Network Diffusion

Information and Influence

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- Population-wide aggregate influence
- Local influence
  - Technology for chatting with friends and colleagues
  - Align political view with your friends

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Local aspects of influence can motivate imitative behavior

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Individual behavior is influenced by the social network

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A simple model to study the behavior of individuals on social networks

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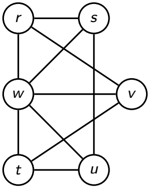
$a = 3$, $b = 2 \rightarrow q = 2/5$



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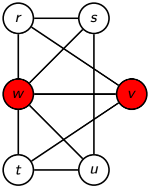

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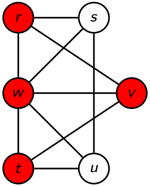

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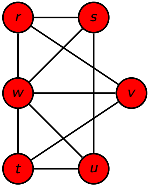

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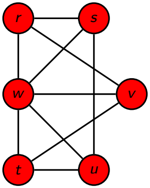

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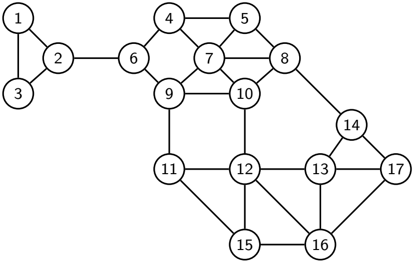

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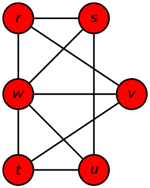

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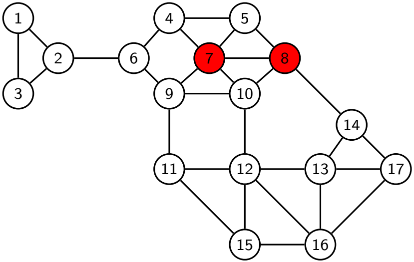

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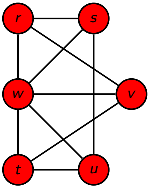

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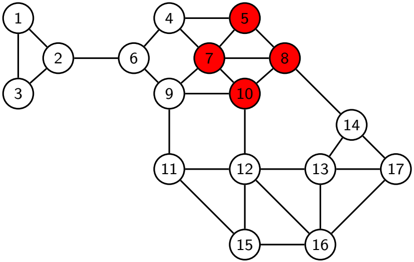

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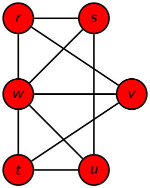

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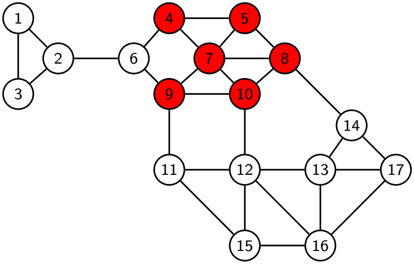

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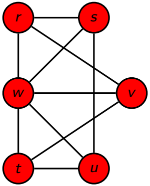

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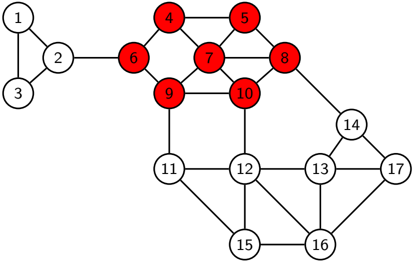

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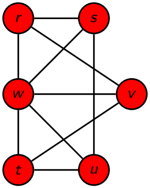

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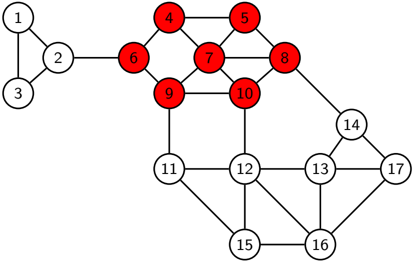

How to increase influence?

