CORSO DI LAUREA MAGISTRALE IN INGEGNERIA INFORMATICA

SOCIAL NETWORKS ANALYSIS A.A. 2021/22

CONGESTION GAMES

Network Effects as Negative Externalities

- In the previous lecture we considered games with positive network effects
  - The value of a good increases with the number of its adopters

- Congestion problems are examples of problems with negative network effects
  - The value of a good decreases with the number of its adopters

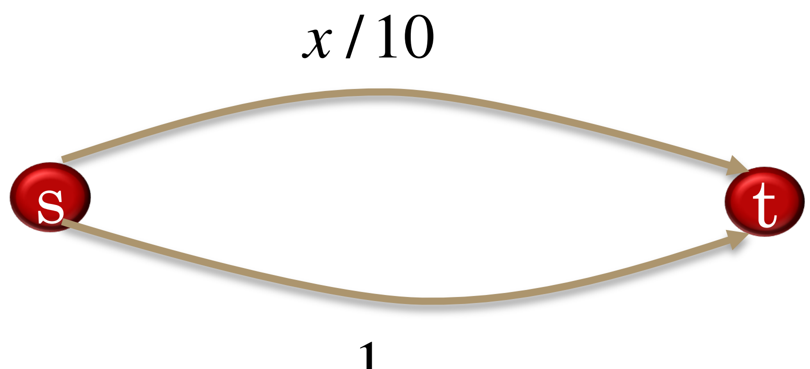
- When moving from positive to negative externalities the nature of the problem changes substantially

The Traffic Problem

- Why traffic should be described as a strategic reasoning setting?
  - Individuals choose their route taking into account decisions of other users and the traffic they can find on different paths
- What can we discover using Game theory?
  - In some cases adding capacity to a network we can create more congestion instead of reducing it
  - The slowdown is no dramatic

Routing Networks

- Assume traffic would flow from a source s to a destination t
  - Consider a simpe case consisting of two disjoint and parallel routes



- Each edge has a latency (delay)
  - Time needed to go through the edge
- Latency depends both on the characteristics of the road and on how many individuals are going through it (traffic)
  - Upper edge has a latency depending on traffic
  - Lower edge has la fixed latency

Routing Games

- Traffic problems are modeled as routing games

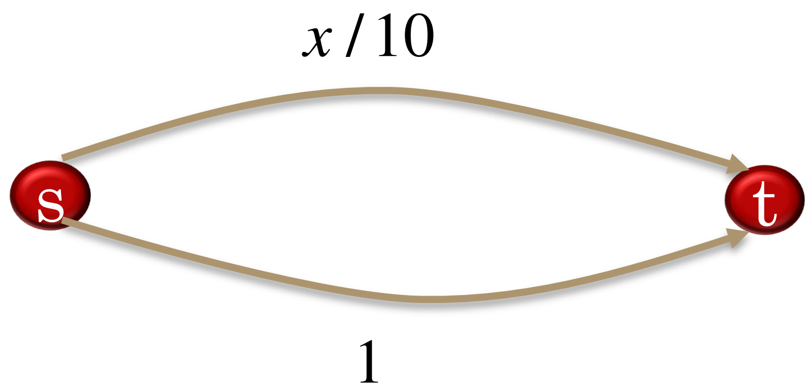
- Each player has to decide autonomously which route to go through
  - Huge number of players (none can influence the congestion of the network)
  - Each player wants to minimize the latency of her route
  - Each player plays strategically reasoning about which routes other players will choose

Routing Games

- Routing Game
  - N players (N very large)
  - Each player has a strategy set consisting of all the possible paths from source to destination
  - The cost for each player is the total latency of her path, given the decisions of all the players
- Routing games are cost games
  - Each player wants to minimize his cost function

AN EXAMPLE

- How players think if there are only 10 players?

