Problem 2(b) (i)

Proof. In order to prove that $f^{-1}(A \bigcap B) = f^{-1}(A) \bigcap f^{-1}(B)$, we first show that $L.H.S \subseteq R.H.S$.

(1)

- $L.H.S \subseteq R.H.S$

Suppose $x \in f^{-1}(A \bigcap B)$, by definition, then $f(x) \in A \bigcap B$, which is $f(x) \in A \wedge f(x) \in B$, that is $x \in f^{-1}(A) \wedge x \in f^{-1}(B)$, the same as $x \in f^{-1}(A) \bigcap x \in f^{-1}(B)$. So $L.H.S \subseteq R.H.S$

(2)

- $R.H.S \subseteq L.H.S$

Suppose $x \in f^{-1}(A) \bigcap f^{-1}(B)$, by definition, then $x \in f^{-1}(A) \wedge x \in f^{-1}(B)$, then it is the same as $f(x) \in A \wedge f(x) \in B$, so $f(x) \in A \bigcap B$, in this case, $x \in f^{-1}(A \bigcap B)$. So $R.H.S \subseteq L.H.S$

Based on the conclusions on (1) and (2), $f^{-1}(A \bigcap B) = f^{-1}(A) \bigcap f^{-1}(B)$. □

Problem 5

Proof. Notice that $x = x + y - y, y = y + x - x$, So

$$|x + x| = |x - y + x + y| \tag{1}$$

$$2|x| \leq |x - y| + |x + y| \quad \textit{By triangle inequality} \tag{2}$$

We can do the same operation on y

$$|y + y| = |y - x + y + x| \tag{3}$$

$$2|y| \leq |y - x| + |y + x| \quad \textit{By triangle inequality} \tag{4}$$

Then, by the properties of absolute values, we know that $|x - y| = |y - x|$

We can do (1)+(2), then we will get

$$2|x| + 2|y| \leq 2|x - y| + 2|x + y|$$

By substracting 2 on both sides, we get

$$|x| + |y| \leq |x - y| + |x + y|, \quad \text{which is exactly what we need to prove.}$$

$\square$