

Auction and Matching Markets

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- ▶ n buyers willing to buy this item
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 - ▶ how much she is willing to pay for that item

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We can run an auction for selling the object

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 - ▶ Sotheby's Fine Art Auctions, E-Bay, ...

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- ▶ Sealed Bid Second-Price Auctions

Descending Auctions vs. First-Price Auctions

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Descending Auctions are equivalent to First-Price Auctions

Ascending Auctions vs. Second-Price Auctions

Ascending Auctions

Second-Price Auctions

Ascending Auctions vs. Second-Price Auctions

Ascending Auctions

- ▶ It does not make sense to **not drop out** when the price is **larger** than your own valuation

Second-Price Auctions

Ascending Auctions vs. Second-Price Auctions

Ascending Auctions

- ▶ It does not make sense to **drop out** when the price is **smaller** than your own valuation

Second-Price Auctions

Ascending Auctions vs. Second-Price Auctions

Ascending Auctions

- ▶ The price b_i at which agent i drops out depends only on your own valuation

Second-Price Auctions

- ▶ The bid b_i that agent i submits depends only on your own valuation

Ascending Auctions vs. Second-Price Auctions

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Ascending Auctions are equivalent to Second-Price Auctions

Second-Price Auctions

Players: n buyers

Strategies: possible bids b_i

Utility: $u_i = v_i - \max_{j \neq i} b_j$ if i is the winner, otherwise $u_i = 0$

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In a sealed-bid second price auction, it is a dominant strategy for each buyer to bid truthfully, i.e. to play the strategy $b_i = v_i$.

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Why truthful bidding is a dominant strategy?

Intuition: Because your utility does not depend on your bid

First-Price Auctions

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Truthfulness?

In a sealed-bid first price auction, bidding truthfully is not a dominant strategy

- ▶ Bidders prefer to **shade** their bids slightly downward
 - ▶ Not too much, otherwise we can lose
 - ▶ Not too few, otherwise shading is almost ineffective

From one item to multiple items

An easy model

- ▶ n items to be sold and n buyers
- ▶ each buyer keeps a list of items at which she is interested
- ▶ a buyer is happy only if she receives an item from his list

From one item to multiple items

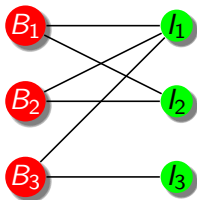
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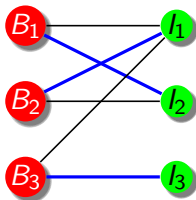
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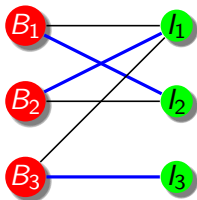
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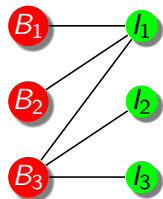
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We need to find a **perfect matching**

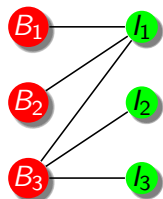
Constricted Sets and Matching Theorem

How can we recognize that there is no perfect matching?



Constricted Sets and Matching Theorem

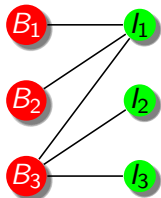
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 - ▶ A set of buyers S such that $|S| > |N(S)|$

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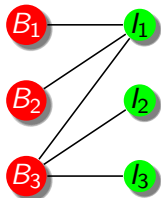
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- ▶ Are there other obstacles to perfect matchings?

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Matching Theorem

If a bipartite graph (with the same number of nodes on both sides) has no perfect matching, then it must contain a constricted set

- ▶ A perfect matching or its obstacle can be efficiently found
 - ▶ For details see Advanced Material at the end of the slides

From one item to multiple items - A more complex setting

Matching markets

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| 7, 5, 2 | B_3 | I_3 |

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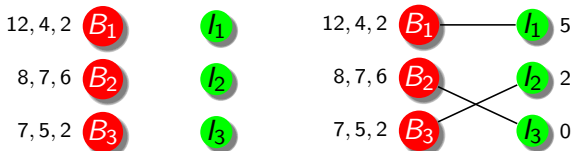
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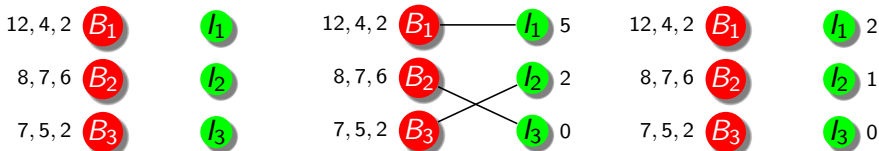
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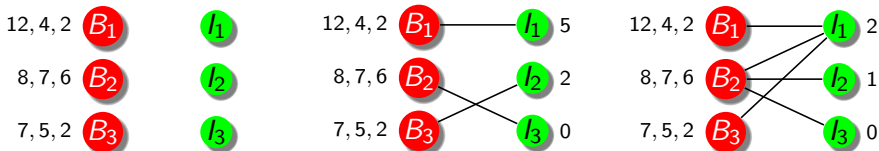
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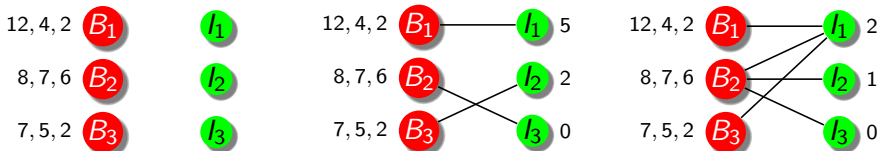
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Goal: **Fix price so to sell all items by maximizing the social welfare**

Market Clearing Prices

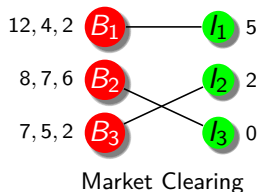
Market Clearing Prices

A set of prices is **market-clearing** if the resulting preferred-item graph has a perfect matching

