Monday May 1

- Today
  - Wilcoxon tests
  - ANOVA
- Tonight
  - practice exam questions
  - will cover them in final practical on May 12 10:45 (CC3)
- Thursday May 4 mini-lecture (13:45 in CC3)
  - topic 1: 2 categorical predictors
  - topic 2: 1 categorical and 1 interval/ratio variable
- Next Monday at 13:45
  - homework due
  - topic: random intercepts

When Assumptions are Broken

- Normality assumptions
- Ordinal data
- You can use… (pp 655 – 673)
  - Independent data
    - Wilcoxon rank-sum test
  - Dependent data
    - Wilcoxon signed-rank test
- Both tests are based on the ranks of your data points and compare medians

Wilcoxon tests in RStudio

Reporting the Results for Wilcoxon rank-sum test (independent samples)

- On average, participants experienced greater anxiety from real spiders (*median* = 50.00, *IQR* = 17), than from pictures of spiders (*median* = 40.00, *IQR* = 12.5). However, according to a Wilcoxon rank-sum test, this difference was not significant, $W = -2.5$, $p = 0.13$; furthermore, it represented a medium effect, $r = -0.31$.

Reporting the Results for Wilcoxon signed-rank test (for dependent samples)

- On average, participants experienced greater anxiety from real spiders (*median* = 50.00, *IQR* = 17), than from pictures of spiders (*median* = 40.00, *IQR* = 12.5). According to a Wilcoxon signed-rank test, this difference was significant, $V = 8$, $p = 0.046$; furthermore, it represented a medium effect, $r = -0.40$.

Comparing Several Means: Analysis of Variance (ANOVA)

Aims

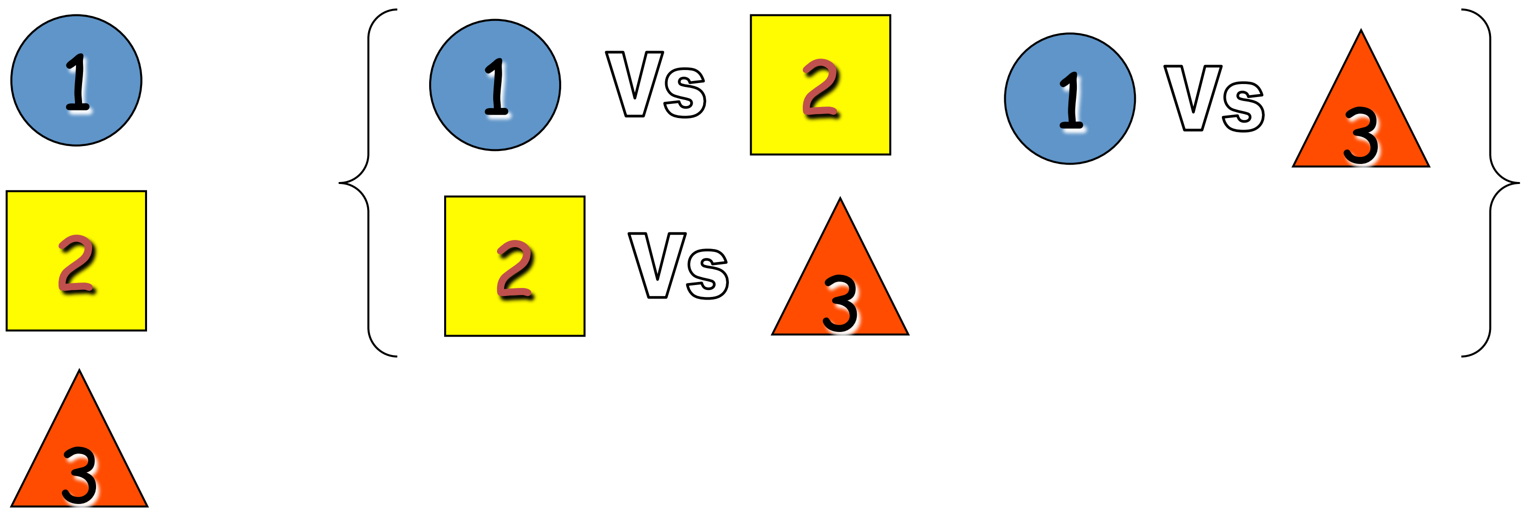
- Understand the basic principles of ANOVA
  - Why it is done?
  - What it tells us?
- Theory of one-way independent ANOVA
- Following up an ANOVA:
  - Planned contrasts/comparisons
  - *Post hoc* tests

When and Why

- When we want to compare means we can use a *t*-test. This test has limitations:
  - You can compare only 2 means: often we would like to compare means from 3 or more groups.
  - It can be used only with one predictor/independent variable.
- ANOVA
  - Compares several means.
  - Can be used when you have manipulated more than one independent variable.
  - It is an extension of regression (the general linear model).

Why Not Use Lots of *t*-Tests?

- If we want to compare several means why don't we compare pairs of means with *t*-tests?
  - Inflates the Type I error rate.



Familywise error

- P-value = .05
    - Under null hypothesis (pattern this strong not in the population), then if you took 100 samples
        - 95 samples would also have not have the observed pattern
        - 5 samples would have a pattern at least as strong
    - or, if 20 samples
        - 19 would have no pattern
        - 1 would have a pattern at least as strong

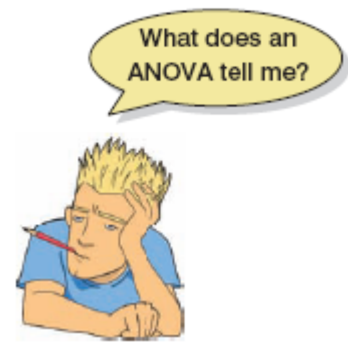
Familywise error

- P-value = .05 across 3 tests
  - 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
    - 5% chance you'll falsely reject null (Type I error)
  - 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
  - 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
- What is the probability of falsely rejecting null in at least 1 test?

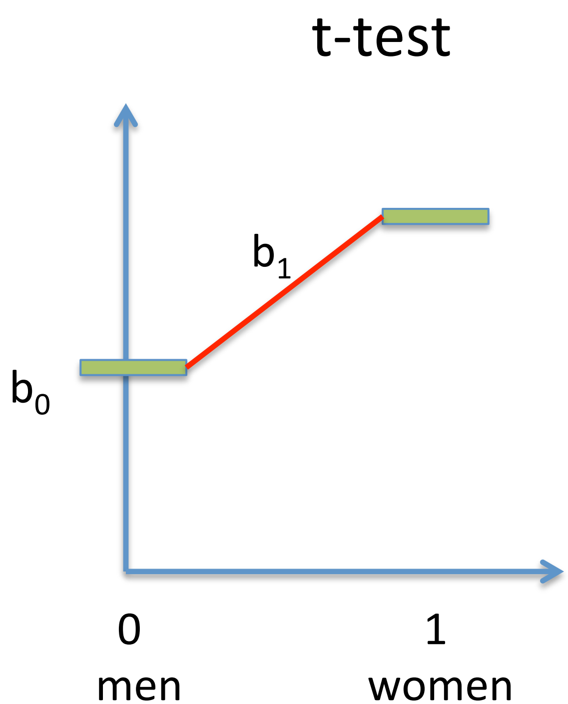
  1 - 0.95*0.95*0.95 = 1 - 0.857 = 0.143 = 14.3%

What Does ANOVA Tell Us?

