Auction and Matching Markets

The setting

- One item must be sold
- n buyers willing to buy this item
- each bidder i has a valuation v_i for the item
  - how much she is willing to pay for that item

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We can run an auction for selling the object

Auctions

- Ascending (English) Auctions
  - Sotheby's Fine Art Auctions, E-Bay, ...

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  - public contracts

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- Sealed Bid First-Price Auctions
  - public contracts
- Sealed Bid Second-Price Auctions

Descending Auctions vs. First-Price Auctions

Descending Auctions

First-Price Auctions

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- No people learn anything about other players' valuation before the winner is announced

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Descending Auctions are equivalent to First-Price Auctions

Ascending Auctions vs. Second-Price Auctions

Ascending Auctions

Second-Price Auctions

Ascending Auctions vs. Second-Price Auctions

Ascending Auctions

- It does not make sense to not drop out when the price is larger than your own valuation

Second-Price Auctions

Ascending Auctions vs. Second-Price Auctions

Ascending Auctions

- It does not make sense to drop out when the price is smaller than your own valuation

Second-Price Auctions

Ascending Auctions vs. Second-Price Auctions

Ascending Auctions

- The price b_i at which agent i drops out depends only on your own valuation

Second-Price Auctions

- The bid b_i that agent i submits depends only on your own valuation

Ascending Auctions vs. Second-Price Auctions

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Ascending Auctions vs. Second-Price Auctions

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- The price b_i at which agent i drops out depends only on your own valuation
- The winner is the agent with the highest b_i
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Ascending Auctions are equivalent to Second-Price Auctions

Second-Price Auctions

Players: n buyers

Strategies: possible bids b_i

Utility: $u_i = v_i - \max_{j \neq i} b_j$ if i is the winner, otherwise $u_i = 0$

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Why truthful bidding is a dominant strategy?

Intuition: Because your utility does not depend on your bid

First-Price Auctions

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Strategies: possible bids b_i

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Truthfulness?

In a sealed-bid first price auction, bidding truthfully is not a dominant strategy

- Bidders prefer to shade their bids slightly downward
  - Not too much, otherwise we can lose
  - Not too few, otherwise shading is almost ineffective

From one item to multiple items

An easy model

- n items to be sold and n buyers
- each buyer keeps a list of items at which she is interested
- a buyer is happy only if she receives an item from his list

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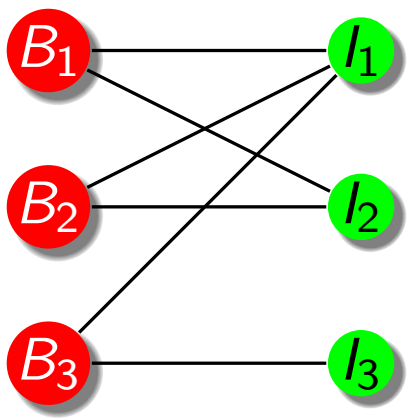

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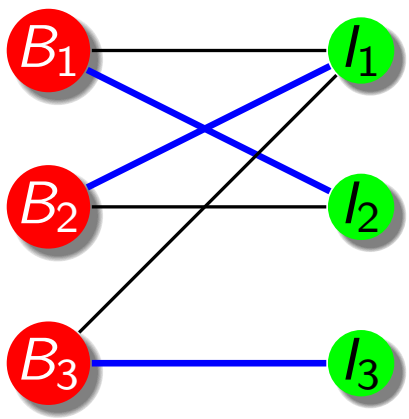

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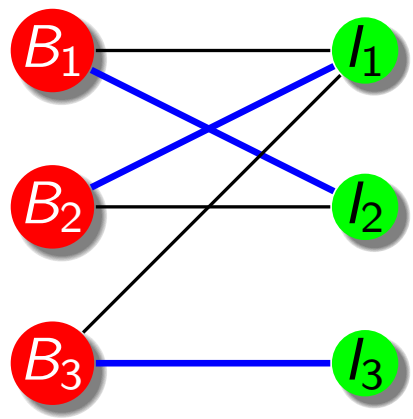
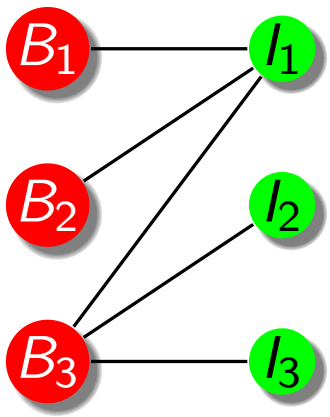


We need to find a perfect matching

Constricted Sets and Matching Theorem

How can we recognize that there is no perfect matching?



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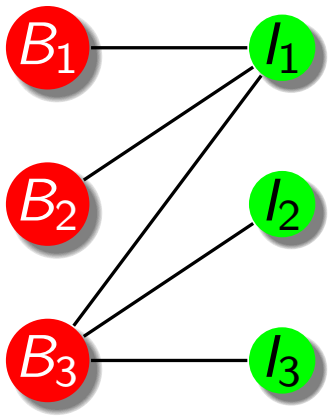

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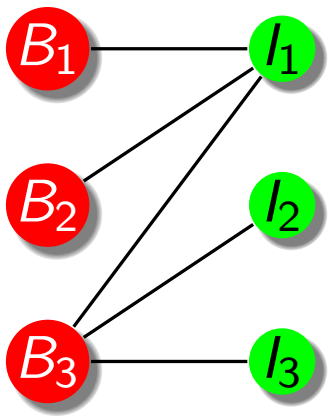

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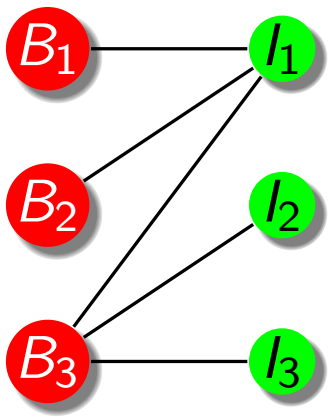

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Matching Theorem

If a bipartite graph (with the same number of nodes on both sides) has no perfect matching, then it must contain a constricted set

- A perfect matching or its obstacle can be efficiently found
  - For details see Advanced Material at the end of the slides

From one item to multiple items - A more complex setting

Matching markets

- n item and n buyers (add dummy items or buyers if necessary)

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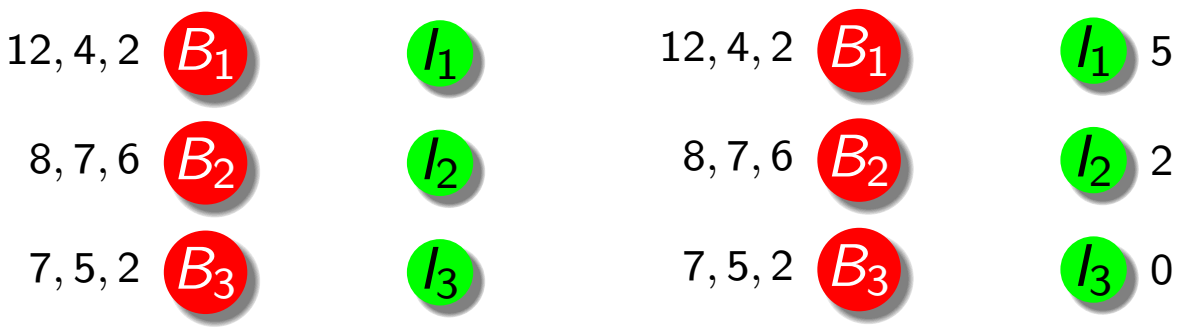

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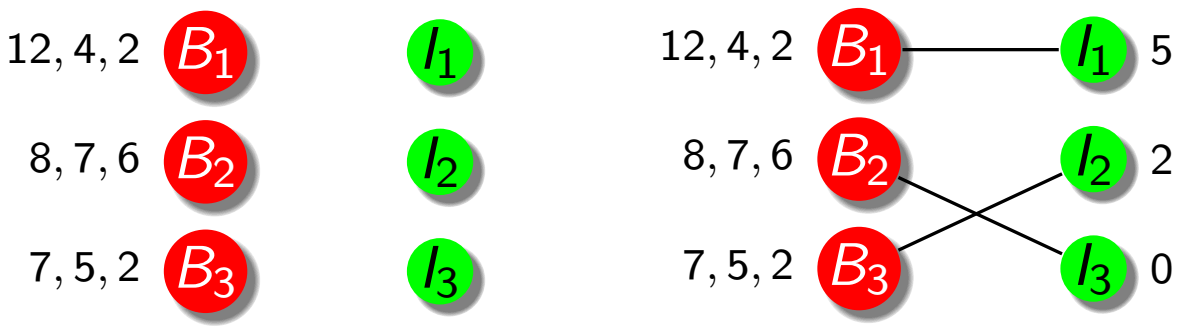

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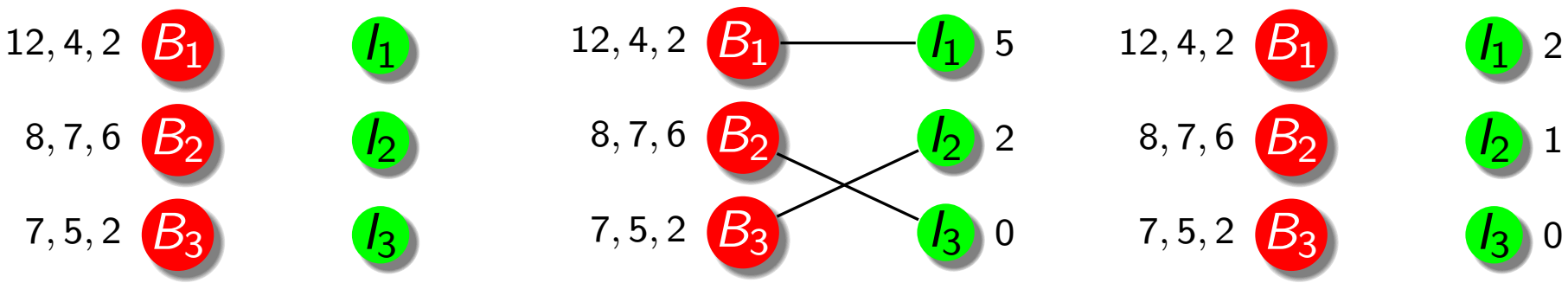