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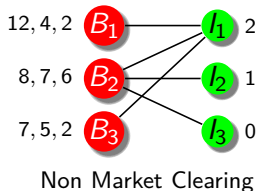
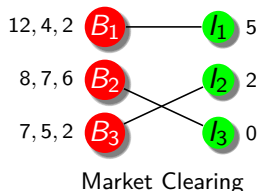
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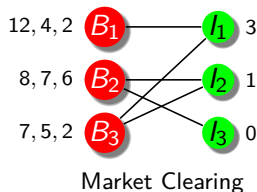
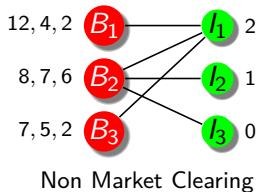
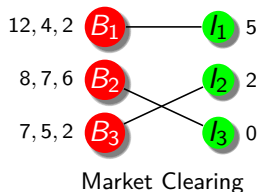
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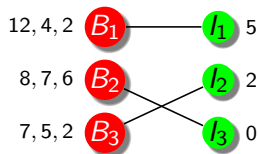
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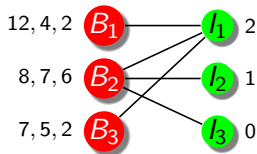
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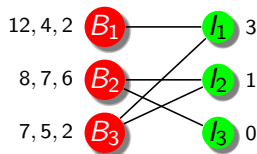
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Market Clearing



Non Market Clearing



Market Clearing

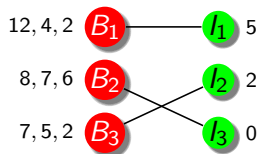
Do market-clearing prices maximize social welfare?

- Buyers buy their best item at current price

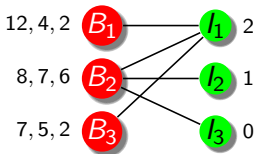
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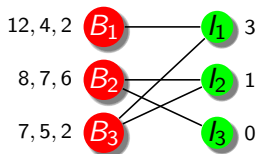
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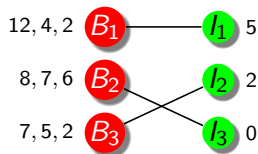
Do market-clearing prices maximize social welfare?

- ▶ Buyers buy their best item at current price
 - ▶ Market clearing prices maximize sum of utilities

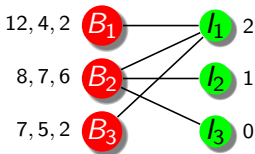
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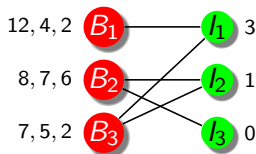
A set of prices is **market-clearing** if the resulting preferred-item graph has a perfect matching



Market Clearing



Non Market Clearing



Market Clearing

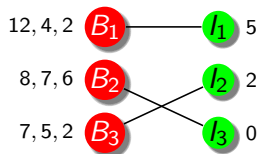
Do market-clearing prices maximize social welfare?

- ▶ Buyers buy their best item at current price
 - ▶ Market clearing prices maximize sum of utilities
- ▶ Sum of utilities = Social Welfare - Sum of Prices

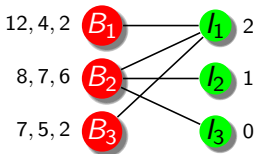
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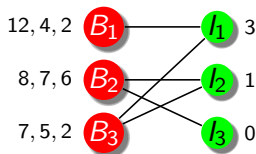
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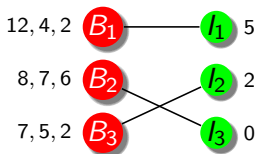
Do market-clearing prices maximize social welfare?

- ▶ Buyers buy their best item at current price
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- ▶ Sum of utilities = Social Welfare - Sum of Prices
- ▶ Prices do not depend on the matching

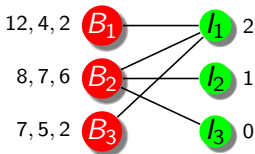
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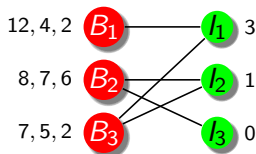
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- ▶ Prices do not depend on the matching
 - ▶ Market clearing prices maximize social welfare

Does market clearing prices always exist?

1. Set price 0 to each item
2. Until the preferred-item graph has not a perfect matching:
 - 2.1 Find the constricted set S
 - 2.2 Raise the price of each item in $N(S)$ by one unit
 - 2.3 Reduce prices so that the smallest price is 0

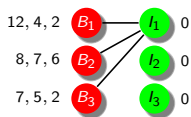
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| | | | |
|----------|-------|-------|---|
| 12, 4, 2 | B_1 | I_1 | 0 |
| 8, 7, 6 | B_2 | I_2 | 0 |
| 7, 5, 2 | B_3 | I_3 | 0 |

