732A96/TDDE15 ADVANCED MACHINE LEARNING

EXAM 2023-10-25

Teacher

Jose M. Peña. He will visit the rooms for questions.

Grades

- For 732A96 (A-E means pass):
  - A=19-20 points
  - B=17-18 points
  - C=14-16 points
  - D=12-13 points
  - E=10-11 points
  - F=0-9 points
- For TDDE15 (3-5 means pass):
  - 5=18-20 points
  - 4=14-17 points
  - 3=10-13 points
  - U=0-9 points

In each question, full points requires clear and well motivated answers and commented code.

Instructions

- This is an individual exam. No help from others is allowed. No communication with others is allowed. Answers to the exam questions may be sent to Urkund.
- This is an anonymous exam. Do not write your name on it.
- The answers to the exam should be submitted in a single PDF file. You can make a PDF from LibreOffice (similar to Microsoft Word). You can also use Markdown from RStudio (no support is provided though). Include important code needed to grade the exam (inline or at the end of the PDF file).

Allowed help

Everything on the course web page. Your individual and group solutions to the labs. This help is available on the corresponding directories of the exam system.

1. Probabilistic Graphical Models (5 p)

This exercise is based on the firing squad example in the book *Causality* by Judea Pearl. Specifically, consider the following scenario. The Captain receives the Court's order to execute a prisoner. The Captain carries out the order with probability 0.9. The Captain orders two riflemen (A and B) to shoot the prisoner. Each rifleman shoots with probability 0.8. When they shoot, the riflemen do not miss and the prisoner always dies.

Now, all we know is that the prisoner is dead, i.e. we do not know if the Court gave the order, if the Captain carried it out, or if the riflemen shot. We want to compute the probability that the prisoner would be dead even if rifleman A had not shot, given that we know that the prisoner is dead. Note that this query concerns the actual world where the prisoner is dead, as well as an hypothetical world where the rifleman A does not shoot and the prisoner may or may not be dead. In other words, we have different pieces of evidence for the actual world (the prisoner is dead) and for the hypothetical world (rifleman A does not shoot). Moreover, the query of interest is about the hypothetical world.

Your task is to build a Bayesian network to answer the question above using probabilistic inference. The Bayesian network should contain the following binary random variables:

- U: Court orders the execution in the actual and hypothetical worlds.
- C: Captain orders fire in the actual world.
- A: Rifleman A shoots in the actual world.
- B: Rifleman B shoots in the actual world.
- D: Prisoner dies in the actual world.
- Ch: Captain orders fire in the hypothetical world.
- Ah: Rifleman A shoots in the hypothetical world.
- Bh: Rifleman B shoots in the hypothetical world.
- Dh: Prisoner dies in the hypothetical world.

Hint: First, construct the Bayesian network for the actual world. Then, duplicate it for the hypothetical world. Finally, connect the two of them through the variable U. In other words, both worlds can be represented by the same model. They just differ in the evidence available. Put differently, there should not be arrows from one world to another, except the ones entering and/or leaving U, because that is the only variable belonging to both worlds. The probabilities not specified in the description above are up to you. The solution to question 2 of the exam in January 2021 should provide you with additional help.

2. Hidden Markov Models (5 p)

(4 p) This exercise is a modification of lab 2 and, thus, you may want to reuse your code. The ring the robot walks has now only five sectors. If the robot is in the sector i, then the tracking device will report that the robot is in the sectors $[i-1, i+1]$ with equal probability. The rest of the sectors receive zero probability. The robot must now spend at least two time steps in sector 1, three time steps in sector 2, two time steps in sector 3, one time step in sector 4, and two time steps in sector 5. The different durations correspond to the robot having to perform different tasks in different sectors. You are asked to implement this modification. In particular, the regime's minimum duration should be implemented explicitly with a decreasing counter. Note that the `HMM` package only allows two types of random variables, namely states and symbols. So, you need to figure out how to incorporate the counter into these variables. Simulate the HMM model built above to confirm that it behaves as expected.

(1 p) Prove that

$$p(z^{T-1}|x^{0:T}) = \sum_{z^T} \frac{p(z^T|z^{T-1})p(z^{T-1}|x^{0:T-1})p(z^T|x^{0:T})}{\sum_{z^{T-1}} p(z^T|z^{T-1})p(z^{T-1}|x^{0:T-1})}$$

and thus it can be computed from smoothed distributions and the transition model.

3. Reinforcement Learning (5 p)

You are asked to implement the value iteration algorithm for the following scenario. A robot moves along a straight corridor that is divided into 10 consecutive sectors. In each sector, the robot chooses between two actions: Either it stays in the same sector, or it moves to the next sector. The result of each action is the intended result, i.e. if the robot chooses the first action then it always stays in the current sector, and if it chooses the second action then it always moves to the next sector. The sectors have rewards associated: The reward is 1 for sector 10, and 0 for the rest of the sectors. Recall that the pseudocode of the value iteration algorithm is available in the course slides. In the algorithm, note that $\mathcal{S}^+$ represents all the states and $\mathcal{S}$ all the non-terminal states. Sector 10 is the only terminal state. Use $\theta = 0.1$ and $\gamma = 0.95$. Initialize the state values to 0. Implement the value iteration algorithm, run it and report the optimal state values and the optimal policy.

4. Gaussian Processes (5 p)

(2 p) This exercise is an extension of lab 4. Specifically, you are asked to compute the covariance between $f(1)$, $f(182)$ and $f(365)$ for both the squared exponential and the locally periodic covariance functions. Use $\sigma_f = 20$ and $\ell = 100$ for the squared exponential covariance function, and $\sigma_f = 20$, $\ell_1 = 1$, $\ell_2 = 100$ and $d = 365$ for the locally periodic covariance function. Discuss the results.

(3 p) Use the log marginal likelihood to select between the two covariance functions above. Use the data for the temperature and time variables from lab 4. Do not scale the data. Recall that the log marginal likelihood is calculated by Algorithm 2.1 in the book by Rasmussen and Williams. So, you may want to reuse your code from the lab.

Finally, let us entertain the idea of selecting the covariance function by computing the mean squared error on some validation subset of the temperature dataset above. Is this possible? If you think it is possible, describe how you would split the data available into training and validation sets. If you think it is not possible, explain why.