Corso di laurea magistrale in Ingegneria Informatica

# Social Networks Analysis
# A.A. 2021/22

# Information Cascades

# The Spread of Influence



- People connected in a network can influence each other's behaviors and decisions
  - In buying products
  - In adopting technologies
  - In forming their political opinions
  - In choosing activities to pursue
  - Etc.

- A large class of social processes can be seen as aggregating local behaviours in population-wide outcomes
  - It is of great interest to study the role networks play in these aggregation processes

- Basic Question: Why this influence occurs and how?

# A SIMPLE EXAMPLE



- Suppose you're visiting a new city and you're looking for a place for dinner
  - Looking on your favourite recomendation site you got very good reviews about place X
  - Arriving at place X you discover that it is almost empty while the next-door place Y is nearly full

- What do you do?
  - Stay with your advisory or join the crowd?
  - Why should you join the crowd?
    - Maybe they are from the place and know both the places much better than you

# JOIN THE CROWD



- Sometimes we join the crowd and imitate their behaviors even if we don't agree
  - Why do we smoke?
  - Why are we fan of a specific football team?
  - Why do we support a specific political party?
  - Why do we love hanging out in a specific club?

- Imitation by conformity
  - I hang out in pub Y because everybody does and I don't want to be different
- Imitation by rational choice
  - I hang out in pub Y because I have limited information and I assume others have more information than me

# Information cascade



- An Information Cascade is the process in which an agent is influenced by the actions of the rest of the population and acts in a different way with respect to her own opinions
  - Also known as herding effect
- Information cascades can occur when
  - Agents take decisions sequentially
  - Each agent can observe the decisions of earlier people and infer from their actions something about what they know
    - In the dinner-place example we infer from the fact the place Y is nearly full that a lot of people prefer Y to X

# The Spread of Trends





- Milgram, Bickman, Berkowitz performed a very interesting experiment
  - A group of x people stands on a street corner staring up in the sky
  - How many passersby stopped and looked up in the sky?

- The number of stopping passersby increased with the size of the initial staring up people
  - The social pressure for conformity grows stronger as the conforming group becomes larger
  - I can rationally decide to imitate the group because there is a (unknown to me) interesting reason for the group actions

- Information cascades can be part of the explanation for many types of imitation in social contexts
  - Fashion and fads, self-enforcing success of bestsellers, spread of technological choices, voting for popular candidates

# Information Cascades are Fragile



- In several cases the assumption that the crowd knows much more than me can be dramatically wrong
  - Information cascades can form on very weak basis
- In the following we will show that sometimes you better not join the crowd

# Information Cascades and Wisdom of Crowds



- Information Cascades is different and (in some sense) opposed to the Wisdom of Crowds

- Information Cascades
  - Sequential decisions
  - Agents are influenced by the earlier decisions

- Wisdom of Crowds
  - Independent choices

# Our Model



- Assume that N agents have to make a decision
- Agents make their decisions sequentially
  - Their choices are public and can be observed by all the other agents
- Each agent has a private information that leads her in her decision
- Agents don't know others' private information
  - but they can infer something from their actions

# A Simple Herding Experiment



- A simple experiment can be used to mathematically explain information cascades
- $n$ players are presented with an urn containing three (red or blue) balls
  - With probability 0,5 the urn contains two red balls and one blue ball (Maj-red)
  - With probability 0,5 the urn contains two blue balls and one red ball (Maj-blue)
  - Players are asked to guess which kind of urn they have in front
- Players, one by one, are called to draw a ball from the urn, privately looking at it and then putting it back in the urn
  - After putting the ball back the agent has to publicly announce her guess

# A First Informal Analysis -- 1



- The first player
  - She has no other information than the color of the ball she observed
  - Her best move is to announce the color she observed

- The second player
  - She knows the color announced by her predecessor and she knows that her best move was to announce the color of the observed ball
  - She knows the color of two balls
  - Her best move is to announce the color of the observed ball
    - If it's equal to the color of her predecessor, obvious
    - If it's different she can choice at random

