Page Rank

Searching in Information Networks

We need a way for filtering the most **important** data

What is important?

An observation

Web search query must return a result very quickly

- We cannot have human experts
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Importance according to PageRank

A web page is important if many important pages are linked to it

Basic PageRank

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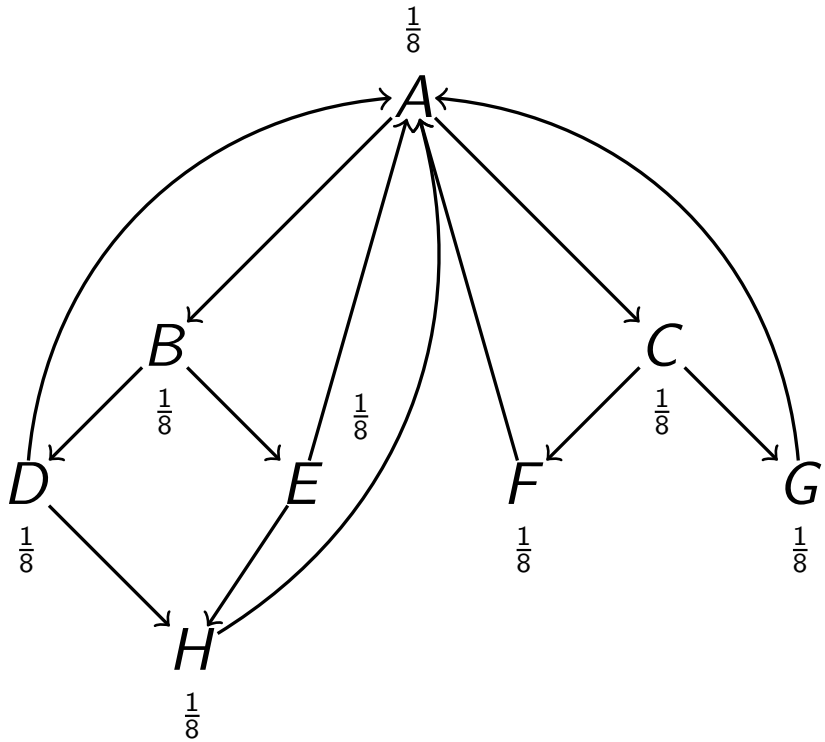

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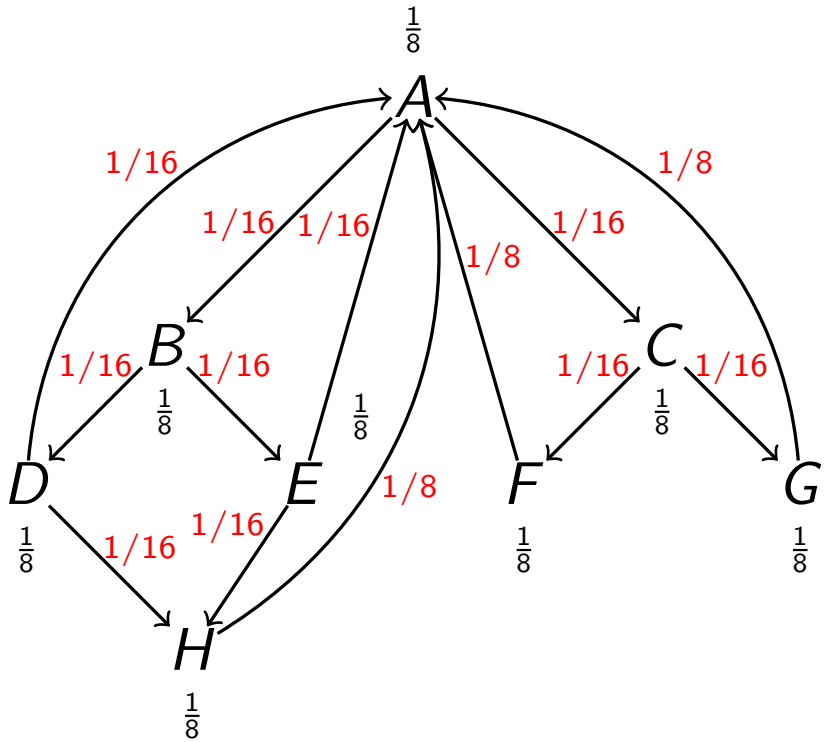

