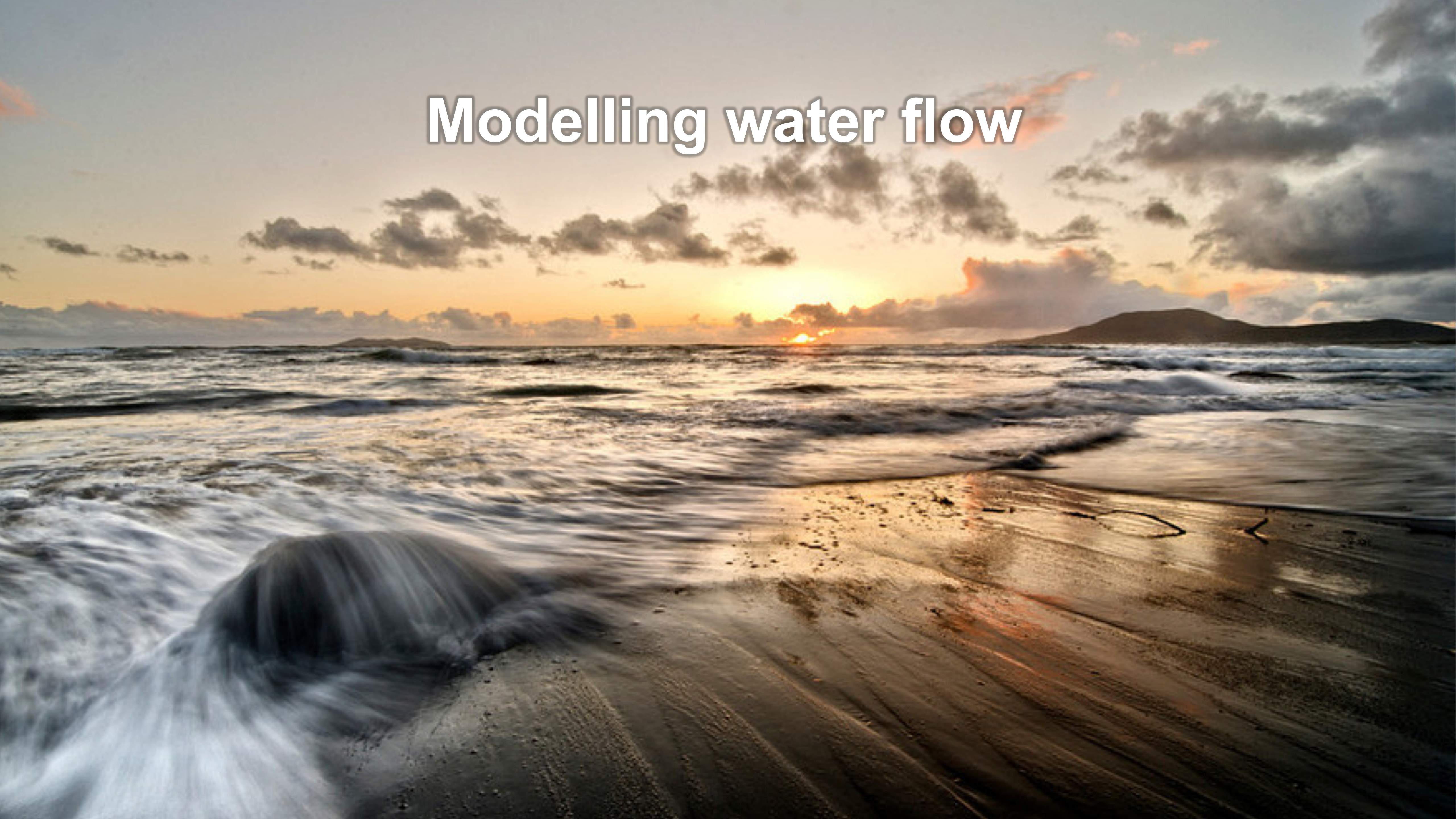
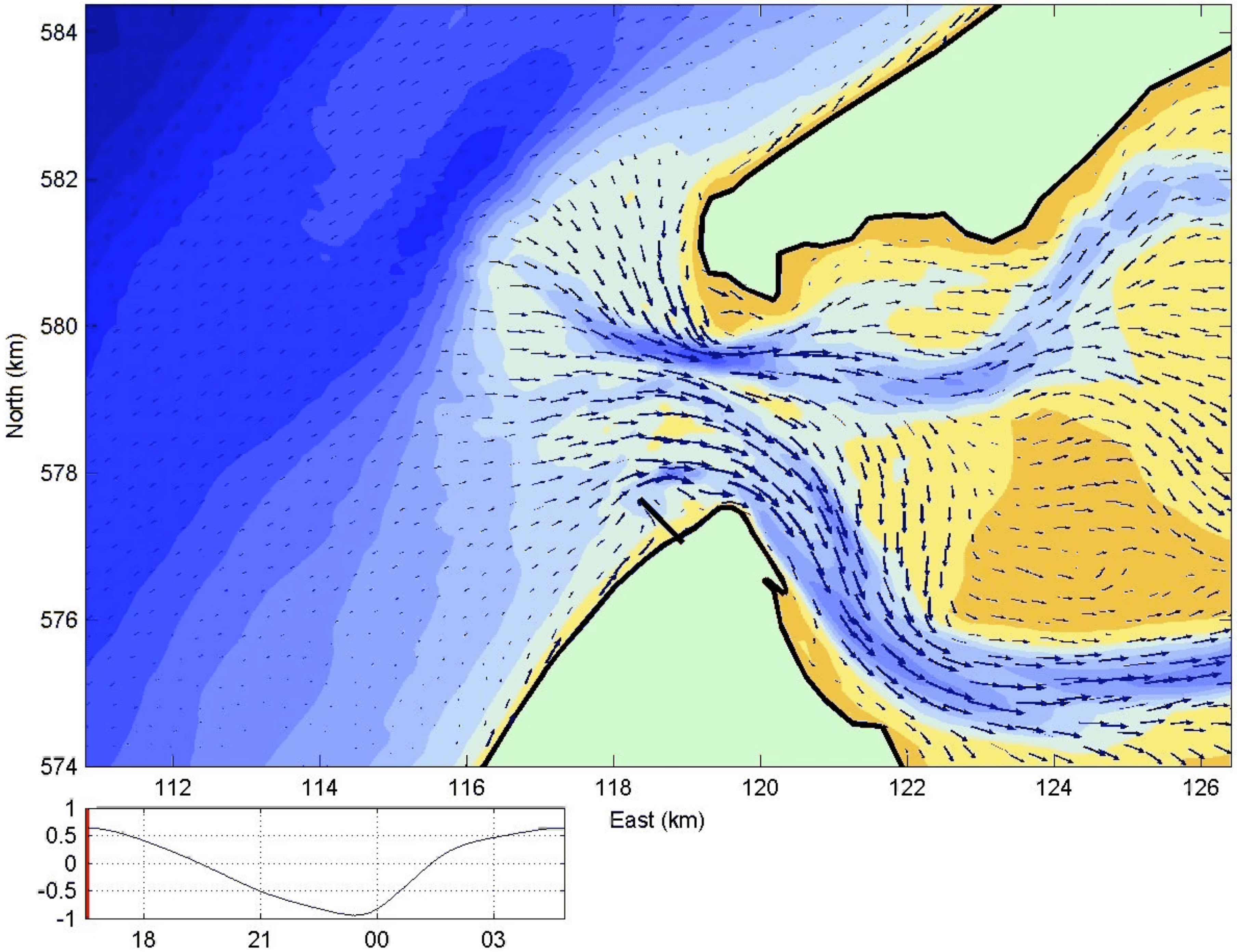