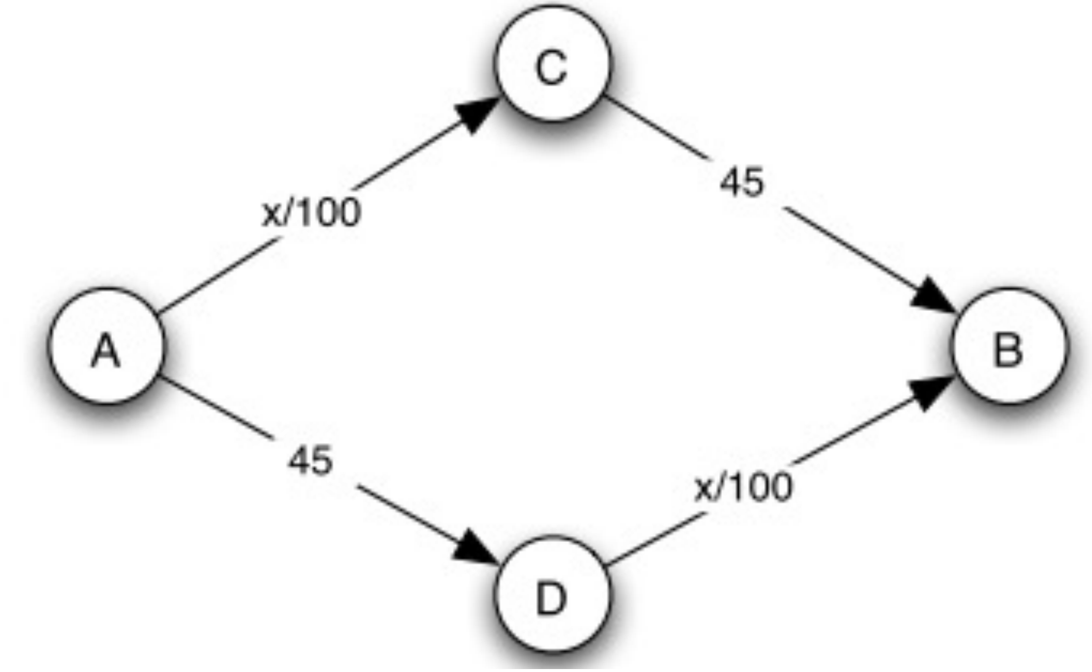


  - In the worst case the latency of the upper path is 1
  - The upper path is no worse than the lower path

- What happens if players are 100?
  - upper path is the best choice only if only a few players are using it

- QUESTION:
  - How is traffic distributed in a Nash Equilibrium?

