General Mathematics

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Contents

Chapter 1

Exponential Function

1.1 Definition of an exponential function

Definition 1.1.1

Let $a > 0$ and $x \in \mathbb{R}$. The function $f(x) = a^x$ is called the exponential function with base a.

Based on the above definition we have:

1. $f(x) = 2^x$ is the exponential function with base 2.
2. $f(x) = e^x$ is the exponential function with base e.

Definition 1.1.2

Consider the exponential function with the form $f(x) = b*a^{x-h}+k$ with $f : \mathbb{R} \longrightarrow \mathbb{R}$; f is an exponential function if and only if $a \in]0, 1[$ or $a \in]1, \infty[$.

Some exponential functions with the form $f(x) = b * a^{x-h} + k$ are:

- $f(x) = 2^{x-1} + 1$
- $f(x) = \frac{1}{2}^{x-1} - 8$

1.2 Properties

Theorem 1.2.1

If the base is negative, the function is not defined for all real numbers.
Let's consider a function $f(x) = (-3)^x$.

At $x = \frac{1}{2}$**.**

$$f\left(\frac{1}{2}\right) = (-3)^{\frac{1}{2}} = \sqrt{-3} = \pm i\sqrt{3} \tag{1.1}$$

Hence, the base of an exponential function cannot be negative.

Theorem 1.2.2

If the function is in the form $f(x) = a^x$, then the function is not transformed, and doesn't have a vertical or horizontal shift.
This also means that the function doesn't have an intersection with the x-axis and the asymptoten is $y = 0$.

Theorem 1.2.3

When $x = 0$ or $x = 1$ the function is constant. It means that isn't an exponential function at these x

values. For this is why the domain of the exponential function is defined like this: $D_f :]0, 1[\cup]1, \infty[$

- If the function base is 1, then the function is constant and $y = 1$.

- If the function base is 0, then the function is constant and equal to 0.

1.3 Graphs

Consider the graph of the function $f(x) = a^x$, $f : \mathbb{R} \to \mathbb{R}^+$.

- The figure 1.1 shows the graph of the function, where a is defined in the interval $]0, 1[$.
- The figure 1.2 shows the graph of the function, where a is defined in the interval $]1, \infty[$.

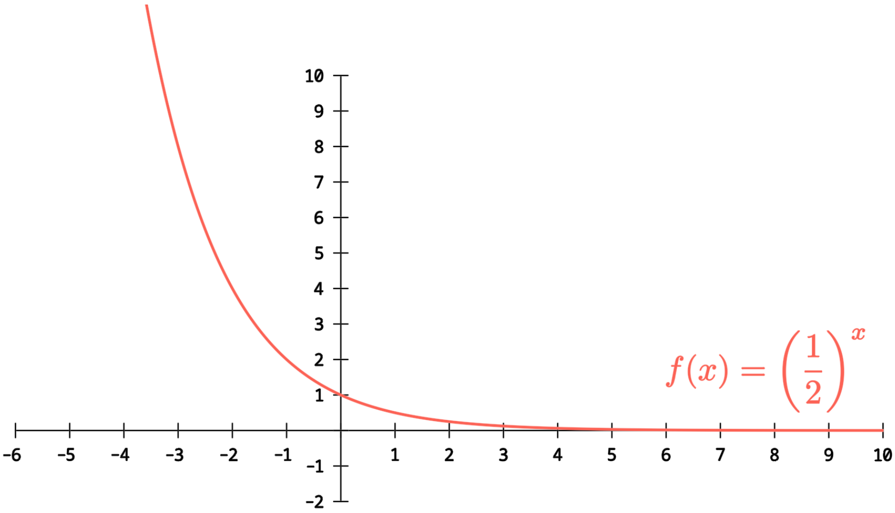

Figure 1.1: Case in which $0 < a < 1$.