Modelling water flow

Why understand water flow?

- Marine biology occurs in water.
- Water flow is more influential for marine systems than air flow is to terrestrial systems.
- Interactions with hydrodynamics are important.
- In many applied modeling project, hydrodynamics dominates because it is more predictable.



584
582
580
578
576
574
North (km)
112
114
116
118
120
122
124
126
East (km)
1
0.5
0
-0.5
-1
18
21
00
03

Modelling water flow

- Only one species: H_2O
- Governing processes: gravity & friction
- Modeling requires you to keep track of mass and energy
- No odd “behavioral” processes: high potential predictability

Why can’t biologists make similar good models?

Today...

1. Basics of Hydrodynamics
2. Interplay of vegetation and water flow
3. Biogeomorphology

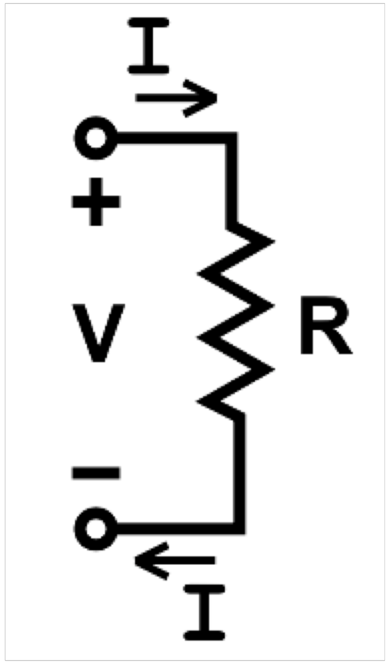
Remember Ohm's law for electricity

- To calculate flow

Georg Simon Ohm
1789 –1854
Germany

$$Flow = \frac{Potential}{Resistence}$$

$$\Rightarrow I = \frac{V}{R}$$



Water flow from first principles

Claude-Louis Navier
(1785-1836)

George Gabriel Stokes
(1819-1903)

The physical laws governing water flow:

- Conservation of mass
- Conservation of momentum

Navier-Stokes equation: a mathematical equation describing water flow:

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\frac{\nabla P}{\rho} + \nu \Delta \boldsymbol{u} \quad \text{where} \quad \boldsymbol{u} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

Here, *u*, *v*, *w* are the flow rate in *x*, *y*, and *z* dimension,
ν is viscosity, *P* is the pressure, and ρ is the fluid density

http://mathworld.wolfram.com/Navier-StokesEquations.html

Processes governing flow

- Pressure (heat) & gravity are the main driving forces
- Conservation of mass and momentum
  - No water gets lost
  - Kinetic energy (moment) doesn’t get lost (mysteriously)
- Momentum is lost through
  - Friction with bottom or other surfaces
  - Viscous turbulence generation
- Additional “forces”: Coriolis force at large scales

Navier-Stokes Equations

Momentum equations

$$\frac{\partial \boldsymbol{u}}{\partial t} = -Adv(\boldsymbol{u}) + Dif(\boldsymbol{u}) - Coriolis(\boldsymbol{u}) + Pressure(\boldsymbol{u}) \qquad \boldsymbol{u} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

$Adv(\boldsymbol{u})$ = Momentum advection (also called the nonlinear terms)

$Dif(\boldsymbol{u})$ = Diffusion of velocity components u, v, w

$Coriolis(\boldsymbol{u})$ = Coriolis force

$Pressure(\boldsymbol{u})$ = Pressure pushes the flow (due to air, height, density differences)

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Navier-Stokes equations

Navier-Stokes equations for an incompressible fluid (~water)

Momentum conservation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - K_x\frac{\partial^2 u}{\partial x^2} - K_y\frac{\partial^2 u}{\partial y^2} - K_z\frac{\partial^2 u}{\partial z^2} = fv - \frac{1}{\rho_0}\frac{\partial P}{\partial x}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} - K_x\frac{\partial^2 v}{\partial x^2} - K_y\frac{\partial^2 v}{\partial y^2} - K_z\frac{\partial^2 v}{\partial z^2} = -fu - \frac{1}{\rho_0}\frac{\partial P}{\partial y}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} - K_x\frac{\partial^2 w}{\partial x^2} - K_y\frac{\partial^2 w}{\partial y^2} - K_z\frac{\partial^2 w}{\partial z^2} = -\frac{1}{\rho_0}\frac{\partial P}{\partial z} - \frac{\rho - \rho_0}{\rho_0}g$$

Mass conservation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Shallow water equations

Simplification: shallow water

- Horizontal scale >> Vertical scale
  (e.g. an estuary of 50 km long, ~10 meters deep)
- Flow in z dimension is ignored
- Pressure only due to water surface elevation difference, multiplied by gravity g.

=> “Shallow water equations”

Shallow water equations

Momentum conservation:

$$\frac{\partial u}{\partial t} = -g\frac{\partial \eta}{\partial x} - u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - bu$$

$$\frac{\partial v}{\partial t} = -g\frac{\partial \eta}{\partial y} - u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} - bv$$

Mass conservation:

$$\frac{\partial h}{\partial t} = -\frac{\partial uh}{\partial x} - \frac{\partial vh}{\partial y}$$

Where surface elevation $\eta = h + s$. Here h is the height of the water column from the bottom to the surface, and s is the elevation of the bottom. b stands for energy loss to turbulence.

