What do we know?

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- How the information spread over these networks

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 - ► The role of fake news

Description of the problem

- Spend your budget to choose a set of influencers
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- Spend your budget to choose a set of influencers
- that are able to influence as many nodes as possible
 - Not only direct influence
 - but also influence by
 - people influenced by influencers
 - people influenced by people influenced by influencers
 - and so on...

Some results

Hardness

It is provably **hard** to design an **efficient** algorithm that computes the **best** set of influencers

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Monotone and Submodular Diffusion Processes

- ► The greedy algorithm
 - returns a set of influencers whose influence is **provably**
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Monotone and Submodular Diffusion Processes

- The greedy algorithm
 - returns a set of influencers whose influence is provably
 - a constant approximation of the optimal influence
- Heuristics based on centrality measures
 - They have been experimentally showed to work in practice
 - No guarantee on approximation
 - Usually faster than greedy algorithm

Some results

Some results

Majority Dynamics and manipulation of the order of updates

► It is possible to efficiently compute a sequence of updates leading a minority to become a majority

Some results

- It is possible to efficiently compute a sequence of updates leading a minority to become a majority
- ► It is possible to efficiently compute a sequence of updates leading a bare majority to become consensus

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- It is possible to efficiently compute a sequence of updates leading a bare majority to become consensus
 - For essentially any network topology
- When the new product enters in a network on which there are already two (or more) competing products, above results do not hold

- ▶ 2016 US presidential election
 - ▶ 92% of Americans remembered pro-Trump false news
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- 2017 French elections
 - automated accounts on Twitter spread considerable amount of political news
- 2018 Italian political election
 - fake news are linked with the content of populist parties that won

- \triangleright Voters $1, 2, \ldots, n$
- ightharpoonup Alternatives X, Y, \dots
- ▶ Preference $X \succ_i Y$

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Complete: either $X \succ_i Y$ or $Y \succ_i X$

Transitive: $X \succ_i Y$ and $Y \succ_i Z$ implies $X \succ_i Z$

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▶ Ranked List $X \succ_i Y \succ_i Z \succ_i W \succ_i \cdots$

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- ▶ Ranked List $X \succ_i Y \succ_i Z \succ_i W \succ_i \cdots$
 - ▶ Ranked list exists iff preferences are complete and transitive

Voting systems

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A function that takes the individual rankings of voters and produces a single group ranking

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Example: Majority Rule

► For two alternatives: the winner is the alternative that is ranked first by the majority of voters

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Example: Majority Rule

- ► For two alternatives: the winner is the alternative that is ranked first by the majority of voters
- For more than two alternatives:
 - ▶ for any pair of alternative X and Y...
 - the majority rule ranks X before Y...
 - if X is preferred to Y by the majority of voters

- \triangleright $X \succ_1 Y \succ_1 Z$
- \triangleright $Y \succ_2 Z \succ_2 X$
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- ► The group preferences are not transitive

Majority Rule and Condorcet Paradox

Example

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Condorcet Paradox

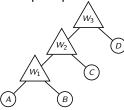
Non transitive group preferences can arise from transitive individual preferences

Tournaments

- Arrange alternatives in some elimination tournament
- Compare pairs accordingly until one alternative is left

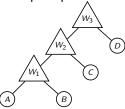
Tournaments

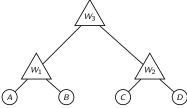
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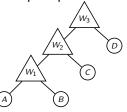
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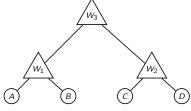




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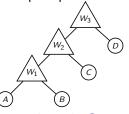


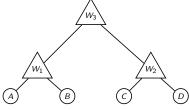


$$\blacktriangleright X \succ_1 Y \succ_1 Z, Y \succ_2 Z \succ_2 X, Z \succ_3 X \succ_3 Y$$

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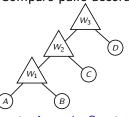


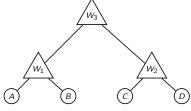


- \blacktriangleright $X \succ_1 Y \succ_1 Z$, $Y \succ_2 Z \succ_2 X$, $Z \succ_3 X \succ_3 Y$
- ▶ There is a tournament in which *X* wins
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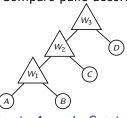