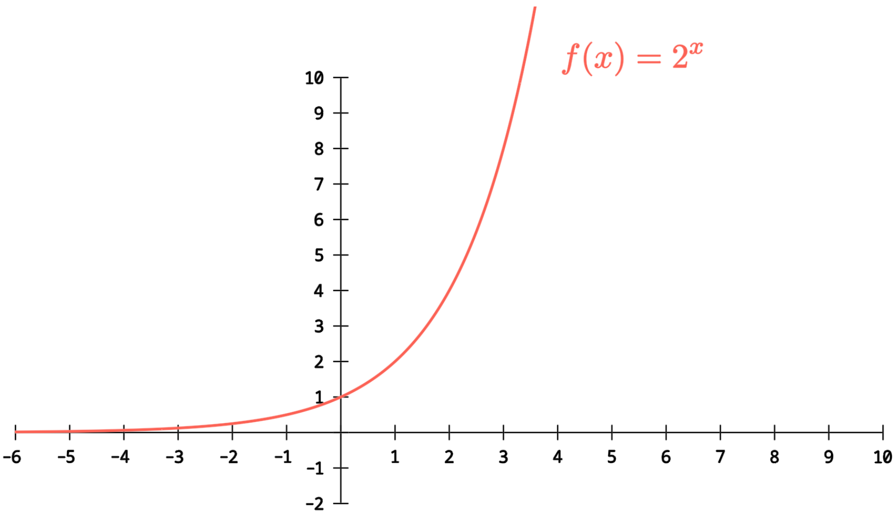

Figure 1.2: Case in which $a > 1$.

1.4 Intercepts and asymptotes

Theorem 1.4.1

The function $f(x) = a^x$ has no intercepts with the x-axis and the asymptoten is $y = 0$.

The function $f(x) = b * a^{x-h} + k$ has an asymptoten in $y = k$, and if k is negative the function has an intersection with the x-axis.

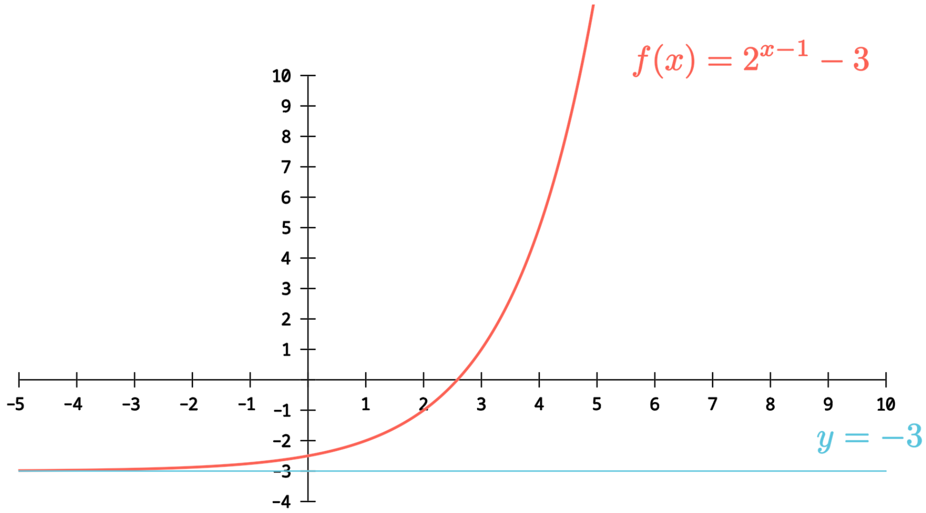

Figure 1.3: Asymptote in $y = -3$ for $f(x) = 2^{x-1} - 3$.

1.5 Solved exercises

Chapter 2

Logarithmic Function

2.1 Definition of a logarithmic function

Definition 2.1.1

Let $a > 0$ and $x \in \mathbb{R}$. The function $f(x) = \log_a(x)$ is called the logarithmic function with base a.

Based on the above definition we have:

1. $f(x) = \log_2(x)$ is the logarithmic function with base 2.
2. $f(x) = \log_e(x)$ is the logarithmic function with base e.