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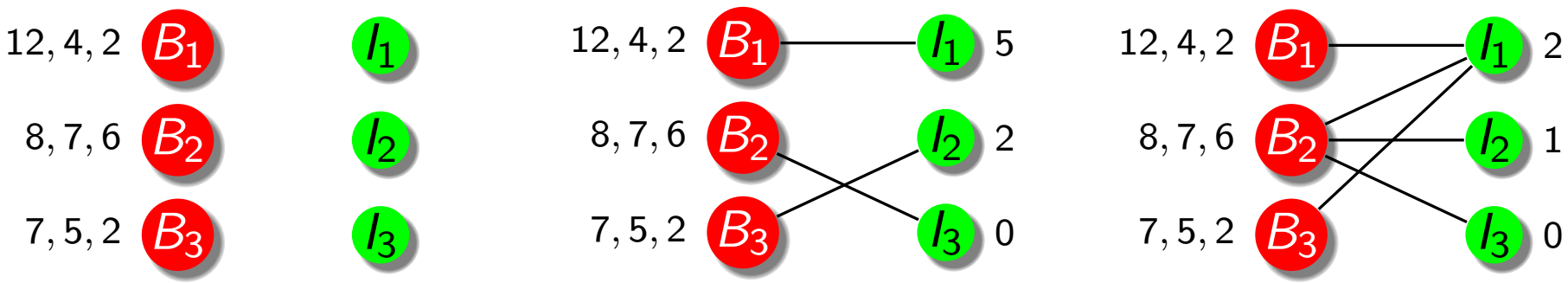

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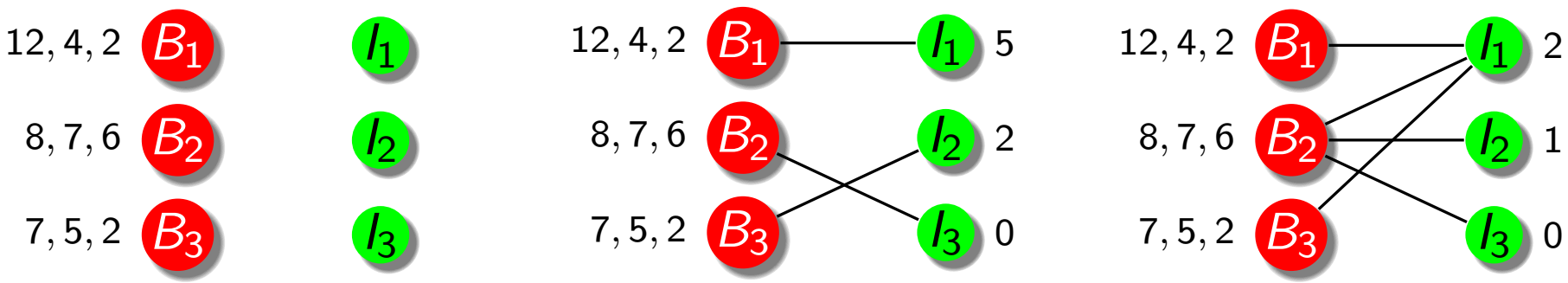

Goal: Fix price so to sell all items by maximizing the social welfare

Market Clearing Prices

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A set of prices is market-clearing if the resulting preferred-item graph has a perfect matching

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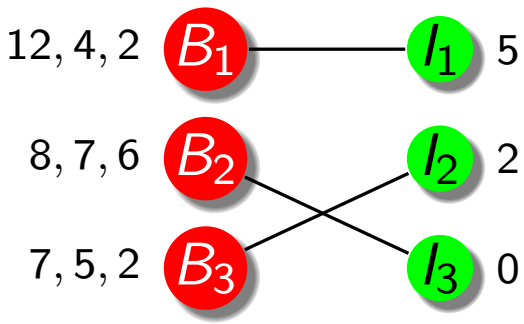

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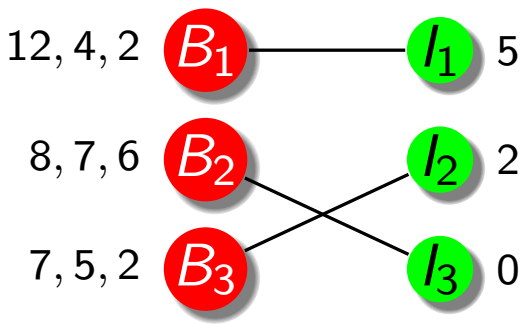

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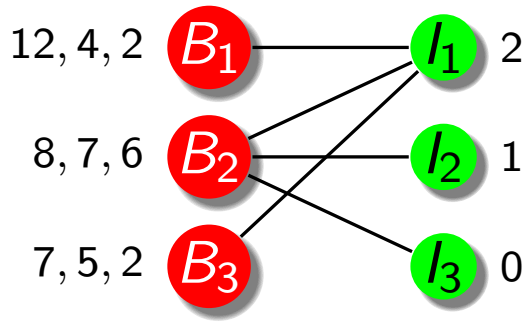

Non Market Clearing

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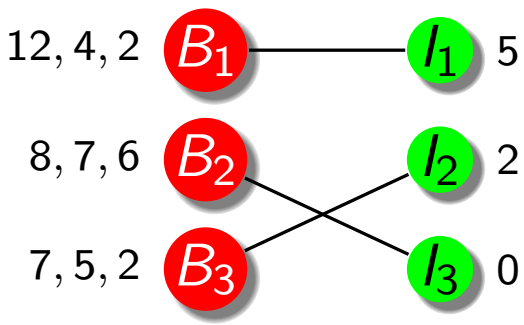

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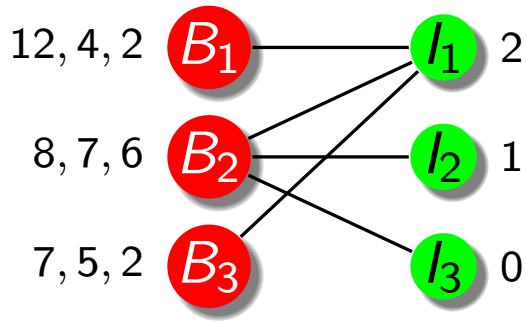

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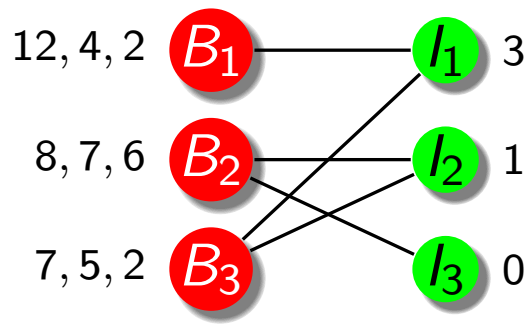

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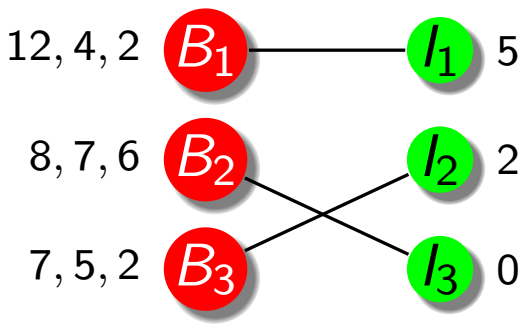

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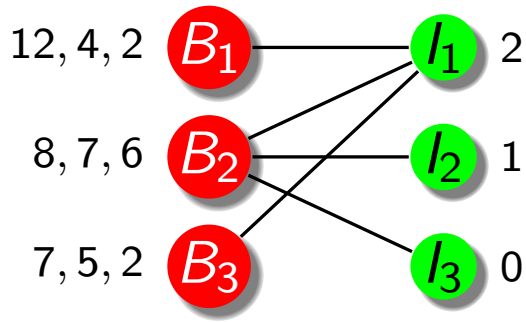

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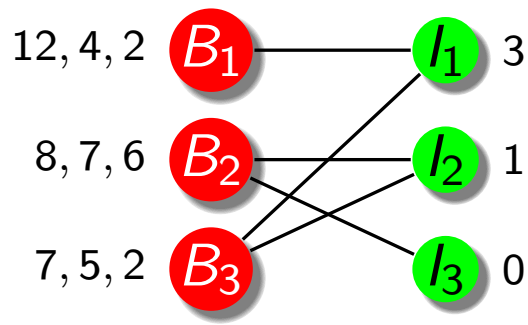

Market Clearing

Do market-clearing prices maximize social welfare?

- Buyers buy their best item at current price

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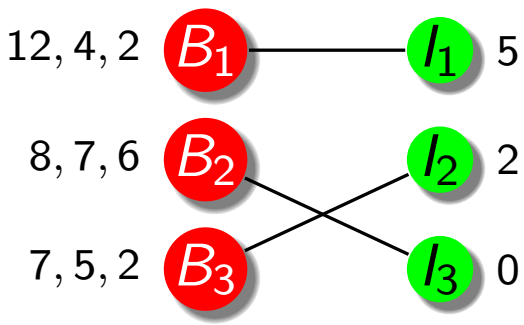

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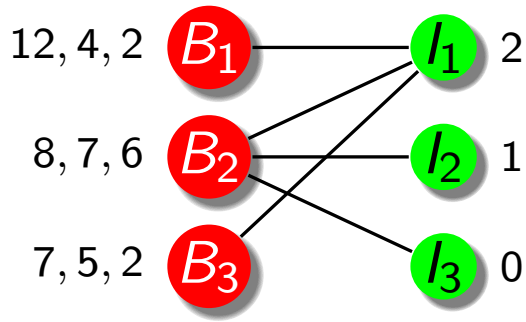
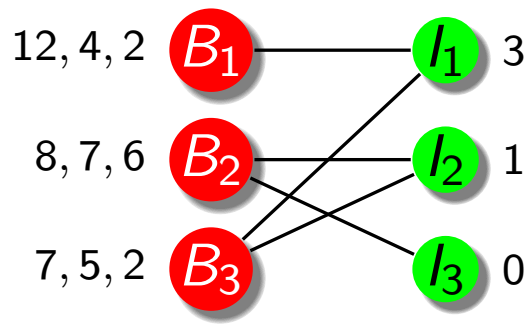


Non Market Clearing



Market Clearing

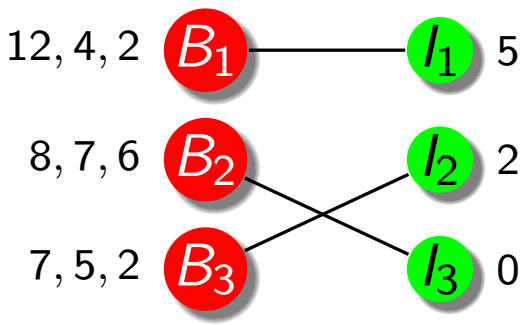
Do market-clearing prices maximize social welfare?

