732A96/TDDE15 Advanced Machine Learning Reinforcement Learning

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Lectures 9: REINFORCE and Deep Q-Learning Algorithms

Contents

- ▶ REINFORCE
- Neural Networks
- ► Example: Grid Worlds
- Deep Q-Learning
- ► Example: CartPole

Literature

Main source

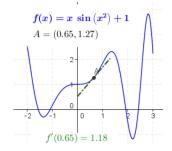
- Sutton, R. S. and Barto, A. G. Reinforcement Learning: An Introduction. The MIT Press, 2018. Chapters 13 and 16.
- Mnih, V. et al. Playing Atari with Deep Reinforcement Learning. NIPS Deep Learning Workshop, 2013.

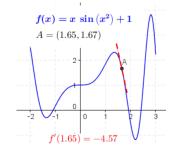
Additional source

- Russel, S. and Norvig, P. Artifical Intelligence: A Modern Approach. Pearson, 2010. Chapters 16, 17 and 21.
- Mnih, V. et al. Human-level Control through Deep Reinforcement Learning. Nature 518, 529–533, 2015.

REINFORCE

- Consider learning a parameterized policy $\pi(a|s,\theta)$ to select actions without consulting state or action values.
- Advantages:
 - The policy function may be simpler to learn than the state and action value functions.
 - Possibility to inject prior knowledge through the parameterization chosen.
 - Possibility to model deterministic and stochastic policies. The latter may be interesting in problems with imperfect knowledge, e.g. bluffing in poker. Learning is also easier as the gradient is smoother.
- Choose the parameter values that maximize the following function for **episodic** tasks: $J(\theta) = v_{\pi_{\theta}}(S_0)$ where S_0 is the initial state. It can also be adapted to continuing tasks.
- ► Stochastic gradient ascent: $\theta^{i+1} \leftarrow \theta^i + \alpha \nabla_{\theta^i} J(\theta^i)$.





REINFORCE

Theorem:

$$\nabla_{\theta} J(\theta) = \nabla_{\theta} v_{\pi_{\theta}}(S_0) \propto E_{\pi_{\theta}} \Big[\sum_{a} \gamma^t q_{\pi}(S_t, a) \nabla_{\theta} \pi(a | S_t, \theta) \Big] \text{ at time } t$$

and thus

$$\begin{split} \nabla_{\theta} J(\theta) &\propto E_{\pi_{\theta}} \Big[\sum_{a} \pi(a|S_{t},\theta) \gamma^{t} q_{\pi}(S_{t},a) \frac{\nabla_{\theta} \pi(a|S_{t},\theta)}{\pi(a|S_{t},\theta)} \Big] = E_{\pi_{\theta}} \Big[\gamma^{t} q_{\pi}(S_{t},A_{t}) \frac{\nabla_{\theta} \pi(A_{t}|S_{t},\theta)}{\pi(A_{t}|S_{t},\theta)} \Big] \\ &= E_{\pi_{\theta}} \Big[\gamma^{t} G_{t} \frac{\nabla_{\theta} \pi(A_{t}|S_{t},\theta)}{\pi(A_{t}|S_{t},\theta)} \Big] = E_{\pi_{\theta}} \Big[\gamma^{t} G_{t} \nabla_{\theta} \ln \pi(A_{t}|S_{t},\theta) \Big]. \end{split}$$

All this results in the REINFORCE algorithm, which asymptotically converges to a local optimum of $J(\theta)$.

REINFORCE: Monte-Carlo Policy-Gradient Control (episodic) for π_*

Input: a differentiable policy parameterization $\pi(a|s,\theta)$

Algorithm parameter: step size $\alpha > 0$

Initialize policy parameter $\boldsymbol{\theta} \in \mathbb{R}^{d'}$ (e.g., to 0)

Loop forever (for each episode):

Generate an episode $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$, following $\pi(\cdot|\cdot, \boldsymbol{\theta})$

Loop for each step of the episode t = 0, 1, ..., T - 1:

$$G \leftarrow \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k$$

$$\theta \leftarrow \theta + \alpha \gamma^t G \nabla \ln \pi (A_t | S_t, \theta)$$
 (G_t)

