2D Ising model, critical exponent $\alpha = 0$?

- For the infinite 2D Ising model we say that for T near T_c, we have function behaviours

$$\langle |m| \rangle \propto |T - T_c|^{\beta} \quad , \quad \beta = 1/8$$

$$C_V \propto |T - T_c|^{-\alpha} \quad , \quad \alpha = 0$$

$$\chi \propto |T - T_c|^{-\gamma} \quad , \quad \gamma = 7/4$$

critical exponents

- What does $C_V \propto |T - T_c|^{-0}$ mean? That C_V is constant ?
- No, $\alpha = 0$ is just the "power-law way" of saying that C_V diverges logarithmically as $T \to T_c$, i.e. slower than $|T - T_c|^{-\alpha}$ for any $\alpha > 0$. So $C_V \propto \ln\left|\frac{T - T_c}{T_c}\right|$ for $T \to T_c$.

- In general: Let $\tilde{\tau} \equiv \frac{T - T_c}{T_c}$. If $f(\tilde{\tau})$ behaves as

$$f(\tilde{\tau}) \propto \tilde{\tau}^{c} \quad \text{when } \tilde{\tau} \to 0$$

then c is the critical exponent. Can find it as

$$\text{crit. exp} = \lim_{\tilde{\tau} \to 0^+} \frac{\ln|f(\tilde{\tau})|}{\ln \tilde{\tau}} = \lim_{\tilde{\tau} \to 0^+} \frac{c \ln \tilde{\tau}}{\ln \tilde{\tau}} = \underline{\underline{c}} \quad , \quad c > 0$$

- Consider case where $f(\tilde{\tau}) \propto \ln|\tilde{\tau}|$

$$\text{crit. exp.} = \lim_{\tilde{\tau} \to 0^+} \frac{\ln|\ln|\tilde{\tau}||}{\ln \tilde{\tau}} = \text{"}\frac{\ln(\infty)}{\infty}\text{"} \overset{\text{L'Hopital}}{\Longrightarrow} \underline{\underline{0}}$$

So $\underline{C_V \propto \ln \tilde{\tau}}$ corresponds to $\underline{C_V \propto \tilde{\tau}^{\alpha} \text{ with } \alpha = 0}$