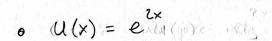
An example error analysis



- o We will use $u_{i+1}-2u_i+u_i$ to approximate the true second der. u_i'' at a point x_i
- · Question & How does the relative error depend on our choice of stepsize h?

e We know the exact answer!
$$u''_i = 4e^{x_i}$$

· Absolute error:

$$\Delta(h) = \left| approx - true \right| = \left| \frac{u_{i+1} - 2u_i + u_{i-1}}{h^2} - u_i'' \right|$$

o Relative euror: $\mathcal{E}(\mathcal{U}) = \left| \frac{\alpha p prox-tune}{tune} \right| = \left| \frac{\Delta(\mathcal{U})}{u_i} \right|$

o Let's model the absolute error as a sum of two contributions:
truncation error and round-off error

$$\Delta(h) = \Delta_{trunc}(h) + \Delta_{RO}(h)$$

The exact
$$u_i^{"}$$

$$\Delta t_{nuc}(h) = \left| \frac{\left(u_{i+1} - 2u_i + u_{i-1}}{h^2} \right) - \left(\frac{u_{i+1} - 2u_i + u_{i-1}}{h^2} + \mathcal{O}(u_i^{c_0} h^2) \right) \right|$$

First look at $\Delta t_{nuc}(h) = \left| \mathcal{O}(u_i^{c_0} h^2) \right|$

$$\Delta t_{nuc}(h) = \left| \mathcal{O}(u_i^{c_0} h^2) \right|$$

First Lectures

o what we want to compute:
$$u_{i+1} - 2u_i + u_{i-1}$$

$$h^2$$

· what we actually compute is something like this ...

for
$$\frac{\int f\left(f\left(f\left(u_{i+1}\right) - f\left(u_{i}\right)\right) - f\left(f\left(u_{i}\right) - f\left(u_{i-1}\right)\right)\right)}{f\left(f\left(u_{i}\right) + f\left(u_{i}\right)\right)}$$

o Consider limit of small hand focus on the subtractions of near identical numbers

o We ran extraste an upper bound

$$fl(u_{i+1}) - fl(u_i) \leq u_{i+1}(1+\delta_m) - u_i(1-\delta_m)$$

$$= (u_{i+1} - u_i) + (u_{i+1} + u_i)\delta_m$$

· In the limit h→ o, i.e. when u;+, → u;

$$fl(u_{i+1}) - fl(u_i) = O(u_i S_m)$$

$$\longrightarrow does \underline{uot} \ go \ to \ zero$$
(Similar contr. from $fl(u_i) - fl(u_{i-1})$) when $u \to 0$

6 Estimate round-off evror in $\frac{u_{i+1}-2u_i+u_{i-1}}{h^2}$ to be $\Delta_{RO}(h) = O\left(\frac{u_i \delta_m}{h^2}\right)$

o Putting it together :

$$\Delta(h) = \left| \Delta_{tounc}(h) + \Delta_{Ro}(h) \right|$$

$$= \left| O(u_i^{(4)}h^2) + O(u_i \frac{\delta_m}{u^2}) \right|$$

o Relative error:

$$\mathcal{E}(h) = \left| \frac{\Delta(h)}{u_i''} \right| = \left| \mathcal{O}\left(\frac{u_i^{(4)}}{u_i''} h^2\right) + \mathcal{O}\left(\frac{u_i \delta_{m}}{u_i''} \frac{1}{h^2}\right) \right|$$

$$\Rightarrow grows when h \Rightarrow \infty$$

In our particular case, with
$$u(x) = e^{2x}$$
, we also have that $O(u_i) \approx O(u_i^n) \approx O(u_i^n)$

· Logio of relative error:

(ook at the limits
$$h \rightarrow 0$$
 and $h \rightarrow \infty$:
$$\log_{10}(\mathcal{E}(h)) \approx \begin{cases} -2\log_{10}(h) + \log_{10}(C_2) & \text{for } h \rightarrow 0 \\ \log_{10}(h) \rightarrow -\infty \end{cases}$$

$$2\log_{10}(h) + \log_{10}(C_1) & \text{for } h \rightarrow \infty$$

$$\log_{10}(h) \rightarrow \infty$$

gradient: -2
$$\frac{\log_{10}(E)}{\text{gradient}}$$
 $\frac{\log_{10}(E)}{\text{gradient}}$ $\log_{10}(A)$