REINFORCE

- REINFORCE has one major disadvantage: It updates the policy only at the end of each episode and, thus, it is not suitable for incremental online learning and may show slow convergence and high variance.
- A particularly interesting advantage of REINFORCE is that updating the policy parameters may change the policy for every action-state pair, and not only for the pairs visited, i.e. generalization.
- Another interesting feature of REINFORCE is that it can be applied to continuous state and action spaces by choosing an appropriate parametric policy model, e.g. $\mathcal{N}(\mu s, \sigma)$:

$$\pi(a|s,\theta) = \pi(a|s,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(a-\mu s)^2}{2\sigma^2}\right)$$

which implies that

$$\nabla_{\mu} \ln \pi(a|s,\mu,\sigma) = \frac{a-\mu s}{\sigma^2} s$$

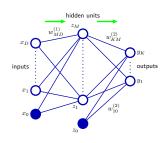
and

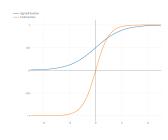
$$\nabla_{\sigma} \ln \pi(a|s,\mu,\sigma) = \left(\frac{(a-\mu s)^2}{\sigma^2} - 1\right) s$$

and, thus, the REINFORCE update has a simple form.

For discrete state and action spaces, we can easily do better than this as we show next.

Neural Networks





- ▶ Consider a multiclass classification problem, i.e. $C \in \{1, 2, ..., K\}$.
- Activations: $a_j = \sum_{i=1}^{D} w_{ji}^{(1)} x_i + w_{j0}^{(1)}$ for all j = 1, ..., M.
- ▶ Hidden units and activation function: $z_j = h(a_j)$ for all j = 1, ..., M.
- Output activations: $a_k = \sum_{j=1}^{M} w_{kj}^{(2)} z_j + w_{k0}^{(2)}$ for all k = 1, ..., K.
- Output : $y_k(\mathbf{x}) = a_k$ for all k = 1, ..., K.
- ► Two-layer NN: $y_k(x) = \sum_{j=1}^{M} w_{kj}^{(2)} h\left(\sum_{i=1}^{D} w_{ji}^{(1)} x_i + w_{j0}^{(1)}\right) + w_{k0}^{(2)}$.
- ► Class probabilities: $p(C = k|x) = \frac{\exp(y_k(x))}{\sum_{k=1}^{K} \exp(y_k(x))}$.
- ▶ All the previous is generalizable to several hidden layers.

Neural Networks

▶ Consider some training data $\{(\mathbf{x}_n, c_n)\}$ of size N. Consider minimizing the cross-entropy or negative log-likelihood loss function:

$$L(\boldsymbol{w}) = \sum_{n=1}^{N} L_n(\boldsymbol{w}) = -\sum_{n=1}^{N} \log p(c_n|\boldsymbol{x}_n, \boldsymbol{w}).$$

- The weight space is highly multimodal and, thus, we have to resort to approximate iterative methods to minimize the previous expression.
- Stochastic gradient descent:

$$\boldsymbol{w}^{i+1} = \boldsymbol{w}^{i} - \alpha \nabla_{\boldsymbol{w}^{i}} L_{n}(\boldsymbol{w}^{i}) = \boldsymbol{w}^{i} + \alpha \nabla_{\boldsymbol{w}^{i}} \log p(c_{n}|\boldsymbol{x}_{n}, \boldsymbol{w}^{i})$$

where n is chosen randomly, and $\nabla_{\mathbf{w}^i} L_n(\mathbf{w}^i)$ can be computed efficiently via backpropagation.

Note the similarities between the update above and the REINFORCE update:

$$\theta^{i+1} = \theta^i + \alpha \gamma^t G \nabla_{\theta^i} \ln \pi (A_t | S_t, \theta^i).$$

Therefore, REINFORCE is straightforward to implement when a NN is used to represent the parameterized policy. We only need to set the sample weights $\gamma^t G$. In other words, the policy can be seen as a classifier whose training involves certain peculiarities: Incremental online, and **delayed labels** via sample weights.