- Buyers buy their best item at current price
  - Market clearing prices maximize sum of utilities

Market Clearing Prices

Market Clearing Prices

A set of prices is market-clearing if the resulting preferred-item graph has a perfect matching



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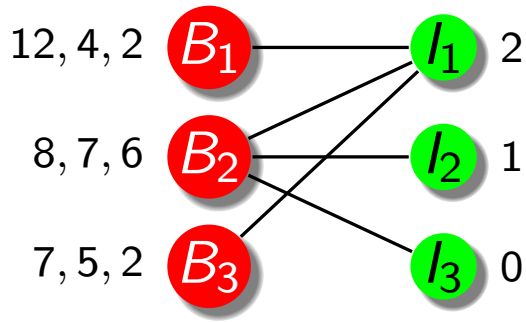

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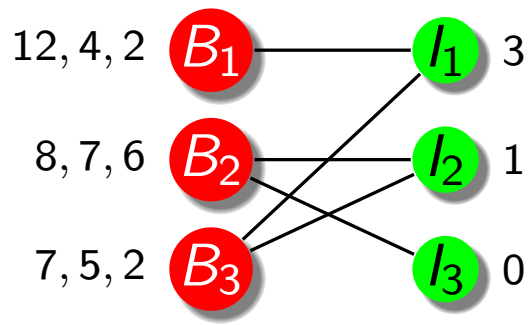

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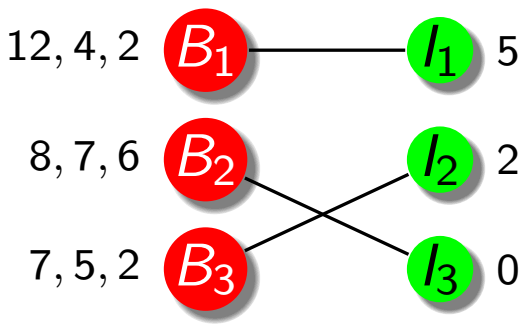

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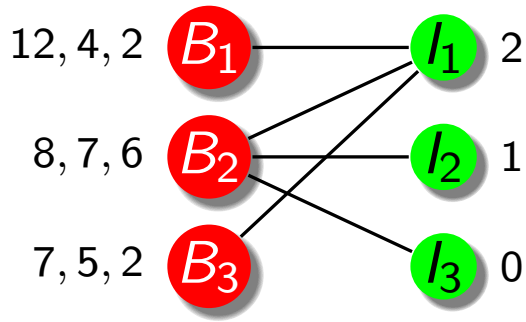

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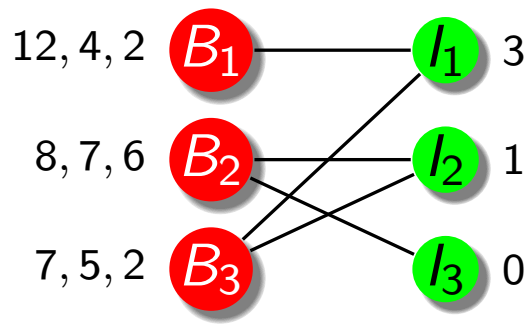

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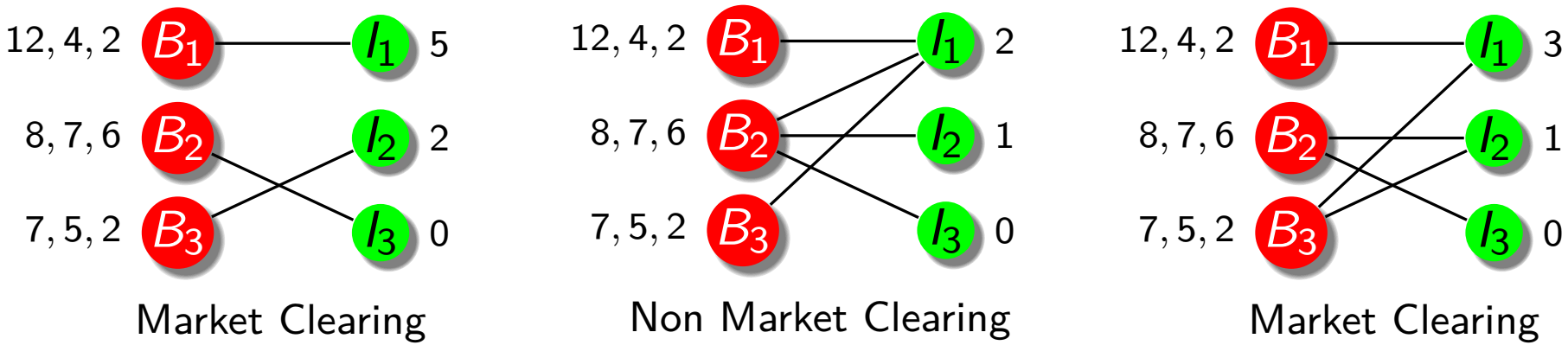

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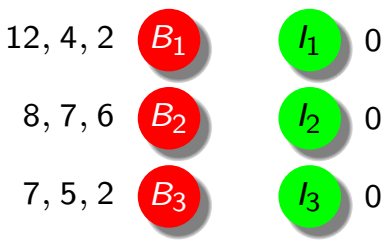

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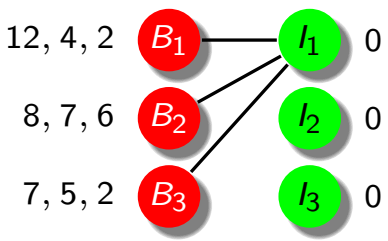

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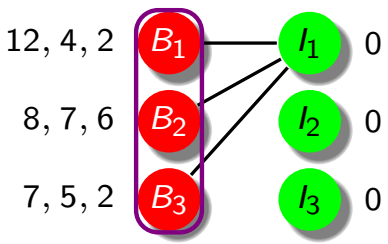

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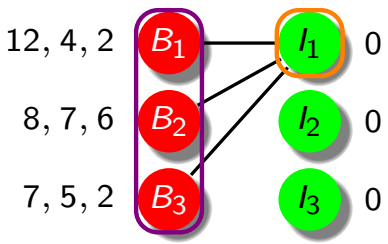

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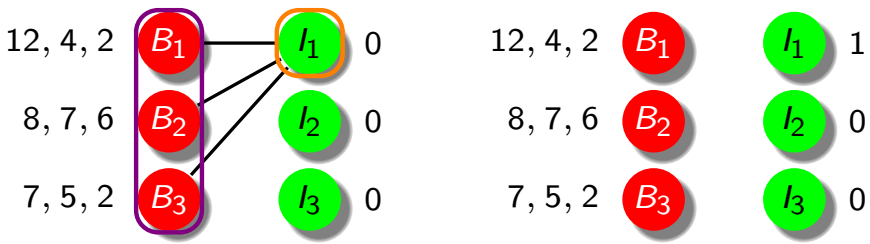

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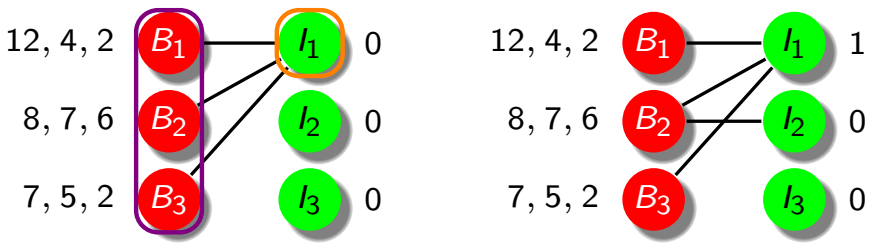

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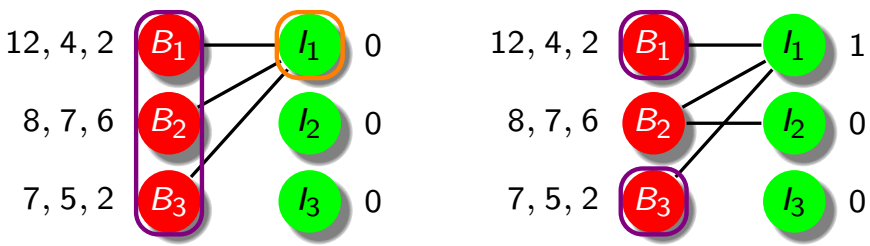

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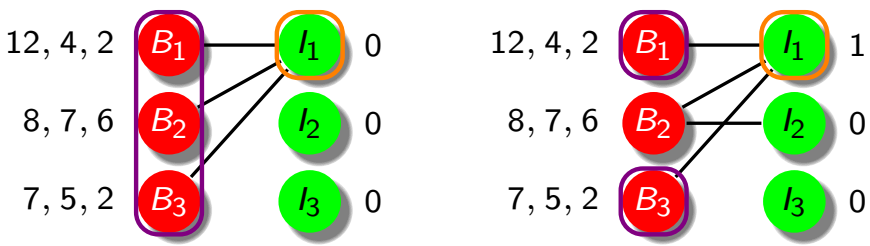

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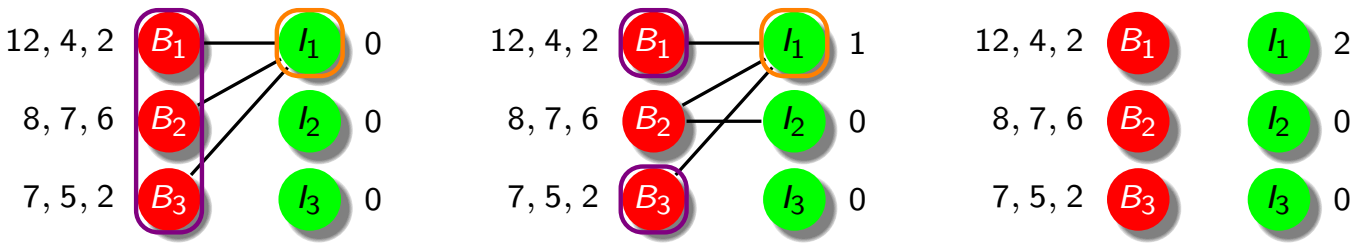

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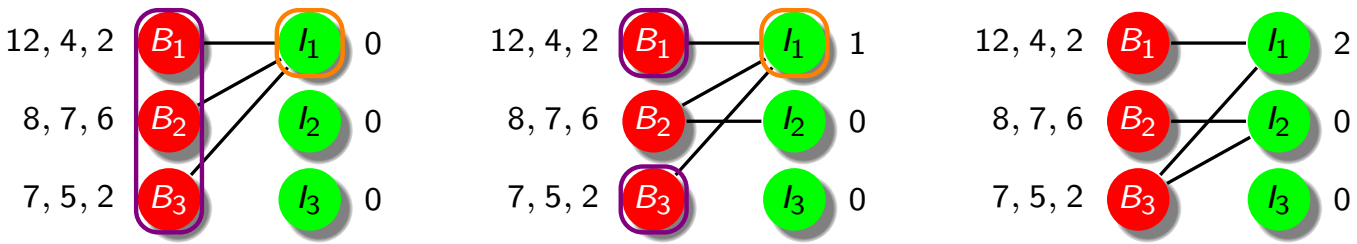