2.2 Properties

Theorem 2.2.1

The logarithmic function is the inverse function of the exponential function.

$$\log_a(x) = y \Leftrightarrow a^y = x \tag{2.1}$$

Example 2.2.1

$$\begin{aligned} \log_2(8) &= y \\ 2^y &= 8 \\ 2^y &= 2^3 \\ y &= 3 \end{aligned} \tag{2.2}$$

$$\therefore \log_2(8) = 3$$

2.3 Domain and Range

Theorem 2.3.1

The domain of the logarithmic function is defined like this: $D_f : \mathbb{R}^+$
The range of the logarithmic function is defined like this: $D_f : \mathbb{R}^+$

2.4 Solved exercises

Question 1

Simplify the following expression:

$$\log_3(5) * \log_2(3) * \log_5(6) * \log_2(6)$$

Solution:

$$\begin{aligned}
\log_3(5) * \log_2(3) * \log_5(6) * \log_2(6) &= \frac{\log(5)}{\log(3)} * \frac{\log(3)}{\log(2)} * \frac{\log(6)}{\log(5)} * \frac{\log(6)}{\log(2)} \\
&= \frac{\log(6)}{\log(2)} * \frac{\log(6)}{\log(2)} \\
&= \frac{\log^2(6)}{\log^2(2)} \\
&= \left[\frac{\log(6)}{\log(2)}\right]^2 \\
&= [\log_2(6)]^2 \\
&= \log_2^2(6)
\end{aligned}$$

😄

Question 2

Check the identity

$$\ln(x * \sqrt[3]{2x+1} * \sqrt[5]{(3x+1)^2}) = \ln(x) + \frac{1}{3}\ln(2x+1) + \frac{2}{5}\ln(3x+1)$$

Solution:

$$\begin{aligned}
\ln(x * \sqrt[3]{2x+1} * \sqrt[5]{(3x+1)^2}) &= \ln(x) + \ln(\sqrt[3]{2x+1}) + \ln(\sqrt[5]{(3x+1)^2}) \\
&= \ln(x) + ln((2x+1)^{\frac{1}{3}}) + ln((3x+1)^{\frac{2}{5}}) \\
&= \ln(x) + \frac{1}{3}\ln(2x+1) + \frac{2}{5}\ln(3x+1)
\end{aligned}$$

😄

Question 3

Check the identity

$$\frac{1}{4}\log(x^2+4x+3) + \frac{1}{4}\log\left(\frac{x+1}{x+3}\right) = \log\sqrt{x+1}$$

Solution:

$$\begin{aligned}
\frac{1}{4}\log(x^2+4x+3) + \frac{1}{4}\log\left(\frac{x+1}{x+3}\right) &= \frac{1}{4}\log\left[(x+1)(x+3)\right] + \frac{1}{4}\log\left(\frac{x+1}{x+3}\right) \\
&= \frac{1}{4}\log\left[(x+1)(x+3)\right] + \frac{1}{4}\left[\log(x+1) - \log(x+3)\right] \\
&= \frac{1}{4}\log(x+1) + \frac{1}{4}\log(x+3) + \frac{1}{4}\log(x+1) - \frac{1}{4}\log(x+3) \\
&= \frac{1}{4}\log(x+1) + \frac{1}{4}\log(x+1) \\
&= \frac{1}{2}\log(x+1) \\
&= \log\sqrt{x+1}
\end{aligned}$$

😄

Question 4

Check the identity

$$\log\left(\frac{x\sqrt{x+1}}{x^2-1}\right) = \log(x) - \frac{1}{2}\log(x+1) - \log(x-1)$$

Solution:

$$\begin{aligned}
\log\left(\frac{x\sqrt{x+1}}{x^2-1}\right) &= \log\left(\frac{x\sqrt{x+1}}{(x+1)(x-1)}\right)\\
&= \log(x\sqrt{x+1}) - \log((x+1)(x-1))\\
&= \log(x\sqrt{x+1}) - \log(x+1) - \log(x-1)\\
&= \log(x) + \log((x+1)^{\frac{1}{2}}) - \log(x+1) - \log(x-1)\\
&= \log(x) + \frac{1}{2}\log(x+1) - \log(x+1) - \log(x-1)\\
&= \log(x) - \frac{1}{2}\log(x+1) - \log(x-1)
\end{aligned}$$