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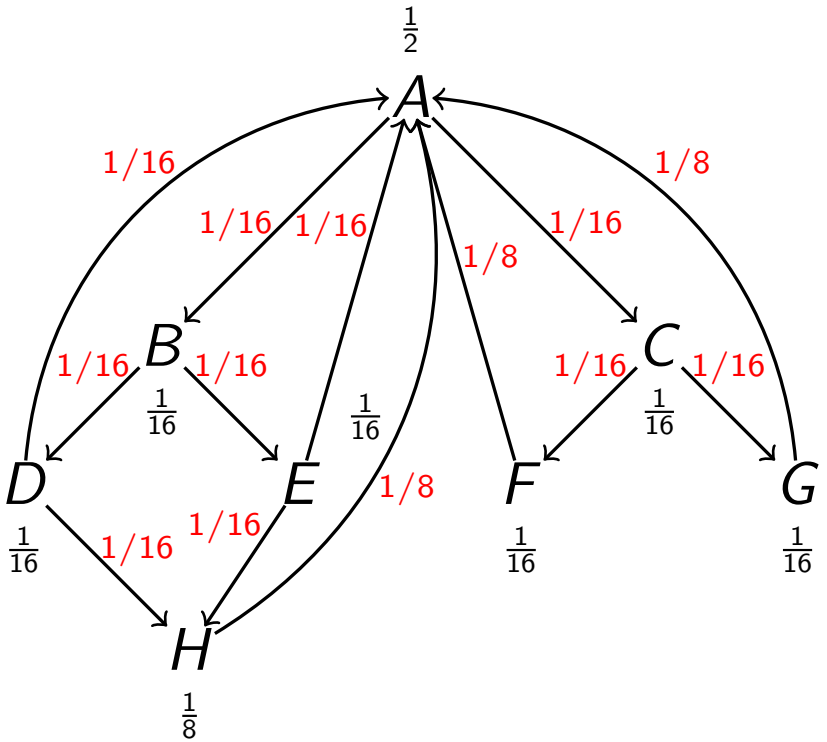

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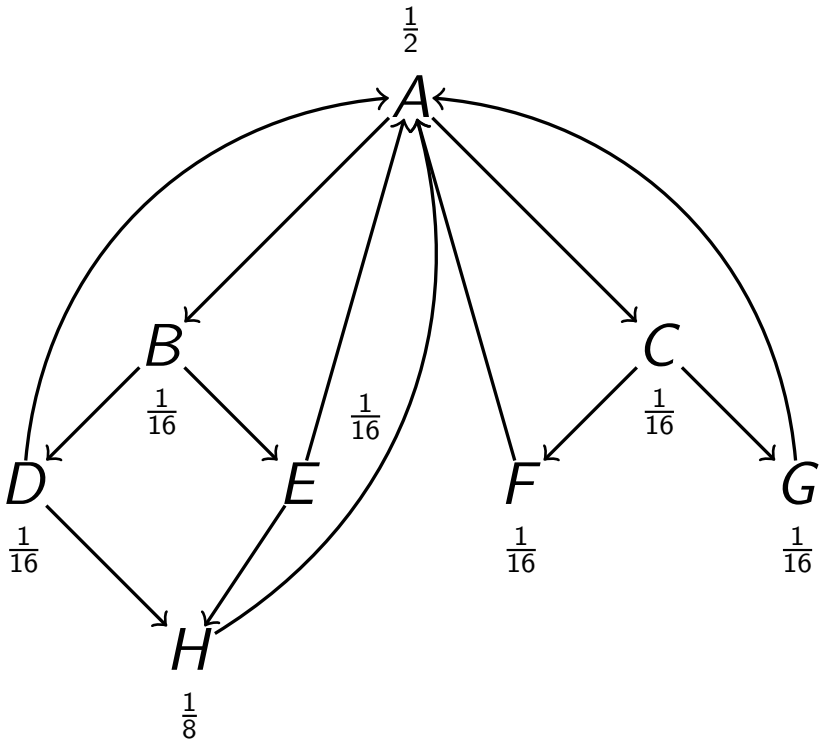