- Null hyothesis:
  - Like a *t*-test, ANOVA tests the null hypothesis that the means are the same.
- Experimental hypothesis:
  - The means differ.
- ANOVA is an omnibus test
  - It test for an overall difference between groups.
  - It tells us that the group means are different.
  - It doesn't tell us exactly which means differ.

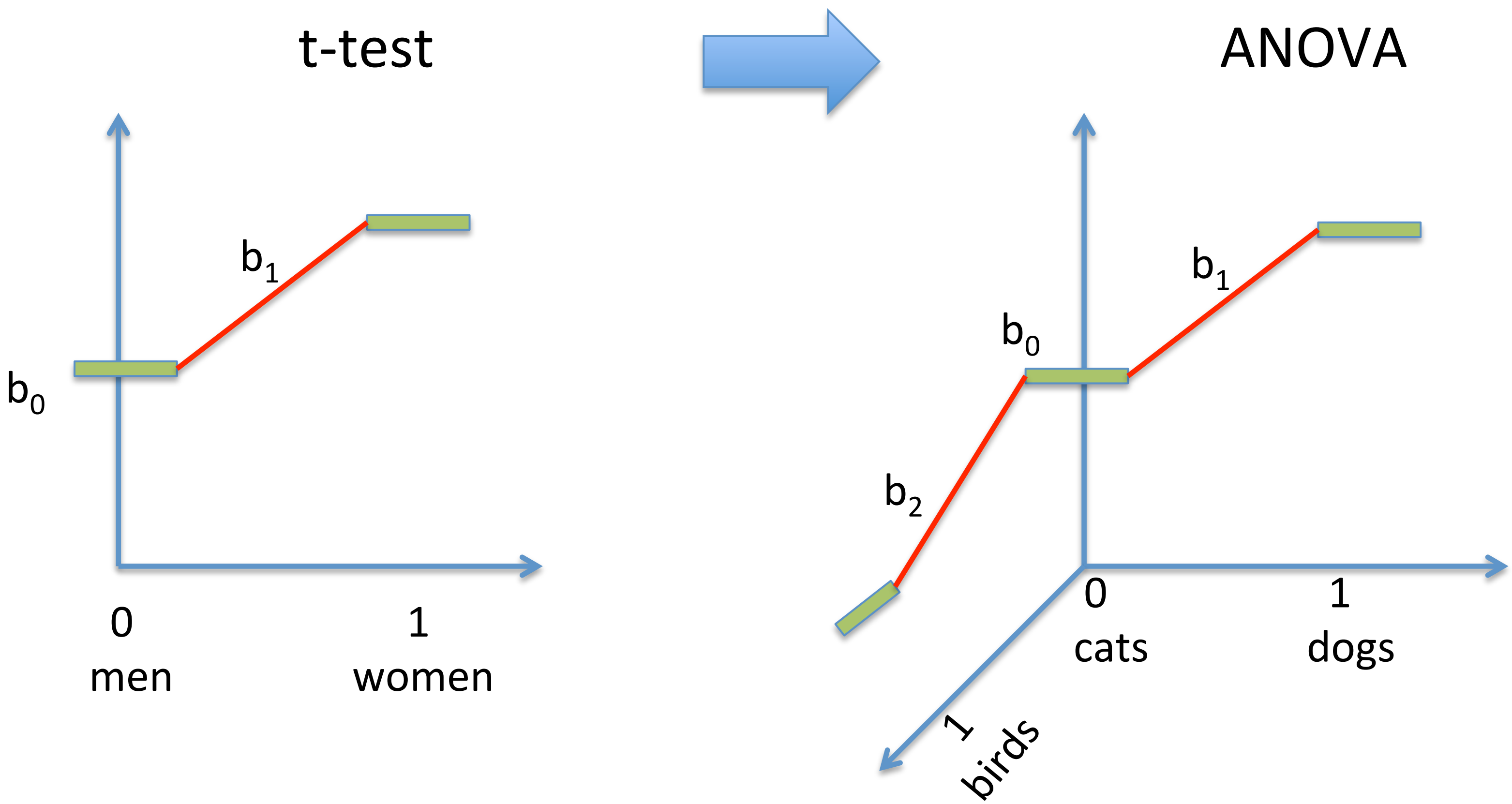


T-test as regression



Binary Variable	Contrast Variable
Men	0
Women	1

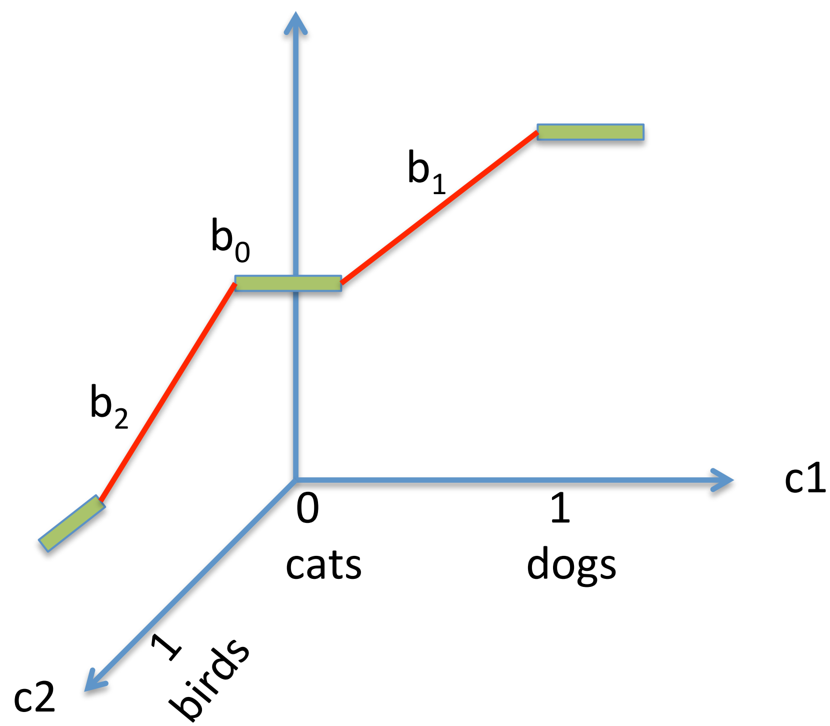
ANOVA as regression



Dummy Contrasts

Nominal Variable	Contrast Variable 1	Contrast Variable 2
Cats	0	0
Dogs	1	0
Birds	0	1