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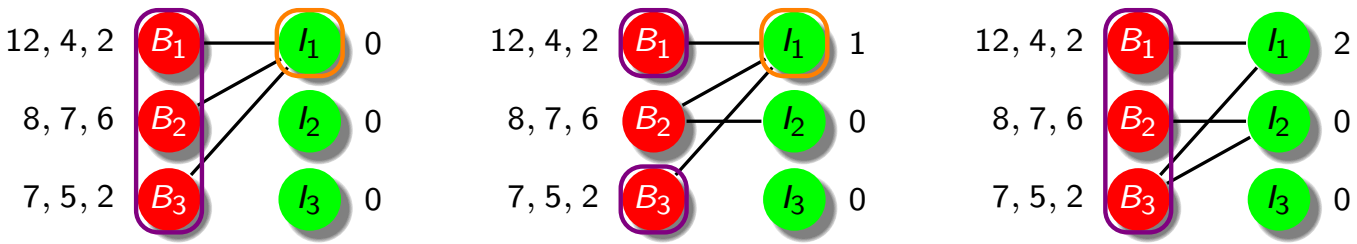

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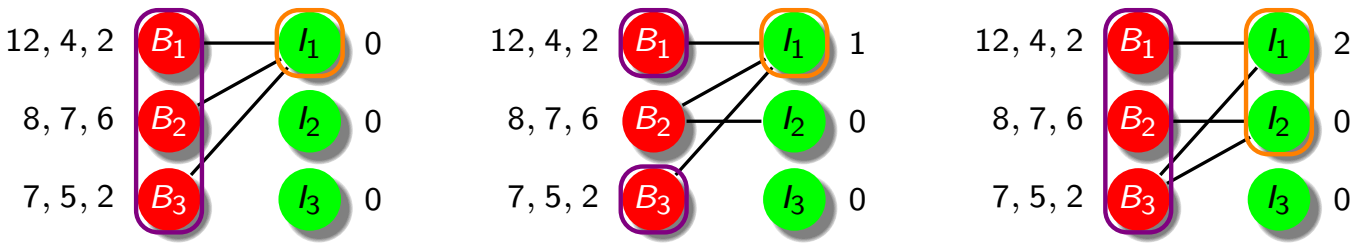

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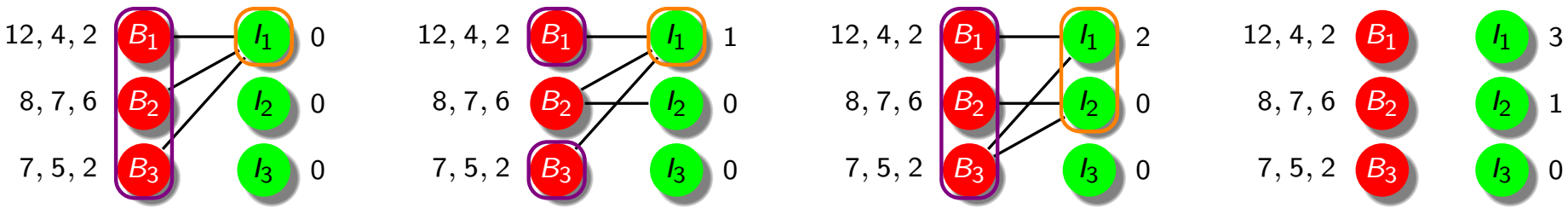

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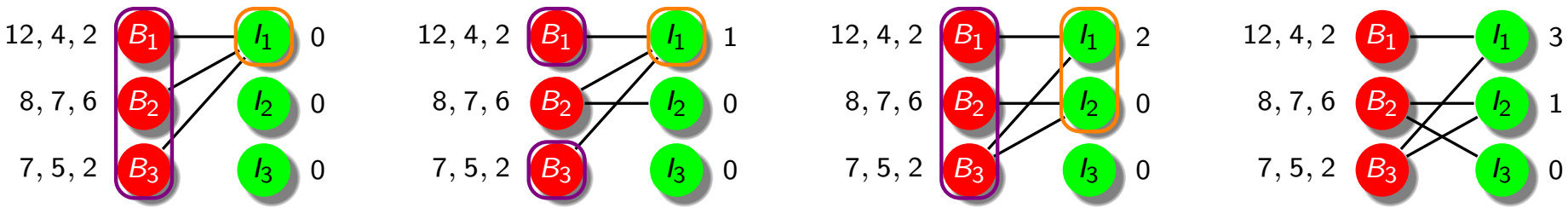

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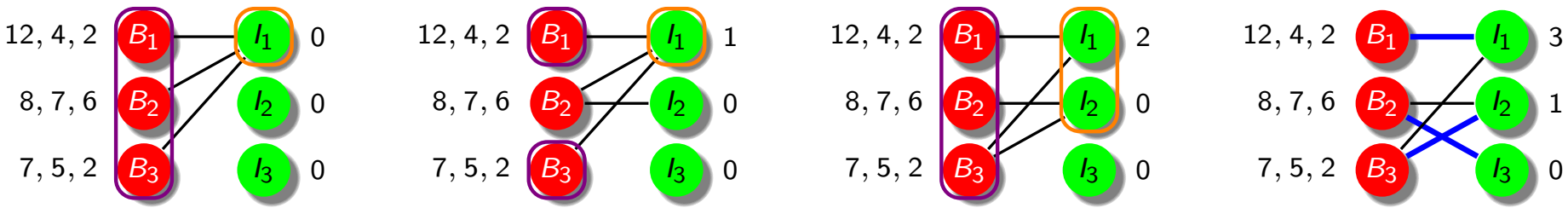

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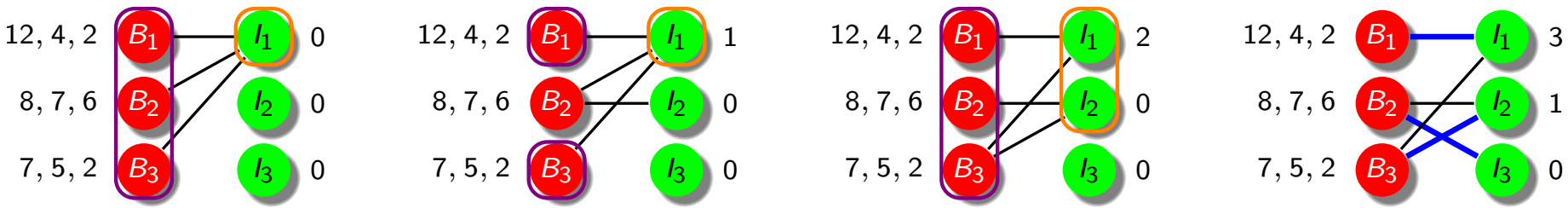

Does the algorithm end?

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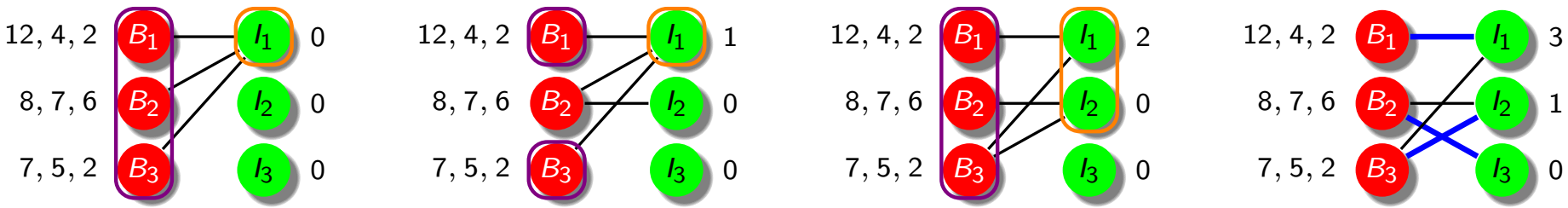

Does the algorithm end?

Yes. More details in Advanced Material

Matching Markets and Auctions

Single item

Multiple items

Matching Markets and Auctions

Single item

- Price = Second highest bid

Multiple items

- Market Clearing Prices

Matching Markets and Auctions

Single item

- Price = Second highest bid
- Price arising from ascending auctions

Multiple items

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Are them really different?

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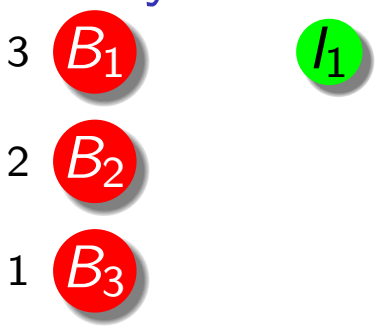

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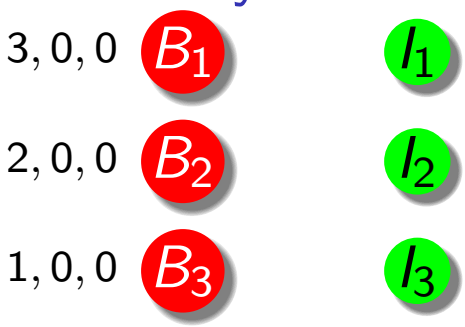

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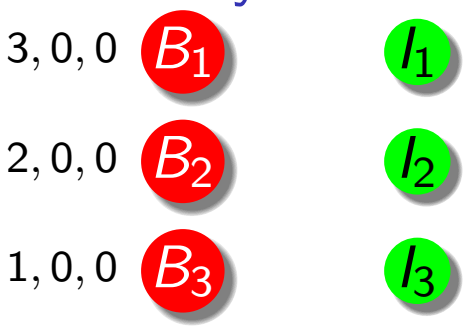

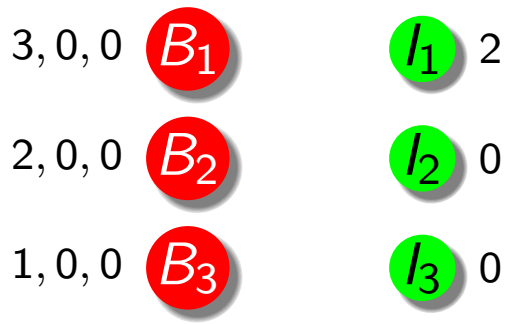

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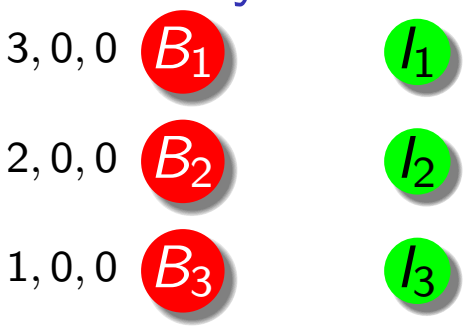

