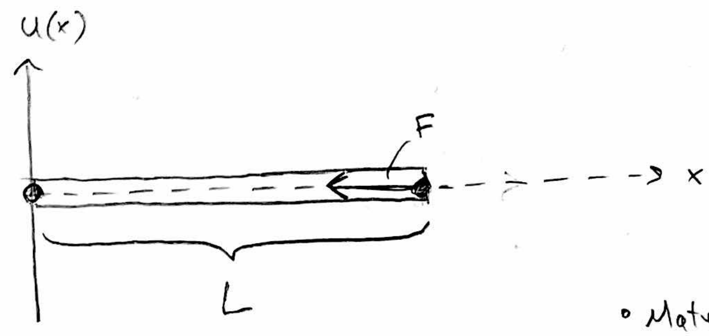
# The physics example in Pr. 2: "Buckling beam"

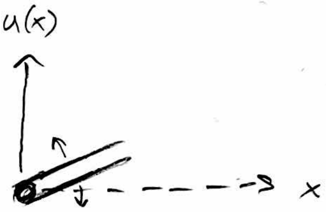



- Material properties collected in constant $\gamma$

- If $F$ is large enough, the configuration is unstable, so any tiny perturbation will cause the beam to buckle (bend) in some shape. What shapes are possible?

- Boundary conditions:
  - $u(0) = 0$
  - $u(L) = 0$

- Pin endpoints:




$\Rightarrow$ We can have solutions with $\frac{du}{dx} \neq 0$ at $x=0$ and $x=L$.

- No dynamics here! (No time-dep) We're looking for the static beam shapes that are theoretically allowed under these conditions.

- Diff. eq:

$$\boxed{\gamma \frac{d^2 u(x)}{dx^2} = - F u(x)}$$

- Scale eq. to use dimensionless $\hat{x} \equiv \frac{x}{L}$
- Define new notation, $\lambda_c = \frac{FL^2}{\gamma}$
- Discretize with $n$ steps ($n+1$ points, including endpoints)

$N = n-1$ interior points

- Result (you'll see this in Pr 7 introduction + problem 7)

Eigenvalue problem

$$\boxed{A\bar{v} = \lambda \bar{v}}$$

- $A$ is $N \times N = (n-1)\times(n-1)$
- $\lambda = \lambda(h)$, should go to $\lambda_c$ in the limit $\lim_{h\to 0} \lambda(h) = \lambda_c$

where $\bar{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_{n-1} \end{bmatrix}$

and elements $v_i$ are approx. to exact $u_i = u(\hat{x}_i)$

- Complete solution $\bar{v}^* = [v_0, \underbrace{\underbrace{v_1, v_2, \dots, v_{n-1}}_{N \text{ points}}, v_n}_{\substack{n+1 \text{ points} \\ (N+2 \text{ points})}}]$

- For each solution $\bar{v}^{(i)}$ a corresponding eigenvalue $\lambda^{(i)}$, which in the cont. limit ($h \to 0$, or $n \to \infty$) eigenvalues $\lambda_c^{(i)}$ and eigenfunctions $u^{(i)}(\hat{x})$ of the cont. differential equation.

- Now $A$ is given by

$$A = \begin{bmatrix} \frac{2}{h^2} & -\frac{1}{h^2} & 0 & \dots \\ -\frac{1}{h^2} & \frac{2}{h^2} & -\frac{1}{h^2} & \dots \\ 0 & -\frac{1}{h^2} & \frac{2}{h^2} & -\frac{1}{h^2} \\ \vdots & & & \end{bmatrix} = \frac{1}{h^2}\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ \vdots & & & \end{bmatrix}$$

General case:

$$\left[\begin{matrix}\text{Some general} \\ \text{operator}\end{matrix}\right] u(x) = \lambda u(x)$$

$$\left[\frac{d^2}{dx^2} + \dots\right] u(x) = \lambda u(x)$$

- Note that $\frac{1}{h^2}$ is kept on the left-hand side. Easier to not get confused in the general case