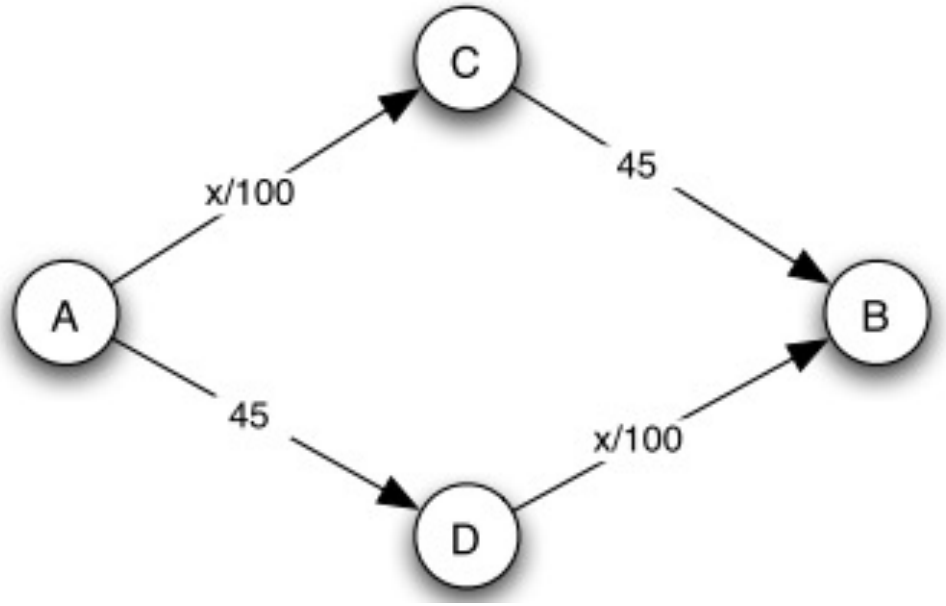
ANOTHER EXAMPLE -- 1



- Two alternative paths from A to B
  - Latencies signed on edges

- Suppose 4.000 players going from A to B
  - What is the average latency for each player?

ANOTHER EXAMPLE -- 2



- If all players choose the upper path
  - Latency is equal to 85
- If all players choose the lower path
  - Latency is equal to 85
- If players eqaully distribute between the two paths
  - Latency is equal to 65 and this is a Nash Equilibrium

- Are there other Nash Equilibria?

Characterization of Nash Equilibria

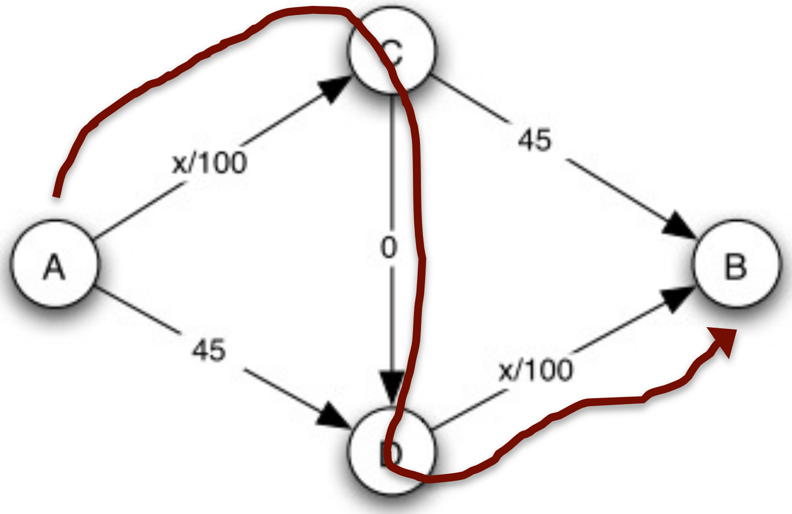
- A solution is a Nash Equilibrium iff all the paths from s to t have the same latency

- If a path would have a lower latency at least a player would change her path

- In our example there exists a unique Nash Equilibrium

BRAESS PARADOX (1968)

- Suppose we can add a new very fast edge to our network



- How the traffic change?
  - We can expect that latencies should decrease …
  - But …. All the players decide to take the path A-C-D-B and their latency is 80
- Adding a new edge to the network we have increased latencies of all players

CONSIDERATIONS ABOUT THE BRAESS PARADOX

- In several games adding a new strategy induces a loss of utility for all the players
  - In the Prisoner Dilemma if players could not confess they all would obtain an advantage
- We can reduce or eliminate the effect of the Braess Paradox discouraging the use of the new edge
  - For example, adding tolls
- Discovering of the Braess Paradox gave rise to several very active research lines
  - How much the addition of the new link can increase social cost of the network (sum of all the players latencies)?
  - Roughgarden and Tardos proved that, if edge latencies are linear, latencies can increase at most of a factor 4/3.

Problems connected to Braess Paradox

- How can we design networks where inefficient equilibria cannot occur?

- How can we design tolls to eliminate inefficient equilibria?