- Raise the quality of the product (relative advantage)
- Increase the seed (viral marketing)

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Cluster C of density p

Any node in C has at least a p fraction of neighbors in C

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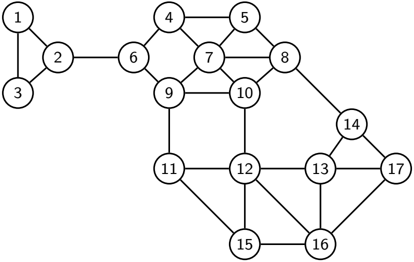

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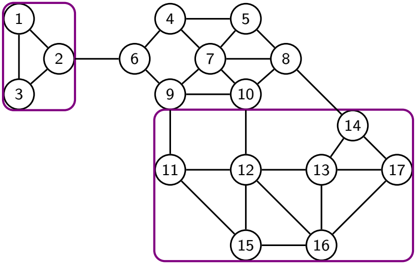

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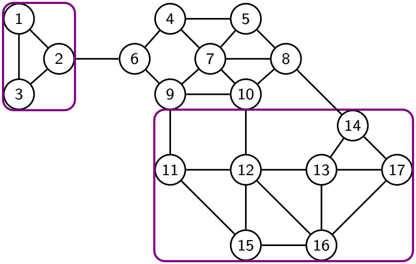

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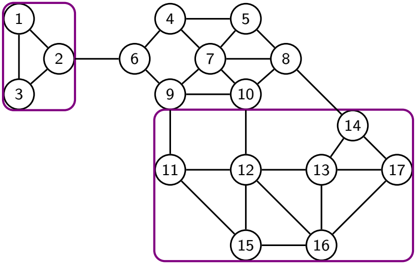

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A cluster of density $1 - q$ is an obstacle to complete cascade

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- Personalized Edge Game Payoffs

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We define simple random models for the spread of epidemics in contact network

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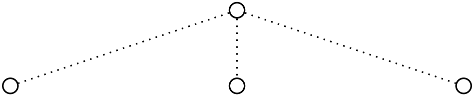

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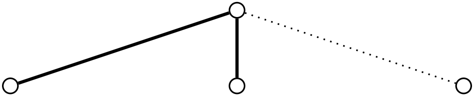

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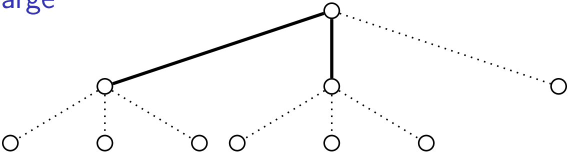

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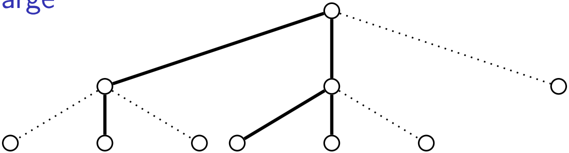

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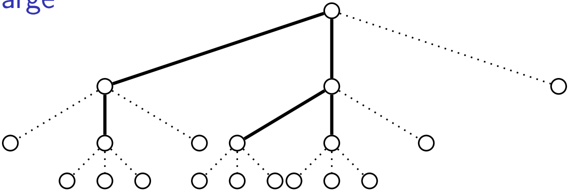

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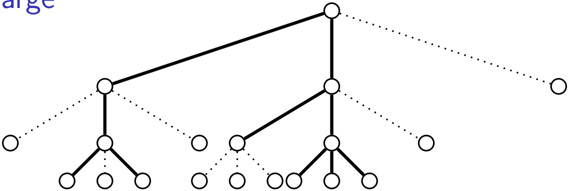

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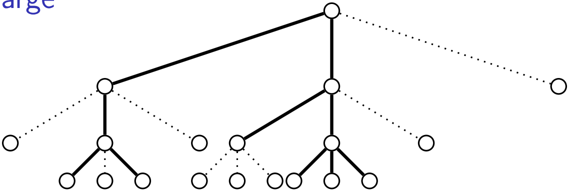