😄

Question 5

Solve in $\mathbb{R}$ the following equation applying power laws:

$$2^x\left(\frac{2^{2x}}{4^{-3+x}}\right)^{x+1} = \sqrt{4^{6-x}}$$

Solution:

$$\begin{aligned}
2^x\left(\frac{2^{2x}}{4^{-3+x}}\right)^{x+1} &= \sqrt{4^{6-x}}\\
2^x\left(\frac{2^{2x}}{(2^2)^{-3+x}}\right)^{x+1} &= \sqrt{(2^2)^{6-x}}\\
2^x\left(\frac{2^{2x}}{2^{-6+2x}}\right)^{x+1} &= \sqrt{2^{12-2x}}\\
2^x(2^{2x-(-6+2x)})^{x+1} &= 2^{\frac{12-2x}{2}}\\
2^x(2^6)^{x+1} &= 2^{6-x}\\
2^x(2^{6x+6}) &= 2^{6-x}\\
2^{x+6x+6} &= 2^{6-x}\\
x+6x+6 &= 6-x\\
7x &= -x\\
8x &= 0\\
x &= 0
\end{aligned}$$

😄

Question 6

Solve in $\mathbb{R}$ the following equation applying power laws:

$$2^{2x} - 6 * 2^x + 8 = 0$$

Solution: With $u = 2^x$ we have

$$\begin{aligned}
(2^x)^2 - 6 * 2^x + 8 &= 0\\
u^2 - 6u + 8 &= 0\\
(u-2)(u-4) &= 0
\end{aligned}$$

For $u = 2$:

$$2^x = 2$$
$$x = 1$$

For $u = 4$:

$$2^x = 4$$
$$2^x = 2^2$$
$$x = 2$$

Therefore, for $u = 2$ we have $x = 1$ and for $u = 4$ we have $x = 2$. 😊

Question 7

Solve in $\mathbb{R}$ the following equation:

$$e^{5x-2} - 7^{1-x} = 0$$

Solution:

$$\begin{aligned} e^{5x-2} &= 7^{1-x} \\ \ln(e^{5x-2}) &= \ln(7^{1-x}) \\ 5x - 2 * \ln(e) &= (1-x)\ln(7) \\ 5x - 2 &= (1-x)\ln(7) \\ 5x - 2 &= \ln(7) - x\ln(7) \\ 5x + x\ln(7) &= \ln(7) + 2 \\ x(5 + \ln(7)) &= \ln(7) + 2 \\ x &= \frac{\ln(7) + 2}{5 + \ln(7)} \end{aligned}$$

$\therefore x = \frac{\ln(7)+2}{5+\ln(7)}$ 😊

Question 8

Solve in $\mathbb{R}$ the following equation:

$$3 * 2^{2x} - 29 * 2^x = -40$$

Solution: With $u = 2^x$ we have:

$$\begin{aligned} 3 * 2^{2x} - 29 * 2^x &= -40 \\ 3u^2 - 29u &= -40 \\ 3u^2 - 29u + 40 &= 0 \\ (3u-5)(u-8) &= 0 \end{aligned}$$

For $3u = 5$:

$$\begin{aligned} 3u &= 5 \\ u &= \frac{5}{3} \\ 2^x &= \frac{5}{3} \\ \log_2(2^x) &= \log_2(\frac{5}{3}) \\ x &= \log_2(\frac{5}{3}) \end{aligned}$$

For $u = 8$:

$$\begin{aligned} u &= 8 \\ 2^x &= 8 \\ 2^x &= 2^3 \quad 😊 \\ x &= 3 \end{aligned}$$

