

CORSO DI LAUREA MAGISTRALE IN INGEGNERIA INFORMATICA



SOCIAL NETWORKS ANALYSIS A.A. 2021/22



NETWORK EFFECTS

- In the previous lection we learned that in making our choices we can be influenced from decisions of other people
 - You decide to follow the crowd since you think they are more informed than you
- Another important factor that can influence our decisions is the explicit benefit/damage that we can incur when aligning our behavior with the behavior of others
 - Compatibility, opportunity to interact, etc.
 - This phenomenon is called **network effect**
- Settings where you can observe network effects
 - Adoption of a new communication technology
 - Registration to a social media or to a file-sharing site
 - Selection of an application software to use

NETWORK EFFECTS AS EXTERNALITIES

- Network effects can be seen as externalities
- An externality is any situation in which the welfare of an individual is affected by the actions of other individuals, without a mutually agreed-upon compensation
 - Positive externality: the value of a choice increases with the number of adopters
 - * Eg., Registration to a social media
 - Negative externality: the value of a choice decreases with the number of adopters
 - ❖ Eg., Traffic and congestion problems
- Not everything is an externality
 - Do we receive a benefit/damage when another person
 - * Registers to our same photo-sharing service?
 - * Takes the same road we are travelling on?
 - * Drinks a can of our favorite drink?

Markets with no Network Effects

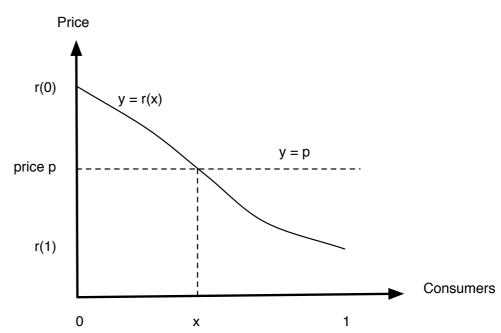
- For sake of simplicity we consider a market for a single good
 - Huge number of potential purchasers, each of whom is small enough relative to the entire market that he or she can make individual decisions without affecting the aggregate behavior
 - * When you buy a loaf of bread you cannot influence the price of bread
 - Many competing producers interested in increasing their revenue
 - * None of them is large enough to be able to influence the market price of the good
 - * Each producer is willing to sell her products to cost price to maintain her share of market
- Each potential consumer
 - Is interested in only a single copy of the good
 - Has her own valuation of the good represented by her reservation price r()
 - Maximum price he's willing to pay for the good

CONTINOUS MODEL

- To maintain computations simple we consider a continous model
 - Potential consumers represented as real numbers in (0, 1), nonincreasingly ordered with respect to their reservation price

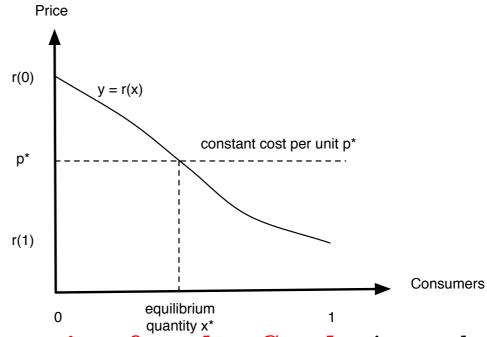
$$\star x < y \rightarrow r(x) \ge r(y)$$

- assume r(x) continous
- Fixed market price of the good *p*
 - x is interested in purchasing the good only if $r(x) \ge p$
 - If x is interested, then all x' < x are interested



THE EQUILIBRIUM QUANTITY OF THE GOOD

- Let p* be the (constant) production cost per unit
- Assume the market can provide any number of units of the good at price p^*
 - Unlimited number of potential producers
- If $p^* > r(0)$
 - No consumer interested
- If $p^* < r(1)$
 - All consumers interested
- If $r(1) < p^* < r(0)$
 - There exists x^* such that $p^* = r(x^*)$
 - A fraction x^* of the population is interested



- x^* is the **Equilibrium Quantity for the Good** given the reservation prices and the cost p^*
 - The quantity of good the market is able to absorb at price p^*

Market Equilibrium

- $o(x^*, p^*)$ is an equilibrium
 - If a fraction x^* of the population purchases the good and none regrets
- No consumer has pressure to change idea
 - $y < x^*$ purchases and is happy since $r(y) > p^*$
 - $y > x^*$ doesn't purchase and is happy since $r(y) < p^*$
- $o(x^*, p^*)$ is socially optimal
 - Utility of x is
 - 0 if she doesn't purchase
 - $r(x) p^*$ if she purchases
 - In (x^*, p^*) all consumers with positive utility are purchasing while no consumer with negative utility is purchasing