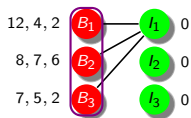
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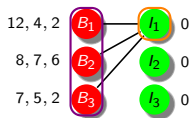
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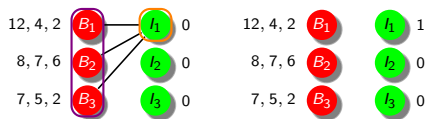
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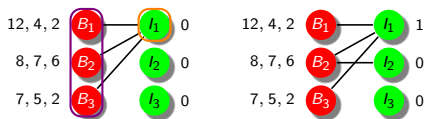
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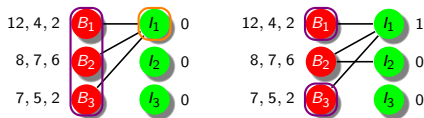
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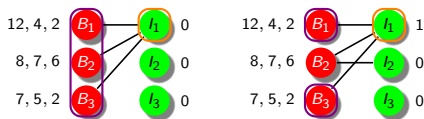
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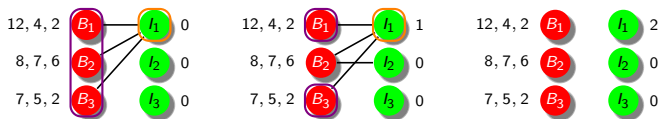
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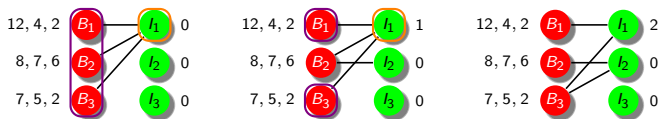
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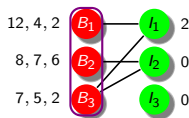
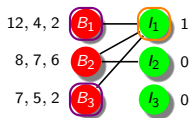
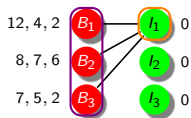
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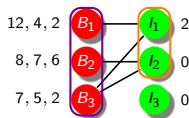
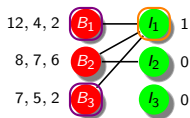
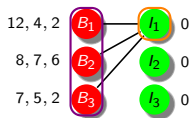
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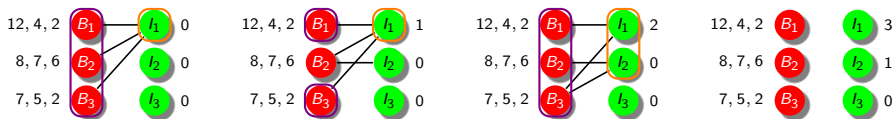
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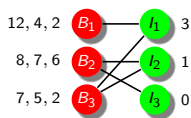
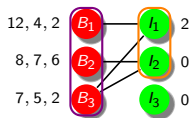
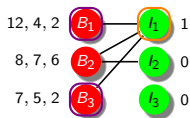
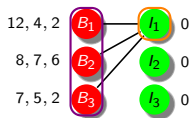
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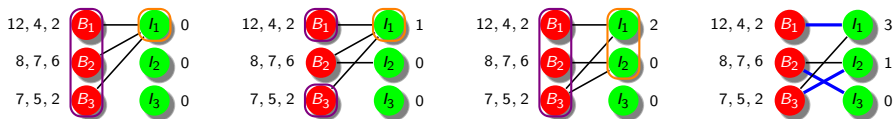
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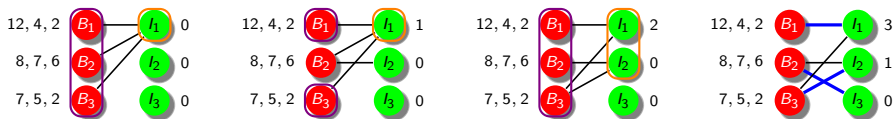
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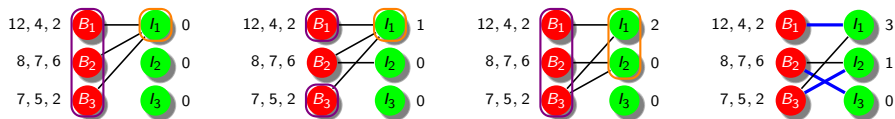
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Does the algorithm end?

Does market clearing prices always exist?

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Does the algorithm end?

Yes. More details in Advanced Material

Matching Markets and Auctions

Single item

Multiple items

Matching Markets and Auctions

Single item

- ▶ Price = Second highest bid

Multiple items

- ▶ Market Clearing Prices

Matching Markets and Auctions

Single item

- ▶ Price = Second highest bid
- ▶ Price arising from ascending auctions

Multiple items

- ▶ Market Clearing Prices
- ▶ Prices computed by the algorithm above

Matching Markets and Auctions

Single item

- ▶ Price = Second highest bid
- ▶ Price arising from ascending auctions

Are them really different?

Multiple items

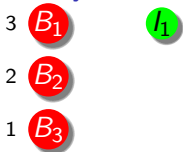
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Matching Markets and Auctions

Single item

- ▶ Price = Second highest bid
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Multiple items

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- ▶ Prices computed by the algorithm above

Are they really different?

| | | |
|---------|-------|-------|
| 3, 0, 0 | B_1 | I_1 |
| 2, 0, 0 | B_2 | I_2 |
| 1, 0, 0 | B_3 | I_3 |

Matching Markets and Auctions

Single item

- ▶ Price = Second highest bid
- ▶ Price arising from ascending auctions

Are they really different?

| | | |
|---------|-------|-------|
| 3, 0, 0 | B_1 | I_1 |
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Multiple items

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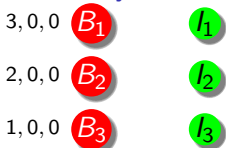
| | | | |
|---------|-------|-------|---|
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Matching Markets and Auctions

Single item

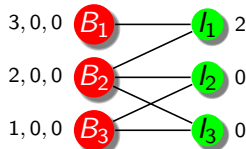
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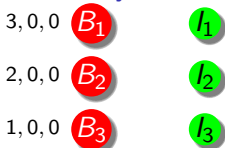


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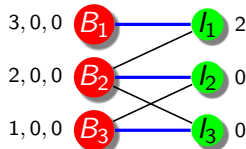
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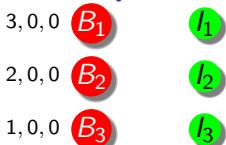


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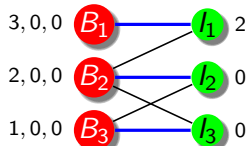
- Price = Second highest bid
- Price arising from ascending auctions

Are they really different?



Multiple items

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- Prices computed by the algorithm above



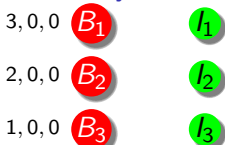
- Second Price Auction produces Market Clearing Prices

Matching Markets and Auctions

Single item

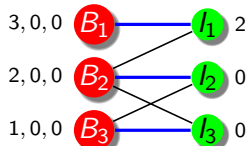
- ▶ Price = Second highest bid
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Are they really different?