Question 9

Solve in $\mathbb{R}$ the following equation:

$$\ln(x\sqrt[3]{4-2x}) = \ln(x) + \frac{1}{3}\ln(4-2x)$$

Solution:

$$\begin{aligned}
\ln(x\sqrt[3]{4-2x}) &= \ln(x) + \frac{1}{3}\ln(4-2x) \\
\ln(x) + \ln(\sqrt[3]{4-2x}) &= \ln(x) + \frac{1}{3}\ln(4-2x) \\
\ln(x) + \frac{1}{3}\ln(4-2x) &= \ln(x) + \frac{1}{3}\ln(4-2x) \\
\ln(x) + \frac{1}{3}\ln(4-2x) - \ln(x) - \frac{1}{3}\ln(4-2x) &= 0 \\
0 &= 0
\end{aligned}$$

Domain of $\ln(x)$ is $\mathbb{R}^+$, so $x \in \mathbb{R}^+$.

Domain of $\sqrt[3]{4-2x}$ is $\mathbb{R}^+$, so for $4-2x > 0$ we have $x \in \mathbb{R}^+$.

$$\begin{aligned}
4-2x &> 0 \\
-2x &> -4 \\
x &< 2
\end{aligned}$$

Therefore, $x \in \mathbb{R}^+$ and $x < 2$, so $D_f :]0,2[$. ☺

Question 10

Solve in $\mathbb{R}$ the following equation:

$$\log_3(x+7) + \log_3(x+1) = 1 + \log_3(11-x)$$

Solution: First, we need the domains of the function:

For $\log_3(x+7)$:

$$\begin{aligned}
\log_3(x+7) &> 0 \\
x+7 &> 0 \\
x &> -7
\end{aligned}$$

For $\log_3(x+1)$:

$$\begin{aligned}
\log_3(x+1) &> 0 \\
x+1 &> 0 \\
x &> -1
\end{aligned}$$

For $\log_3(11-x)$:

$$\begin{aligned}
\log_3(11-x) &> 0 \\
11-x &> 0 \\
x &< 11
\end{aligned}$$

So, $D_f :]-1,11[$.

$$\begin{aligned}
\log_3(x+7)+\log_3(x+1) &= 1+\log_3(11-x)\\
\log_3((x+7)(x+1))-\log_3(11-x) &= 1\\
\log_3\left(\frac{(x+7)(x+1)}{11-x}\right) &= 1\\
3^1 &= \frac{(x+7)(x+1)}{11-x}\\
3(11-x) &= (x+7)(x+1)\\
33-3x &= x^2+7x+x+7\\
33-3x &= x^2+8x+7\\
0 &= x^2+8x+7+3x-33\\
0 &= x^2+11x-26\\
0 &= (x-2)(x+13)
\end{aligned}$$

Now we have $x = 2$ and $x = -13$, but $2 \in]-1, 11[$ and $-13 \notin]-1, 11[$, so $x = 2$ is the only solution. 😄

Question 11

Solve in $\mathbb{R}$ the following equation:

$$\log_2(5y-6)-\log_2(5y+1) = 3$$

Solution: First we need the domains of the function:

For $\log_2(5y-6)$:

$$\begin{aligned}
\log_2(5y-6) &> 0\\
5y-6 &> 0\\
y &> \frac{6}{5}
\end{aligned}$$

For $\log_2(5y+1)$:

$$\begin{aligned}
\log_2(5y+1) &> 0\\
5y+1 &> 0\\
y &> -\frac{1}{5}
\end{aligned}$$

So, $D_f :]\frac{6}{5}, \infty[$. Now, we can solve the equation:

$$\begin{aligned}
\log_2(5y-6)-\log_2(5y+1) &= 3\\
\log_2\left(\frac{5y-6}{5y+1}\right) &= 3\\
2^3 &= \frac{5y-6}{5y+1}\\
8(5y+1) &= 5y-6\\
40y+8 &= 5y-6\\
35y &= -14\\
y &= -\frac{14}{35}\\
y &\approx -0.4
\end{aligned}$$

But $-0.4 \notin]\frac{6}{5}, \infty[$, so there is no solution.
$\therefore$S=$\emptyset$ 😄

