Probability

Question: How many have done a course on prob. and/or statistics?

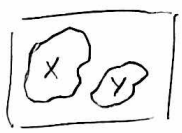
- [Discuss coin flip or dice throw]
- What does the statement $P(X) = 10\%$ mean?
- We don't know! Or at least, we don't agree...
- [Show article "Interpretations of Probability", Stanford Encyclopedia of Philosophy]
- Bertrand Russel, 1929: "Probability is the most important concept in modern science, especially as nobody has the slightest notion what it means"
- Two main interpretations
  - Frequentist: $$P(X) \equiv \lim_{n \to \infty} \frac{n_x}{n}$$ Probability defined as long-run relative frequency
  - Bayesian: $P(X) \equiv$ degree of belief / knowledge that X is true
    - Subjective Bayesian
    - Objective Bayesian

Bruno Di Finetti: PROBABILITY DOES NOT EXIST "Theory of Prob", 1974

Reasoning under uncertainty

- Both satisfy the Kolmogorov axioms that defines the mathematical prop. of the function $P(X)$ $\Rightarrow$ Use the same math!

Basically: $0 \leq P(x) \leq 1$



P is additive: $P(X \cup Y) = P(X) + P(Y)$ when $X \cap Y = \emptyset$

- Has several important consequences:

  - Different prob interpretations give rise to different approaches to statistics.

    Example:

    - Bayesian statistics can ask

      $P(\text{parameter value} \mid \text{data})$?

    - Frequentist inference can not ask this question, since probability of a parameter value does not make sense. (Can only ask questions related to repeatable trails.)

  - What does <u>randomness</u> mean?
    - what's the connection between randomness and probability?
    - Is anything random (⇒ metaphysics, determinism, ...)
      ↳ <u>Apparent randomness</u> vs <u>true randomness</u>.
  - Probabilities in physics — what do they mean?

    In particular: Interpretations of quantum mechanics.

No <u>necessary</u> link between probabilities and randomness! Can simply use prob. to express our uncertainty!

↳ Example: statistical physics (E.T. Jaynes)