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- ▶ Prices computed by the algorithm above



- ▶ Second Price Auction produces Market Clearing Prices
- ▶ The algorithm for computing Market Clearing Prices is a generalization of an Ascending Price Auctions

Keyword Advertising

The screenshot shows a Google search interface. The search bar contains the text "keuka lake". To the right of the search bar are links for "Advanced Search" and "Preferences". Below the search bar, a message states "Customized based on recent search activity. [More details](#)". The search results are categorized under "Web Books". The first result is "Welcome to The Keuka Lake Wine Trail", which includes a brief description and links to "Cached" and "Similar pages". The second result is "A complete guide to the Keuka Lake Wine Country", also with a description and links. The third result is "Keuka Lake - Wikipedia, the free encyclopedia", providing a summary of the lake's geography and links. The fourth result is "Seneca Lake (New York) - Wikipedia, the free encyclopedia", with a similar summary and links. On the right side of the page, there is a "Sponsored Links" section with four advertisements: "Keuka Lake Lodging", "Keuka Lake Real Estate", "Finger Lakes Real Estate", and "Finger Lakes Real Estate", each with a brief description and a link.

Google

keuka lake

Advanced Search
Preferences

Customized based on recent search activity. [More details](#)

Web Books

Results 1 - 10 of about 381,000 for **keuka lake** [definition]. (0.19 seconds)

Welcome to The Keuka Lake Wine Trail
Information about seven wineries on **Keuka Lake** in the Finger Lakes district. Offers a trail map, event calendar, winery descriptions, tourist services, ...
[www.keukawinetrail.com/](#) - 13k - [Cached](#) - [Similar pages](#) - [Note this](#)

A complete guide to the Keuka Lake Wine Country
your own, follow the **Keuka Lake** Wine Trail, or book a wine tour and leave the driving to a pro. From casual to gourmet, holidays to haute cuisine, ...
[www.keukalake.com/](#) - 24k - [Cached](#) - [Similar pages](#) - [Note this](#)

Keuka Lake - Wikipedia, the free encyclopedia
Keuka Lake is an unusual member of the Finger Lakes because it is Y-shaped instead of long and narrow. Because of its shape, it was referred to in the past ...
[en.wikipedia.org/wiki/Keuka_Lake](#) - 26k - [Cached](#) - [Similar pages](#) - [Note this](#)

Seneca Lake (New York) - Wikipedia, the free encyclopedia
The two main inlets are Catharine Creek at the southern end and the **Keuka Lake** Outlet.

Sponsored Links

Keuka Lake Lodging
Lakeside vacation rentals on the Finger Lakes in upstate New York.
[FingerLakesPremierProperties.com](#)

Keuka Lake Real Estate
Looking for information about **Keuka Lake** Real Estate?
[www.MarkMalcolm.com](#)
New York

Finger Lakes Real Estate
Find your dream home; Lakefront, Lakeview, Cottage, Land or Farm!
[www.winefrlproperties.com](#)
New York

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www.MarkMalcolm.com
New York

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www.winevalproperties.com
New York



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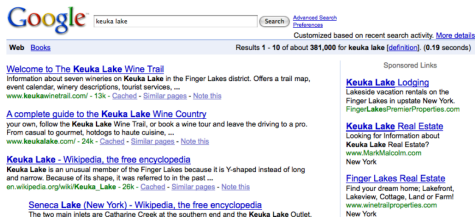
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New York



Pay-per-Impression

Keyword Advertising



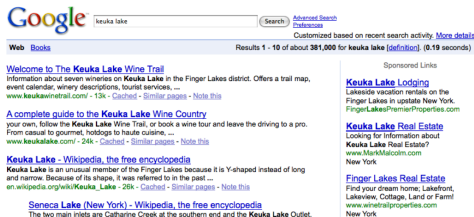
The screenshot shows a Google search for "keuka lake". The search bar is at the top with the Google logo. Below it, the search results are categorized into "Web" and "Books". The "Web" results include links to "Welcome to The Keuka Lake Wine Trail", "A complete guide to the Keuka Lake Wine Country", "Keuka Lake - Wikipedia, the free encyclopedia", and "Seneca Lake (New York) - Wikipedia, the free encyclopedia". The "Books" results include "Keuka Lake Lodging", "Keuka Lake Real Estate", and "Finger Lakes Real Estate". The page also shows a "Sponsored Links" section with ads for "Keuka Lake Lodging", "Keuka Lake Real Estate", and "Finger Lakes Real Estate".



Pay-per-Impression

- Ads not relevant to the page

Keyword Advertising



The screenshot shows a Google search for "keuka lake". The search bar is at the top with the Google logo. Below it, the search results are displayed. The first result is "Welcome to The Keuka Lake Wine Trail" with a description and a link. The second result is "A complete guide to the Keuka Lake Wine Country" with a description and a link. The third result is "Keuka Lake - Wikipedia, the free encyclopedia" with a description and a link. The fourth result is "Seneca Lake (New York) - Wikipedia, the free encyclopedia" with a description and a link. On the right side, there are "Sponsored Links" for "Keuka Lake Lodging", "Keuka Lake Real Estate", and "Finger Lakes Real Estate".

