z, \theta \mid A)}$$

Updates on theta stay the same!

Updating μ $\qquad \mu \in \mathbb{R}^{N \times \tilde{K}}$

$$\mathcal{L}(q, \theta, \mu) \propto \sum_z q(z) \log p(z)$$

$$\log p(z)$$

$$= \log \left(\prod_t \prod_{i<j} \mu_i^T \mu_j^{\,z_{ij}^t} (1 - \mu_i^T \mu_j)^{1 - z_{ij}^t} \right)$$

$$= \sum_t \sum_{i<j} z_{ij}^t \log(\mu_i^T \mu_j)$$

$$+ (1 - z_{ij}^t) \log(1 - \mu_i^T \mu_j)$$

$$\Rightarrow \mathcal{L} \propto \sum_{z} q(z) \sum_{t} \sum_{i<j} z_{ij}^{t} \log(\mu_i^T \mu_j)$$

$$+ (1 - z_{ij}^{t}) \log(1 - \mu_i^T \mu_j)$$

$$= \sum_{t} \sum_{i<j} \log(\mu_i^T \mu_j) \; Q_{ij}^{t}$$

$$+ \log(1 - \mu_i^T \mu_j) \; (1 - Q_{ij}^{t})$$

$$= \sum_{i<j} \log(\mu_i^T \mu_j) \left(\sum_{t} Q_{ij}^{t} \right)$$

$$+ \log(1 - \mu_i^T \mu_j) \left(T - \sum_{t} Q_{ij}^{t} \right)$$

$$\frac{\partial}{\partial \mu_{kl}} \mathcal{L}(\mu) = \sum_{i<j} \left(\sum_t Q_{ij}^t\right) \frac{\partial}{\partial \mu_{kl}} \log(\mu_i^T \mu_j)$$

$$+ \left(T - \sum_t Q_{ij}^t\right) \frac{\partial}{\partial \mu_{kl}} \log(1 - \mu_i^T \mu_j)$$

$$= \sum_{i<j} \left(\sum_t Q_{ij}^t\right) \frac{\delta_{k,i}\,\mu_{j,l} + \delta_{k,j}\,\mu_{i,l}}{\mu_i^T \mu_j}$$

$$+ \left(T - \sum_t Q_{ij}^t\right) \frac{\delta_{k,i}\,\mu_{j,l} + \delta_{k,j}\,\mu_{i,l}}{1 - \mu_i^T \mu_j}$$

$$= \sum_{i \neq k} \mu_{il} \left(\frac{\sum_t Q_{ik}^t}{\mu_i^T \mu_k} + \frac{T - \sum_t Q_{ij}^t}{1 - \mu_i^T \mu_k} \right) \overset{!}{=} 0$$

$/\partial \mu_{kl}$

<u>Different notation in code:</u>

$$\partial \mu_{ik} \quad \begin{matrix} k \to i \\ l \to k \\ i \to j \end{matrix} \quad \sum_{j \neq i} \mu_{jk} \; \frac{\sum_t Q_{ji}^t}{\mu_j^T \mu_i} + \frac{T - \sum_t Q_{ji}^t}{1 - \mu_j^T \mu_i} \Big)$$

$$\mu_j^T \mu_i = \sum_l \mu_{jl} \cdot \mu_{il}$$

$$= \mu_{jk} \mu_{ik} + \sum_{l \neq k} \mu_{jl} \mu_{il}$$

$$Q_{ij}^t = p(z_{ij}^t = 1 \mid \theta, A)$$

$$= p(z_{ij}^t = 1 \mid \theta, A_{ij}^t, A_{ji}^t)$$

$$= \frac{p(z_{ij}^t = 1)\, p(A_{ij}^t \mid \underbrace{z_{ij}^t}_{=1}, \theta)\, p(A_{ji}^t \mid \underbrace{z_{ij}^t}_{=1}, \theta)}{\sum_z \quad \ldots}$$

$$\frac{\mu_i^T \mu_j \quad \mathrm{Pois}(A_{ij}^t; \lambda_{ij})\ \mathrm{Pois}(A_{ji}^t; \lambda_{ji})}{\mu_i^T \mu_j \mathrm{Pois}(A_{ij}^t)\ \mathrm{Pois}(A_{ji}) + (1 - \mu_i^T \mu_j)\, \delta(A_{ij}^t)\, \delta(A_{ji}^t)}$$