MARKETS WITH POSITIVE NETWORK EFFECTS

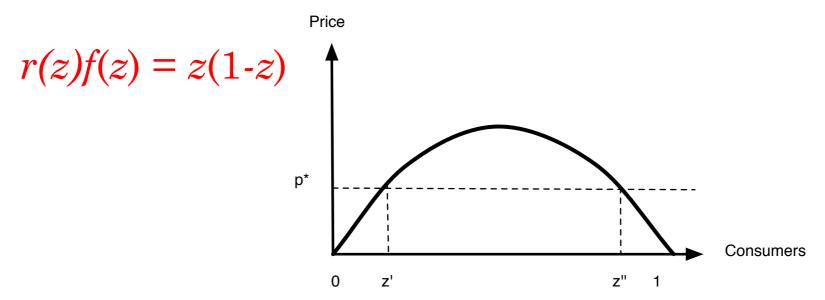
- In a market with network effects the willingness of a consumer to purchase the good depends on
 - her reservation price
 - Market price
 - the number of other people using the good
 - * The larger the user population, the more
 - This value can only be estimated
- We model the network effect as a multiplicative factor f() for the reservation price
 - If a fraction z of the population purchases the good its value would be f(z)r(z)
- Assume f(z) increasing, always positive and f(0) = 0
 - You are not interested in purchasing a good that none uses
- If a fraction z of the population purchases the good, x is willing to purchase only if $r(x)f(z) \ge p^*$

EQUILIBRIA WITH NETWORK EFFECTS AND PERFECT PREDICTIONS

- Suppose that consumers are able to predict the fraction z of the population that will purchase the product
- o (z, p^*) is a self-fulfilling expectations equilibrium if
 - $r(z)f(z) = p^*$
- o if everyone expects that a z fraction will purchase the product, then this expectation is in turn fulfilled by people's behavior
 - * z purchases since $r(z)f(z) = p^*$
 - * y < z purchases since $r(y)f(z) > p^*$
 - * y > z doesn't purchase since $r(y)f(z) < p^*$
- \circ (0, p^*) is always an equilibrium
 - if everyone expects that a z = 0 fraction of the population will purchase then none will want to purchase

AN EXAMPLE

$$r(x) = 1-x$$
$$f(z) = z$$



- The function has its maximum at z = 0.5 and f(0.5) = 0.25
- \circ $(0, p^*)$ is an equilibrium
- If $p^* \ge 0.25$ there are no other equilibria
- Is $0.25 > p^* > 0$ there are other two equilibria

Self-Fullfilling Equilibrium

- Self-fullfilling equilibrium is an equilibrium concept related to "consumers confidence"
 - If the population has no confidence in the success of the good, no one will want it
 - if the population is confident of its success, then it is possible for a significant fraction of the population to decide to purchase it
- As the price diminishes, the two equilibria tend to move to the extremes of the interval (0, 1)
 - Characteristic of all the markets with positive network effetcs

PREDICTIONS NOT IN EQUILIBRIUM

- What happens when consumers have a prediction *z* that is not in equilibrium?
- Let 0, z', z'' be the predictions in equilibrium with respect to the price p^*
 - If 0 < z < z'
 - * $r(z)f(z) < p^*$ and consumer z wishes not to have purchased it
 - * There is a downward pressure to use the good
 - If z' < z < z''
 - * $r(z)f(z) > p^*$ the successor of z wishes to have purchased it
 - * There is an upward pressure to use the good
 - If z > z"
 - * $r(z)f(z) < p^*$ and consumer z wishes not to have purchased it
 - * There is a downward pressure to use the good

EQUILIBRIA STABILITY

• An equilibrium z is **stable** if consumers have a prediction $z\pm\epsilon$ and we can expect that a fraction z of the population will purchase the good

• In our example

- z" is a stable equilibrium
 - * If z = z"- ϵ the upward pressure brings the demand at the equilibrium
 - * If z = z"+ ε the downward pressure brings the demand at the equilibrium
- z' is an unstable equilibrium
 - * If $z = z' \varepsilon$ the downward pressure brings the demand to 0
 - * If $z = z' + \varepsilon$ the upward pressure brings the demand to z''
 - * z' is feasible only when the prediction is exactly z'
- 0 is trivially stable