Google
keuka lake
Advanced Search
Preferences
Customized based on recent search activity. [More details](#)

Web Books
Results 1 - 10 of about 381,000 for keuka lake [definition]. (0.19 seconds)

Welcome to The Keuka Lake Wine Trail
Information about seven wineries on **Keuka Lake** in the Finger Lakes district. Offers a trail map, event calendar, winery descriptions, tourist services, ...
[www.keukawinetrail.com/](#) - 13k - [Cached](#) - [Similar pages](#) - [Note this](#)

A complete guide to the Keuka Lake Wine Country
your own, follow the **Keuka Lake Wine Trail**, or book a wine tour and leave the driving to a pro. From casual to gourmet, holidays to haute cuisine, ...
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Keuka Lake - Wikipedia, the free encyclopedia
Keuka Lake is an unusual member of the Finger Lakes because it is Y-shaped instead of long and narrow. Because of its shape, it was referred to in the past ...
[en.wikipedia.org/wiki/Keuka_Lake](#) - 26k - [Cached](#) - [Similar pages](#) - [Note this](#)

Seneca Lake (New York) - Wikipedia, the free encyclopedia
The two main inlets are Catharine Creek at the southern end and the **Keuka Lake Outlet**.

Sponsored Links

Keuka Lake Lodging
Lakeside vacation rentals on the Finger Lakes in upstate New York.
[FingerLakesPremierProperties.com](#)

Keuka Lake Real Estate
Looking for information about **Keuka Lake Real Estate**?
[www.MarkMalcolm.com](#)
New York

Finger Lakes Real Estate
Find your dream home; Lakefront, Lakeview, Cottage, Land or Farm!
[www.winevailproperties.com](#)
New York



Pay-per-Impression

- ▶ Ads not relevant to the page
- ▶ Pay even if ad is not useful

Keyword Advertising

The screenshot shows a Google search for "keuka lake". The search bar is at the top with "keuka lake" entered. Below the search bar, there are links for "Advanced Search" and "Preferences". A message states: "Customized based on recent search activity. [More details](#)". Below this, it says "Results 1 - 10 of about 381,000 for keuka lake [definition]. (0.19 seconds)".

Under the "Web" tab, the first organic result is "Welcome to The Keuka Lake Wine Trail" with a description of seven wineries in the Finger Lakes district. The second organic result is "A complete guide to the Keuka Lake Wine Country" with a description of wine tours and dining. The third organic result is "Keuka Lake - Wikipedia, the free encyclopedia" with a description of the lake's shape and location. The fourth organic result is "Seneca Lake (New York) - Wikipedia, the free encyclopedia" with a description of the lake's inlets.

On the right side, under "Sponsored Links", there are four ads: "Keuka Lake Lodging" (Lakeside vacation rentals), "Keuka Lake Real Estate" (Looking for information about Keuka Lake Real Estate?), "Finger Lakes Real Estate" (Find your dream home), and "Finger Lakes Real Estate" (Find your dream home).



The Web Search approach

Pay-per-Impression

- ▶ Ads not relevant to the page
- ▶ Pay even if ad is not useful

Keyword Advertising

The screenshot shows a Google search for "keuka lake". The search bar is at the top with the Google logo. Below the search bar, it says "Customized based on recent search activity. More details". The results are divided into "Web" and "Books" sections. Under "Web", there are several organic search results from "The Keuka Lake Wine Trail", "A complete guide to the Keuka Lake Wine Country", "Keuka Lake - Wikipedia, the free encyclopedia", and "Seneca Lake (New York) - Wikipedia, the free encyclopedia". To the right of these results, under the heading "Sponsored Links", there are four paid advertisements: "Keuka Lake Lodging", "Keuka Lake Real Estate", "Finger Lakes Real Estate", and "Finger Lakes Real Estate".



The Web Search approach

► Keyword-Based Advertising

Pay-per-Impression

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Keyword Advertising

The screenshot shows a Google search for "keuka lake". The search bar includes the text "keuka lake" and a "Search" button. Below the search bar, it says "Customized based on recent search activity. [More details](#)". The results are categorized under "Web Books".

Organic Search Results:

- Welcome to The Keuka Lake Wine Trail**
Information about seven wineries on **Keuka Lake** in the Finger Lakes district. Offers a trail map, event calendar, winery descriptions, tourist services, ...
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- A complete guide to the Keuka Lake Wine Country**
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New York
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The Web Search approach

- ▶ **Keyword-Based Advertising**
- ▶ **Pay-per-Click**

Pay-per-Impression

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The Web Search approach

- ▶ **Keyword-Based Advertising**
- ▶ **Pay-per-Click**

Does it work?

Pay-per-Impression

- ▶ Ads not relevant to the page
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Keyword Advertising

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The Web Search approach

- ▶ **Keyword-Based Advertising**
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Pay-per-Impression

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Does it work?

- ▶ The top spot for *calligraphy pens* costs \$1.50
- ▶ The top spot for *calligraphy pens* costs \$0.60

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The Web Search approach

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- ▶ The top spot for *loan consolidation* costs \$50

How the prices are chosen?

Preliminary definitions

- ▶ Clickthrough rates r_j of spot j

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Preliminary definitions

- ▶ **Clickthrough rates** r_j of spot j
 - ▶ Clickthrough rates are known
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Problem: we do not know who is the best advertiser

How can we solve this problem?

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Truthfulness

- ▶ Each advertiser i submit a bid b_i
- ▶ It is a dominant strategy for an advertiser i to bid truthfully, i.e. $b_i = v_i$

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Can we run second price auctions?

- ▶ They have been defined for single item auctions
- ▶ Now we have multiple slots to sell

Second Price Auctions revisited

- ▶ The item is allocated to the highest bidder

Second Price Auctions revisited

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Examples

- ▶ n agents with valuation $v_1 \geq \dots \geq v_n$

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 - ▶ remaining player have utility 0

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 - ▶ agent 2 wins the item and has utility v_2
 - ▶ remaining player have utility 0

Second Price Auctions revisited

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- ▶ The winner is charged the second highest bid
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 - ▶ she wins the item
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 - ▶ remaining player have utility 0
- ▶ The harm caused by agent 1 amounts to v_2

VCG Auctions

Vickrey-Clarke-Groves Principle

- ▶ Agents submit **bids**
- ▶ **Allocation** maximizes the social welfare
- ▶ **Prices** are the harm to other bidders