Shallow water equations

Momentum conservation:

$$\frac{\partial u}{\partial t} = -g\frac{\partial \eta}{\partial x} - u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - bu$$

$$\frac{\partial v}{\partial t} = -g\frac{\partial \eta}{\partial y} - u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} - bv$$

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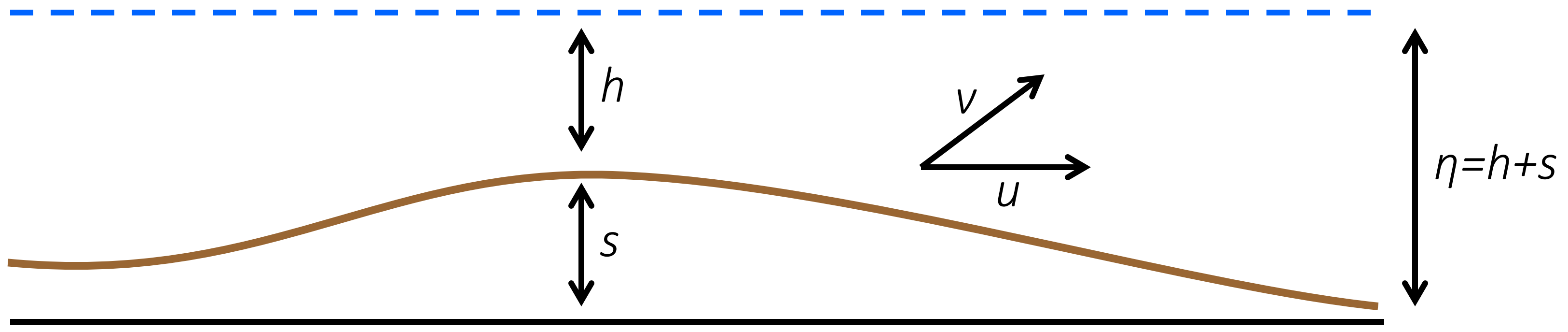
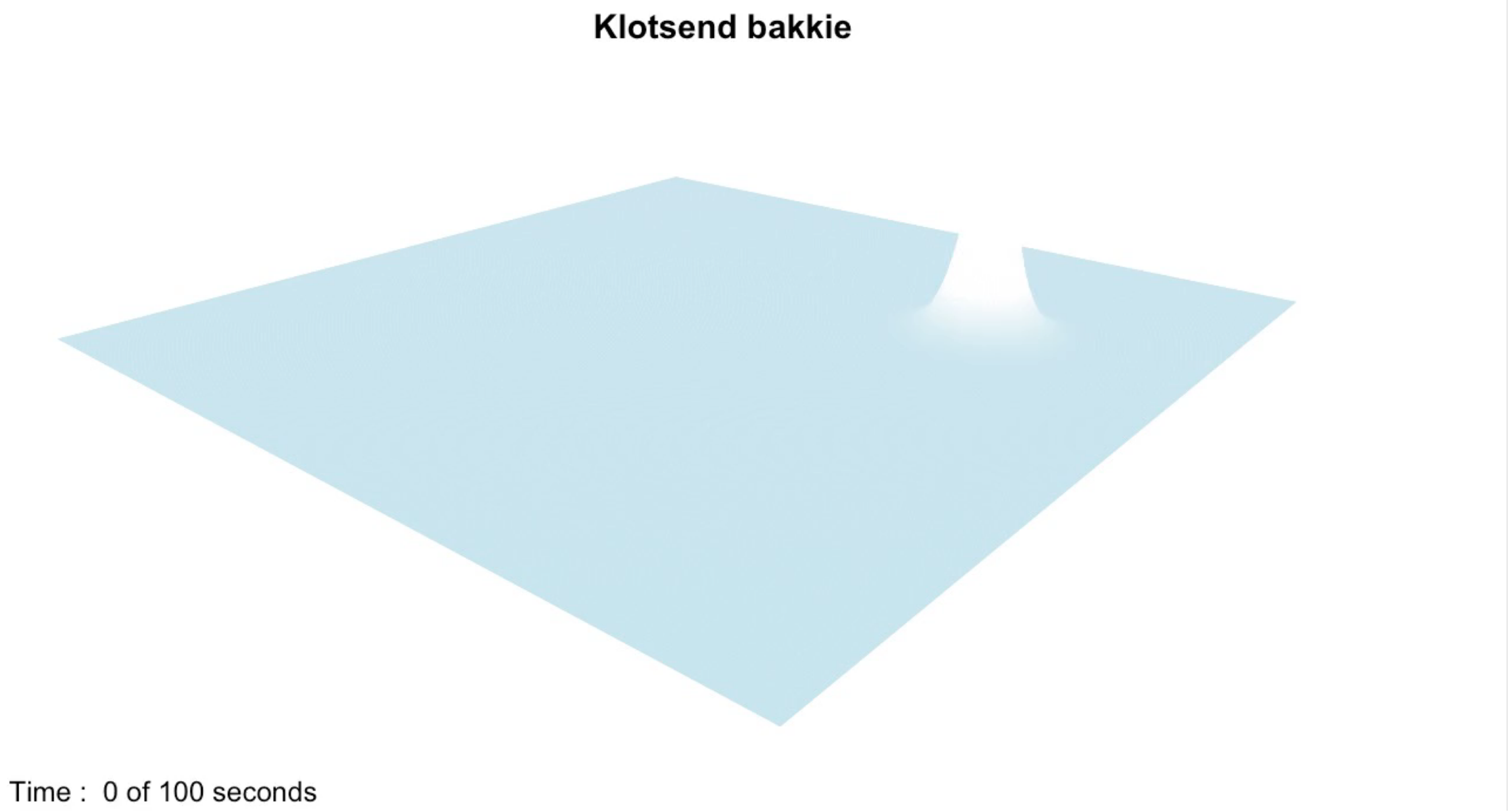
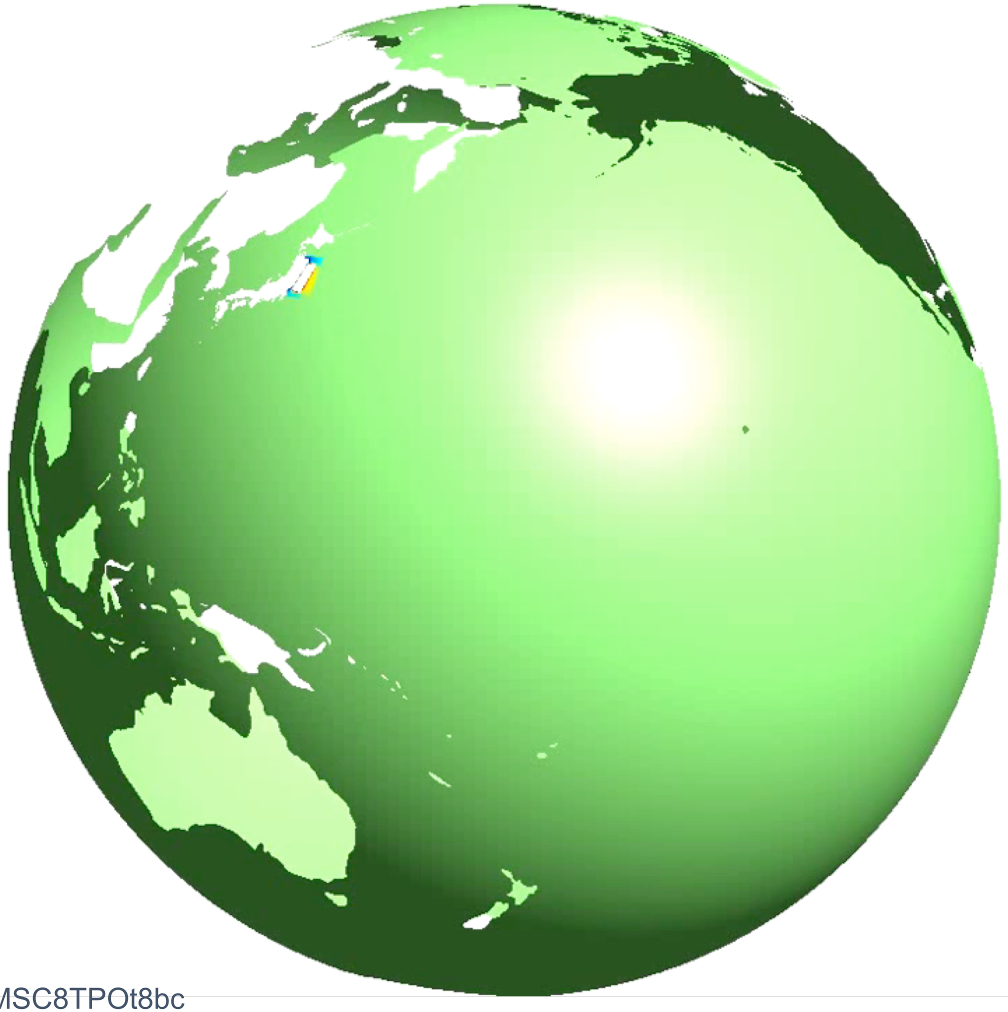