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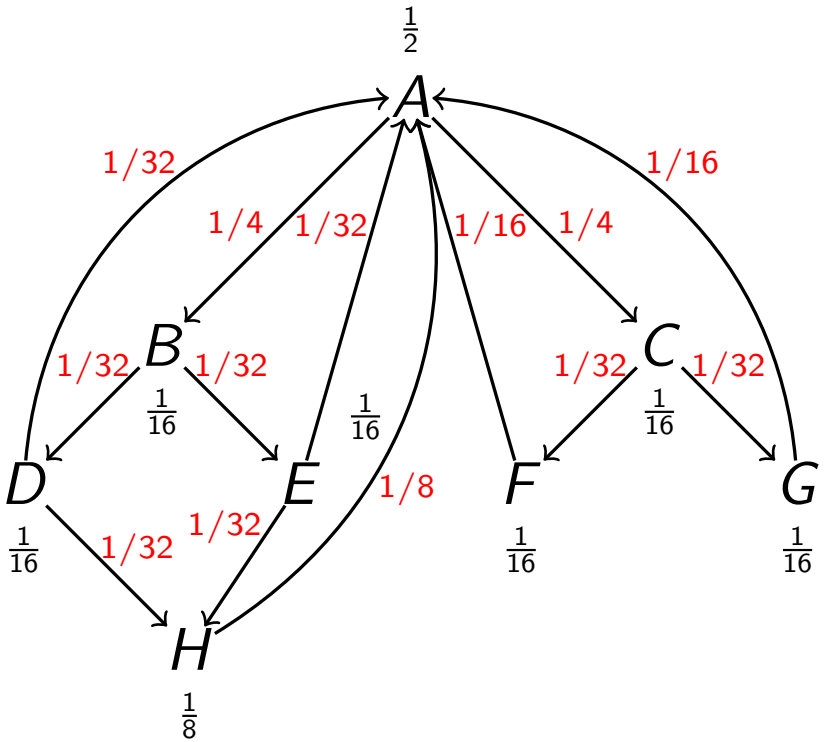

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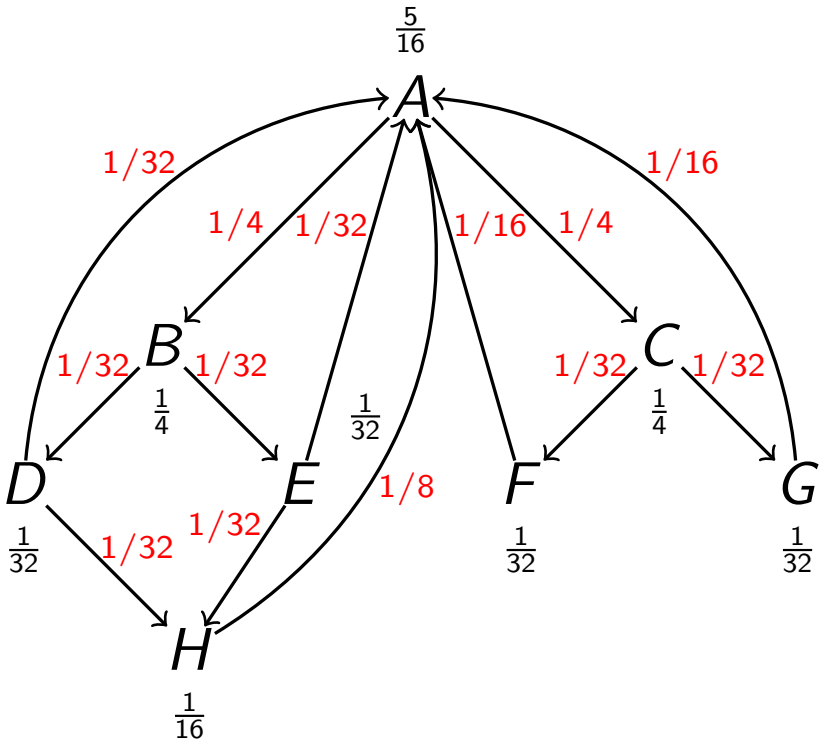