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VCG Prices

- ▶ $SW(A)$ = maximum social welfare with all bidders and slots
- ▶ $SW(A_{-j}^{-i})$ = maximum social welfare without j and its slot i
- ▶ $SW(A_{-j})$ = maximum social welfare without j (but all slots)

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VCG Auctions are truthful See Advanced Material for details

VGC Auctions - Example

- ▶ 3 slots with Clickthrough Rates 10, 5 and 2
- ▶ 3 advertisers with Revenue per Click 3, 2 and 1
- ▶ Assume bids = valuations

VGC Auctions - Example

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- ▶ 3 slots with Clickthrough Rates 10, 5 and 2
- ▶ 3 advertisers with Revenue per Click 3, 2 and 1
- ▶ Assume bids = valuations
- ▶ Optimal assignment (welfare = 42)
- ▶ Welfare without first advertiser and first slot: 12

VGC Auctions - Example

- ▶ 3 slots with Clickthrough Rates 10, 5 and 2
- ▶ 3 advertisers with Revenue per Click 3, 2 and 1
- ▶ Assume bids = valuations
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VGC Auctions - Example

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- ▶ The first advertiser must pay $25 - 12 = 13$

VGC Auctions - Example

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- ▶ Welfare without second advertiser and second slot: 32

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- ▶ Assume bids = valuations
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- ▶ Welfare without the second advertiser but with second slot: 35

VGC Auctions - Example

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- ▶ The second advertiser must pay $35 - 32 = 3$

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- ▶ Welfare without third advertiser and third slot: 40

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- ▶ The first advertiser must pay $25 - 12 = 13$
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- ▶ Welfare without the second advertiser but with second slot: 35
- ▶ The second advertiser must pay $35 - 32 = 3$
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- ▶ Welfare without second advertiser and second slot: 32
- ▶ Welfare without the second advertiser but with second slot: 35
- ▶ The second advertiser must pay $35 - 32 = 3$
- ▶ Welfare without third advertiser and third slot: 40
- ▶ Welfare without the third advertiser but with third slot: 40
- ▶ The third advertiser must pay $40 - 40 = 0$

VGC Auctions: Market Clearing Prices

VGC prices

Personalized Prices

VGC Auctions: Market Clearing Prices

VGC prices

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Market Clearing prices

Posted Prices

Let us model sponsored search as Matching Markets

VGC Auctions: Market Clearing Prices

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3 x

2 y

1 z

1 10

2 5

3 2

VGC Auctions: Market Clearing Prices

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Let us model sponsored search as Matching Markets

| | | | | |
|-----------|---|---|---|----|
| 30, 15, 6 | 3 | x | 1 | 10 |
| 20, 10, 4 | 2 | y | 2 | 5 |
| 10, 5, 2 | 1 | z | 3 | 2 |

VGC Auctions: Market Clearing Prices

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30, 15, 6 **3** **x**

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1 **10** 13

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VGC Auctions: Market Clearing Prices

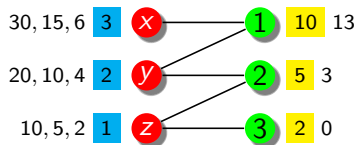
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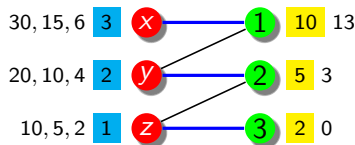
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VGC Auctions: Market Clearing Prices

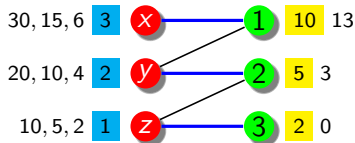
VGC prices

Personalized Prices

Market Clearing prices

Posted Prices

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VCG prices vs. Market Clearing Prices

VCG prices are market clearing prices of minimum total sum

VGC Auctions: Market Clearing Prices

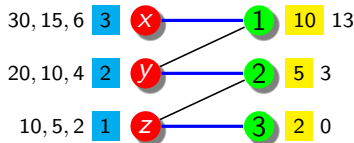
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Observations

- VGC Auctions are a generalization of Second-Price Auctions

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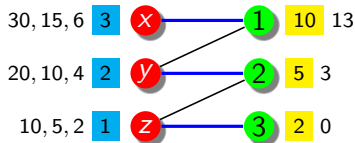
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- ▶ VGC Auctions are a generalization of Second-Price Auctions
- ▶ Market-clearing prices given by generalized ascending auctions

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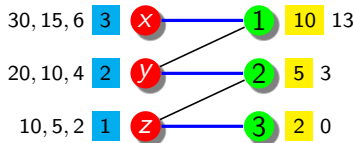
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Observations

- ▶ VGC Auctions are a generalization of Second-Price Auctions
- ▶ Market-clearing prices given by generalized ascending auctions
- ▶ Ascending Auctions are equivalent to Second-Price Auctions

Advanced Material

Matching Theorem: Sketch of the proof

How to find a perfect matching?

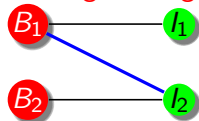
Use **augmenting paths** until you have a perfect matching

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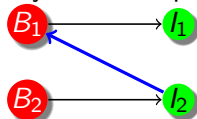


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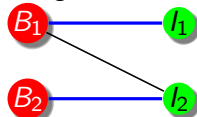
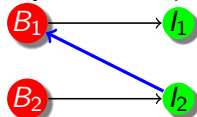


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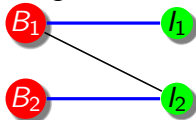
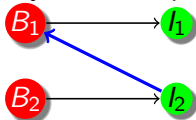


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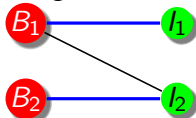
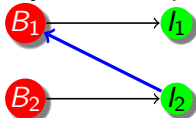
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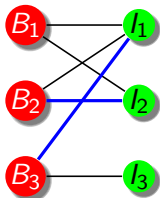
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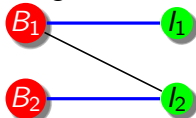
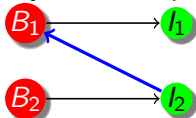
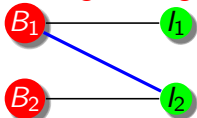