Example: Grid Worlds

					Act	ion probabilities	after 0 iterat	ions				
		0.03						0.03			0.03	
41	0.6	←	0.1		Goal		0.5	←	0.1	0.44	←	0.1
		0.27						0.37			0.43	
		0.04			0.04			0.04			0.04	
1.	0.51	←	0.13	0.48	←	0.14	0.42	←	0.13	0.36	1	0.12
		0.31			0.35			0.41			0.49	
*		0.07			0.06			0.05			0.04	
1-	0.37	Ţ	0.16	0.39	Ţ	0.16	0.34	Ţ	0.14	0.3	Ţ	0.14
		0.4			0.4			0.47			0.51	
		0.09			0.08			0.06			0.05	
	0.26	1	0.17	0.27	1	0.16	0.29	1	0.16	0.27	1	0.16
		0.48			0.5			0.43			0.51	
					1							
						n probabilities a					4	
					7440							
					0			0			0	
41	0	\rightarrow	0.63	0	\downarrow	0.3	0.01	1	0.00	0.05	\downarrow	0.00
		0.37			0.7			0.91			0.92	
					0.01			0.01			0.02	
11		\rightarrow	0.79	0.01	\rightarrow	0.56	0.06	1	0.23	0.32	1	0.06
		0.2			0.42			0.71			0.6	
*		0.02			0.04						0.09	
21	0.01	\rightarrow	0.9	0.02	\rightarrow	0.77		Goal		0.69	←	0.07
		0.08			0.17						0.15	
		0.06			0.18			0.39			0.2	
1-	0.01	\rightarrow	0.66	0.05	\rightarrow	0.71	0.24	1	0.31	0.73	←	0.06
		0.05			0.06			0.05			0.02	

Neural Networks

Now, consider a unidimensional regression problem. Consider also some training data $\{(x_n, c_n)\}$ of size N. Consider minimizing the mean squared-error loss function:

$$L(\mathbf{w}) = \sum_{n=1}^{N} L_n(\mathbf{w}) = \sum_{n=1}^{N} \frac{1}{2} (c_n - y(\mathbf{x}_n))^2.$$

▶ The above implies the following stochastic gradient descent update:

$$\mathbf{w}^{i+1} = \mathbf{w}^{i} - \alpha \nabla_{\mathbf{w}^{i}} L_{n}(\mathbf{w}^{i}) = \mathbf{w}^{i} + \alpha (c_{n} - y(\mathbf{x}_{n})) \nabla_{\mathbf{w}^{i}} y(\mathbf{x}_{n})$$

where n is chosen randomly, and $\nabla_{\mathbf{w}^i} y(\mathbf{x}_n)$ can be computed efficiently via **backpropagation**.

If the transition model were so that s and a are always followed by s' and r, then $q_*(s,a) = r + \gamma \max_{a'} q_*(s',a')$ by the Bellman optimality equation and, thus, $0 = r + \gamma \max_{a'} q_*(s',a') - q_*(s,a)$. We can try to enforce this constraint by executing π one step from s and a and, then, updating the estimate of $q_*(s,a)$ as

$$q_*(s,a) \leftarrow q_*(s,a) + \alpha(r + \gamma \max_{s'} q_*(s',a') - q_*(s,a)).$$

where $\alpha > 0$ is the learning rate.

The reasoning above also applies to stochastic transition models, since the number of times that s and a are followed by s' and r in the sampled episodes is proportional to the transition probability.