ANOVA



Dummy Contrasts

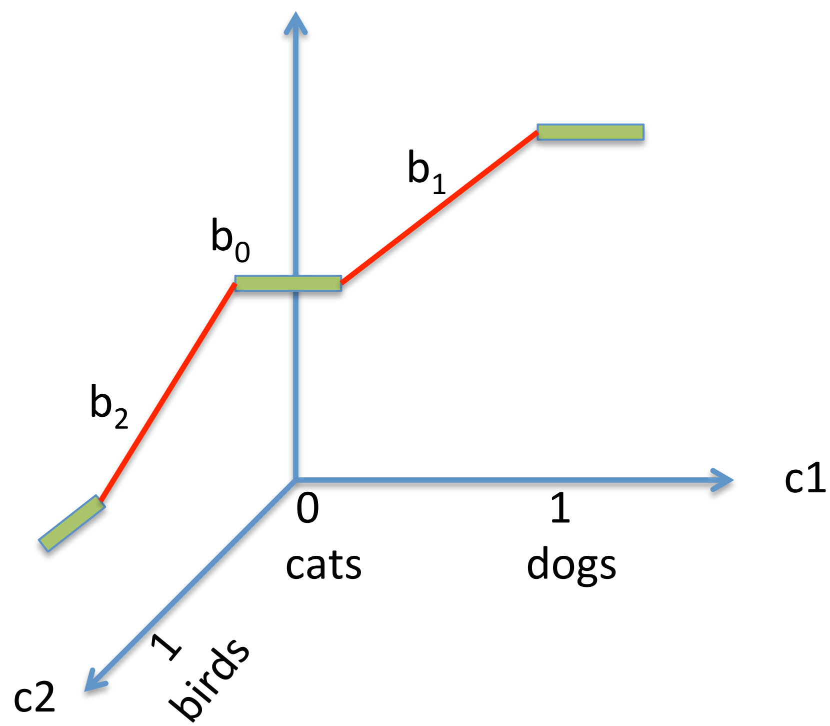
Nominal Variable	Contrast Variable 1	Contrast Variable 2
Cats	0	0
Dogs	1	0
Birds	0	1

$$Predicted_{birds} = b0 + b1 * c1_{birds} + b2 * c2_{birds}$$

$$Predicted_{birds} = b0 + b1 * 0 + b2 * 1$$

$$Predicted_{birds} = b0 + b2$$

ANOVA



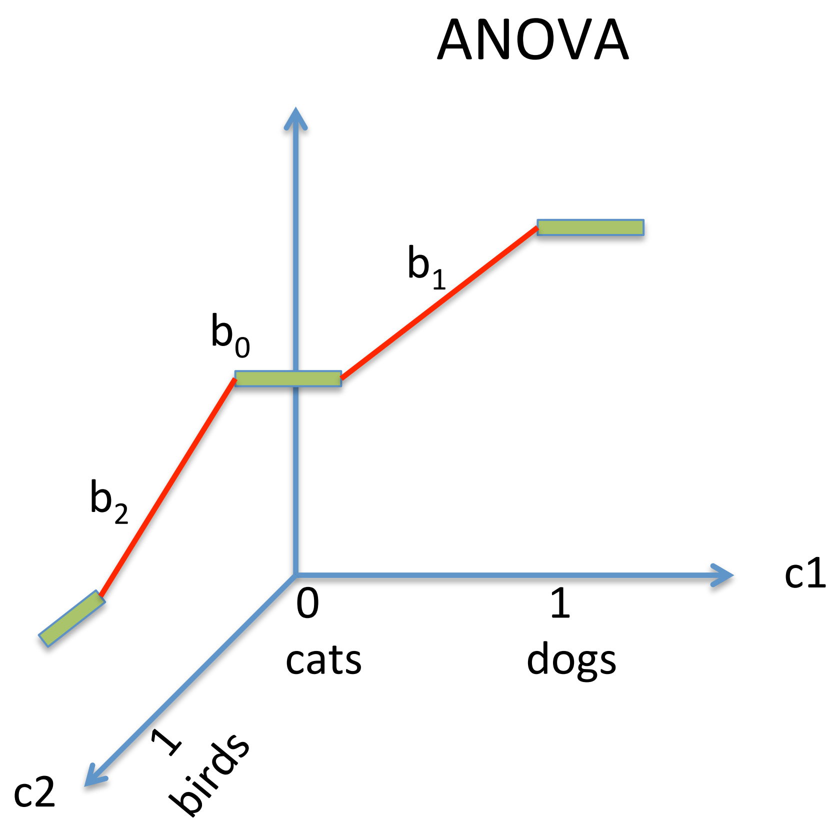
Dummy Contrasts

Nominal Variable	Contrast Variable 1	Contrast Variable 2
Cats	0	0
Dogs	1	0
Birds	0	1

$\text{Predicted}_{cats} = b0 + b1 * c1_{cats} + b2 * c2_{cats}$

$\text{Predicted}_{cats} = b0 + b1 * 0 + b2 * 0$

$\text{Predicted}_{cats} = b0$



Dummy Contrasts

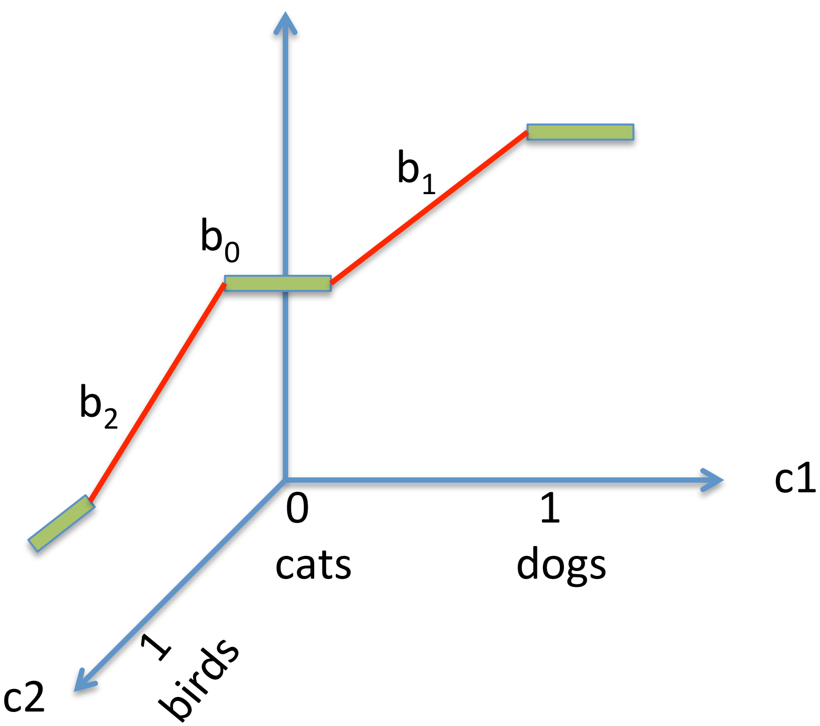
Nominal Variable	Contrast Variable 1	Contrast Variable 2
Cats	0	0
Dogs	1	0
Birds	0	1