TIPPING POINTS

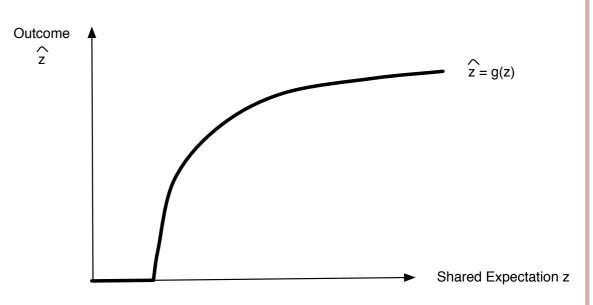
- \circ The equilibrium z' is a tipping point in the success of the good
 - If the consumers' prediction is < z then the downward pressure will drive the sales to 0
 - If the consumers' prediction is > z' then the upward pressure will make the sales increase to z''
- The value z' is the hump the firm must get over in order to succeed
 - Reducing p^* you can lower z'
- A possible marketing strategy for the firm to conquer the market
 - Reduce p^* in order to lower the tipping point
 - Convince the market that a fraction z > z' of the population is interested in purchasing the good
 - Use the market pressure to conquer a fraction z" of the market

Uncorrect Predictions

- What happens when consumers are not able to make correct predictions?
 - Consumers prediction z
 - The fraction of population that is willing to purchase the good is ζ tale che $r(\zeta) = p^*/f(z)$
 - * Since r() is continous a solution always exists and is unique

o Let $g(z) = \zeta$

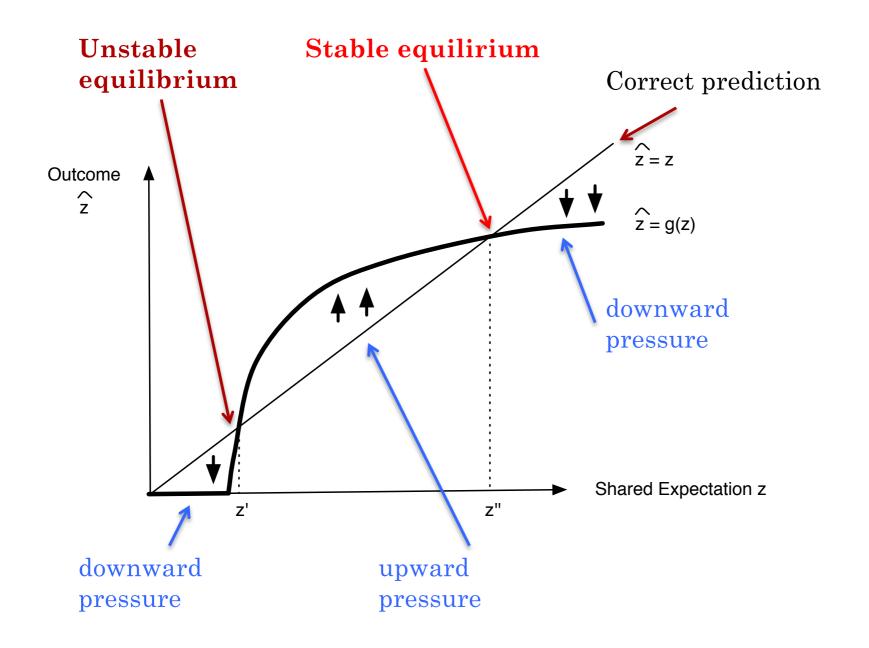
• g(z) is the fraction of population that will purchase the good when the prediction is z



HOW TO USE THE FUNCTION G()

- When g(z) = z you have a self-fullfilling equilibrium
 - if g(z) > z there is upward pressure
 - if g(z) < z there is downward pressure
- An equilibrium is stable when the pressure goes from upward to downward
- An equilibrium is unstable when the pressure goes from downward to upward

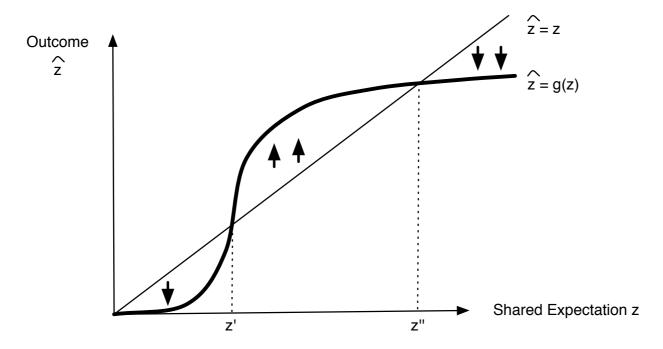
A GRAPHIC REPRESENTATION



A GENERAL MODEL

• The graphics in the previous slide depends on the particular choices of r() and f()

