

732A66 Decision Theory

# Assignment 4

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## 1 Question 1

Assume some budget calculations depend on whether a certain cost will be at least SEK 120 000 or lower than this amount. A reasonable model for this cost is a normal distribution with standard deviation SEK 12 000 (independent of the mean) and a mean that can be modelled as normally distributed with mean 115 000 (SEK) and standard deviation 9 000 (SEK). No trend is anticipated for this cost and for the 6 previous periods the average cost was SEK 121 000. Note that the hypotheses are about the actual cost, not the expected cost.

### 1.1 a

Show that the prior odds for the hypothesis that the cost will exceed SEK 120 000 (against the alternative that it will not) is about 0.59. [hint: write the observed variable as a sum of two independent random variables  $\tilde{x} = \tilde{\mu} + \tilde{\epsilon}$ ]

### 1.2 b

Show that the Bayes factor (considering the average cost for the previous 6 periods) for the hypothesis that the cost will exceed SEK 120 000 (against the alternative that it will not) is about 1.63 [Not about 1.60 or about 1.70].

### 1.3 c

If the loss of accepting the hypothesis that the cost will be lower than SEK 120 000 while the opposite will be true is SEK 4 000, and the loss of accepting the hypothesis that the cost will be at least SEK 120 000 while the opposite will be true is SEK 6 000, which decision should be made for the budget (according to the rule of minimizing the expected loss)?

## 2 Question 2

Consider a big box filled with an enormous amount of poker chips. You know that either 70% of the chips are red and the remainder blue, or 70% are blue and the remainder red. You must guess whether the big box has 70% red / 30% blue or 70% blue / 30% red. If you guess correctly, you win US dollar 5 . If you guess incorrectly, you lose US \$ 3 . Your prior probability that the big box contains 70% red / 30% blue is 0.40, and you are risk neutral in your decision making (i.e. your utility is linear in money).

### 2.1 a

If you could purchase sample information in the form of one draw of a chip from the big box, how much should you be willing to pay for it? Assume now that the cost of sampling is US\$0.25 (i.e. 25 US cents) per draw.

### 2.2 b

What is the ENGSR for a sample of 10 chips using a single-stage sampling plan.