ANOVA

$$\text{Predicted}_{dogs} = b0 + b1 * c1_{dogs} + b2 * c2_{dogs}$$

$$\text{Predicted}_{dogs} = b0 + b1 * 1 + b2 * 0$$

$$\text{Predicted}_{dogs} = b0 + b1$$

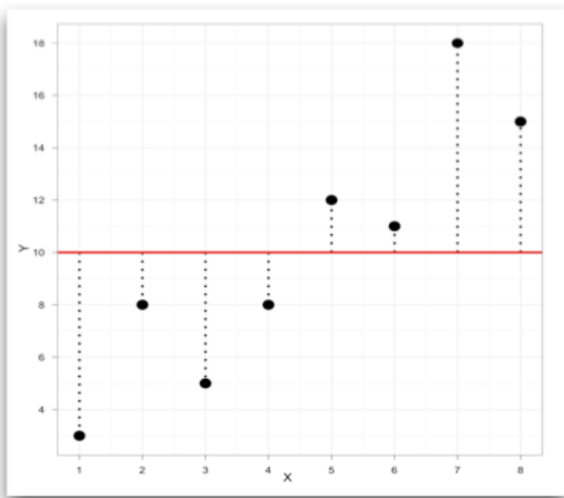


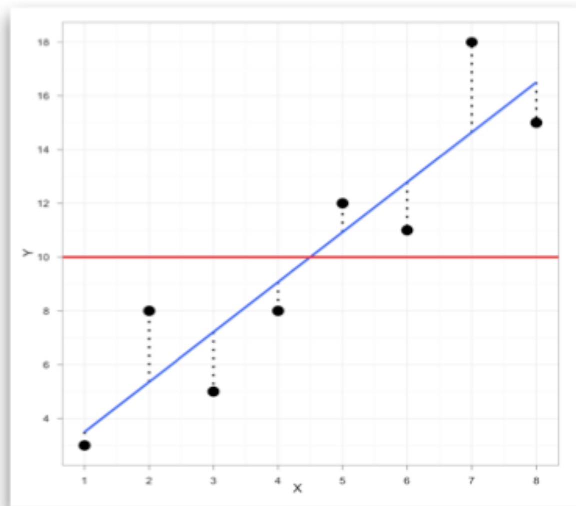
Regression Output

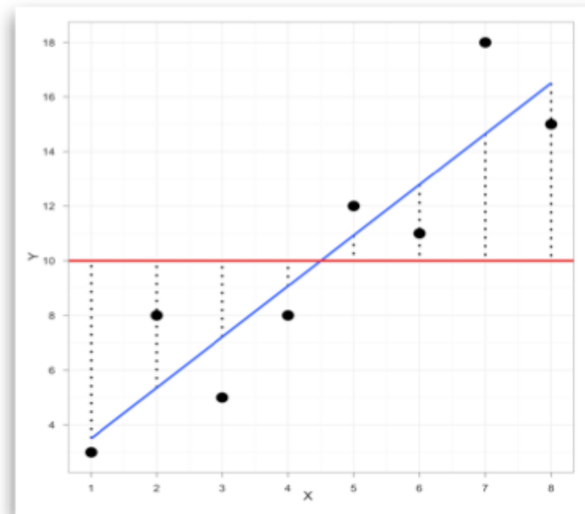
```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)     5.060      0.280   18.07 4.54e-10 ***
animalsdogs     4.100      0.396   10.35 2.46e-07 ***
animalsbirds   -2.360      0.396   -5.96 6.61e-05 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.6261 on 12 degrees of freedom
Multiple R-squared:  0.9578,    Adjusted R-squared:  0.9508
F-statistic: 136.3 on 2 and 12 DF,  p-value: 5.621e-09
```

Sums of Squares in Regression

Total Sum of Squares (SSt) - Residual Sum of Squares (SSr) = Model Sum of Squares (SSm)







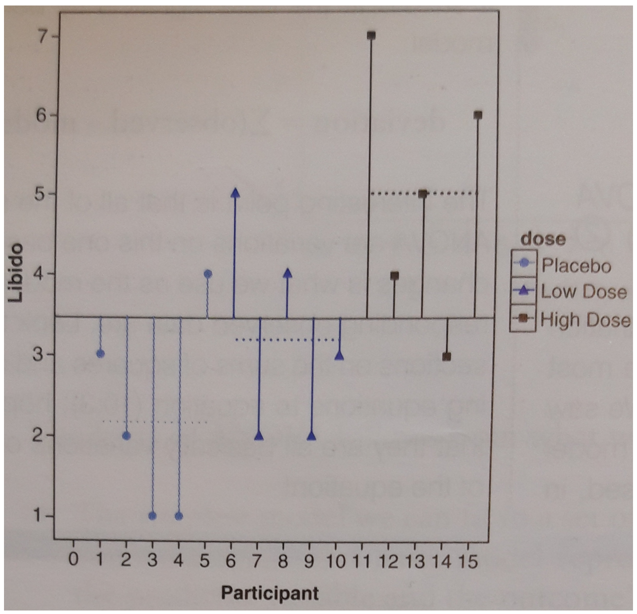
Residual Mean Squares (MSr)

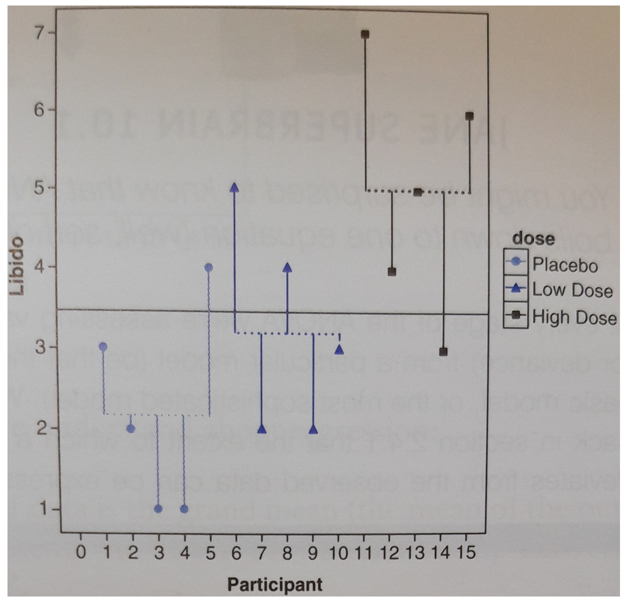
Mean Squares for the Model (MSm)

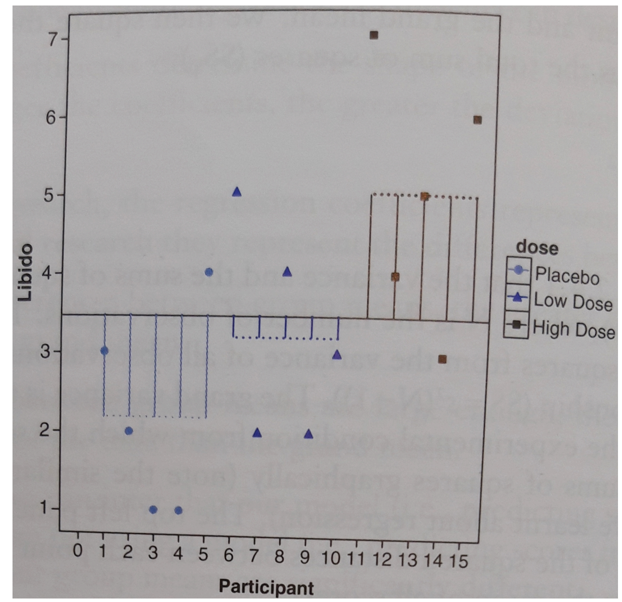
$$\frac{\text{Mean Squares for the Model}}{\text{Residual Mean Squares}} = \frac{\text{Explained Variance}}{\text{Unexplained Variance}} = F$$

Sums of Squares in ANOVA

Total Sum of Squares (SSt) - Residual Sum of Squares (SSr) = Model Sum of Squares (SSm)







Residual Mean Squares (MSr)

Mean Squares for the Model (MSm)

$$\frac{\text{Mean Squares for the Model}}{\text{Residual Mean Squares}} = \frac{\text{Explained Variance}}{\text{Unexplained Variance}} = F$$

Theory of ANOVA

- We calculate how much variability there is between scores
  - Total sum of squares (SS_T).
- We then calculate how much of this variability can be explained by the model we fit to the data
  - How much variability is due to the experimental manipulation, model sum of squares (SS_M)...
- … and how much cannot be explained
  - How much variability is due to individual differences in performance, residual sum of squares (SS_R).