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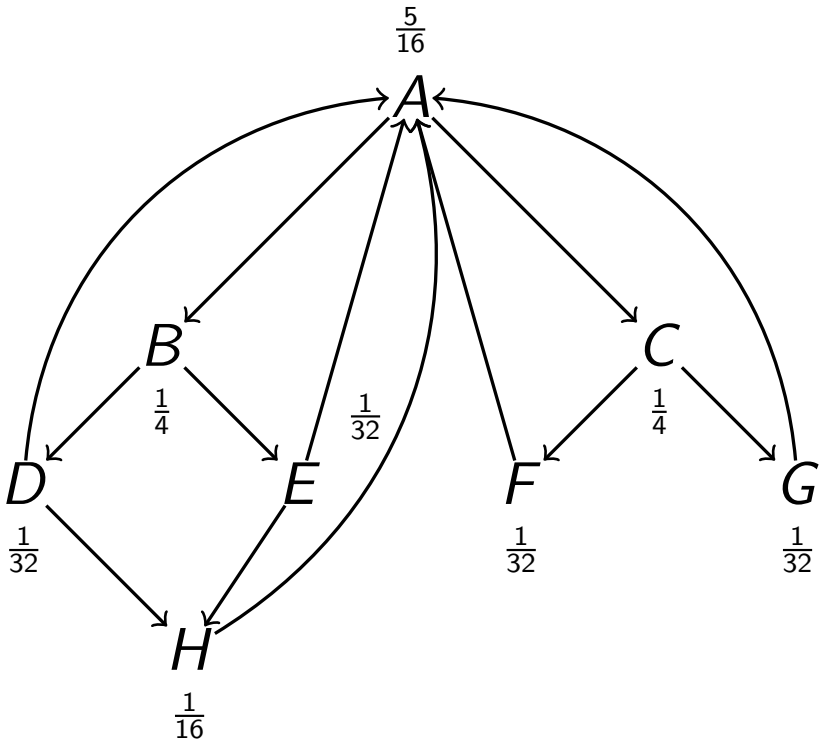
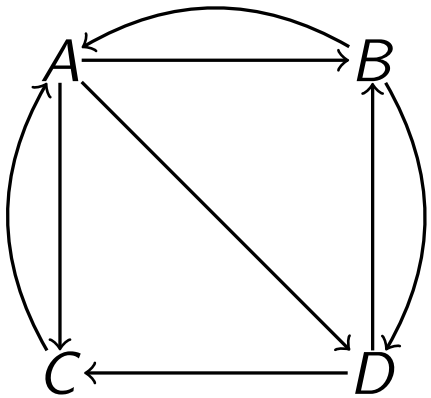


Math enters...