The bathtub model



https://en.wikipedia.org/wiki/Shallow_water_equations

Example: a tsunami



https://www.youtube.com/watch?v=MSC8TPOt8bc

What about bottom friction?

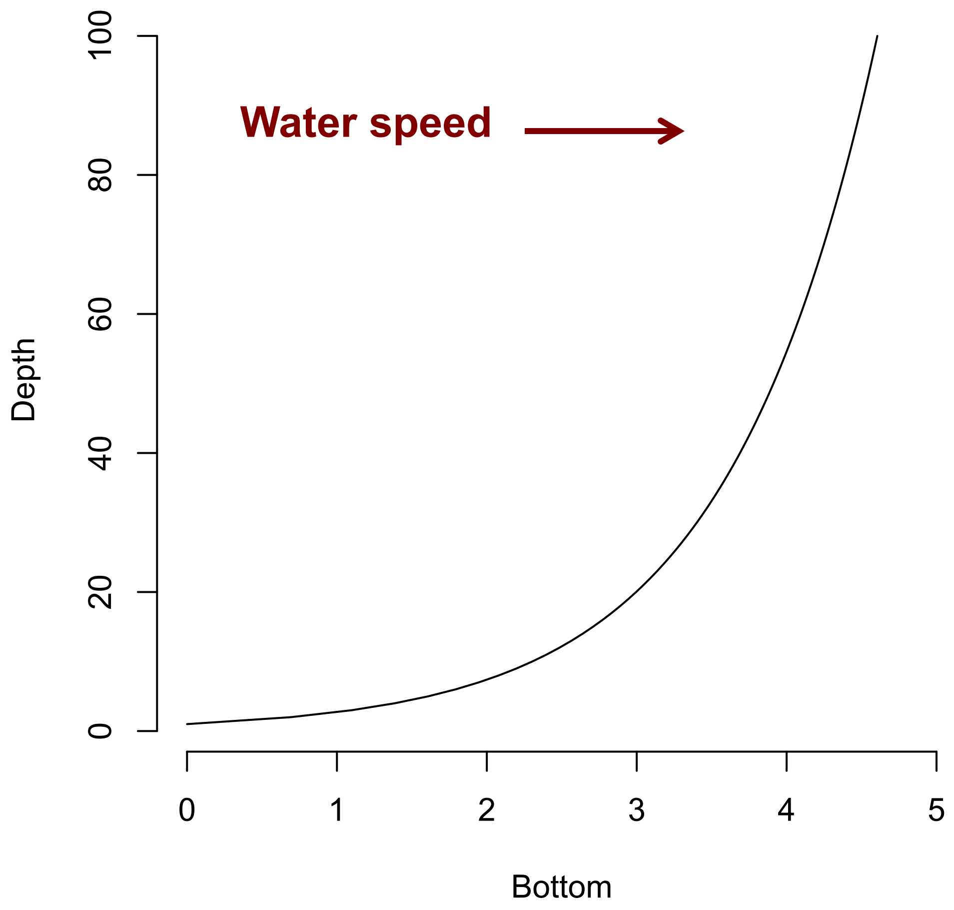
Bottom friction:

- The momentum transfer at the lower boundary of the water column to the solid earth by friction with the bottom.

$$\frac{\partial \boldsymbol{u}}{\partial t} = -g\nabla\eta - \boldsymbol{u}\cdot\nabla\boldsymbol{u} + K\Delta\boldsymbol{u} - \frac{\tau_b}{\rho h} \qquad \boldsymbol{u} = \binom{u}{v}$$

T_b = Stress imposed by the bottom
ρ = Density of the water
h = Water height

What about bottom friction?