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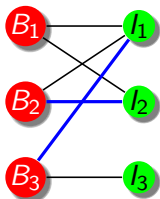
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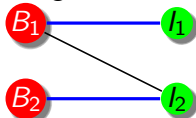
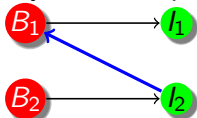


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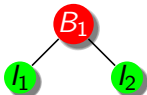
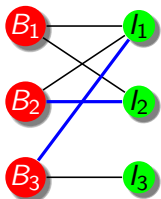
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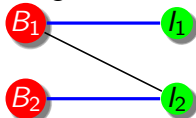
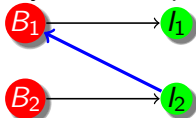


Advanced Material

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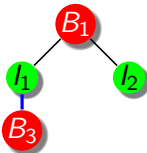
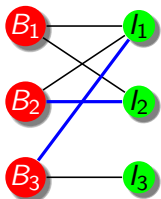
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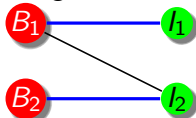
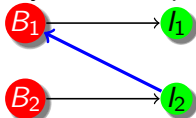


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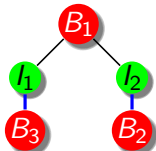
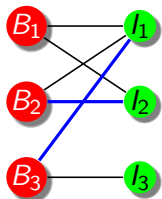
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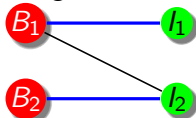
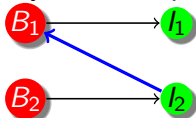


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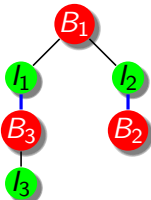
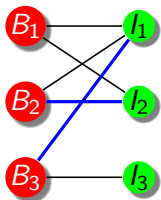
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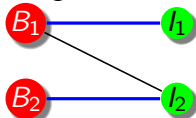
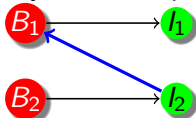


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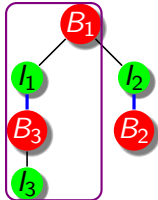
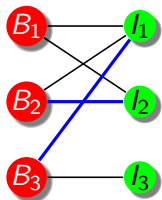
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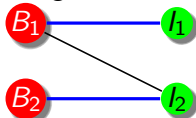
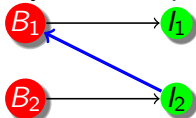


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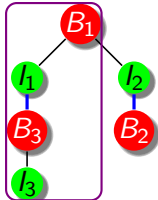
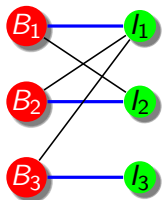
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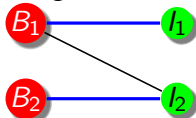
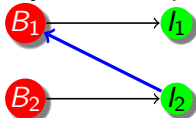


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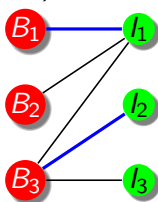
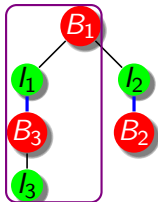
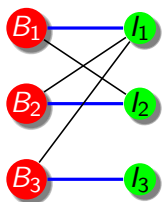
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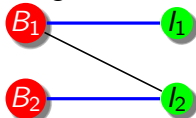
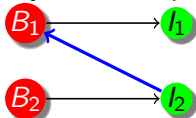
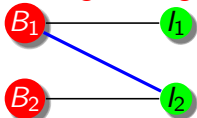


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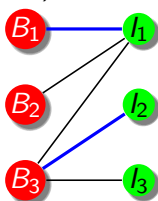
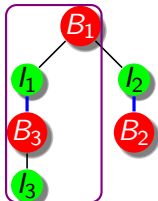
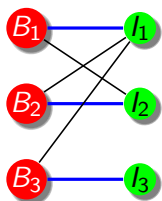
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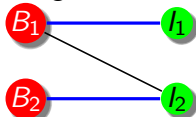
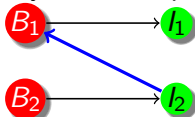


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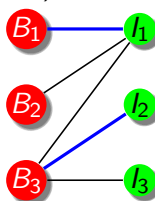
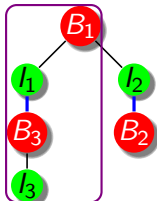
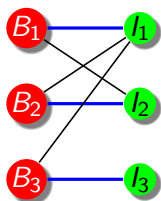
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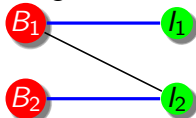
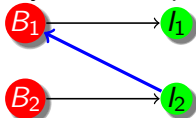


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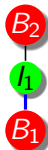
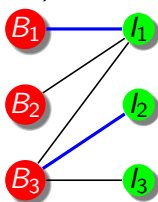
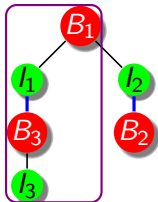
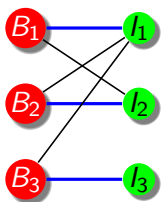
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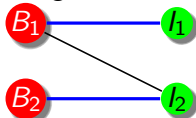
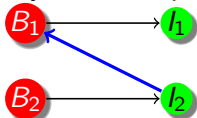