- Transition matrix of the web



$$M = \begin{bmatrix} 0 & 1/2 & 1 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix}$$

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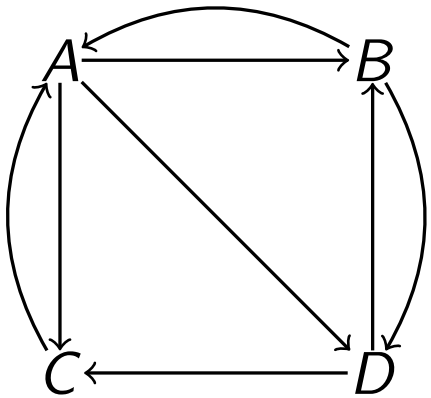

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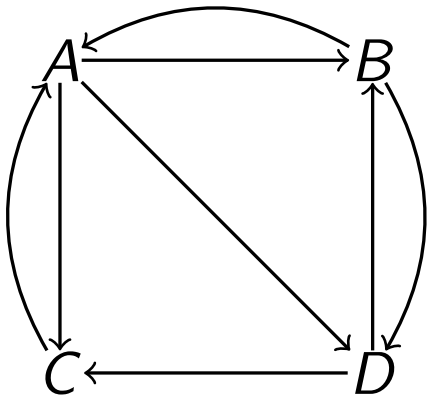

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- Update process

$$\mathbf{v}' = M\mathbf{v}$$

Page Rank corresponds to a Markov chain

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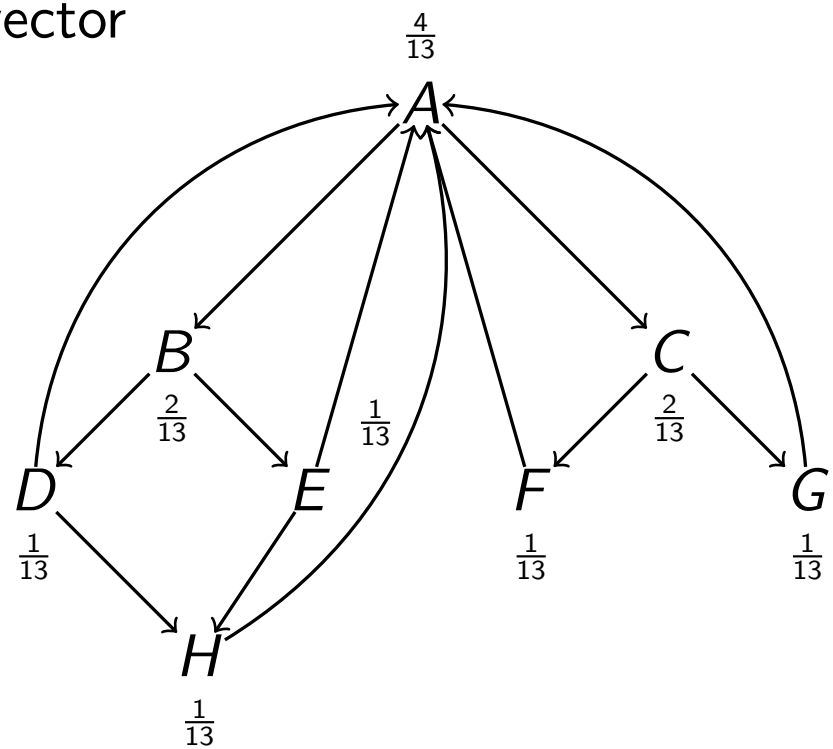
- If the graph is strongly connected, then there is a unique limiting rank vector