- The third player
  - She knows colors announced by her two predecessors and she knows they correspond to the observed colors
  - She knows the colors of three balls
  - Her best move is to announce the color observed twice
    - If the two predecessors observed the same color, the third player follows their announcement independently from the color of the ball she observed

# A First Informal Analysis -- 2



- The fourth player knows the colors announced by her three predecessors
  - She can be in one of two different situations

- She heard the same color three times
  - She knows the first two announcements correspond to “real” observations while the third is simply a copy
  - She is exactly in the same situation ad the third player

- She heard different colors
  - She knows they all correspond to “real” observations
  - Her best move is to announce the color she observed (same situation as player 2)

- All the following players are in the same situation as player 3

# A First Informal Analysis -- 3

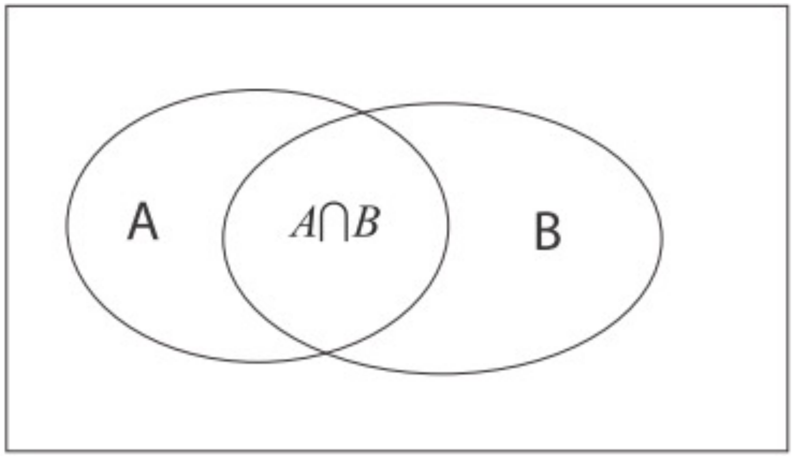


- If the two first players draw balls with the same color an information cascade starts
  - All the following players announce the same color independently from their "real" observations
- Is the result always correct?
  - No
- The first two players could have drawn the ball with the minority color
  - probability 1/9!!!!
- The mistake probability doesn't change if the players are 2, 10, 100 o 1000 players

# A Mathematical Model for Information Cascades

- Each player decision can be reduced to a problem of maximizing a conditional probability
  - Which is the decision with the largest probability to be right, given her private information and the information inferred from the actions of the other players?

- Pr[A] = Probability of event A
- Pr[AB] = Probability of events A and B
- Pr[A | B] = Probability of event A given B




Pr[A | B] = Pr[AB] / Pr[B]

# The Bayes Formula



- Pr[A] is the a priori probability of event A
- Pr[A|B] is the a posteriori probability of event A given event B

- The Bayes Formula allows to compute the a posteriori probability Pr[A|B] in terms of the a priori probability Pr[B|A]

$$\Pr[B \mid A] = \frac{\Pr[B \cap A]}{\Pr[A]} = \frac{\Pr[A \cap B]}{\Pr[A]}$$

$$\Pr[A \mid B] \cdot \Pr[B] = \Pr[A \cap B] = \Pr[B \mid A] \cdot \Pr[A]$$

$$\boxed{\Pr[A \mid B] = \frac{\Pr[A] \cdot \Pr[B \mid A]}{\Pr[B]}}$$

# How to Use the Bayes Formula -- 1



- Suppose you are investigating on a crime involving a taxi
  - 80% of taxi in the city are black and the remaining 20% are yellow
  - The reliability of witnesses is 80% on average
- Which is the probability that the involved taxi is yellow given that a witness declared to have seen a yellow taxi?
  - "True" = real color of the vehicle
  - "Report" = color declared by the witness
- Pr[true = Y | report = Y]?