Q-learning (off-policy TD control) for estimating $\pi \approx \pi_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$

Initialize Q(s, a), for all $s \in \mathbb{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(terminal, \cdot) = 0$

Loop for each episode:

Initialize S

Loop for each step of episode:

Choose A from S using policy derived from Q (e.g., ε -greedy)

Take action A, observe R, S'

$$Q(S, A) \leftarrow Q(S, A) + \alpha [R + \gamma \max_{a} Q(S', a) - Q(S, A)]$$

$$S \leftarrow S'$$

until S is terminal

- Q-learning has a major disadvantage: Representing the action value function q(s, a) as a look-up table is not feasible for large state and/or action spaces, due to storage space and time to convergence.
- A solution is to represent it as a **parameterized function** $q(s, a|\theta)$. This brings advantages such as fewer parameters than table entries and thus easier to learn and reach convergence, less storage space, **generalization** to unvisited state-action pairs, etc.
- It also comes with the challenge of selecting an appropriate parameterized function: It has to be expressive and allow for incremental online training. A NN is typically used.

For the i-th iteration (i.e., episode step), the NN is trained by minimizing the mean squared-error loss function:

$$L(\theta^{i}) = E_{s,a \sim \rho(s,a)}[(y^{i} - q(s,a|\theta^{i}))^{2}]$$

where $\rho(s,a)$ is the behaviour distribution induced by ϵ -greedy, and

$$y^{i} = E_{s' \sim p(s'|s,a)}[r + \gamma \max_{a'} q(s', a'|\theta^{i-1})|s, a].$$

The gradient is given by

$$\nabla_{\theta^i} L(\theta^i) = -E_{s,a \sim \rho(s,a);s' \sim p(s'|s,a)} \left[\left(r + \gamma \max_{a'} q(s',a'|\theta^{i-1}) - q(s,a|\theta^i) \right) \nabla_{\theta^i} q(s,a|\theta^i) \right]$$

and stochastic gradient descent is typically considered:

$$\theta^{i+1} = \theta^{i} + \alpha (r + \gamma \max_{a'} q(s', a'|\theta^{i-1}) - q(s, a|\theta^{i})) \nabla_{\theta^{i}} q(s, a|\theta^{i}).$$

▶ Note the similarities between the update above and the NN update:

$$\mathbf{w}^{i+1} = \mathbf{w}^i + \alpha(c_n - y(\mathbf{x}_n)) \nabla_{\mathbf{w}^i} y(\mathbf{x}_n)$$

which implies that deep Q-learning is straightforward to implement when a NN is used to represent the parameterized state value function. In other words, the action value function can be seen as a regressor whose training involves certain peculiarities: Incremental online, and **delayed targets** via bootstrapping on the model from the previous iteration.

Deep Q-learning with experience replay for estimating $\pi \approx \pi_*$

Algorithm parameters: small $\epsilon > 0$

Initialize $Q(S, A|\theta)$ with random weights, and initialize \mathcal{D} with N random experiences

Loop for each episode:

Initialize S

Loop for each step of episode:

Choose A from S using policy derived from Q (e.g., ϵ -greedy)

Take action A, observed R, S'

Store transition (S, A, R, S') in \mathcal{D}

Sample random minibatch of transitions (S_j, A_j, R_j, S_j') from \mathcal{D}

$$Y_j \leftarrow \begin{cases} R_j & \text{for terminal } S_j' \\ R_j + \gamma \max_a Q(S_j', a | \theta) & \text{for non-terminal } S_j' \end{cases}$$

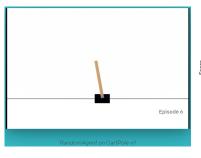
Perform a gradient descent step on $(Y_j - Q(S_j, A_j | \theta))^2$

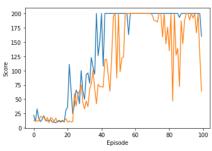
 $S \leftarrow S'$

until S' is terminal

- Learning a parameterized state value function may resemble supervised learning but the learning data is **not i.i.d.** due to highly correlated states and changing agent behaviour. Experience replay alleviates these problems by smoothing the training distribution over many past behaviours.
- ▶ All action values at once: $S \to NN \to (Q(S, A_1), \dots, Q(S, A_k))$.

Example: CartPole





- ▶ Open Al's gym
- ► PilcoLearner

Summary

- ▶ REINFORCE
- Neural Networks
- ► Example: Grid Worlds
- Deep Q-Learning
- ► Example: CartPole
- ▶ Interested in more ? Check out AlphaGo The Movie.

Thank you