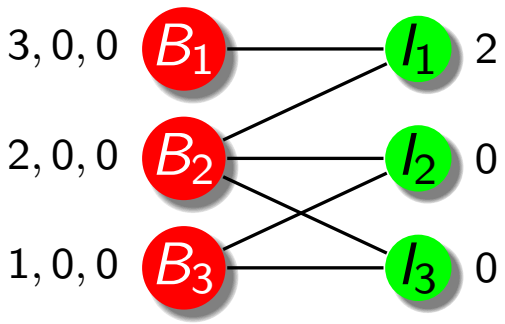

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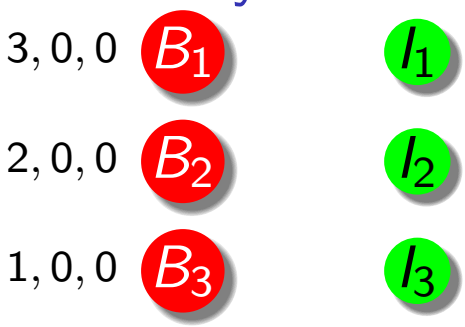

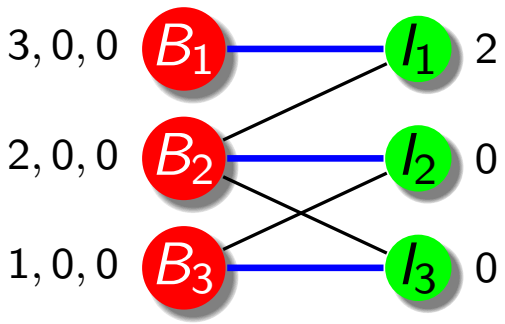

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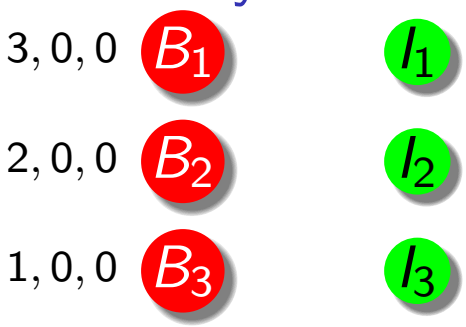

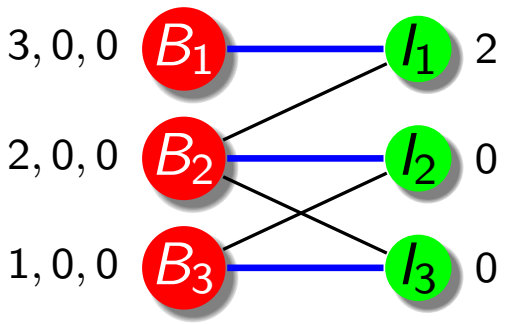

- Second Price Auction produces Market Clearing Prices

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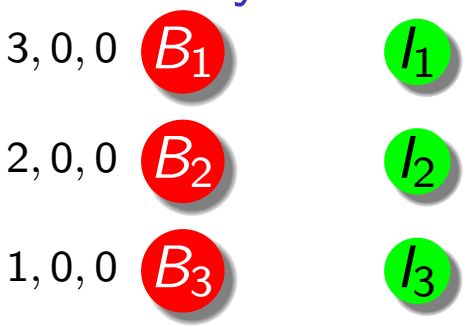
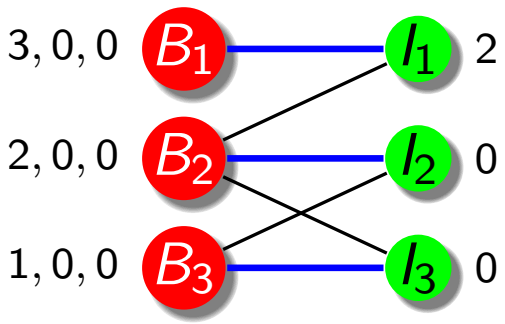
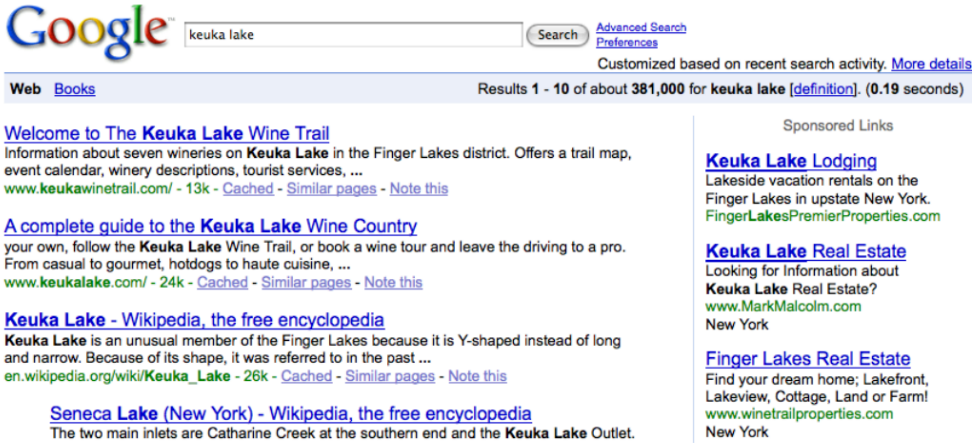




- Second Price Auction produces Market Clearing Prices
- The algorithm for computing Market Clearing Prices is a generalization of an Ascending Price Auctions

Keyword Advertising



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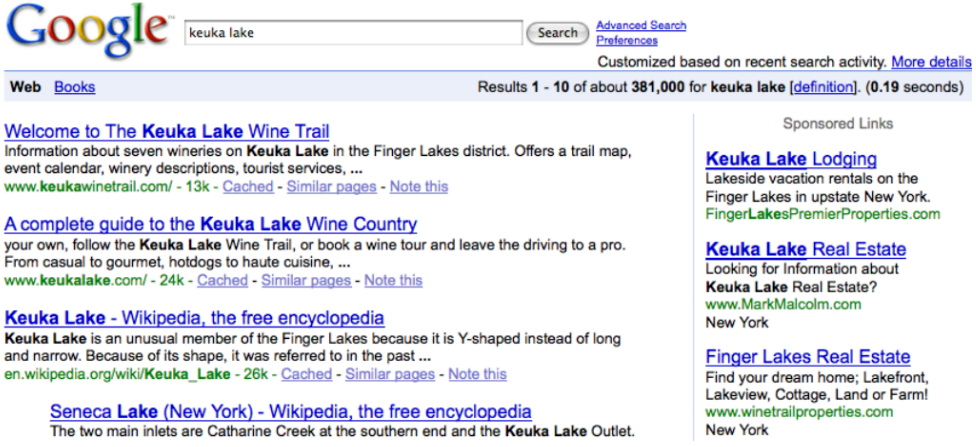

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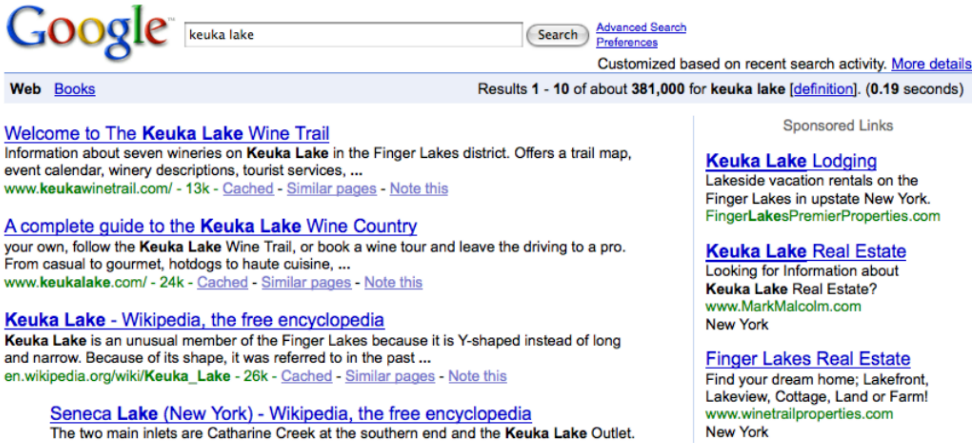

Pay-per-Impression

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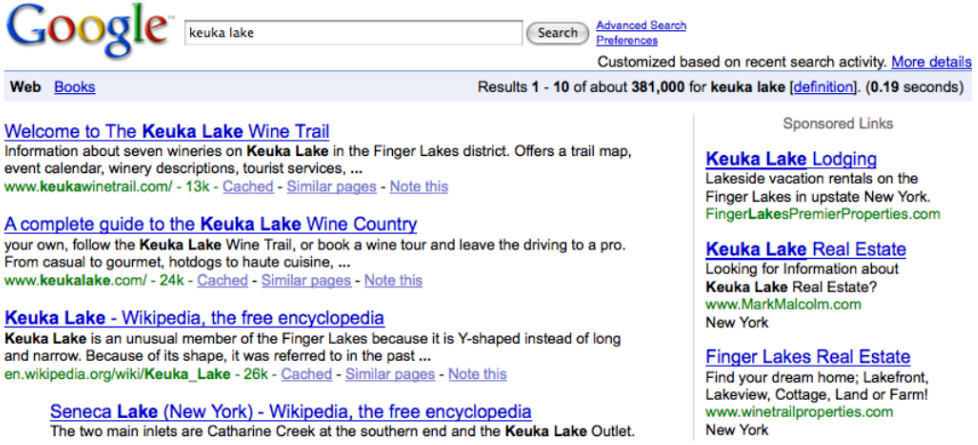
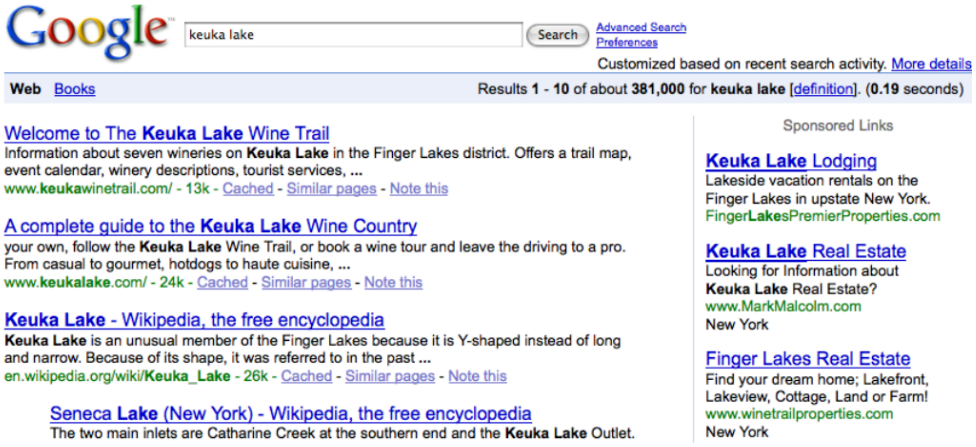