# How to Use the Bayes Formula -- 2





$$\Pr[true = Y \mid report = Y] = \frac{\Pr[true = Y] \cdot \Pr[report = Y \mid true = Y]}{\Pr[report = Y]}$$

- If the witness declared to have seen a yellow taxi we have to distinuguish two cases
  - The real color of the taxi is yellow

$$\Pr[true = Y] \cdot \Pr[report = Y \mid true = Y] = 0.2 \cdot 0.8 = 0.16,$$

  - The real color of the taxi is back

$$\Pr[true = B] \cdot \Pr[report = Y \mid true = B] = 0.8 \cdot 0.2 = 0.16.$$

  - then

$$\begin{aligned}\Pr[report = Y] &= \Pr[true = Y] \cdot \Pr[report = Y \mid true = Y] + \\ &\quad \Pr[true = B] \cdot \Pr[report = Y \mid true = B] \\ &= 0.2 \cdot 0.8 + 0.8 \cdot 0.2 = 0.32.\end{aligned}$$

# How to Use the Bayes Formula -- 3





$$\Pr\left[true = Y \mid report = Y\right] = \frac{\Pr\left[true = Y\right] \cdot \Pr\left[report = Y \mid true = Y\right]}{\Pr\left[report = Y\right]}$$

- Substituting in the Bayes formula we have

$$\begin{aligned}
\Pr\left[true = Y \mid report = Y\right] &= \frac{\Pr\left[true = Y\right] \cdot \Pr\left[report = Y \mid true = Y\right]}{\Pr\left[report = Y\right]} \\
&= \frac{0.2 \cdot 0.8}{0.32} \\
&= 0.5.
\end{aligned}$$

- Even if the witness says to have seen a yellow taxi, the probability that the taxi was yellow is ½
  - Her deposition is totally unreliable

# Come back to our Herding Experiment



- Each player wants to maximize her utility
  - Guess the majority color in the urn

  - Maj-red = urn with a majoirity of red balls
  - Maj-blue = urn with a majoirity of blue balls
  - Red = the drawn ball is red
  - Blue = the drawn ball is blue
- A player announces blue if
  - Pr[Maj-Blue | colors observed and heard] > 1/2
  - Otherwise she announces red
- The a priori probability
  - Pr[Maj-Red] = Pr[Maj-Blue] = 1/2
- The a posteriori conditional probability
  - Pr[Red | Maj-Red] = Pr[Blue | Maj-Blue] = 2/3

# First Player





$$\Pr\left[\textit{majority-blue} \mid \textit{blue}\right] = \frac{\Pr\left[\textit{majority-blue}\right] \cdot \Pr\left[\textit{blue} \mid \textit{majority-blue}\right]}{\Pr\left[\textit{blue}\right]}$$

$$\begin{aligned}\Pr\left[\textit{blue}\right] &= \Pr\left[\textit{majority-blue}\right] \cdot \Pr\left[\textit{blue} \mid \textit{majority-blue}\right] + \\ &\quad \Pr\left[\textit{majority-red}\right] \cdot \Pr\left[\textit{blue} \mid \textit{majority-red}\right] \\ &= \frac{1}{2} \cdot \frac{2}{3} + \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{2}.\end{aligned}$$

$$\Pr\left[\textit{majority-blue} \mid \textit{blue}\right] = \frac{1/3}{1/2} = \frac{2}{3}.$$

- The best move of the first player is to announce the color of the ball she drew

# Second Player





- Similar to first player
  - Assume that if the a posteriori conditional probability is ½ she announces the color she observed
- Best move of the second player is to announce the color of the ball she drew