Question 12

Solve in $\mathbb{R}$ the following equation:

$$\log_2(x) + 4 + \log_2(x+1) = \log_2(x+2) + 5$$

Solution: First we need the domains of the function:

For $\log_2(x)$:

$$\begin{aligned} \log_2(x) &> 0 \\ x &> 0 \end{aligned}$$

For $\log_2(x+1)$:

$$\begin{aligned} \log_2(x+1) &> 0 \\ x+1 &> 0 \\ x &> -1 \end{aligned}$$

For $\log_2(x+2)$:

$$\begin{aligned} \log_2(x+2) &> 0 \\ x+2 &> 0 \\ x &> -2 \end{aligned}$$

So, D_f :$]0, \infty[$. Now, we can solve the equation:

$$\begin{aligned} \log_2(x) + 4 + \log_2(x+1) &= \log_2(x+2) + 5 \\ \log_2(x) + \log_2(x+1) - \log_2(x+2) &= 1 \\ \log_2\left(\frac{x(x+1)}{x+2}\right) &= 1 \\ 2^1 &= \frac{x(x+1)}{x+2} \\ 2(x+2) &= x(x+1) \\ 2x+4 &= x^2+x \\ 2x &= x^2+x-4 \\ 0 &= x^2-x-4 \end{aligned}$$

We need to solve the quadratic equation $x^2 - x - 4 = 0$.

$$\begin{aligned} x^2 - x - 4 &= 0 \\ (x - \frac{1+\sqrt{17}}{2})(x - \frac{1-\sqrt{17}}{2}) &= 0 \end{aligned}$$

Now we have $x = \frac{1+\sqrt{17}}{2}$ and $x = \frac{1-\sqrt{17}}{2}$, but $\frac{1-\sqrt{17}}{2} \notin]0, \infty[$, so $\frac{1+\sqrt{17}}{2}$ is the only solution.

$\therefore$S=$\left\{\frac{1+\sqrt{17}}{2}\right\}$ ☺

Chapter 3

Trigonometric Ratios

3.1 Introduction

3.1.1 Pythagorean Theorem

Sea ΔABC un triángulo rectángulo, recto en B, como se muestra en la siguiente figura, entonces se cumple que $h^2 = a^2 + b^2$.

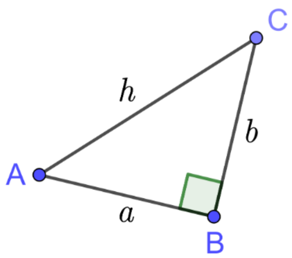

Figure 3.1: Triángulo rectángulo

3.1.2 Fundamental Identities

$$\begin{aligned}
\tan\theta &= \tfrac{\sin\theta}{\cos\theta} \\
\cot\theta &= \tfrac{\cos\theta}{\sin\theta} \\
\cot\theta &= \tfrac{1}{\tan\theta} \\
\sec\theta &= \tfrac{1}{\cos\theta} \\
\csc\theta &= \tfrac{1}{\sin\theta}
\end{aligned}$$

Chapter 4

Triangulos rectangulos

4.1 Angulos

Question 13: Problema 2

Desde un punto sobre el suelo a 500 pies de la base de un edificio, un observador encuentra que el angulo de elevacion hasta la parte superior del edificio es 24° y que el angulo de elevacion a la parte superior de un aste de bandera sobre el edificio es 27°. Determine la altura del edificio y la longitud del asta.

Solucion: Altura del edificio:

$$\begin{aligned} \tan(24) &= \frac{h}{500} \\ 500\tan(24) &= h \\ h &\approx 222,61 \end{aligned}$$

Longitud del asta:

$$\begin{aligned} \tan(27) &= \frac{k}{500} \\ 500\tan(27) &= k \\ k &\approx 254,76 \end{aligned}$$

$$\begin{aligned} x &= k - h \\ x &= 254,76 - 222,61 \\ x &\approx 32,15 \end{aligned}$$

$\therefore$ la medida del asta es de $32,15$ pies. ☺

Question 14: Problema 3

Una escalera de 40° pies esta apoyada en un edificio. Si la base de la escalera esta separada 6° pies de la base del edificio, cual es el angulo que forman la escalera y el edificio?