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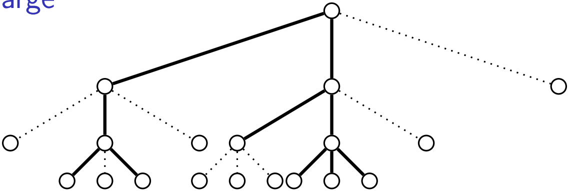

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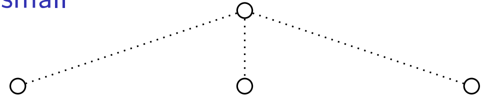

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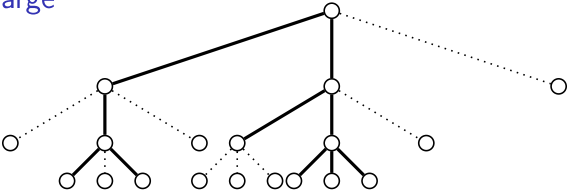

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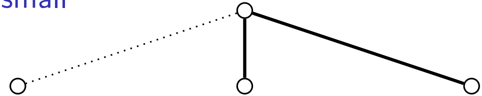

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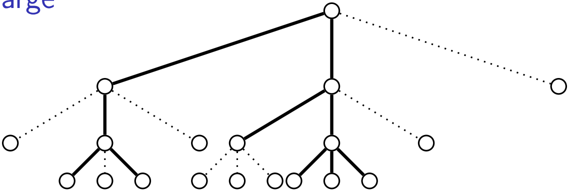

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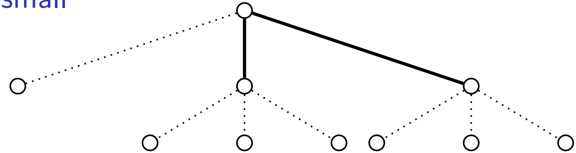

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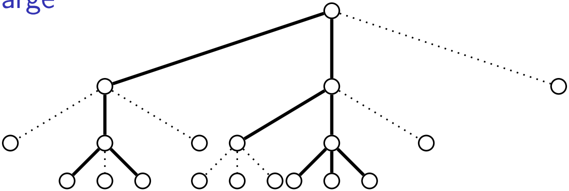

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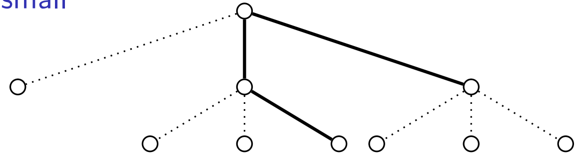

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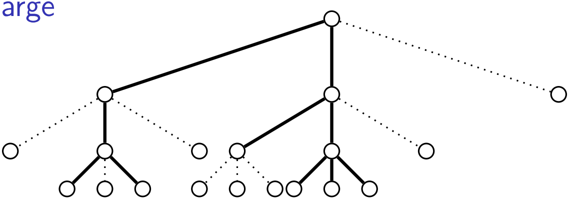

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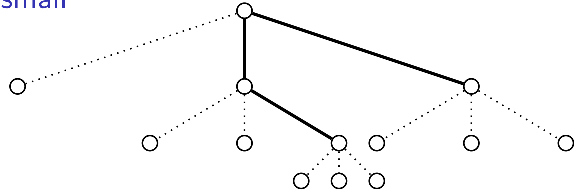

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The dichotomy for branching process

The dichotomy

- If $R < 1$, then with probability 1, the disease dies out after a finite number of steps
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- Better sanitary practices (reduces p)

A more complex model

Node states

- Susceptible (S): the node has not been infected and it can be infected
- Infected (I): the node is actually infected and can infect other nodes
- Removed (R): the node has been infected and now it neither can be infected nor infect

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The contact network

Direct graph modeling contacts

- It allows to represent both mono-directional and bidirectional epidemics

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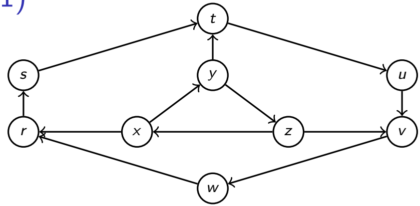

