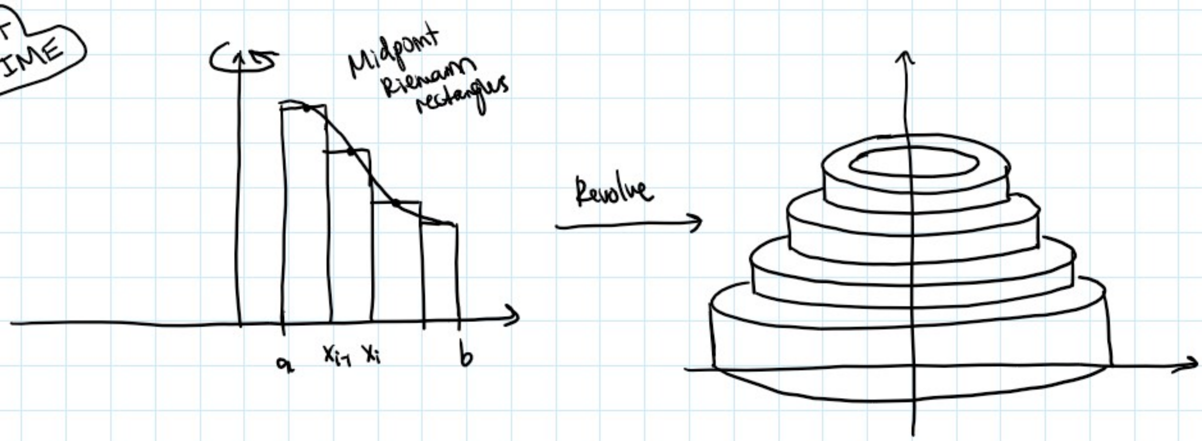
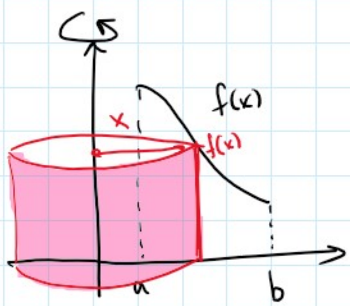
February 17, 2023 11:17 AM

LAST TIME

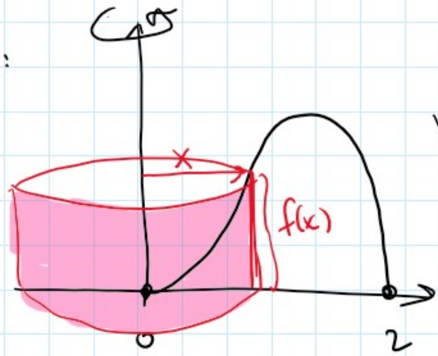




$$V = \int_a^b \underbrace{(2\pi x)}_{\substack{\text{circumference} \\ \text{(radius x)}}} \overbrace{f(x)}^{\text{height}} \underbrace{dx}_{\text{thickness.}}$$

<u>Ex</u>: Find the volume of the SoR obtained by revolving about the y axis the region bounded by $y = 2x^2 - x^3$ and $y = 0$.

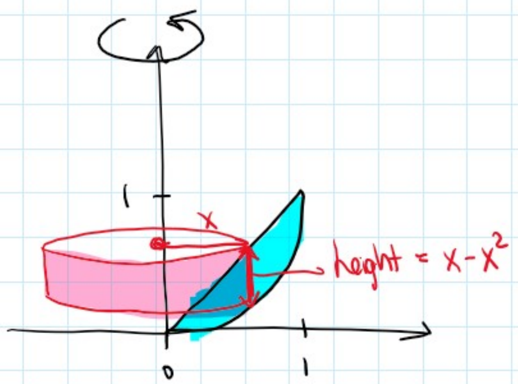
Sol:



$$V = \int_0^2 2\pi x\,(2x^2 - x^3)\,dx$$
$$= 2\pi \int_0^2 (2x^3 - x^4)\,dx$$
$$= \dots$$
$$= \frac{16\pi}{5}$$

Ex 2 Find the volume of the solid obtained by revolving about the y-axis the region between $y=x$ and $y=x^2$

Sol:

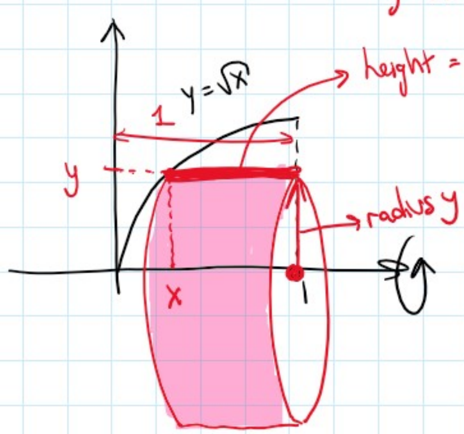


$$V = \int_0^1 (2\pi x)(x - x^2)\,dx$$
$$= 2\pi \int_0^1 (x^2 - x^3)\,dx$$
$$= \frac{\pi}{6}$$

Ex 3: Use cylindrical shells to find the volume of the solid obtained by rotating about the x-axis the region under $y=\sqrt{x}$ from $x=0$ to $x=1$.