Theory of ANOVA

- We compare the amount of variability explained by the model (MSm), to the error in the model (MSr)
  - This ratio is called the *F*-ratio.
- If the model explains a lot more variability than it can't explain, then the experimental manipulation has had a significant effect on the outcome.

Why Use Follow-Up Tests?

- The *F*-ratio tells us only that the experiment was successful
  - i.e. group means were different
- It does not tell us specifically which group means differ from which.
- We need additional tests to find out where the group differences lie.

How?

- Planned comparisons
  - Hypothesis driven
  - Planned beforehand
  - e.g., dummy contrasts, but often not what we want
- *Post hoc* tests
  - Not planned (no hypothesis)
  - Compare all pairs of means

Planned Comparisons/Contrasts

- Basic idea:
  - The variability explained by the model (experimental manipulation, SS_M**)** is due to participants being assigned to different groups.
  - This variability can be broken down further to test specific hypotheses about which groups might differ.
  - We break down the variance according to hypotheses made before the experiment.
  - It's like cutting up a cake

Rules When Choosing Contrasts

- Independent
  - Contrasts must not interfere with each other (they must test unique hypotheses).
- Only two chunks
  - Each contrast should compare only two chunks of variation.
- K–1
  - You should always end up with one less contrast than the number of groups.

Generating Hypotheses

- Example: Testing the effects of Viagra on libido using three groups:
  - Placebo (sugar pill)
  - Low dose viagra
  - High dose viagra
- Outcome variable was an objective measure of libido.
- Intuitively, what might we expect to happen?

	Placebo	Low Dose	High Dose
	3	5	7
	2	2	4
	1	4	5
	1	2	3
	4	3	6
Mean	2.20	3.20	5.00

How do I Choose Contrasts?

- Big hint:
  - In most experiments we usually have one or more control groups.
  - The logic of control groups dictates that we expect them to be different from groups that we've manipulated.
  - The first contrast will always be to compare any control groups (chunk 1) with any experimental conditions (chunk 2).

Hypotheses

- Hypothesis 1:
  - People who take Viagra will have a higher libido than those who don't.
  - placebo ≠ (low, high)
- Hypothesis 2:
  - People taking a high dose of Viagra will have a greater libido than those taking a low dose.
  - low ≠ high