Pay-per-Impression

- Ads not relevant to the page

Keyword Advertising



Pay-per-Impression

- Ads not relevant to the page
- Pay even if ad is not useful

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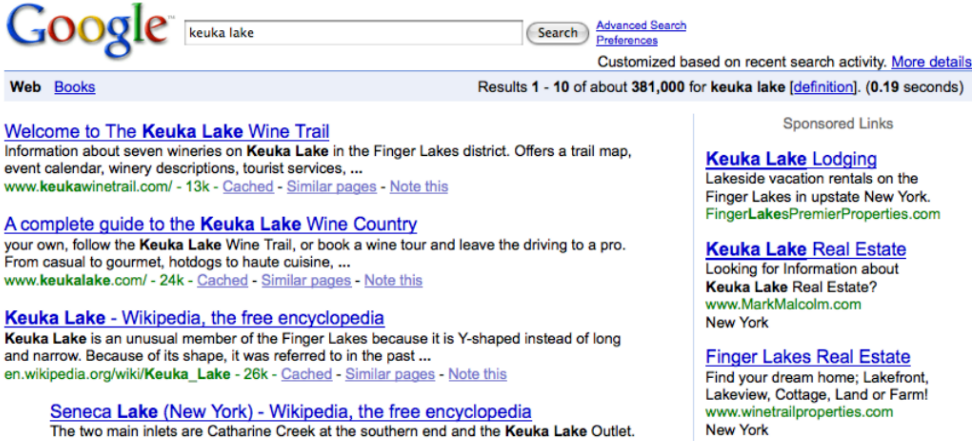

The Web Search approach

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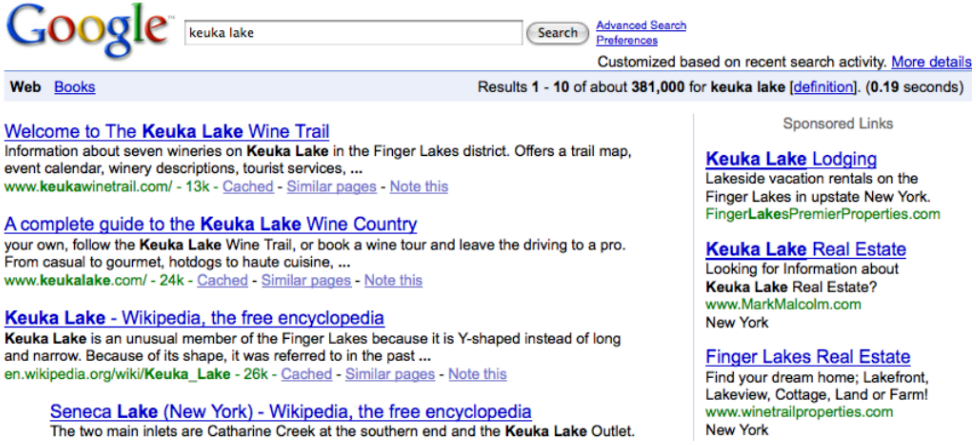

The Web Search approach

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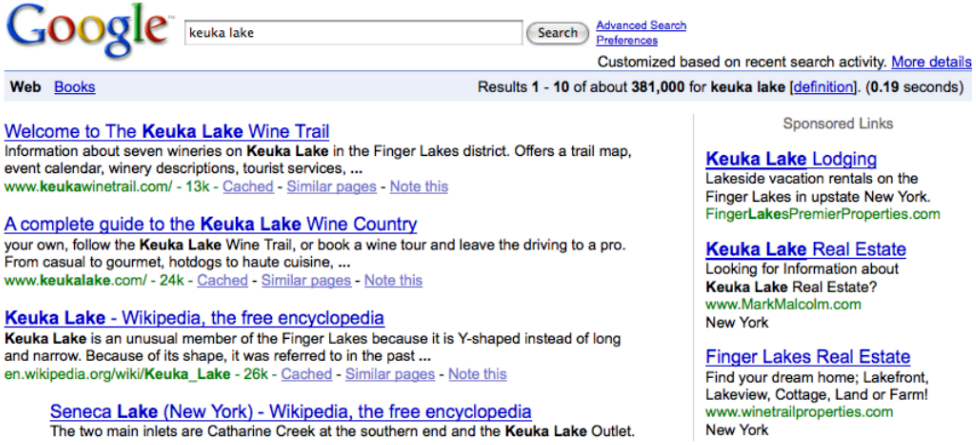

The Web Search approach

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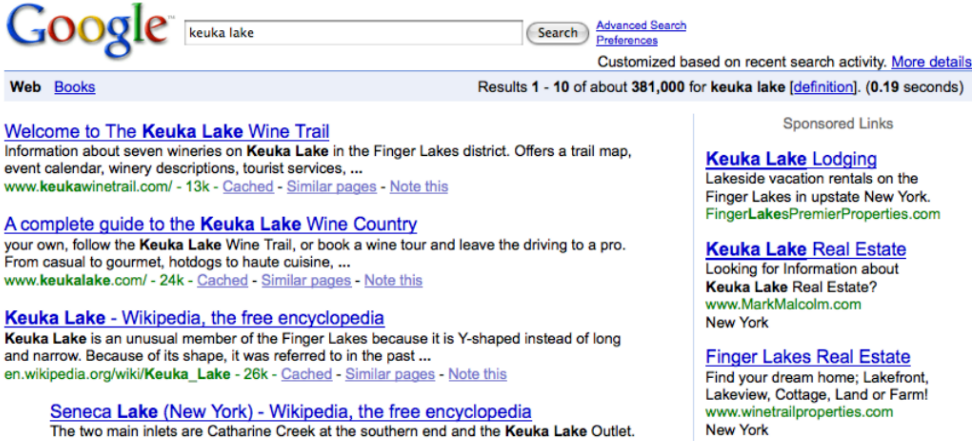

The Web Search approach

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Does it work?

Pay-per-Impression

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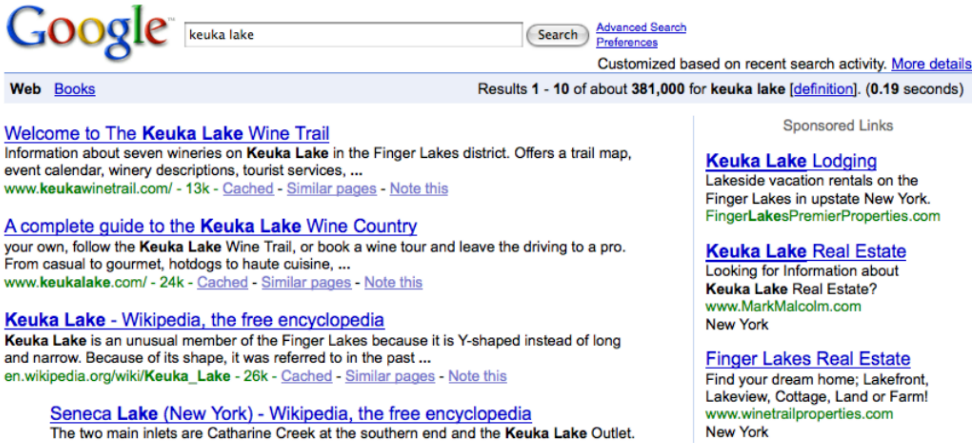

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Does it work?

- The top spot for *calligraphy pens* costs \$1.50
- The top spot for *calligaphy pens* costs \$0.60

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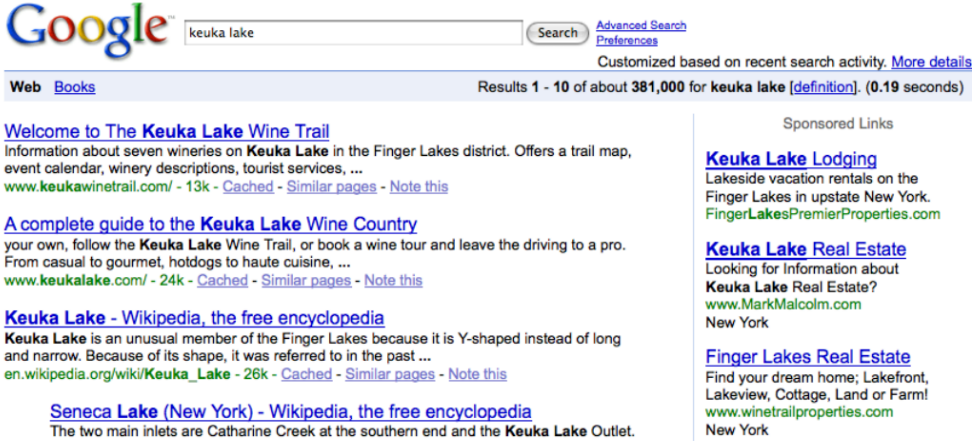

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- The top spot for *calligraphy pens* costs \$1.50
- The top spot for *calligaphy pens* costs \$0.60
- The top spot for *loan consolidation* costs \$50

How the prices are chosen?

Preliminary definitions

- Clickthrough rates r_j of spot j

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Problem: we do not know who is the best advertiser

How can we solve this problem?

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Truthfulness

- Each advertiser i submit a bid b_i
- It is a dominant strategy for an advertiser i to bid truthfully, i.e. $b_i = v_i$

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Can we run second price auctions?

- They have been defined for single item auctions
- Now we have multiple slots to sell

Second Price Auctions revisited

- The item is allocated to the highest bidder

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VCG Auctions

Vickrey-Clarke-Groves Principle

- Agents submit bids
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VCG Prices

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- $SW(A^{-i}_{-j})$ = maximum social welfare without j and its slot i
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VCG Auctions are truthful See Advanced Material for details

VGC Auctions - Example

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- 3 advertisers with Revenue per Click $3, 2$ and 1
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VGC Auctions: Market Clearing Prices

VGC prices

Personalized Prices

VGC Auctions: Market Clearing Prices

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Let us model sponsored search as Matching Markets

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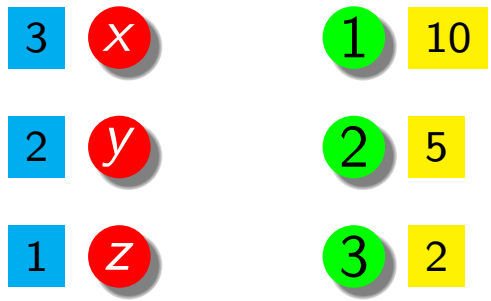

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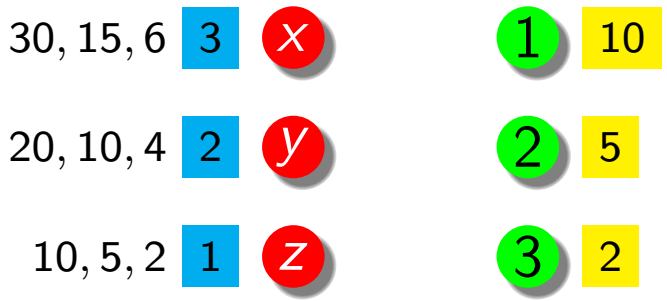