- Useful references can be found in the Roughgarden PhD thesis about game-theoretic models for network traffic problems

El Farol Bar Problem

- Famous problem with negative externalities used to model individual behaviours

- The El Farol is a famous bar in Santa Fe where each Thursday night there is a music live show

- The bar has seating for only 60 people
  - Showing up for the music is enjoyable only when at most 60 people are in the bar
  - When the bar is overcrowded it's better to stay home or go to another bar

- The bar has 100 friends on FB that are weekly invited to its shows
  - They all agree that it's worthwhile to go to the show only if attendance is no greater than 60

- How does each person reason about whether to go or to stay home, knowing that each FB friend is reasoning about this decision as well?

Reasoning about Negative Externalities

- Reasoning about negative externalities is much more complicated than the positive case

- With positive externalities individuals tend to follow the majority
  - If enough players adopt a strategy I'm incentivized to follow them

- With negative externalities individuals tend to distinguish from the majority
  - If too many players adopt a strategy I'm incentivized to take a different strategy

Differences between Positive and Negative Case

- Problems with positive and negative externalities differ with respect to some relevant aspects

- In the positive case there always exist self-fulfilling equilibria where all players have the same expectations about the choices of the population
  - If players believe all players buy then they buy
  - If players believe that none buys then none buys

- In the negative case there are no self-fullfilling equilibria where all players have the same expectations about the choices of the population
  - We need more complex equilibria that can explain why players can take different strategies
  - Some players go to the bar and other stay away

Nash Equilibria for the El Farol Bar Game

- Definition: the El Farol Bar Game
  - 100 players
  - Each player has two strategies (Go, Stay)
  - The payoff is
    - 0 if she plays Stay
    - x>0 if she plays Go and there are <= 60 people attending the show
    - -y<0 if she plays Go and there are > 60 people attending the show

- There is a unique Pure Nash Equilibrium
  - Exactly 60 players play Go
  - It's not clear how this group can be selected

- There is a Mixed Nash Equilibrium where all players play the same strategy
  - play Go with probability p and Stay with probability $(1-p)$
  - take p so that Pr[attendants <= 60] = $\boldsymbol{x}/(\boldsymbol{x}+\boldsymbol{y})$

Mixed Nash Equilibria for the El Farol Bar Game

- If $x=y$ then $p = 0{,}6$

- If $x<<y$ then $p << 0{,}6$
  - When the bar is overcrowded it' really unbearable to stay
  - In this case the bar is expected to be not full

- If all players have beliefs only on the number of attendants there are no self-fulfilling equilibria

- If all players believe that each player plays a mixed strategy to decide if to show up at the bar there exists a self-fulfilling equilibrium

2-Players Games with Externalities

- 2-players games with externalities reduce to well-knonwn games
  - Negative externalities: Anti-coordination games (Hawk-Dove)
  - Positive externalities: Coordination games (Battle of Sex)

Repeated El Farol Bar Games

- How can a group converge to a mixed-strategy equilibrium?

- Suppose to repeatedly play the El Farol Bar Game
  - Every Tuesday each player has to decide if to book a seat for the Thursday show
  - The player payoff is the sum of the payoffs received in each week
  - Each player knows how many people attended the shows of previous weeks

- The strategy of each player consists of two parts:
  - a forecasting rule that maps the past history of the play to a prediction about the actions that all other players will take in the future
  - a choice rule that decides if to book a seat depending on the forecast

Forecasting Rules

- If all players use the same forecasting rule then everyone will make very bad predictions
  - If they predict less than 60 people will show up then they all will show up
  - If they predict more than 60 people will show up then they all all will stay home
- We need for players to use a diversity of different forecasting rules
- A long line of active research is considering how a group behaves when they use different classes of forecasting rules and this makes the system to converge to an equilibrium
  - Both analytical models and simulations
  - Results show that, under a variety of conditions, the system converges to a state where the average attendance varies around 60
  - The bar's managers cannot optimize the utilization of the bar

Mixed Externalities

- In most real cases we have to deal with settings where there are both positive and negative externalities
  - Predicting that a certain number of people will attend the show reassures about the quality of the show
  - Predicting that no more than 60 people will attend the show reassures about the pleasantness of the bar
- It's an open question to understand how these two different effects interact