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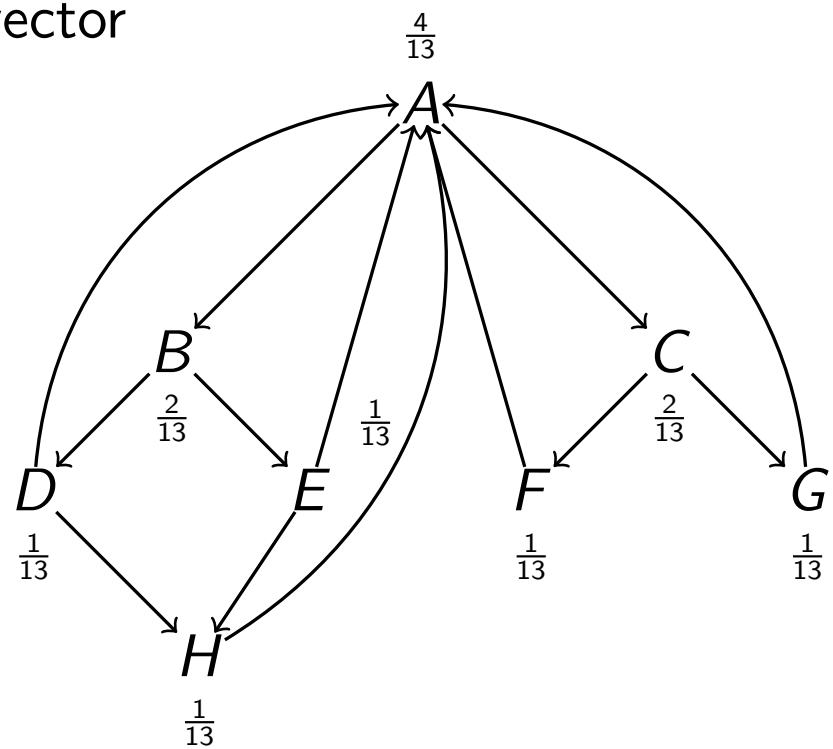

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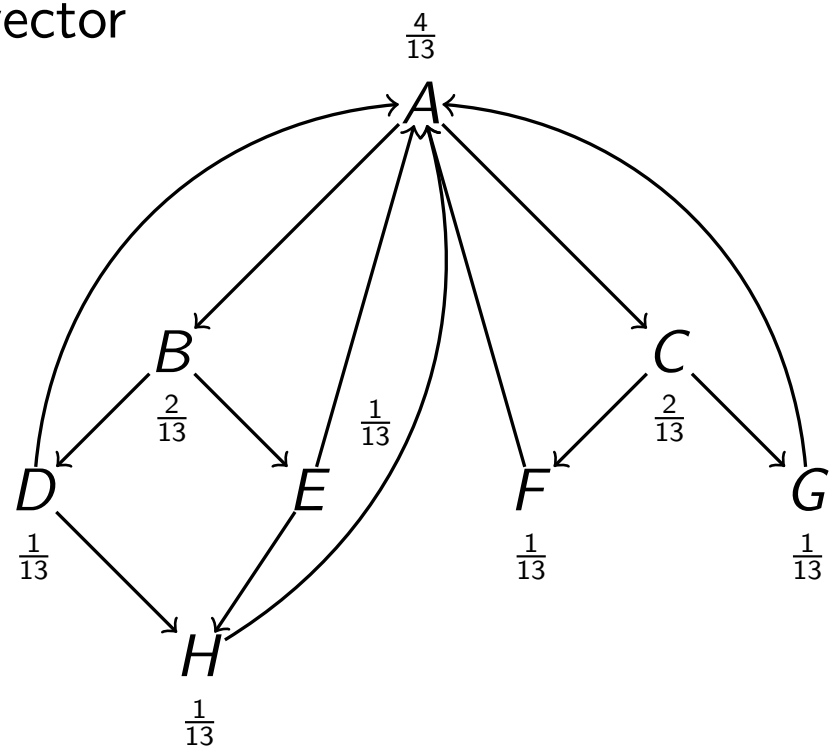

  - The rank does not depend on the starting vector
  - Experimentally 50-75 repetition are sufficient to converge (within the error limits of double-precision arithmetic)

Page Rank and the experience of a random surfer

An alternative view of Page Rank

- The behavior of a random surfer...
  - Once he lands on a web page...
  - he follows a random link of that page