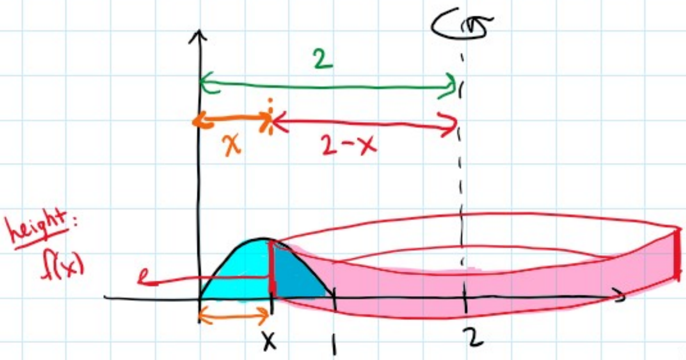
$y=\sqrt{x}$ means $y^2 = x$.

Sol:



$$V = \int_0^1 (2\pi y)(1-y^2)\,dy$$
$$= 2\pi \int_0^1 (y - y^3)\,dy$$
$$= \dots$$
$$= \frac{\pi}{2}$$

Minute Math / Ex 4: Rotate the region bounded by $y = x - x^2$ and $y = 0$ around the line $x = 2$ and calculate the volume of the resulting solid.



circumference: $2\pi(2-x)$

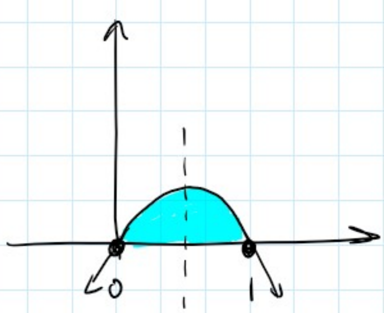
$$V = \int_0^1 \overbrace{\left(2\pi(2-x)\right)}^{\text{circumference}} \underbrace{\left(x - x^2\right)}_{\text{height}} \overbrace{dx}^{\text{thickness}}$$

$$= \dots$$

$$= \frac{\pi}{2}$$

How did we get the region:

$$x - x^2 = 0$$
$$x(1-x) = 0$$
$$\hookrightarrow x = 0, \quad x = 1$$



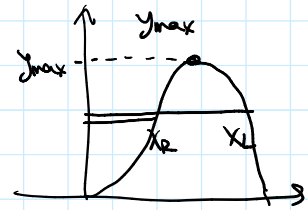
When to use Disk/washers and when to use cylindrical shells?

Main Q: Ask yourself, is the region better described by top/bottom boundaries $\left(\overset{y_T, y_B}{y = f(x)}\right)$ or right/left boundaries $\left(\underset{x_L, x_R}{x = g(y)}\right)$

Sub questions

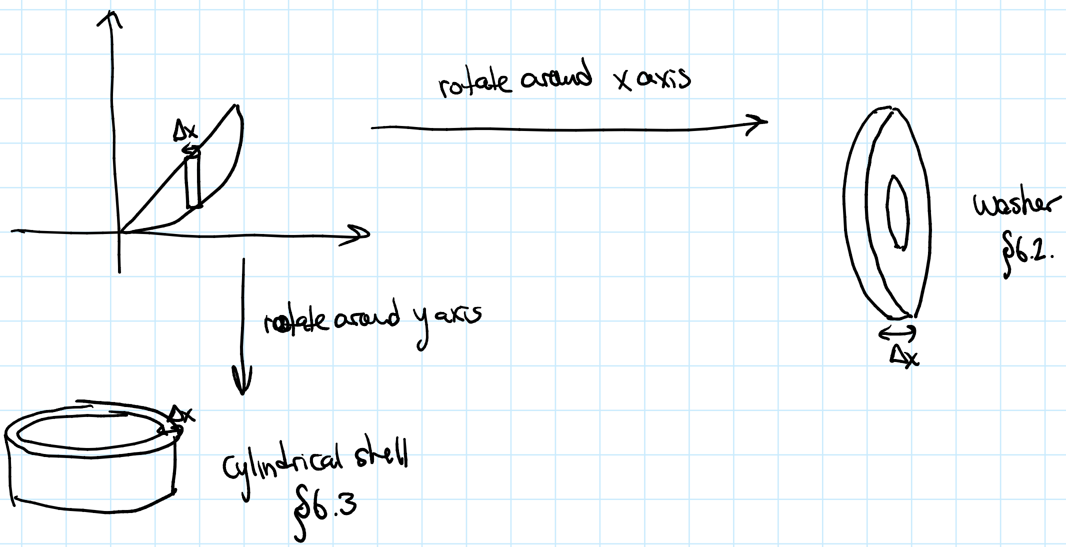
x_L, x_R

Sub questions
- ↳ which choice is easier to work with
- ↳ which has easier bounds of integration?
    - ↳ Do I need to split the integration into multiple pieces?
    - ↳ Do I need to find a maximum/minimum.
- ↳ Are the resulting integrals easier to work with.