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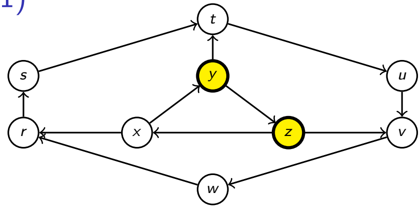

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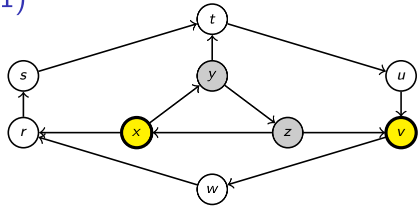

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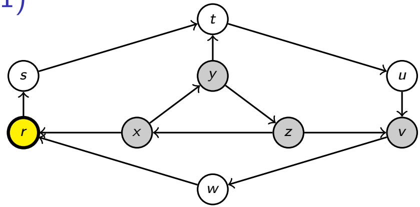

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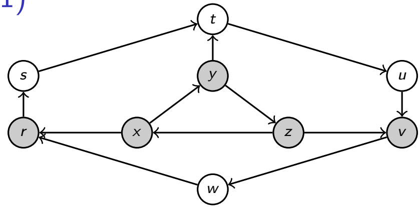

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  - Contagion more probable between certain pairs of nodes

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  - Early, middle and late periods of infection
- Time-mutating pathogen
  - Probabilities and infection periods change during the infection spread

The Basic Reproductive Number reloaded

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The expected number of new cases of the disease caused by a single individual

- Alternative definition: The expected number of new cases caused by a randomly chosen individual

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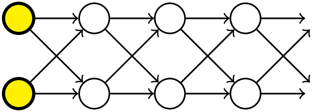

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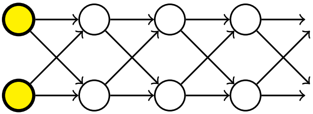

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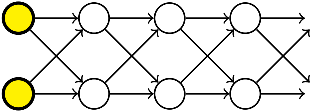

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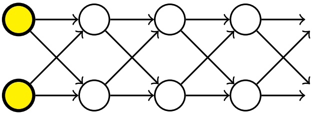

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Different network structure can be more or less conductive

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Adding the possibility of being reinfected

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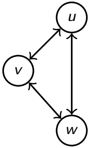

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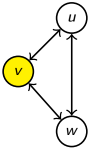

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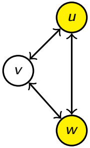

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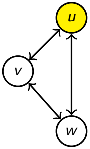

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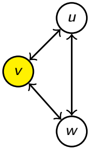

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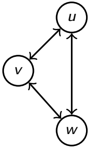

SIS vs. SIR

The Life Cycles

- SIR ends after a relatively small number of steps

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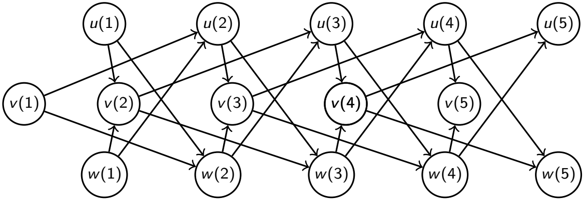

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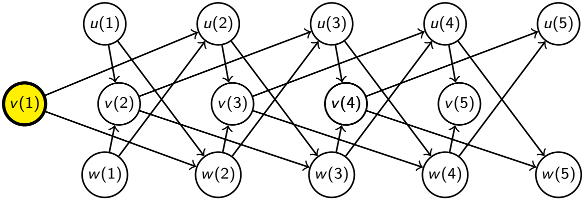

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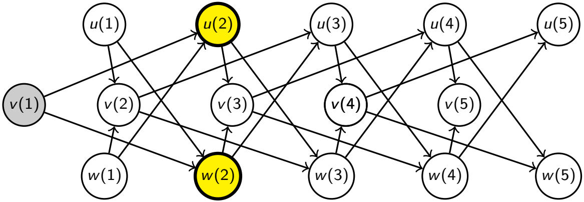