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- The behavior of a random surfer...
  - Once he lands on a web page...
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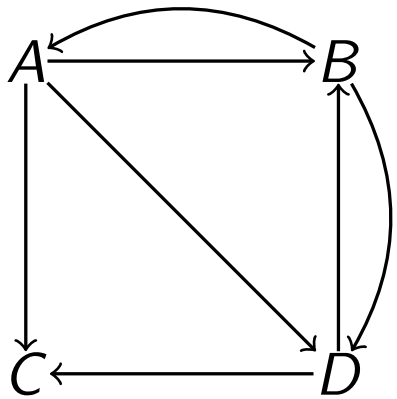
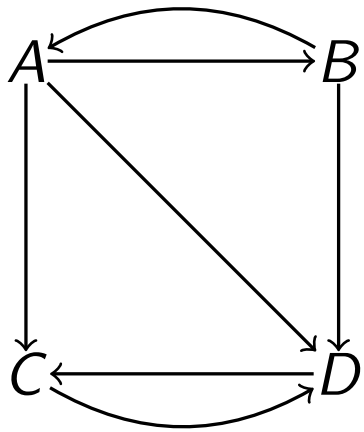
- This process is called random walk
- Page rank measures how often the random surfer can be found on a given page

Some problems

The problem

- There are dead ends or spider traps





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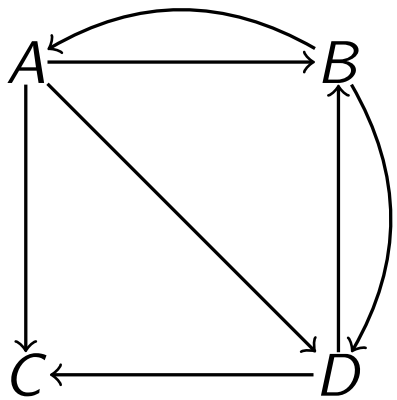

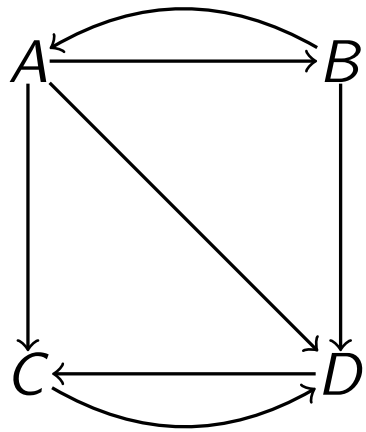

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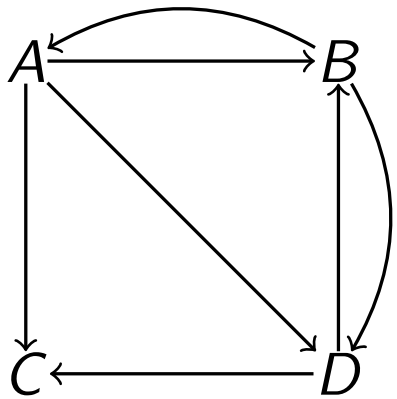

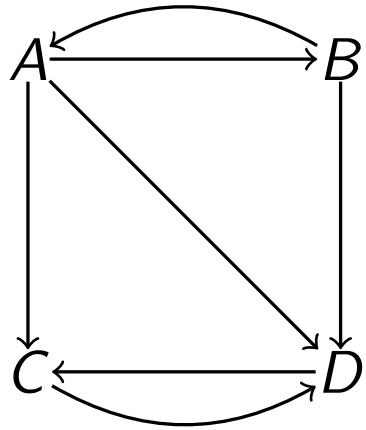

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The solution: The taxation principle

At each step the random surfer has a probability to be teleported to a random page

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  - The rank depends on s
    - Usually s is between 0.8 and 0.9
    - It reduces sensitivity to addition or deletion of pages

Alternatives to PageRank: Hubs and authorities

- Authorities: Valuable pages providing infos about a topic
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- An extension of HITS is used by Google and other search engines (e.g., Ask.com)