Planned Comparisons

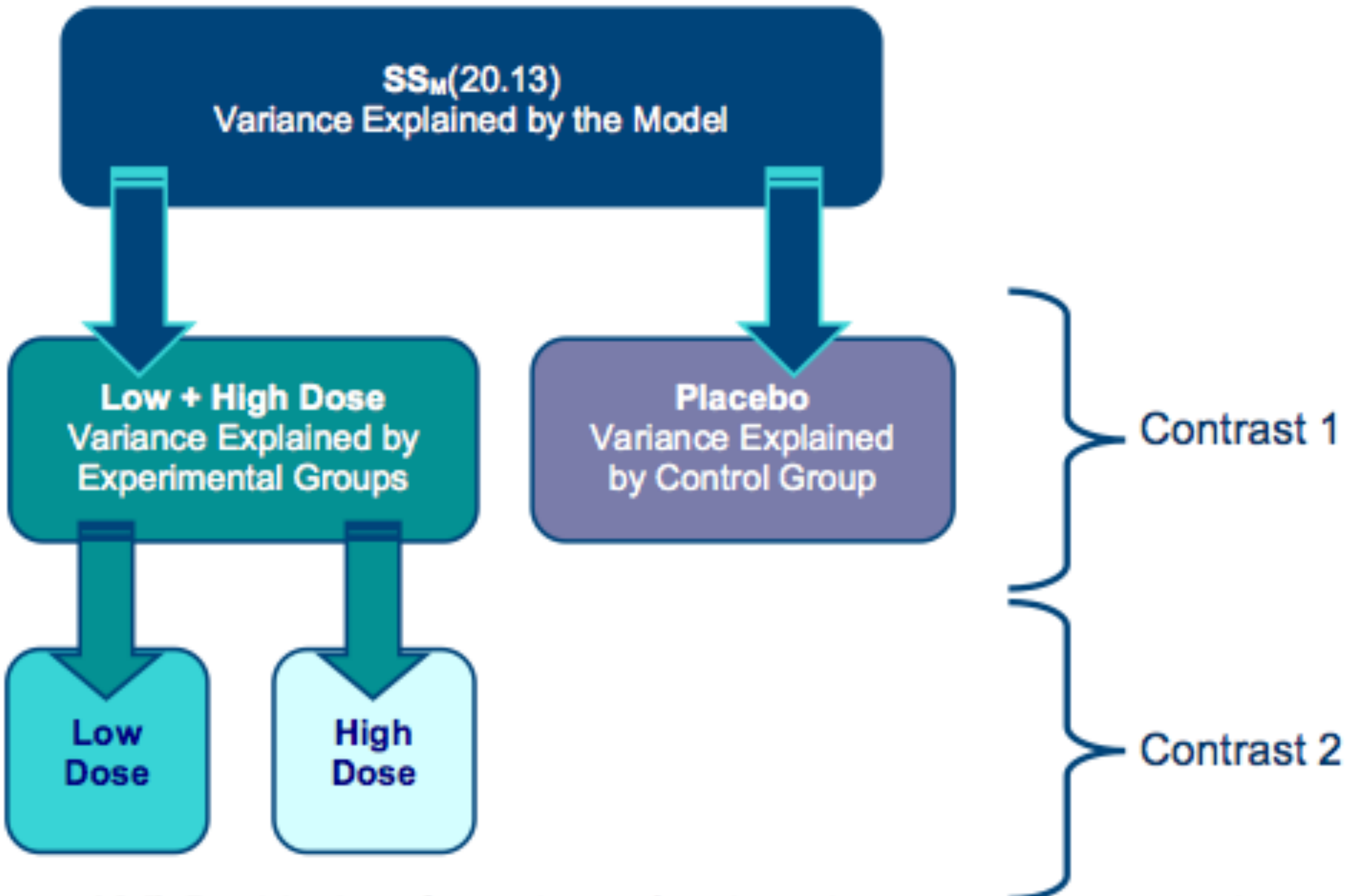


Figure 10.5: Partitioning of experimental variance into component comparisons

Another Example

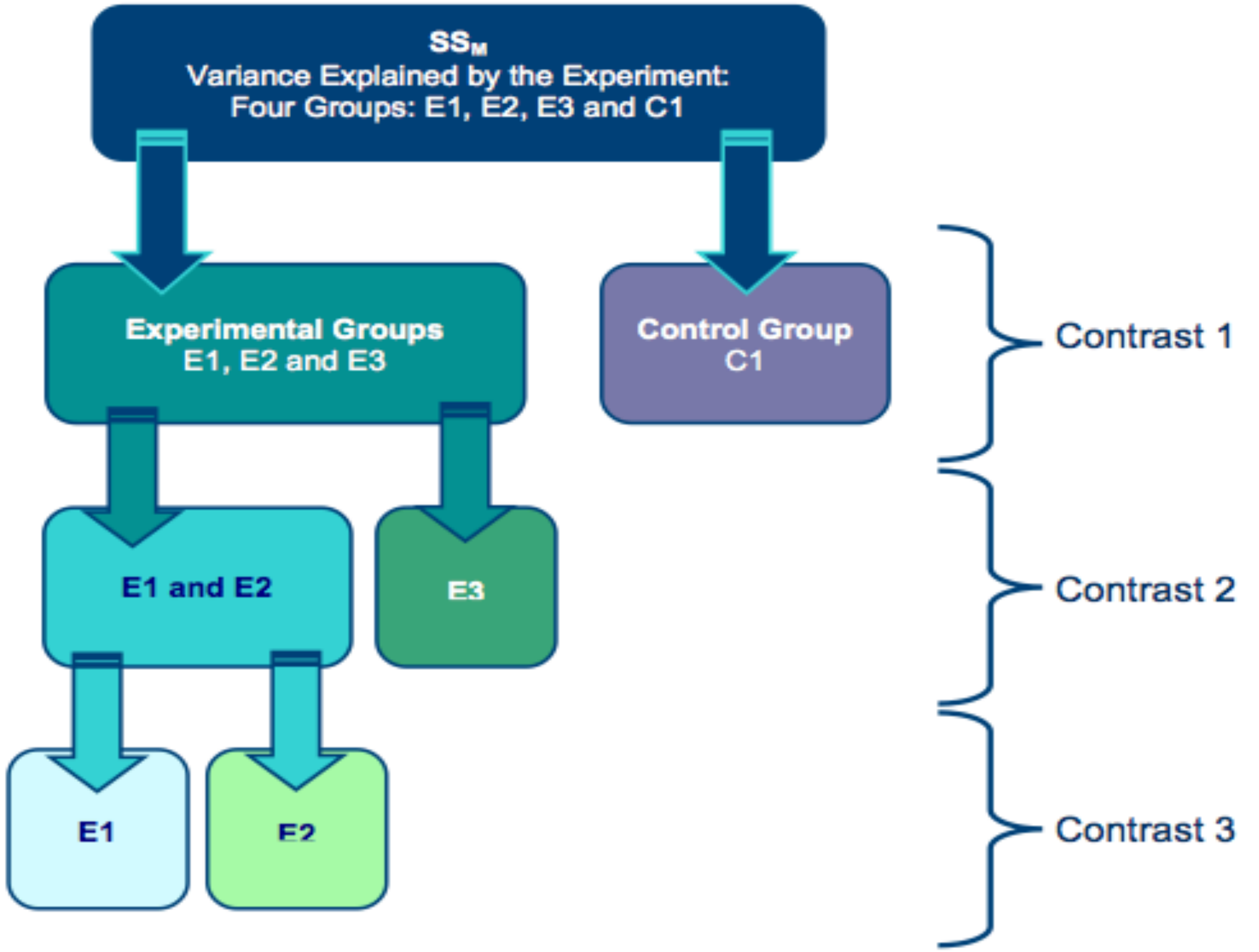


Figure 10.6: Partitioning variance for planned comparisons in a four-group experiment using one control group

Coding Planned Contrasts: Rules

- Rule 1
  - Groups coded with positive weights compared to groups coded with negative weights.
- Rule 2
  - The sum of weights for a comparison should be zero.
- Rule 3
  - If a group is not involved in a comparison, assign it a weight of zero.
- Rule 4
  - For a given contrast, the weights assigned to the group(s) in one chunk of variation should be equal to the number of groups in the opposite chunk of variation.
- Rule 5
  - If a group is singled out in a comparison, then that group should not be used in any subsequent contrasts.

Group	Contrast 1	Contrast 2
Placebo	-2	0
Low Dose	1	-1
High Dose	1	1
Total	0	0

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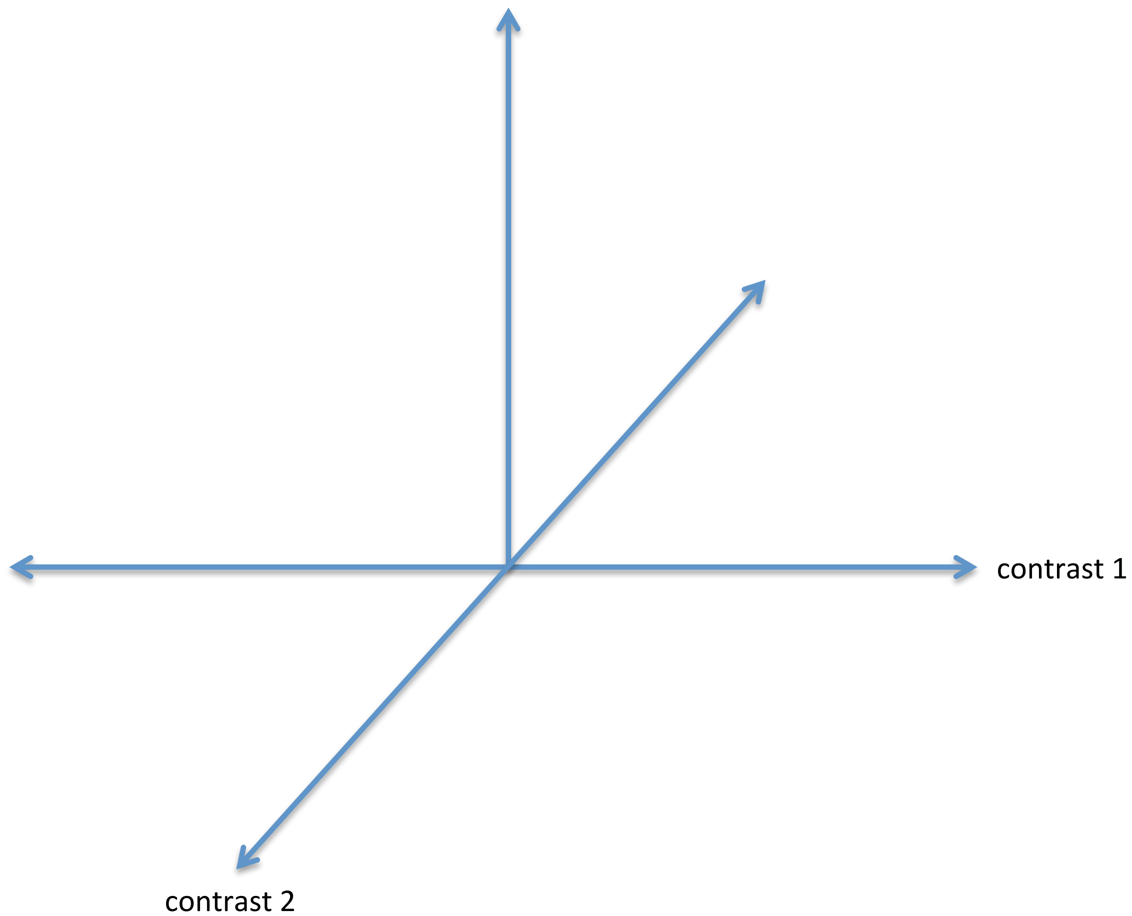