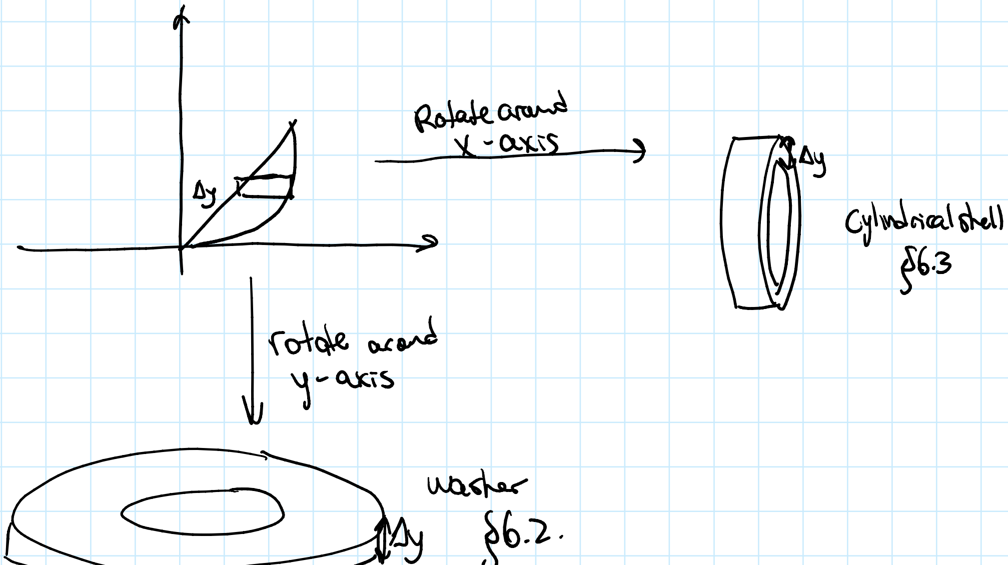


Once you've answered the main question, draw a simple rectangle:

if integrating in x:



If integrating in y:



We're done §6.3. Let's do one more §6.2 example which isn't a SoR.

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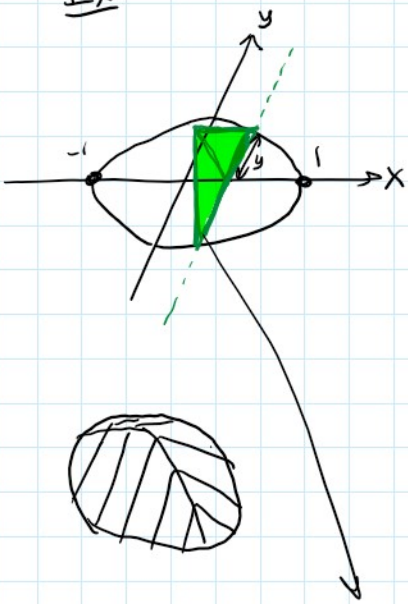
Ex:

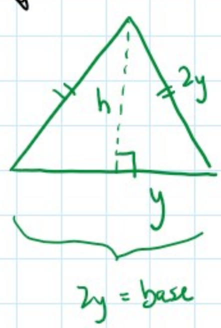


This solid has a circle for it's base, and parallel cross sections to the base are equilateral triangles.

we learned that the volume of this shape is

$$V = \int_{-1}^{1} A(x)\,dx$$

$A(x)$ is the area of the cross section, ie the area of this triangle.



$$A(x) = \tfrac{1}{2} h (2y) \qquad h = \sqrt{(2y)^2 - y^2} = \sqrt{4y^2 - y^2} = \sqrt{3}\,y$$

$$= hy$$

$$= \sqrt{3}\,y^2 \longleftarrow y^2 + x^2 = 1 \text{ on the circle}$$

$$= \sqrt{3}(1 - x^2)$$

$$V = \int_{-1}^{1} \sqrt{3}(1-x^2)\,dx = 2\int_{0}^{1} \sqrt{3}(1-x^2)\,dx = \cdots = \frac{4\sqrt{3}}{3}$$