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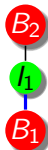
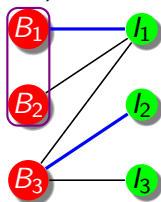
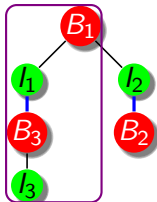
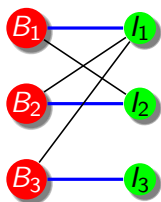
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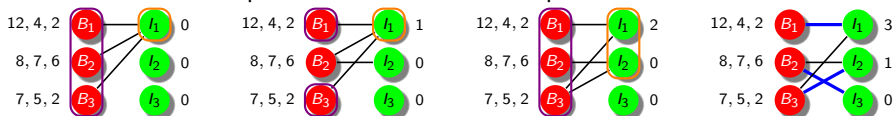
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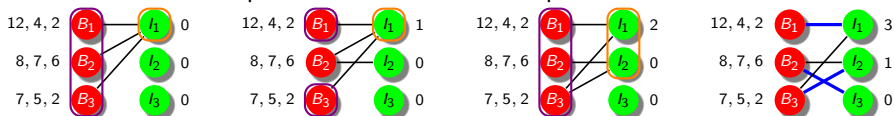
1. Set price 0 to each item
2. Until the preferred-item graph has not a perfect matching:
 - 2.1 Find the constricted set S
 - 2.2 Raise the price of each item in $N(S)$ by one unit
 - 2.3 Reduce prices so that the smallest price is 0



Does the algorithm end?

Advanced Material

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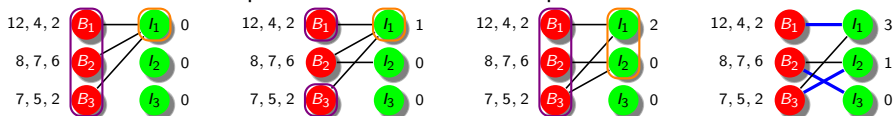


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- **Potential function:** sum of price + sum of buyers' utilities

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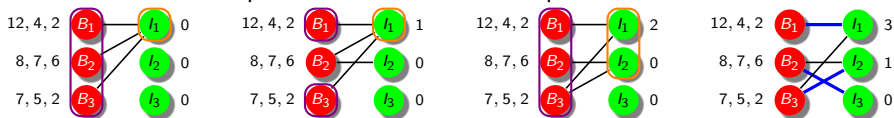


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- ▶ **Potential function:** sum of price + sum of buyers' utilities
- ▶ Potential function is always at least 0 (**price reduction**)

Advanced Material

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Does the algorithm end?

- ▶ **Potential function:** sum of price + sum of buyers' utilities
- ▶ Potential function is always at least 0 (**price reduction**)
- ▶ At each step, we raise the price of $N(S)$ items

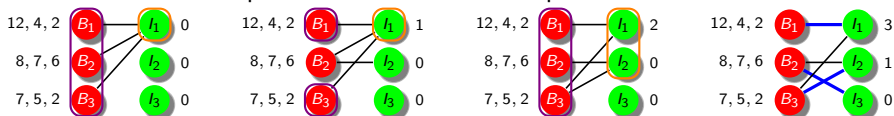
Advanced Material

1. Set price 0 to each item
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2.1 Find the constricted set S

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Does the algorithm end?

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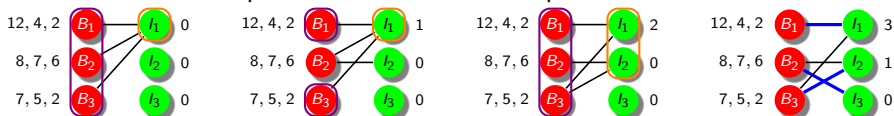
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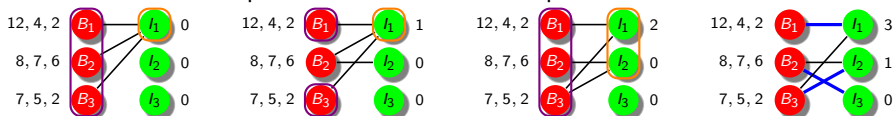
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27

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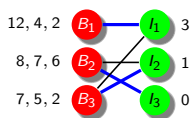
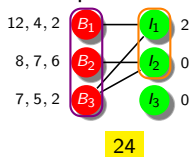
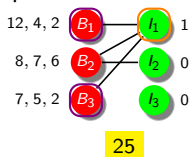
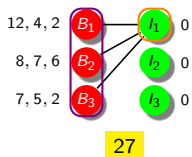


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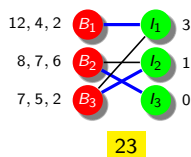
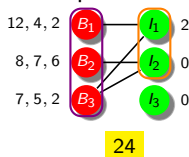
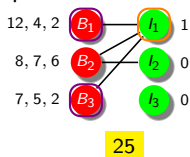
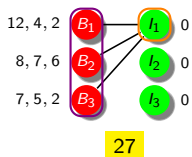


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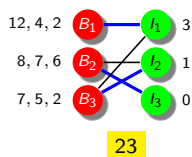
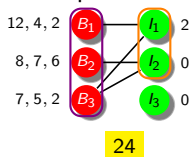
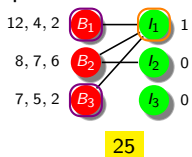
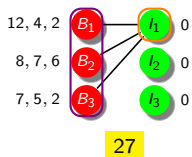


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 - ▶ At each step, the potential function decreases
 - ▶ **Number of steps are finite**

Advanced Material

VCG Auctions: Truthfulness

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Advanced Material

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$$v_{ij} - p_{ij} \stackrel{?}{\geq} v_{ik} - p_{ik}$$

$$v_{ij} - [SW(A_{-i}) - SW(A_{-i}^{-j})] \stackrel{?}{\geq} v_{ik} - [SW(A_{-i}) - SW(A_{-i}^{-k})]$$

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$$v_{ij} + SW(A_{-i}^{-j}) \stackrel{?}{\geq} v_{ik} + SW(A_{-i}^{-k})$$

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$$SW(A) \stackrel{?}{\geq} v_{ik} + SW(A_{-i}^{-k})$$

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$$SW(A) \stackrel{?}{\geq} v_{ik} + SW(A_{-i}^{-k}) \quad \text{YES}$$