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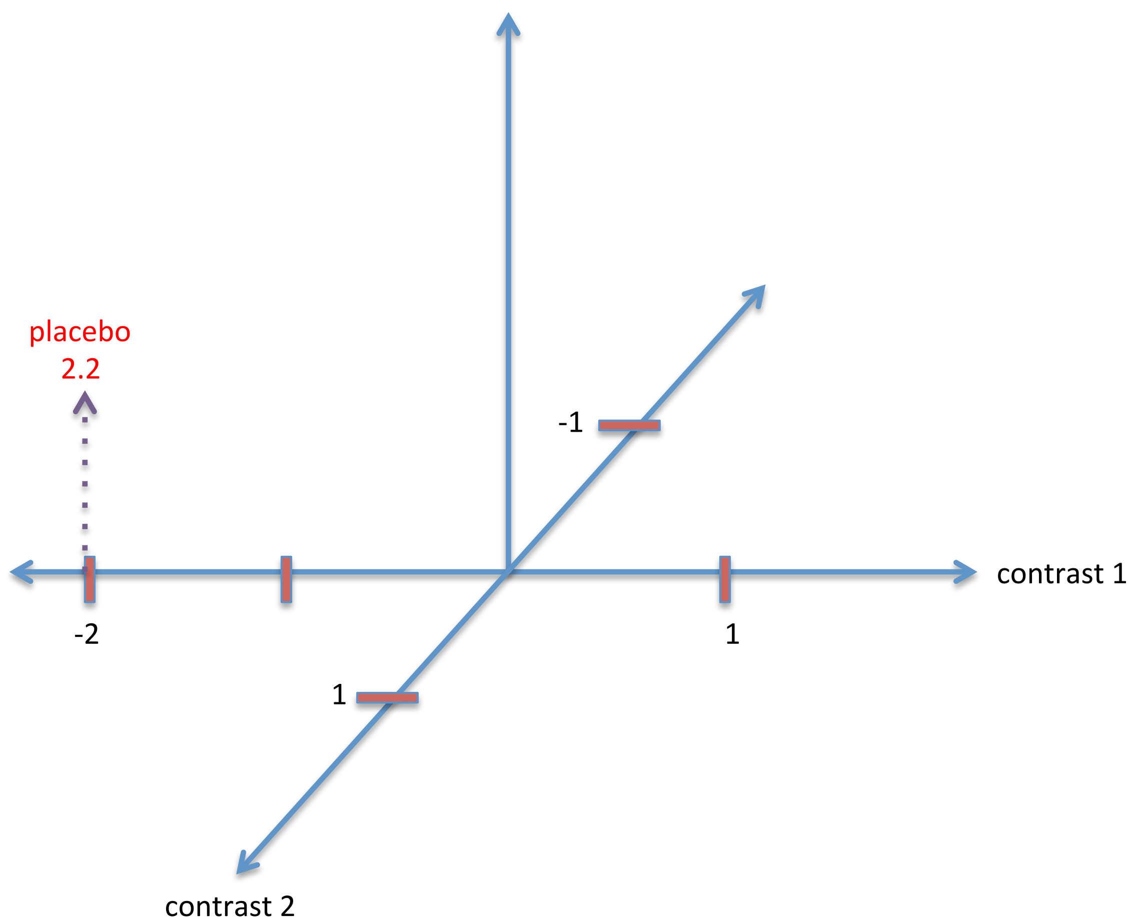

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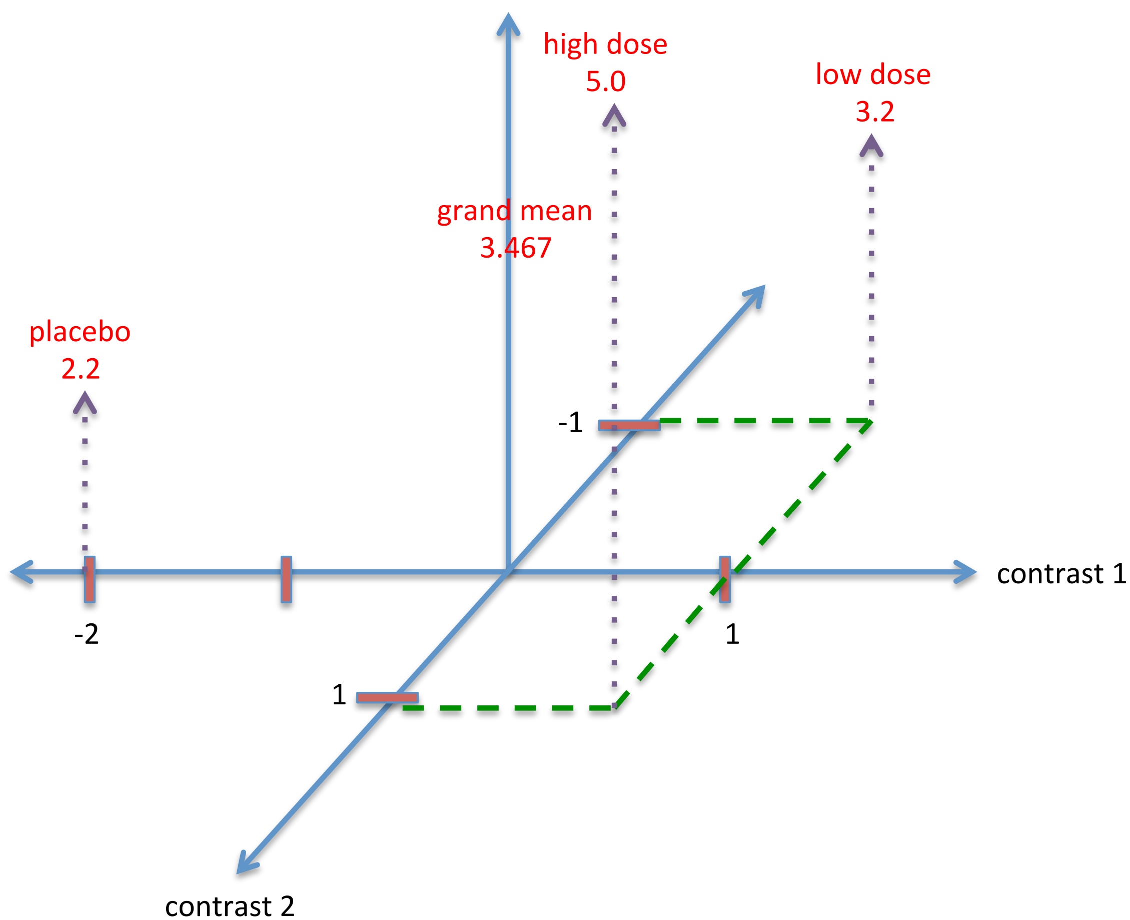