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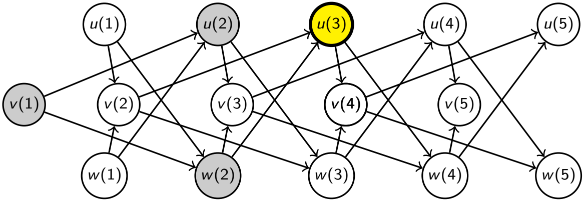

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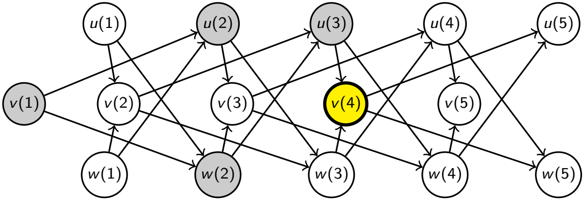

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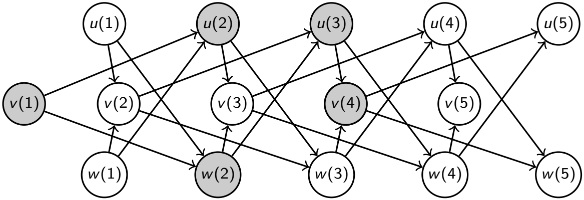

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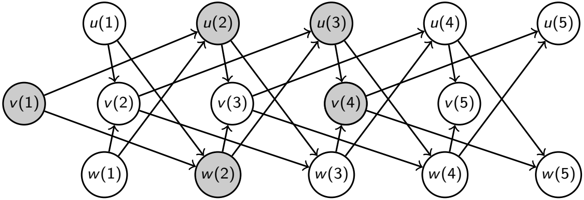

SIS model = SIR model on the time-expanded contact network

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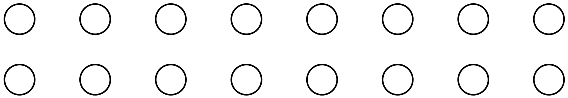

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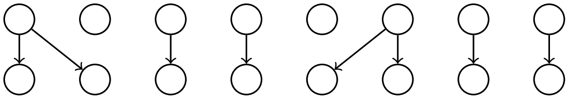

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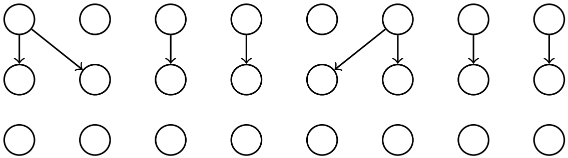

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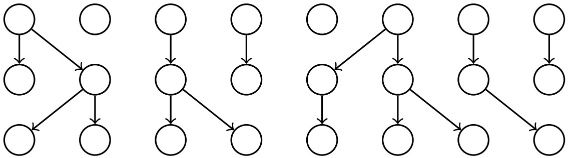

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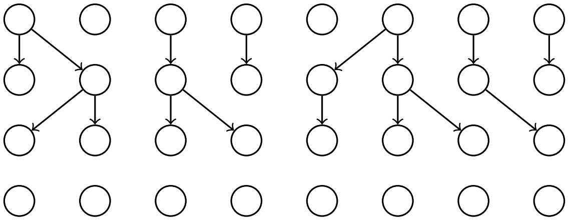

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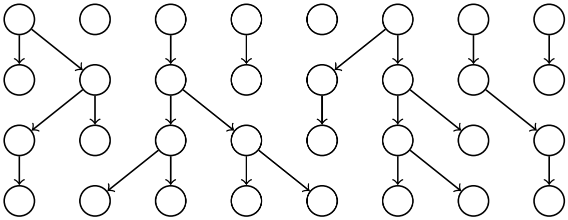