Solucion:

$$\begin{aligned} \sin(\theta) &= \frac{6}{40} \\ \arcsin(\sin(\theta)) &= \arcsin(\frac{6}{40}) \\ \theta &= \arcsin(\frac{6}{40}) \\ \theta &= \end{aligned}$$

☺

Question 15: Problema 4

Una mujer parada sobre una colina ve un asta de bandera que sabe que tiene 60 pies de altura. El angulo de depresion respecto de la parte inferior del asta es de 14° y el angulo de elevacion repecto de la parte superior del asta es de 18°. Encuentre la distancia x desde el asta.

Solucion:

$$\tan(14) = \frac{y}{x}$$
$$\tan(18) = \frac{60 - y}{x}$$

$$y = \tan(14)x$$

$$\tan(18) = \frac{60 - \tan(14)x}{x}$$
$$x\tan(18) = 60 - \tan(14)x$$
$$x\tan(18) + \tan(14)x = 60$$
$$x = \frac{60}{\tan(18) + \tan(14)}$$
$$x \approx 104,48$$

😄

4.2 Leyes de los senos y cosenos

Theorem 4.2.1 Ley de senos

Considere el $\triangle ABC$ un triangulo cualquiera y considere $a = \overline{BC}$, $b = \overline{AC}$ y $c = \overline{AB}$, $a = m\measuredangle BAC$, $b = m\measuredangle ABC$ y $\theta = m\measuredangle ACB$ tal como se muestra en la siguiente figura.

$\therefore \frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\theta}{c}$

Question 16: Problema 1

Los puntos A y B estan separados por un lago. Para hallar la distancia entre ellos el topografo localiza un punto C sobre el suelo, tal que $m\measuredangle CAB = 48,6°$. Ademas, toma la mediada de $\overline{AC}$ en metros, donde $\overline{AC} = 312m$ y $\overline{CB} = 527m$. Encuentre la distancia de A a B.

Solucion:

$$
\begin{aligned}
\frac{\sin(\theta)}{x} &= \frac{\sin(48,6)}{527} = \frac{\sin\beta}{312} \\
\frac{\sin(\theta)}{x} &= \frac{\sin(48,6)}{527}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\sin(48,6)}{527} &= \frac{\sin\beta}{312} \\
312\frac{\sin(48,6)}{527} &= \sin\beta \\
\arcsin(312\frac{\sin(48,6)}{527}) &= \beta \\
\beta &= 26,36°
\end{aligned}
$$

$$
\begin{aligned}
\theta &= 180 - 48,6 - 26,36 \\
\theta &= 105,04°
\end{aligned}
$$

$$
\begin{aligned}
\frac{x}{\sin(105,04)} &= \frac{527}{\sin(48,6)} \\
x &= 527\frac{\sin(105,04)}{\sin(48,6)} \\
x &\approx 679,49
\end{aligned}
$$

😄

Question 17: Problema 2

Una torre de 125m de altura se encuentra sobre una base de concreto en la orilla de un rio, como se muestra en la figura. Desde lo alto de la torre, el angulo de depresion a un punto en la orilla opuesta es de 42° y desde la base de la torre el angulo de depresion al mismo punto es de 13,3°. Hallar la medida del ancho del rio y la medida de la base donde descansa la torre.

Solucion:

$$
\tan(42) = \frac{125 + y}{x}
$$

$$
\tan(13,3) = \frac{y}{x}
$$

$$
y = \tan(13,3)x
$$

$$
\begin{aligned}
\tan(42) &= \frac{125 + \tan(13,3)x}{x} \\
x\tan(42) &= 125 + \tan(13,3)x \\
x\tan(42) - \tan(13,3)x &= 125 \\
x &= \frac{125}{\tan(42) - \tan(13,3)} \\
x &\approx 188,24
\end{aligned}
$$

