Binary representation

> A bit is not neces. related to computers. concept from information theory

- Basic element : a bit 1/0 on/off , true/false , ...
- This gives us two digits (0,1) we can represent physically, so better use a number sys. that only uses two different digits

  $\Rightarrow$ Binary (base 2) system (fewer different digits, used more places)

- Integers

Example: 137 in base 10 and base 2

$$(137)_{10} = (10001001)_2$$

- $$(137)_{10} = \frac{10^2 \quad 10^1 \quad 10^0}{1 \quad 3 \quad 7} = (1\times10^2) + (3\times10^1) + (7\times10^0)$$
$$= 100 + 30 + 7$$

- $$(10001001)_2 = \frac{2^7\, 2^6\, 2^5\, 2^4\, 2^3\, 2^2\, 2^1\, 2^0}{1\;0\;0\;0\;1\;0\;0\;1} = (1\times2^7) + (0\times2^6) + \ldots + (1\times2^3) + \ldots + (1\times2^0)$$
$$= 128 + 0 + 0 + 0 + 8 + 0 + 0 + 1$$

Easy way to find rep:

Integer division and record remainders

	Remainder	Position	
137\2 = 68	1	2^0	(1×2^0)
68\2 = 34	0	2^1	
34\2 = 17	0	2^2	
17\2 = 8	1	2^3	(1×2^3)
8\2 = 4	0	2^4	
4\2 = 2	0	2^5	
2\2 = 1	0	2^6	
1\2 = 0	1	2^7	(1×2^7)

> The more bits (0's and 1's) we use, the larger the integer we can store!

> + One bit for the sign! $(-1)^0$ or $(-1)^1$

Floating point numbers

- How to represent real numbers ($\mathbb{R}$) in binary?
- Effectively need three pieces of info
  - The sign
  - The digits appearing in the number
  - The location of the point

  Ex: $\boxed{-235.713664}$

- Strategy: use <u>scientific notation in base 2</u>

- In decimal (base 10):

$$-9.90625 \times 10^{0} \qquad -0.990625 \times 10^{1}$$

[integer exp.]

General: $\pm \left[\text{number in } (\tfrac{1}{10}, 1)\right] \times 10$

- In binary (base 2):

$$\boxed{\pm \left[\text{number in } (\tfrac{1}{2}, 1)\right] \times 2^{[\text{integer exp}]}}$$

- Names: $[\text{sign}] \times [\text{mantissa}] \times 2^{[\text{exponent}]}$

- Binary repr. of mantissa

$$\overset{2^0\;2^{-1}\;2^{-2}\;2^{-3}\;2^{-4}}{(0.1001)_2} = (0 \times 2^0) + (1 \times 2^{-1}) + (0 \times 2^{-2}) + (0 \times 2^{-3}) + (1 \times 2^{-4})$$

$$= (0 + 0.5 + 0 + 0 + 0.0625)_{10}$$

$$= (0.5625)_{10}$$

- "Single precision" : 32 bits in total (4 bytes)
  - Sign : 1 bit
  - Exponent : 8 bits
  - Mantissa : 23 bits

Ex: $-3.25 = (-1)^{①} \times (0.8125) \times 2^{②}$

Different standards exist!

In memory, something like this:

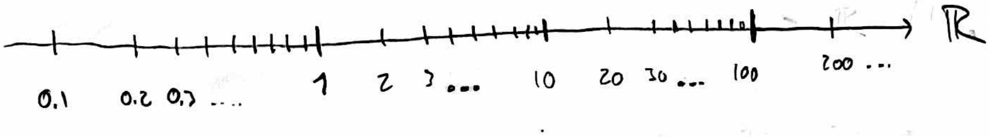
Sign bit	8-bit exponent (2)	23-bit mantissa (0.8125)
1	00000010	01101000 000000

- "Double precision" : 64 bits (8 bytes)
  - Sign : 1 bit
  - Exp : 11 bits
  - Mant. : 52 bits

11-bit exponent
↓
exponent range (−1024, 1024)
since $2^{11} = 2048$
↓
$2^{1024} \approx 10^{308}$, so
max range is $\sim (10^{-308}, 10^{308})$

- Source of unavoidable problems
  - Limited number of bits for exp. ⇒ limited range of $\mathbb{R}$ can be repr. $\sim (10^{-308}, 10^{308})$
  - Limited number of bits for mantissa ⇒ limited "resolution" in representation of the cont. $\mathbb{R}$ (only discrete set can be written exact) (~ 15 digits in decimal syst.)

- Intuitive example:
  - Base 10
  - Assume we only had memory for 1 digit in exp. and one digit in mantissa
  - Could repr. numbers ..., $1\times10^{-1}, 2\times10^{-1}, \ldots, 9\times10^{-1}, 1\times10^{0}, 2\times10^{0}, 3\times10^{0}, \ldots,$
  - Range: 10^{-9} to 10^{9}



Only numbers we could use!