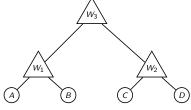


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- ► Assigns a weight to each position in voters' ranked list
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Arrow's Impossibility Theorem

```
Desiderata
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Unanimity: if $X \succ_i Y$ for any i, then $X \succ Y$

Independence of Irrelevant Alternatives (IIA): Preference between X and Y do not depend on Z

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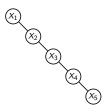
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Arrow's Impossibility Theorem

If there are at least three alternatives, then there is no voting system that satisfies Unanimity, IIA and No-Dictatorship

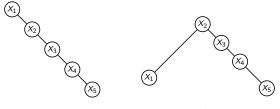
- ▶ Alternative are ordered $X_1, X_2, ..., X_k$
- No voter has an alternative X_s such that...
- ▶ both X_{s-1} and X_{s+1} are ranked above X_s

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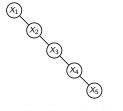
$$X_1 \succ_i X_2 \succ_i X_3 \succ_i X_4 \succ_i X_5$$

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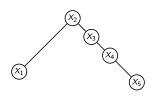


 $X_2 \succ_i X_3 \succ_i X_4 \succ_i X_1 \succ_i X_5$

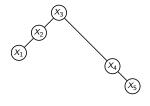
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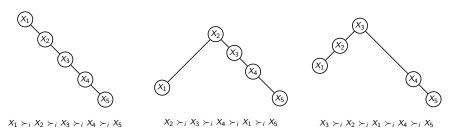
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Single-Peaked Preferences

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- No voter has an alternative X_s such that...
- ▶ both X_{s-1} and X_{s+1} are ranked above X_s



Theorem

With single-peaked preferences, the majority rule always produces a group ranking that is complete and transitive

- ► There are *m* candidates
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Election Manipulation vs. Viral Marketing

- ▶ Being influenced is not sufficient
 - need to alter rankings

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Election Manipulation vs. Viral Marketing

- ▶ Being influenced is not sufficient
 - need to alter rankings
- Promote candidate c may be insufficient
 - need to reduce votes of strong candidates

A first setting

The setting

- Plurality Voting Rule
- Independent Cascade Model Diffusion Process
- ▶ Ranking update: each message increases the rank by one

A first setting

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The result

The problem is monotone and submodular

A first setting

The setting

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- Ranking update: each message increases the rank by one

The result

The problem is monotone and submodular

- Greedy algorithm returns a constant approximation
- Heuristics based on centrality measures work in practice

A second setting

The setting

- ► Any scoring Based Voting Rule
- ► Linear Threshold Model Diffusion Process
- Ranking update: improves of more than one position if influence is much larger than the threshold

A second setting

The setting

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The result

The problem is monotone and submodular

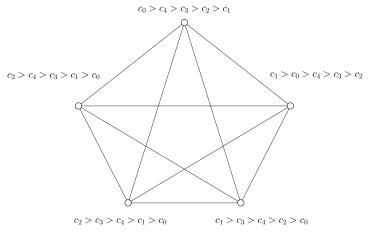
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A limitation in previous results

Only one message may be spread over the network

A limitation in previous results

Only one message may be spread over the network



A third setting

The setting

- ► Plurality Voting Rule
- ► Independent Cascade Model Diffusion Process

A third setting

The setting

- Plurality Voting Rule
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- Different messages may be sent over the networks
 - Number of sent messages limited by budget
- Ranking update: ranking may improve of more positions

A third setting

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Computing efficiently the optimal choice of messages is hard

- Even efficiently computing a good approximation is hard
 - Unless budget is very high
- Greedy algorithms and Centrality based heuristics may fail
 - even in simple networks

Future directions

- Understanding when manipulation is feasible
 - How it depends on network topology
 - How it depends on assumption on rankings
 - ▶ How it depends on assumption of network diffusion

Future directions

- Understanding when manipulation is feasible
 - How it depends on network topology
 - How it depends on assumption on rankings
 - How it depends on assumption of network diffusion
- Understanding how to limit manipulation
 - Budget limitations
 - Forcing a network topology
 - Blocking fake news diffusion