The Chézy equation for bottom friction

$\tau_b = C_b|\boldsymbol{u}|\boldsymbol{u}$ where $|u|$ is the absolute flow velocity in all dimensions. C_b and $|u|$ are given by:

$$C_b = \frac{g}{C^2}\ ;\ |\boldsymbol{u}| = \sqrt{u^2 + v^2};$$

C = Chézy roughness coefficient (44 for the river Seine)
g = gravity.

$$C = \frac{R^{\frac{1}{6}}}{n}$$

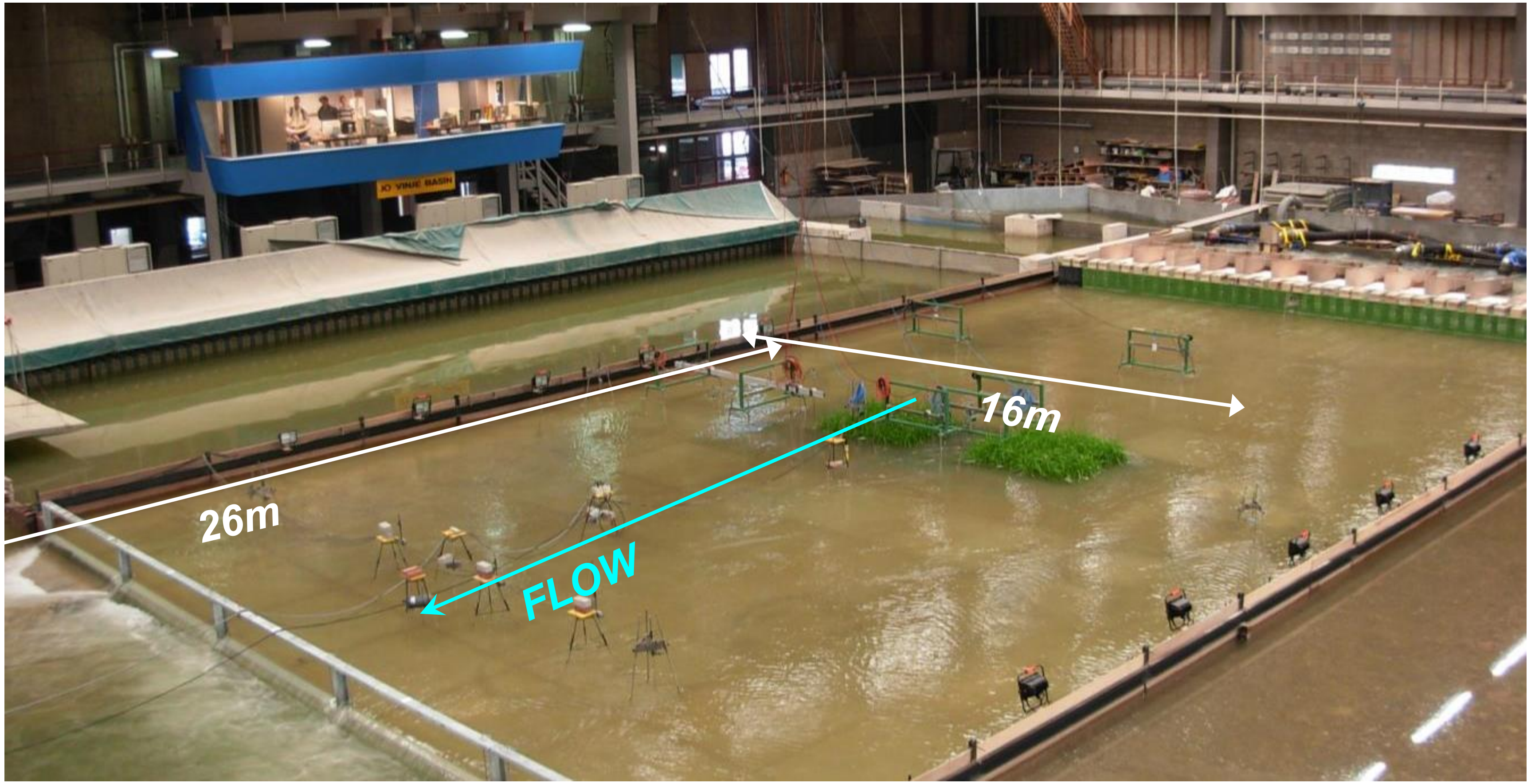
R = hydraulid radius (of a pipe), which can be replace by h
n = Manning's coefficient for bottom roughness