# Third Player

- Suppose the third player saw and "blue-blue-red"

$$\Pr[\textit{majority-blue} \mid \textit{blue, blue, red}] = \frac{\Pr[\textit{majority-blue}] \cdot \Pr[\textit{blue, blue, red} \mid \textit{majority-blue}]}{\Pr[\textit{blue, blue, red}]}.$$

$$\Pr[\textit{blue, blue, red} \mid \textit{majority-blue}] = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3} = \frac{4}{27}$$

$$\begin{aligned}\Pr[\textit{blue, blue, red}] &= \Pr[\textit{majority-blue}] \cdot \Pr[\textit{blue, blue, red} \mid \textit{majority-blue}] + \\ &\quad \Pr[\textit{majority-red}] \cdot \Pr[\textit{blue, blue, red} \mid \textit{majority-red}] \\ &= \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{2}{3} = \frac{6}{54} = \frac{1}{9}.\end{aligned}$$

$$\Pr[\textit{majority-blue} \mid \textit{blue, blue, red}] = \frac{\frac{4}{27} \cdot \frac{1}{2}}{\frac{1}{9}} = \frac{2}{3}.$$

- The best move of the third player is to announce blue even if she observed a red ball

# Following Players





- All the players are in the same situation as player 3
  - All their announcements give no information

- In this case we have an information cascade

- What triggered the information cascade?
  - A wrong announcement of the first two players
  - In the general case, an information cascade is triggered when the difference between the number of players announcing the two colors > 1

# A General Model



- A group of people make decisions sequentially
  - Each individual has to *accept* o *reject* a given option
    - Buy the last model of cell
    - Read a book
- Our model has three main ingredients
  - State of the World
  - Payoffs
  - Signals

# State of the World



- (I) State of the World (a priori)
  - A priori the World can be in one of two states
    - The proposed option is really a good idea (G) or a bad idea (B)
  - The World is in the state G with probability $p$ and in the state B with probability 1-$p$
  - These probabilities are commonly known

# Payoffs





- (II) Payoffs
  - In case of reject: payoff = 0
  - In case of accept of a good option: $v_g > 0$
  - In case of accept of a bad option: $v_b < 0$
  - Expected payoff: $v_g\, p + v_b\, (1\text{-}p)$
    - Assume the expected payoff is 0

# SIGNALS



- (III) Signals
  - Model effects of the private information
  - Signal High (H) suggests that accept is a good idea
  - Signal Low (L) suggests that accept is a bad idea
- Signals H have greater probability to occur when the State of the World is G
  - Similarly for signals L in state B

| | | States | |
|---|---|---|---|
| | | $B$ | $G$ |
| Signals | $L$ | $q$ | $1-q$ |
| | $H$ | $1-q$ | $q$ |

$q > \frac{1}{2}$

# Cascade General Model



- Consider an individual making her decision
  - Suppose her decision is based only on her personal information and the a priori probabilities
- If she receives a signal H then her best move is to accept since her expected payoff is

$$v_a \Pr[G \mid H] + v_b \Pr[B \mid H]$$

$$\begin{aligned}
\Pr[G \mid H] &= \frac{\Pr[G] \cdot \Pr[H \mid G]}{\Pr[H]} \\
&= \frac{\Pr[G] \cdot \Pr[H \mid G]}{\Pr[G] \cdot \Pr[H \mid G] + \Pr[B] \cdot \Pr[H \mid B]} \\
&= \frac{pq}{pq + (1-p)(1-q)} \\
&> p,
\end{aligned}$$

- similarly, if she receives signal L she has to reject

# Multiple Signals





- What if the individual receives multiple signals?
  - for example, information inferred from the actions of all the earlier players?
- Suppose she receives a sequence S of a signals H and b signals L
- Observe that

(i) the posterior probability $\Pr[G \mid S]$ is greater than the prior $\Pr[G]$ when $a > b$;