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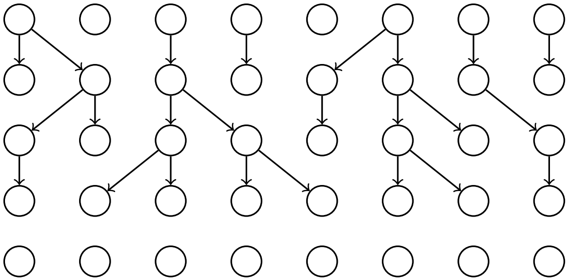
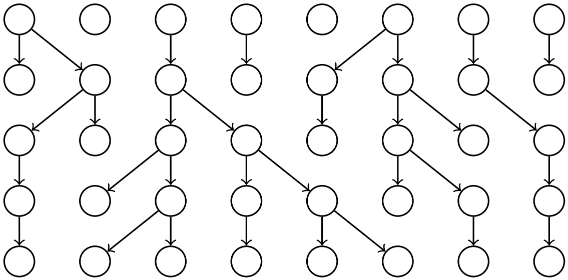


The mitochondrial Eve



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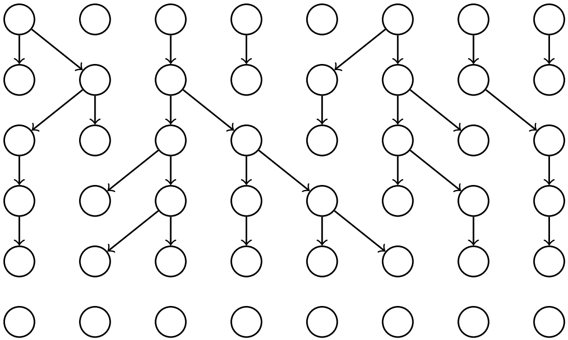

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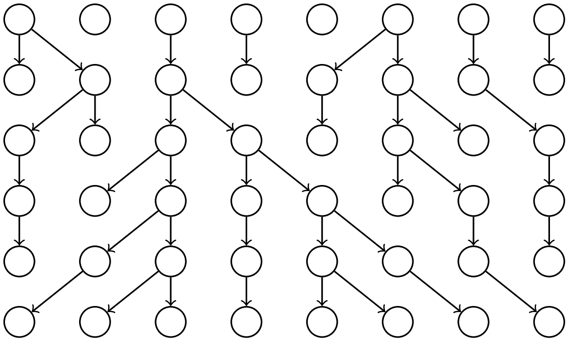

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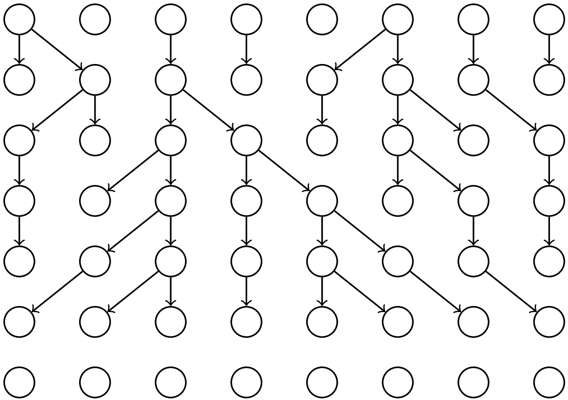

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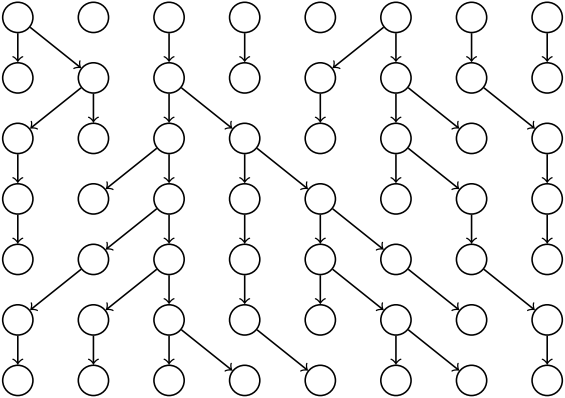