For values, see: http://www.engineeringtoolbox.com/mannings-roughness-d_799.html

Tussock effects

How bio-physical feedbacks lead to SELF-ORGANISATION in tidal wetlands

Vegetation patches in a lab flume study

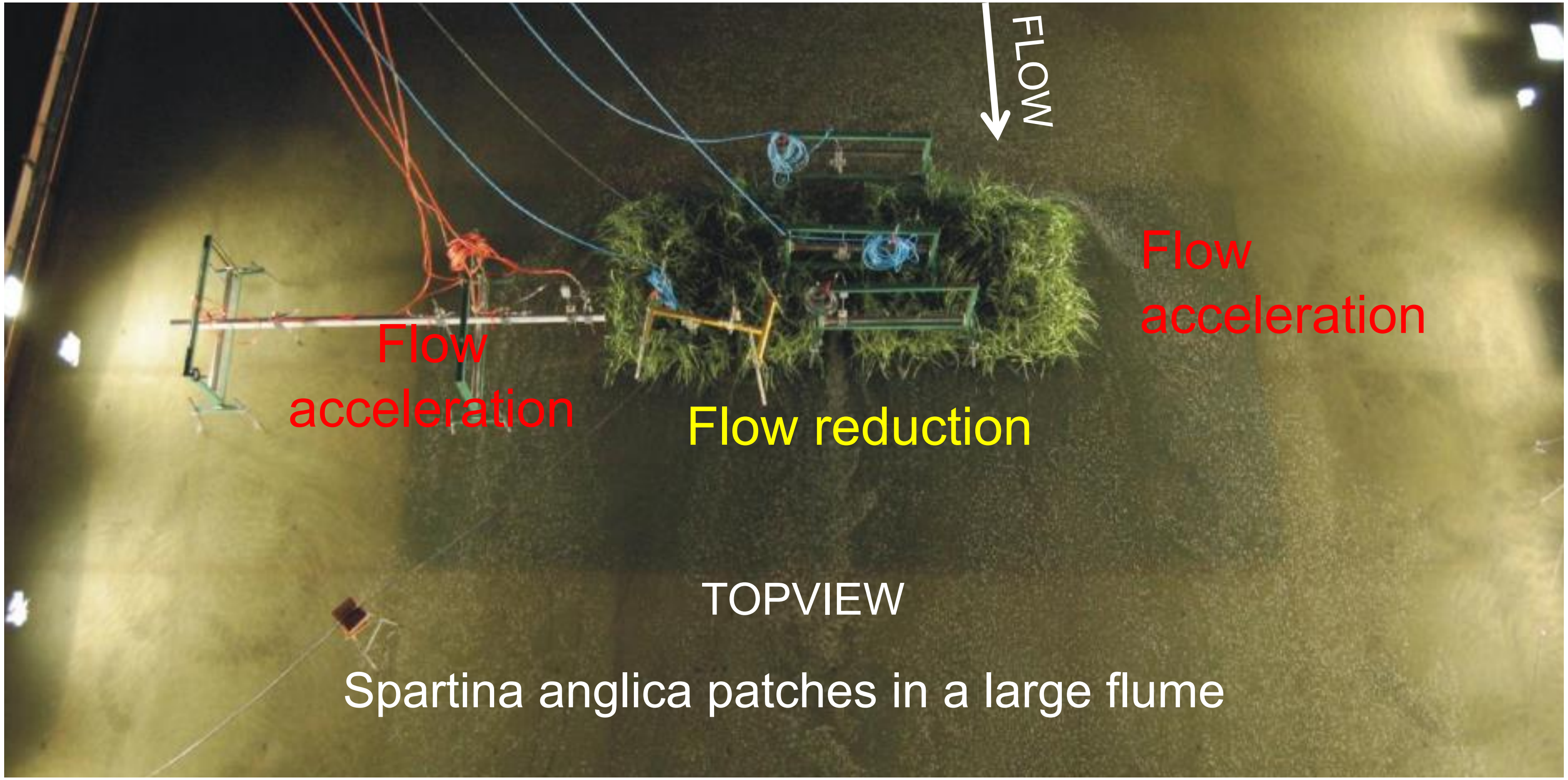


How bio-physical feedbacks lead to SELF-ORGANISATION in tidal wetlands

Vegetation patches in a lab flume study

Vandenbruwaene at al. (2011) J.Geophys.Res.

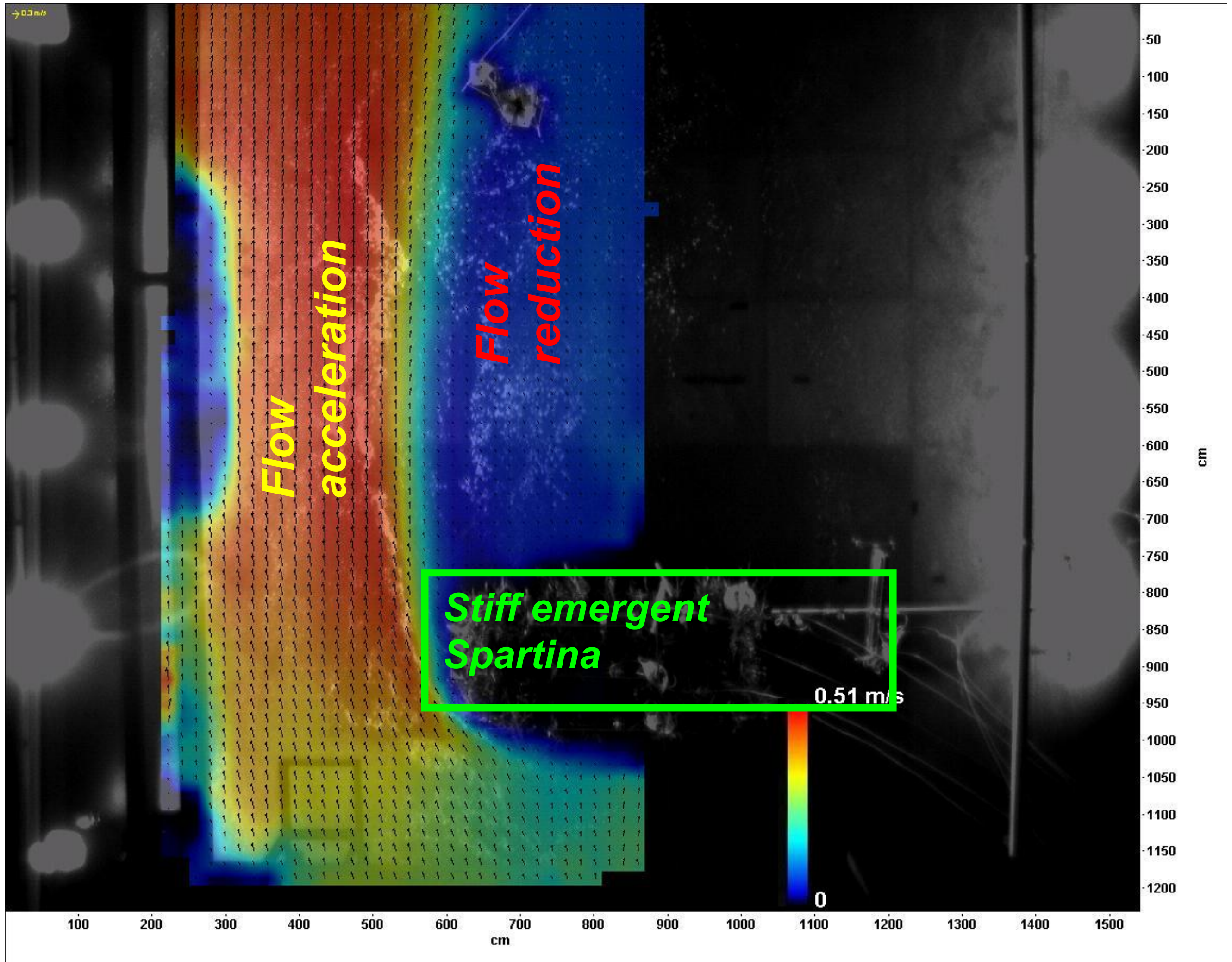
How bio-physical feedbacks lead to SELF-ORGANISATION in tidal wetlands



Bouma et al (2013) Geomorphology

How bio-physical feedbacks lead to SELF-ORGANISATION in tidal wetlands

Top view



FLOW DIRECTION

Incoming flow velocity =0.3 m/s

Bouma et al (2013) Geomorphology

Shallow water equations

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$$\frac{\partial v}{\partial t} = -g\frac{\partial \eta}{\partial y} - u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} - bv$$

Mass conservation:

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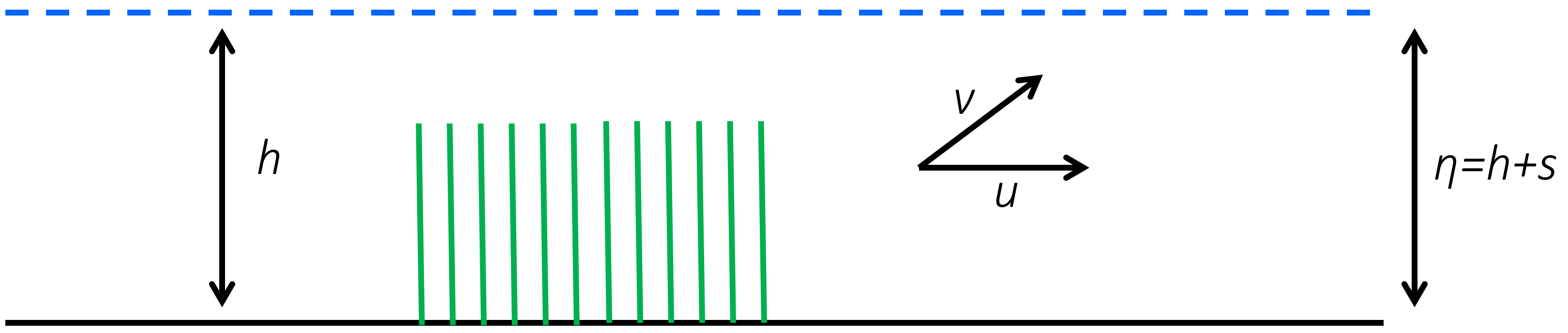


Friction by vegetation

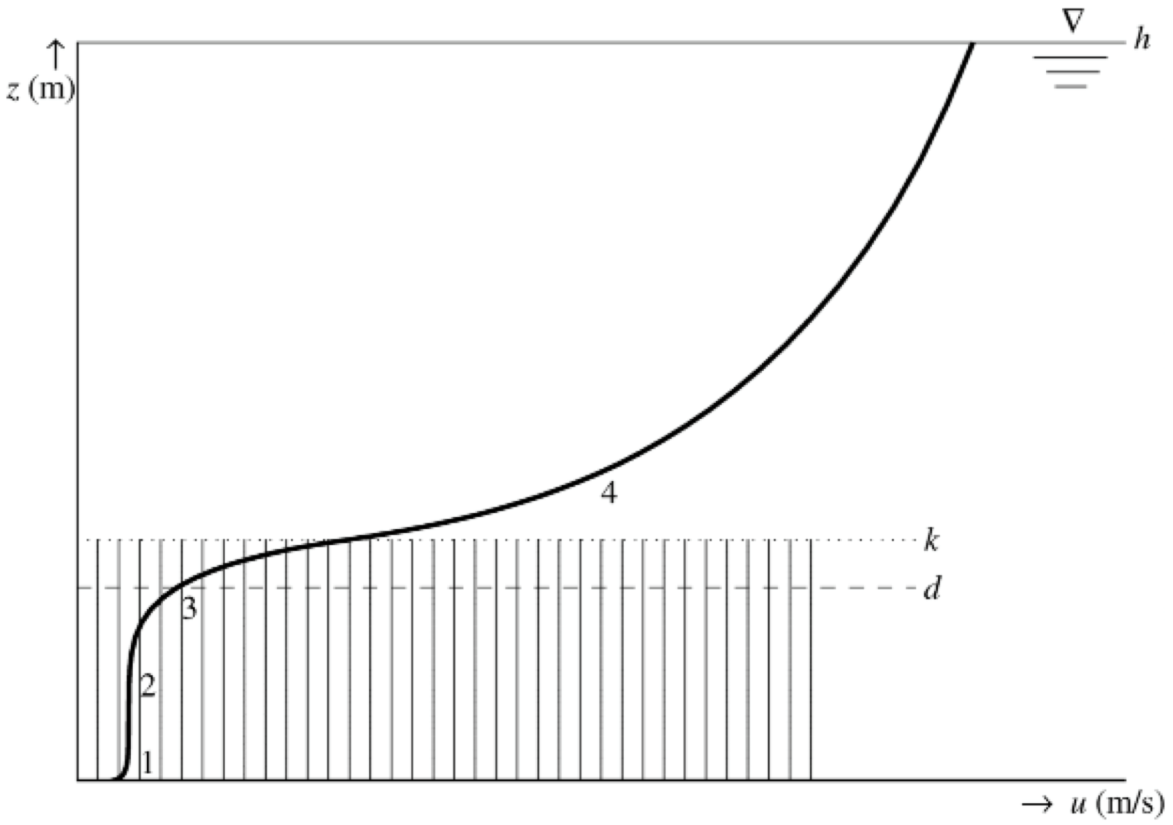


Figure 1 Four zones in the vertical profile for horizontal velocity, $u(z)$, through and over vegetation, h = water depth (m), k = vegetation height (m), d = zero-plane displacement (m).

Baptist, et al. 2007. On inducing equations for vegetation resistance. Journal of Hydraulic Research 45:435-450.