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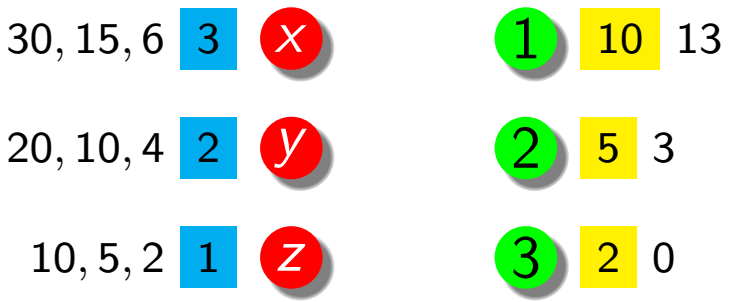

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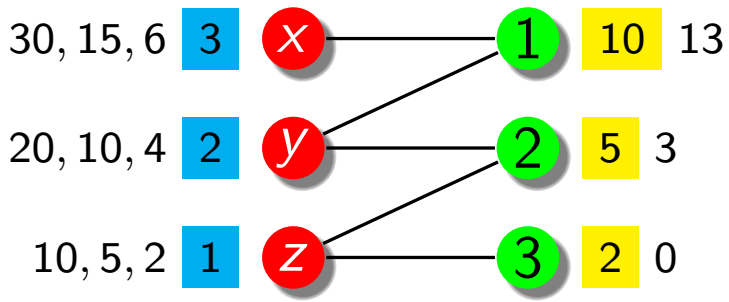

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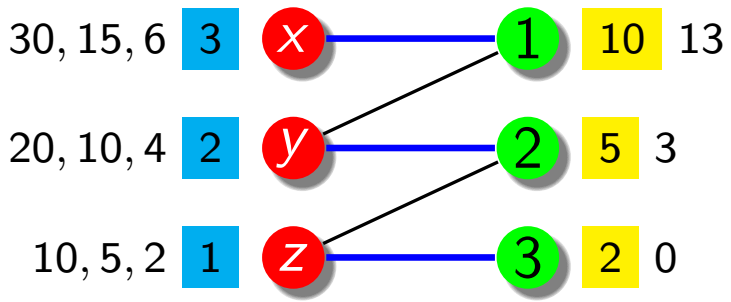

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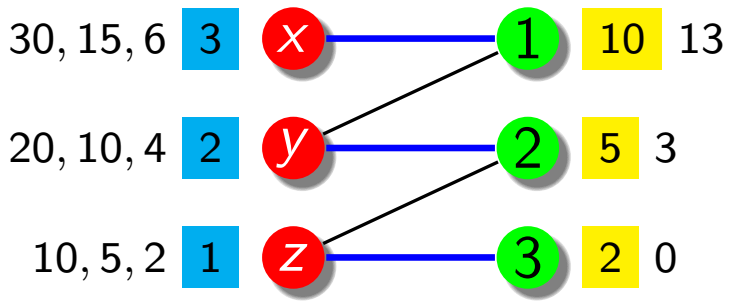

VCG prices vs. Market Clearing Prices

VCG prices are market clearing prices of minimum total sum

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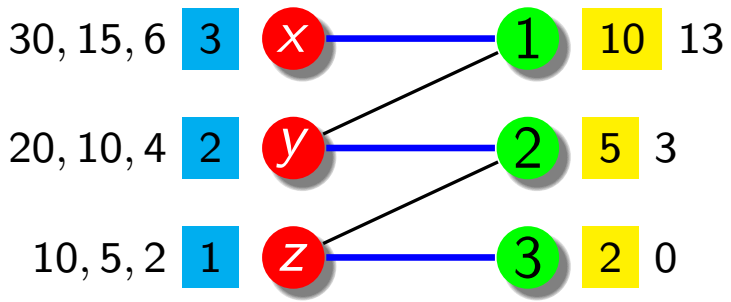

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Observations

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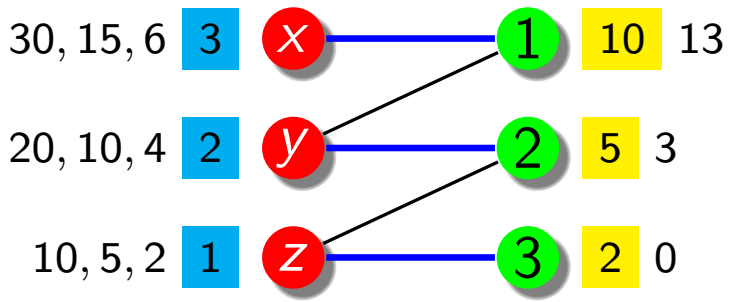

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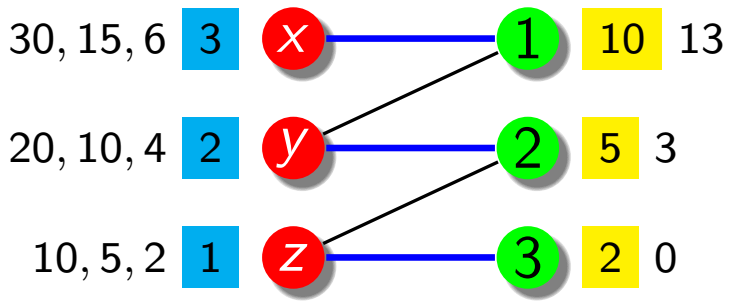

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Advanced Material

Matching Theorem: Sketch of the proof

How to find a perfect matching?

Use augmenting paths until you have a perfect matching

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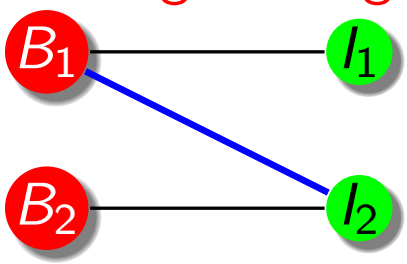

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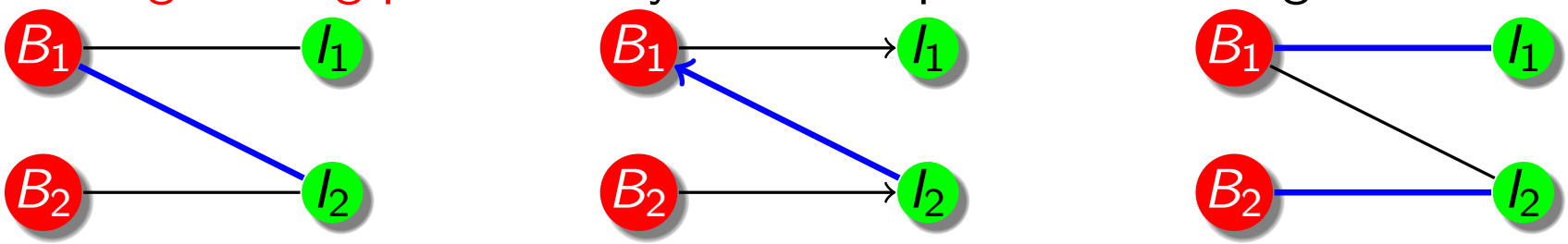

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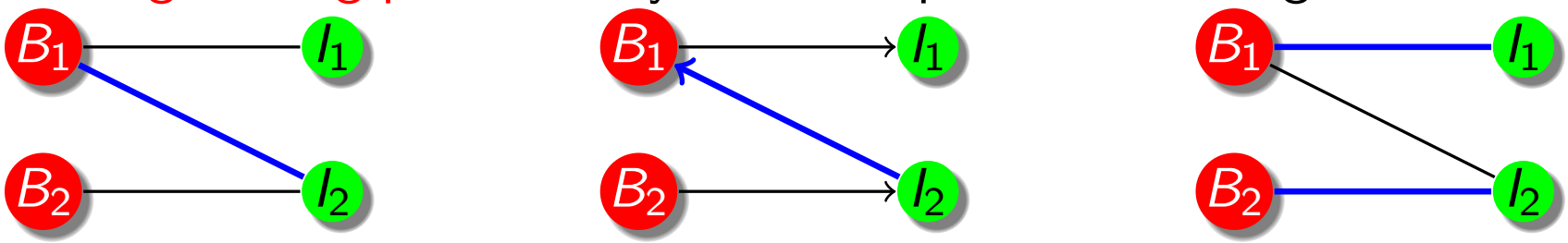

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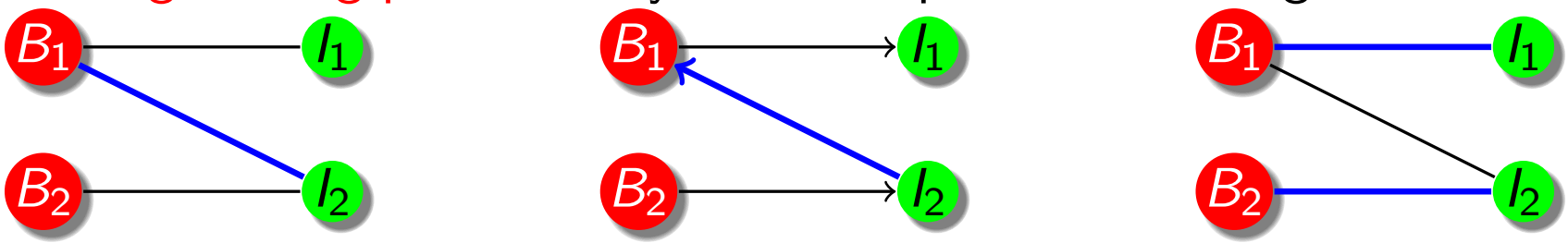

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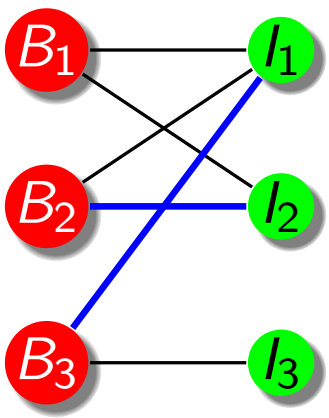

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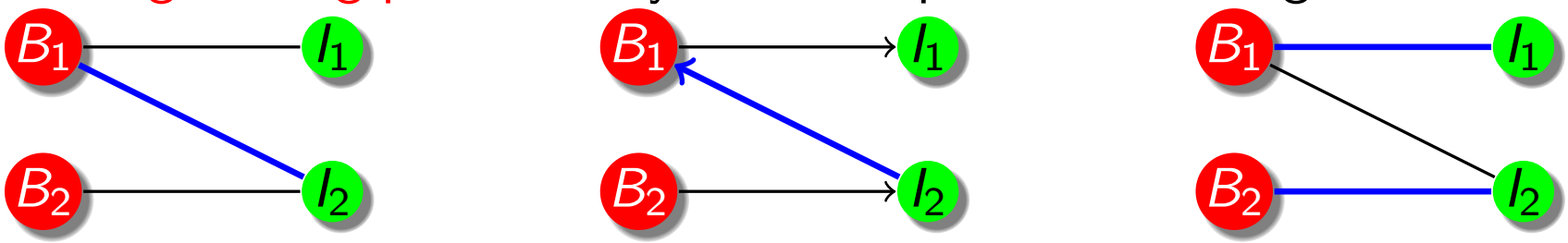