• In a market with positive network effects the relation between the expected number and the real number of purchasers is always similar to the one in the previous slide



THE DYNAMIC BEHAVIOR OF THE POPULATION -- 1

- In 1970 Granovetter and Schelling studied how a population might react dynamically to a network effect
 - They studied how the number of people participating in a given activity with network effects would tend to grow or shrink over time
 - Their case study was the example illustrated in the previous slides
- Problem: an individual has to decide if register at the site of a social media
 - Her valuation of the participation is expressed r()
 - The network effect is represented by *f*()
 - The registration has a cost p^*
 - Configuration work needed for the registration

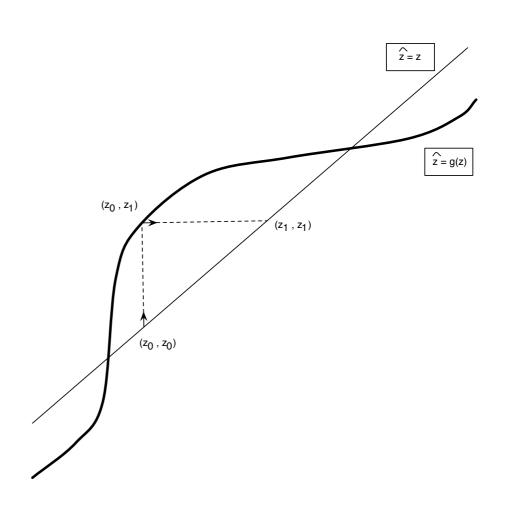
THE DYNAMIC BEHAVIOR OF THE POPULATION -- 2

- The experiment proceeds in fixed time slots t = 0, 1, 2, 3, ...
- At time t = 0 a fraction z_0 of the population (audience size) is registered at the site
- At time t > 0 people decides if to register in the site depending on the number of users at time t-1
 - At t = 1 partecipants are $z_1 = g(z_0)$
 - At t = 2 partecipants are $z_2 = g(z_1)$
 - ...
 - At generic time t partecipants are $z_t = g(z_{t-1})$
- The model is myopic
 - Assumes that future will be the same as the present
 - Corresponds to the concept of sulf-fullfilling equiibrium

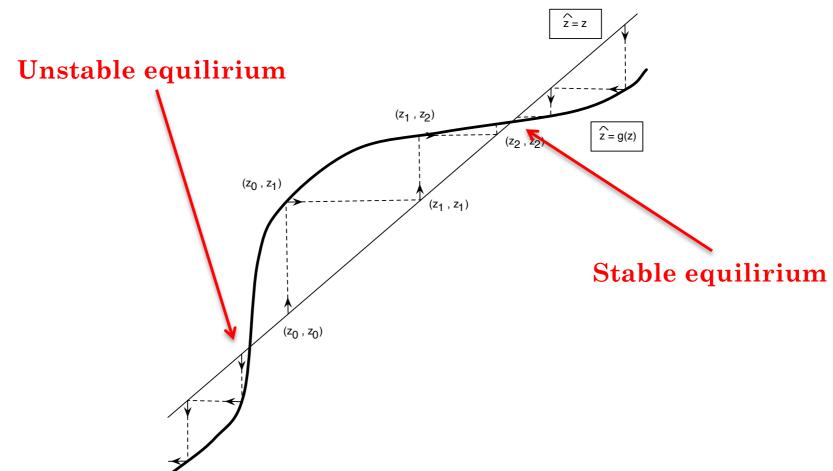
Analyzing of the Dynamics -- 1

• We can analyze the behaviour of the dynamics tracing the points (z_t, z_t) on the line $z = \zeta$

- $o If z_t < g(z_t)$
 - $z_t < z_{t+1}$
 - Upward pressure
- $o If z_t > g(z_t)$
 - $z_t > z_{t+1}$
 - Downward pressure



Analyzing the Dynamics -- 2



- When the curve is over the line points tend to move towards right
- When the curve is under the line points tend to move towards left

EVOLUTION OF A MARKET WITH NETWORK EFFECTS

A typical evolution of a market with network effects with respect to the price p^*