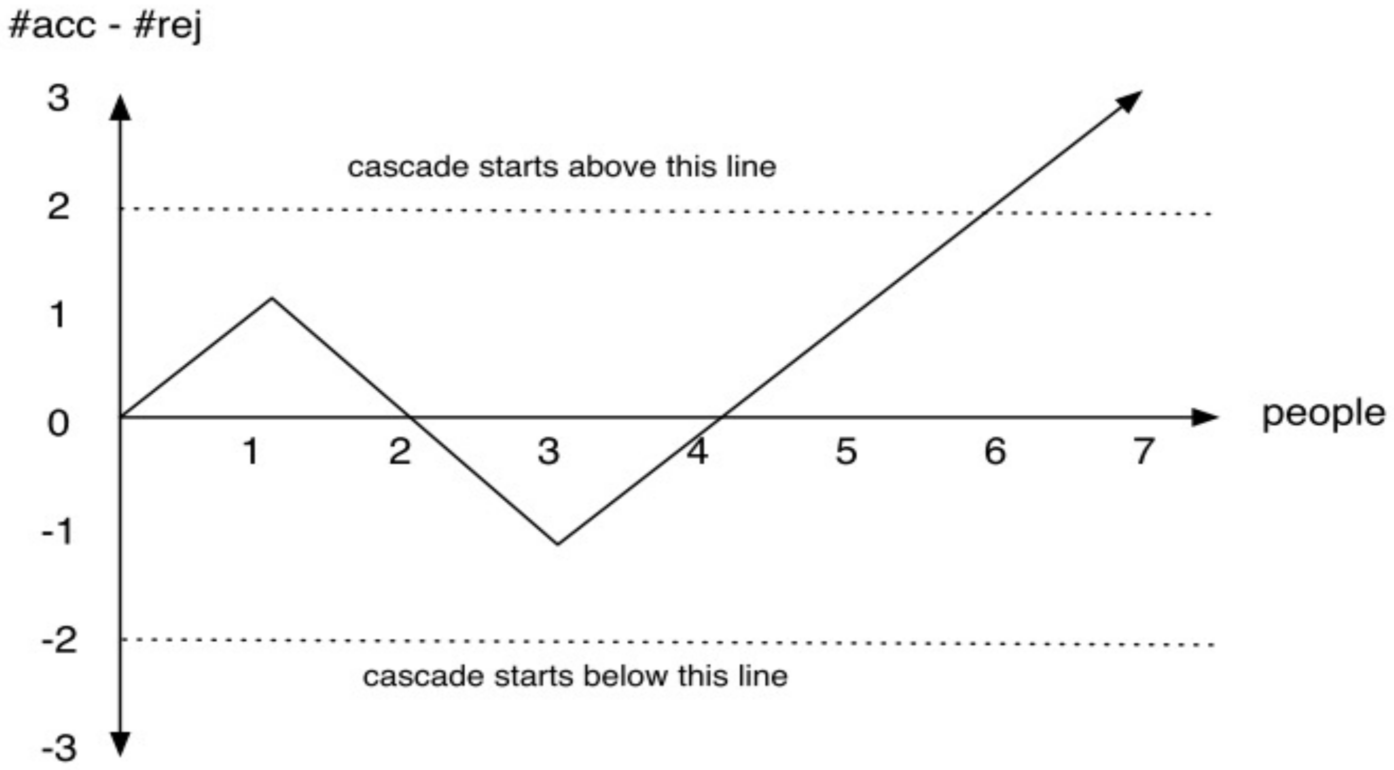
(ii) the posterior $\Pr[G \mid S]$ is less than the prior $\Pr[G]$ when $a < b$; and

(iii) the two probabilities $\Pr[G \mid S]$ and $\Pr[G]$ are equal when $a = b$.

- Her best move is
  - accept if $a > b$
  - reject if $a < b$

# The General Model Applied to the Herding Experiment

Apply the general model to our experiment




- When a player receives a set of signals where the difference between H and L is greater than 1 an information cascade starts

# Lessons to Learn from Cascades





Some things we must take in mind from cascades

- Information Cascades can be wrong
  - Accepting an option can be wrong even if other players have done it
- Information Cascades can be triggered by very little information
  - Once a cascade started all players rationally decide to ignore their personal information
- Information Cascades are fragile
  - Very little additional information can be sufficient to stop a cascade