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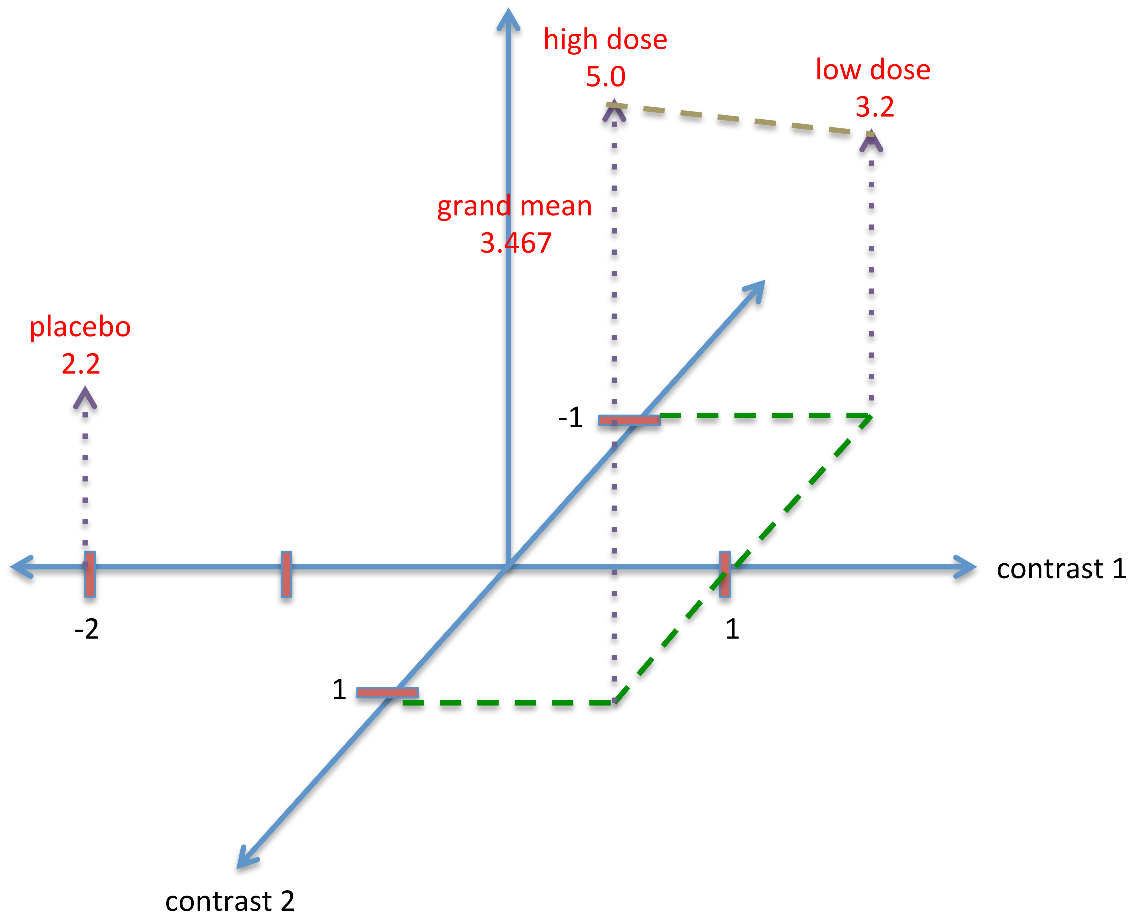

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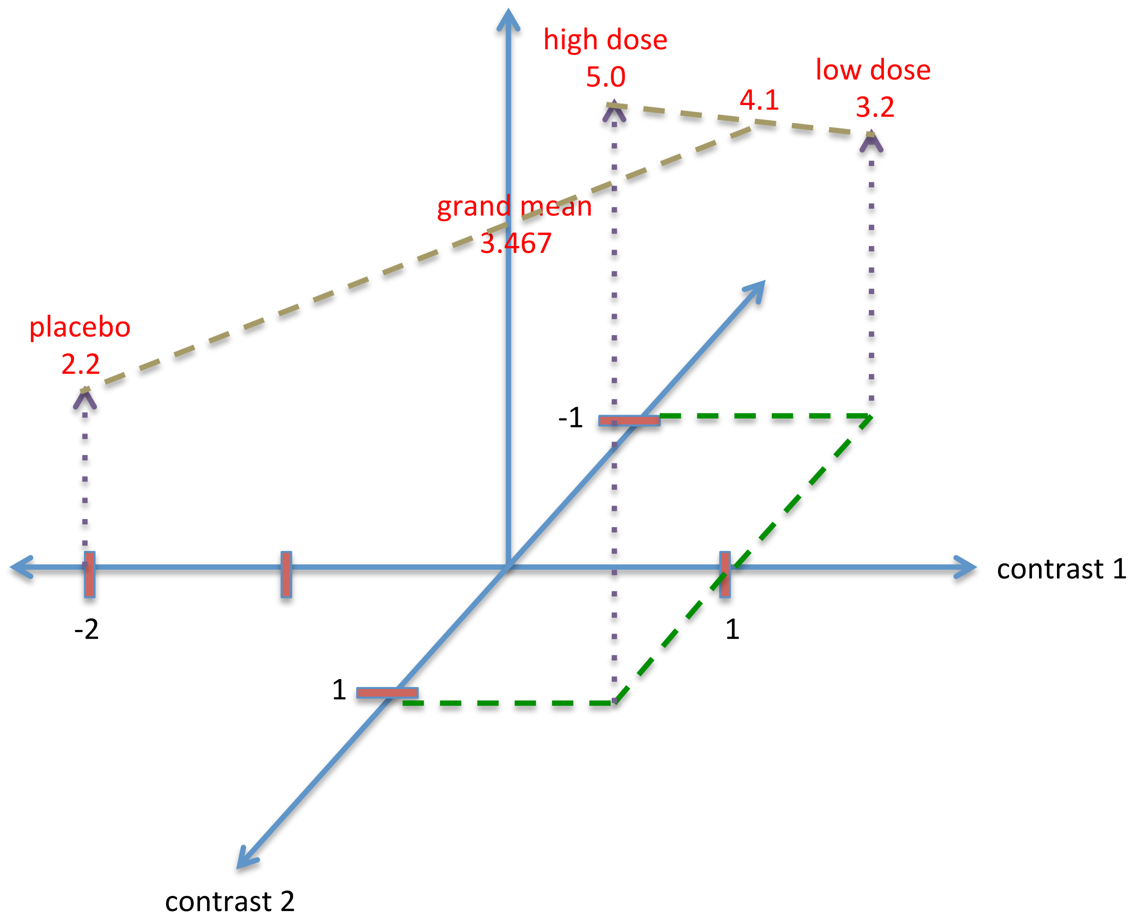

ANOVA in RStudio