- \circ Initially p^* could be too high to have non trivial equilibria
 - No one is confident that people will use the good and she doesn't purchase it
- When p^* lowers two new equilibria appear but the tipping point is too much high
 - All the consumers are confident that a very few people will use the good and she doesn't purchase it
- When p^* continues to lower the tipping point lowers and some consumers starts purchasing the good
 - If the sells exceed the tipping point the market rapidly moves to the stable equilibrium

Marketing a Good with Network

EFFECTS

- How can a firm that wants to sell a product with a network effect use these insights to market its product?
- Starting small and hoping to grow slowly is unlikely to succeed
 - unless the good is widely used it has little value to any potential purchaser
- You need to convince a large initial group to adopt your good before others will be willing to buy it
- How would you do this?
 - Set an initial low, introductory price for the good
 - Distribute it at a loss (maybe for free) to a bunch of initial (influential) adopters to reach a critical mass and go over the tipping point
 - * Possible initial losses can be recovered by making future profits

SOCIAL OPTIMALITY IN MARKETS WITH NETWORK EFFECTS

- In markets with no network effects the equilibrium is socially optimal
- o In markets with positive network effects equilibria are typically non optimal
 - Markets provide less of the good than is socially optimal
 - There are potential consumers that would be willing to purchase the good but they don't do it

NETWORK EFFECTS AND COMPETITION

- In previous slides we considered a market for a single good
- What might happen if multiple firms develop competing new products, each of which has its own network effects?
 - Two competing social media
 - Two alternative technologies
- It is likely that one product will dominate the market and its concurrent will disappear
- Who wins?
 - Not the best ...
 - ... but the first one that goes over the tipping point

Individual Effects and Network Effects

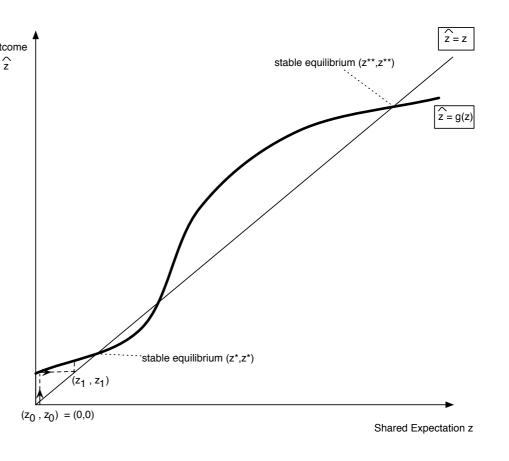
- Till now we considered only network effects
 - The interest in a good depends only on the number of its adopters
 - f(0) = 0
- In some scenarios individuals could have their own valuations of a good, independent from the number of its adopters
 - f(0) > 0
 - 0 is not necessarily an equilibrium
- Granovetter described some interesting phenomena that can occur when individual and network effects combine

AN EXAMPLE -- 1

$$r(x) = 1-x$$
$$f(z) = 1+az^2$$

$$r(x)f(z) = (1-x)(1+az^2).$$

$$g(z) = 1 - \frac{p^*}{1 + az^2}.$$



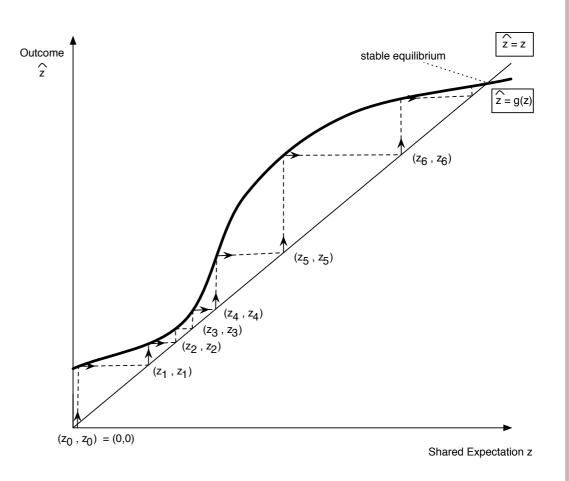
- The dynamics starts with an audience size equal to 0 and reaches the first equilibrium
 - You cannot go over since the equilibrium is stable

AN EXAMPLE -- 2

$$r(x) = 1-x$$
$$f(z) = 1+az^2$$

$$r(x)f(z) = (1-x)(1+az^2).$$

$$g(z) = 1 - \frac{p^*}{1 + az^2}.$$



 \circ Lowering p^* we can move the curve upward until the blocking equilibrium disappears