Friction by vegetation-abstracted

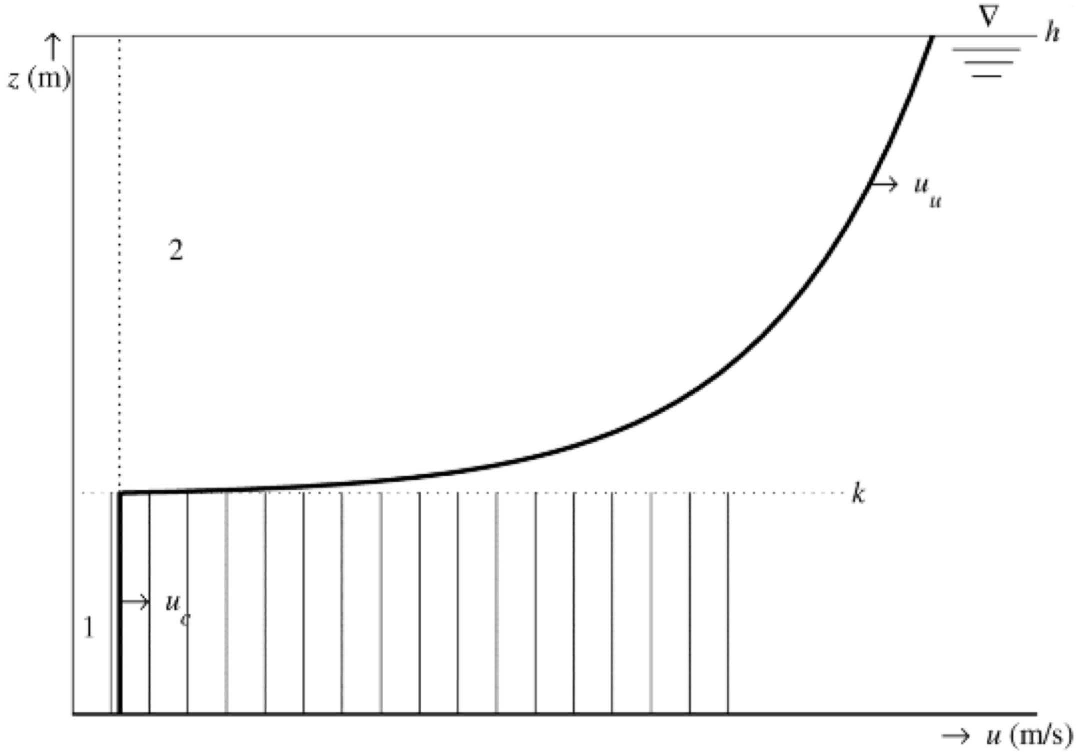


Figure 2 Representation of the vertical velocity profile in two zones for the method of effective water depth, h = water depth (m), k = vegetation height (m), u_c = uniform flow velocity profile (m/s), u_u = logarithmic flow velocity profile (m/s).

Baptist, et al. 2007. On inducing equations for vegetation resistance. Journal of Hydraulic Research 45:435-450.

Calculating friction of vegetation

When the vegetation is emerging from the water (like, e.g. with reeds), the entire water collum is occupied by water. In that case, the Chézy coefficient is given by:

$$C_r = \sqrt{\frac{1}{C_b^{-2} + (2g)^{-1} C_d D_v h}}$$

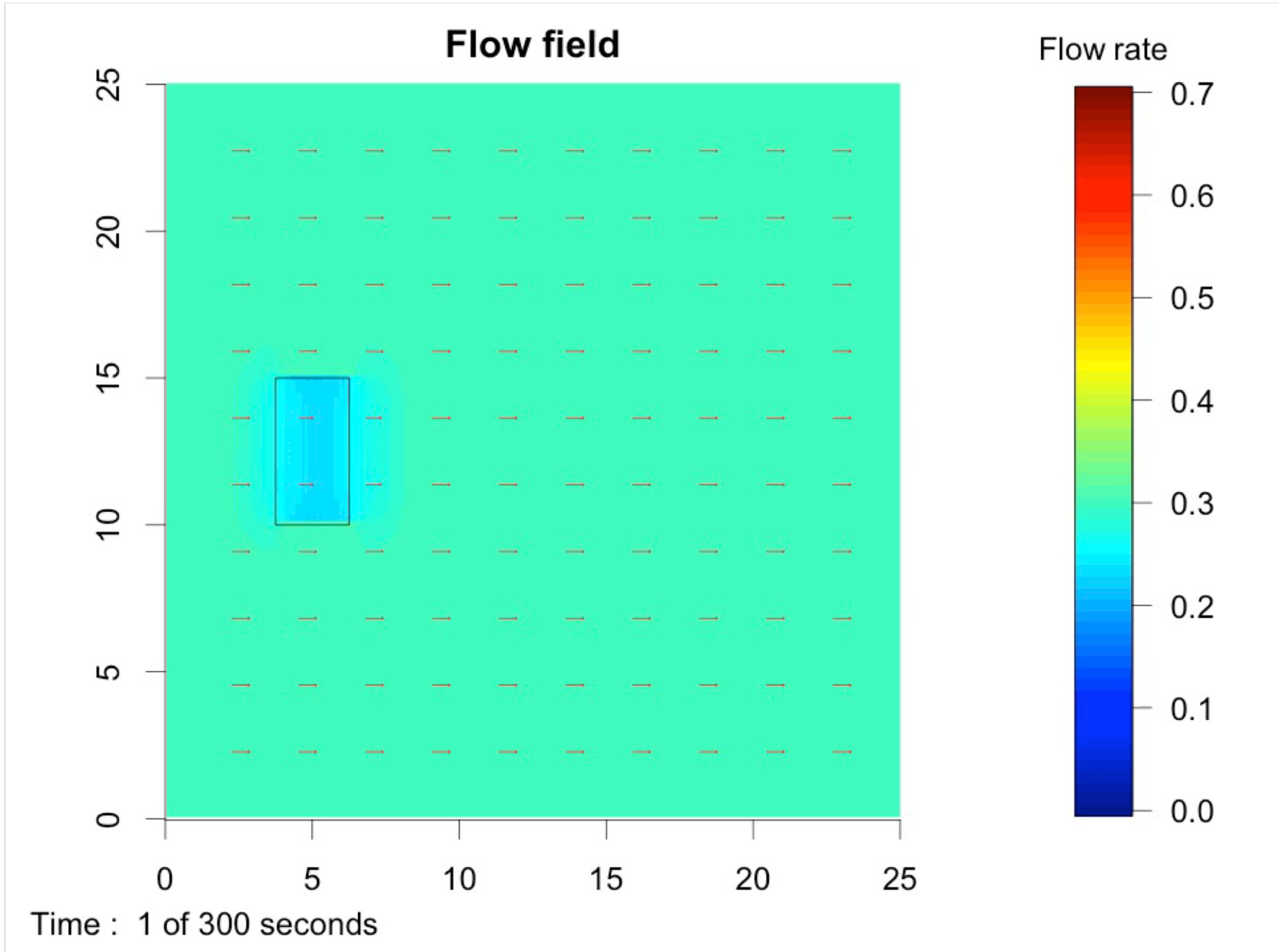
here C_b is the Chezy roughness of the bare bed, C_d is the drag coefficient of the vegetation, D_v is the vertical vegetation density.

When the vegetation is submerged, part of the water column is free, and the Chézy coefficient is given by:

$$C_r = \sqrt{\frac{1}{C_b^{-2} + (2g)^{-1} C_d D_v H_v}} + \sqrt{\frac{g}{\kappa} ln \frac{h}{H_v}}$$

Here H_v is the vegetation height, and κ is the Von Kármàn constant (0.4).

A simple model



Assignments

Taking the shallow water equation

- Make a model of wave formation (maybe in 1D) and how it hits the coasts, comparing with or without tidal flat or salt marsh (easy).
- Make a model of a pipe from a nuclear power plant pumping warm water in a flowing body of water

Make a model that takes a vertical slice rather then a horizontal slice, and do one of the following

- What happens of the wind makes the top water layer water move
- What happens to the warm water originating from vulkanic seeps