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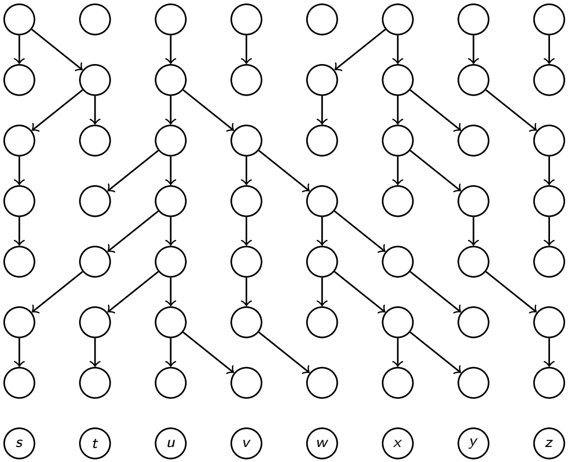

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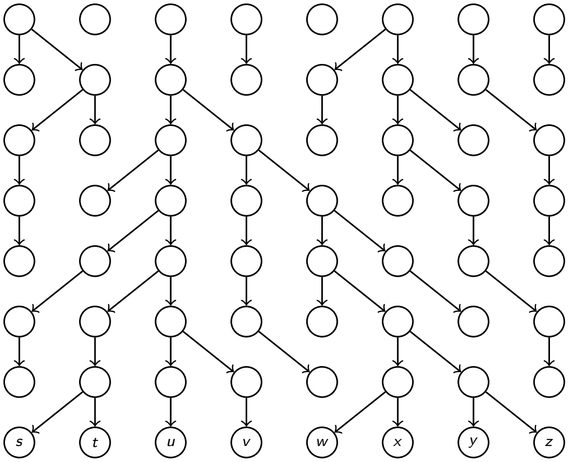

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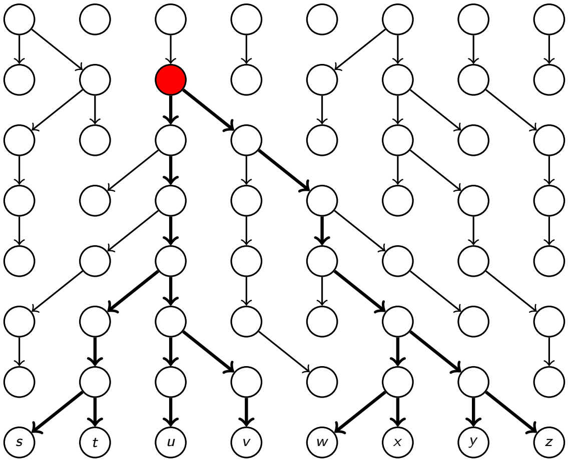
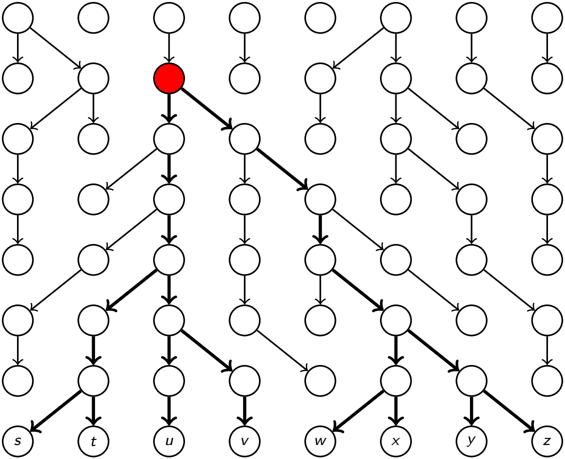


The mitochondrial Eve



Does a common ancestor always exist?

The mitochondrial Eve

Wright-Fisher model as a branching process with multiple roots

The mitochondrial Eve

Wright-Fisher model as a branching process with multiple roots

- **Branching process**: from the root and proceed to children
- **Wright-Fisher model**: from children and proceed to root

The mitochondrial Eve

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The coalesce process

- Start with a population of size N
- At each time step, every member chooses a parent randomly
- If two members choose the same parent, their lineage coalesce

The mitochondrial Eve

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- j parents are different with prob. $(1 - \frac{1}{N}) \cdots (1 - \frac{j-1}{N})$

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There is a mitochondrial Eve that lived 100,000 - 200,000 years ago