$$
\begin{aligned}
y &= \tan(13,3)188,24 \\
y &\approx 44,49
\end{aligned}
$$

😄

Question 18: Problema 3

Un barco zarpo de un muelle y navego $150km$ en direccion $N30°22'0''O$ para reunirse con otra embarcacion. Luego de la reunion, el barco viro con rumbo $S20°45'30''O$ y navego toda la noche. En un determinado instante, el encargado del cuarto de maquinas dio el aviso al capitan sobre un problema que los dejaria a la deriva en aproximadamente $220km$ por lo que deberan regresar a puerto de inmediato. El navegante determino que el rumbo a seguir es $E28°15'0''N$ para llegar al mismo muelle donde habia partido. A que distancia esta el barco del muelle?, Que distancia recorrio el barco desde la reunion hasta la deteccion del problema?, Lograran llegar antes de quedar a la deriva?

Solucion: ☺

4.3 Ley de cosenos

Question 19: Problema 1

Solucion: Aplicando la ley de cosenos tenemos que:

$$
\begin{aligned}
x^2 &= 543^2 + 425^2 - 2 \cdot 543 \cdot 425 \cdot \cos(72.5°) \\
x^2 &= 294849 + 180625 - 461550 \cdot \cos(72.5°) \\
x^2 &= 475474 - 138790.7618 \\
x^2 &= 336683.2382 \\
x &= \sqrt{336683.2382} \\
x &= 580.24
\end{aligned}
$$

☺

Question 20: Problema 2

Desde el borde de un acantilado de $60m$ de altura una persona observa un barco con un angulo de depresion de $17°$, en el mismo instante observa un helicoptero que vuela $145m$ de altura sobre el nivel del mar con un angulo de elevacion de $26°$. Si se sabe que el observador, el barco y el helicoptero se encuentran en el mismo plano vertical, determine la distancia entre el barco y el helicoptero en dicho instante.

Solucion:

$$
\begin{aligned}
(17) &= \frac{60}{x} \\
x &= \frac{60}{(17)} \\
x &= 205,22
\end{aligned}
$$

$$
\begin{aligned}
(26) &= \frac{85}{y} \\
y &= \frac{85}{(26)} \\
y &= 193,90
\end{aligned}
$$

$$
\begin{aligned}
x^2 &= (193,90)^2 + (205,22)^2 - 2 \cdot 193,90 \cdot 205,22 \cdot \cos(43) \\
x^2 &= 21508,1743 \\
x &= \sqrt{21508,1743} \\
x &= 146,65
\end{aligned}
$$

$\therefore$ La distancia entre el barco y el helicoptero es de $146,65m$ aproximadamente. ☺

Question 21: Problema 3

Una camara de vigilancia instalada en la parte superior de un poste vertical de $8m$ de alto ubicado sobre un parque plano, enfoca un punto **A** sobre el suelo con un angulo de depresion de 61°. Si se sabe que el angulo entre las lineas de vision qde la camara en el punto **A** y el punto **B** es de 43°, determine la distancia entre **A** y **B**.

Solucion:

$$
\begin{aligned}
(61) &= \frac{8}{x} \\
x &= \frac{8}{(61)} \\
x &= 9,14
\end{aligned}
$$

$$
\begin{aligned}
(48) &= \frac{8}{y} \\
y &= \frac{8}{(48)} \\
y &= 10,76
\end{aligned}
$$

$$
\begin{aligned}
AB^2 &= (9,14)^2 + (10,76)^2 - 2 \cdot 9,14 \cdot 10,76 \cdot \cos(43) \\
AB &= 7,44
\end{aligned}
$$

∴ La distancia entre **A** y **B** es de $7,44m$ aproximadamente. 😆

Question 22: Problema 4

La ciudad A esta ubicada directamente al sur de la ciudad B, pero no hay vuelos de la ciudad A a la ciudad B. Los aviones primero vuelan $143km$ de la ciudad A a una ciudad C, la cual esta 51° al norte, 51° este de A y de ahi vuela $212km$ hasta la ciudad B. Determine la distancia (en linea recta) entre las ciudades A y B.