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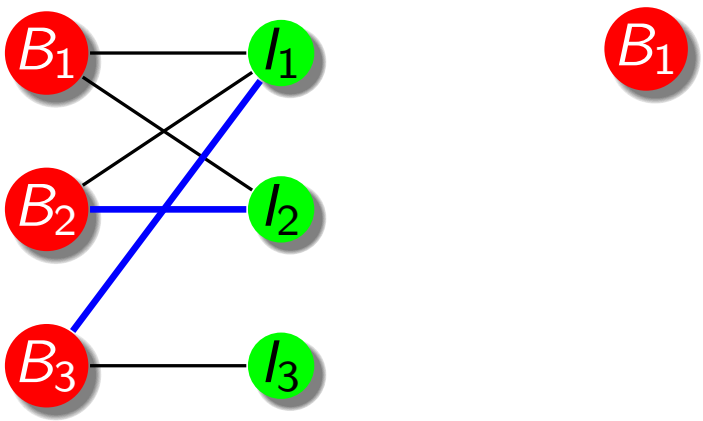

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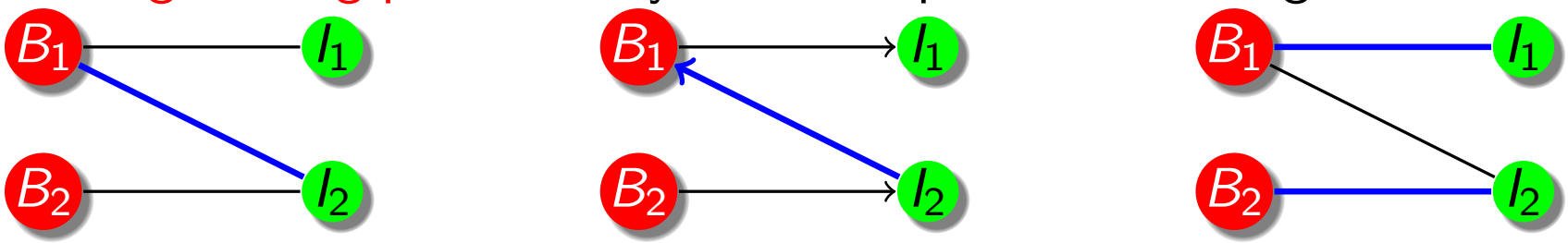

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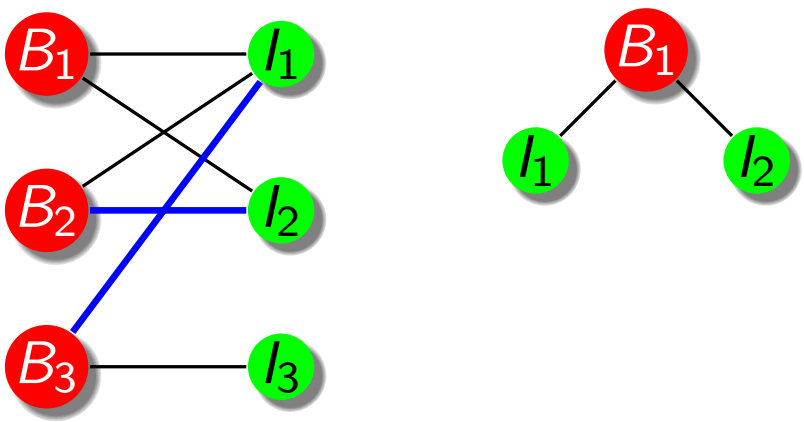

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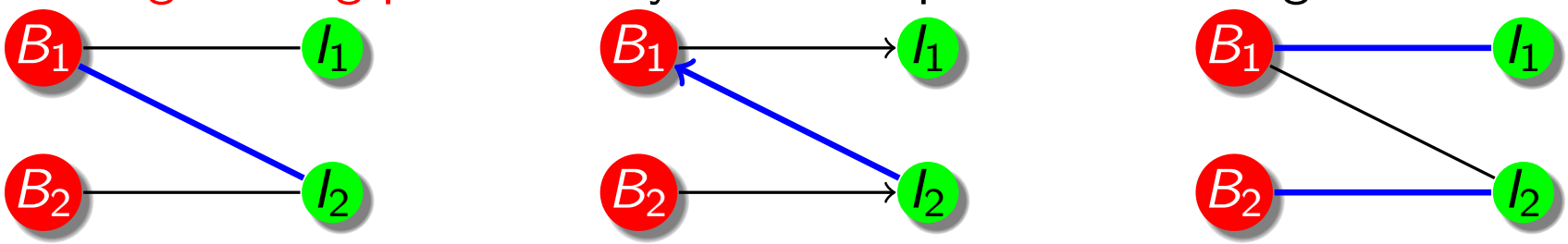

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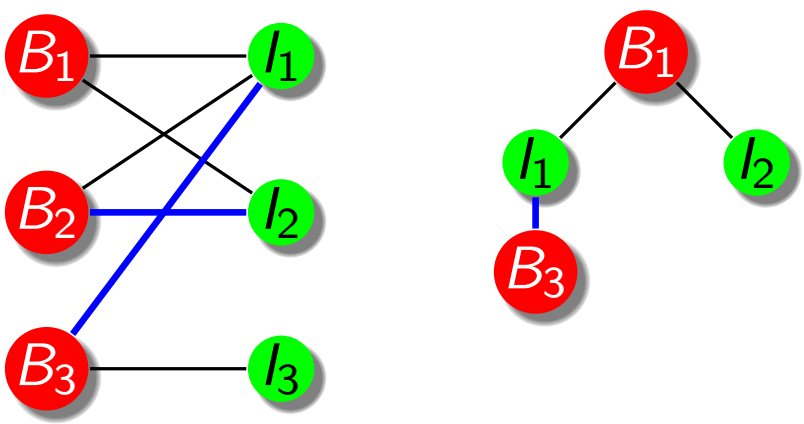

Advanced Material

Matching Theorem: Sketch of the proof

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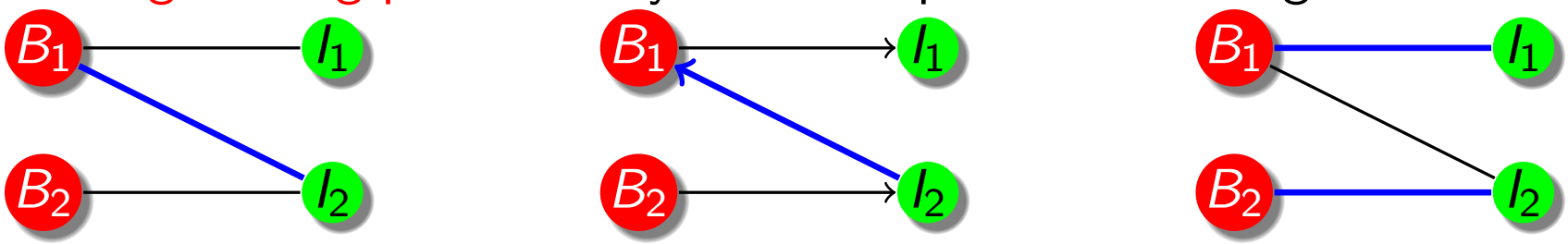

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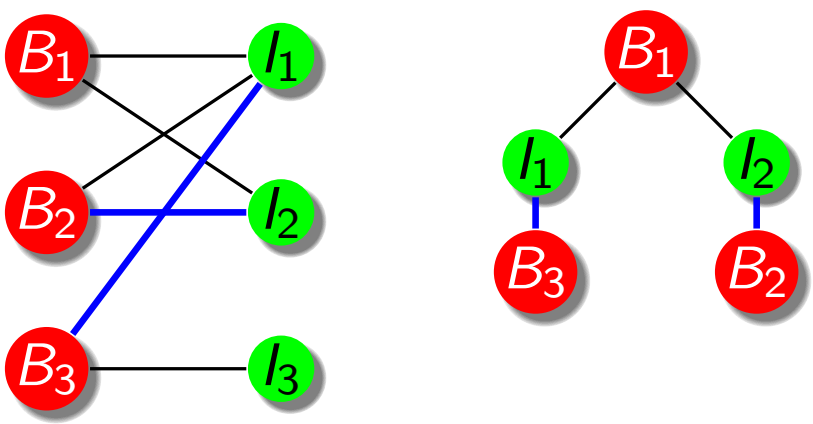

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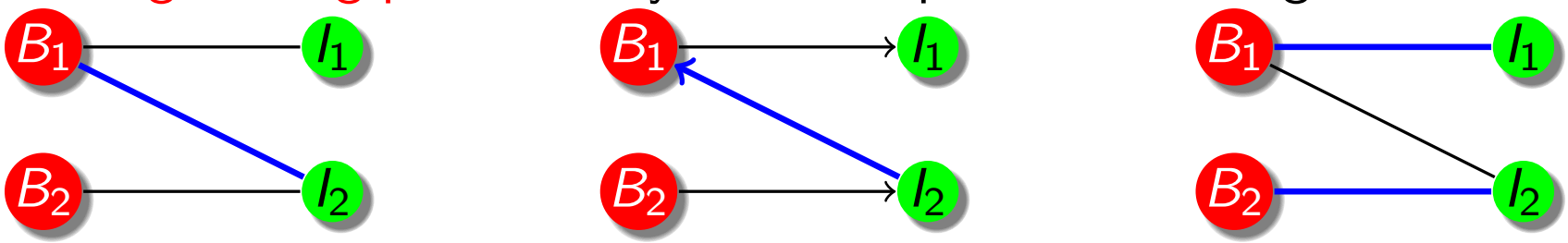

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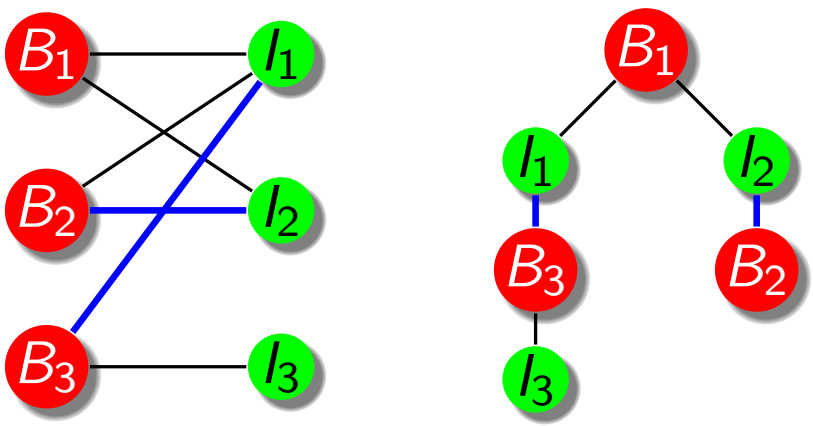

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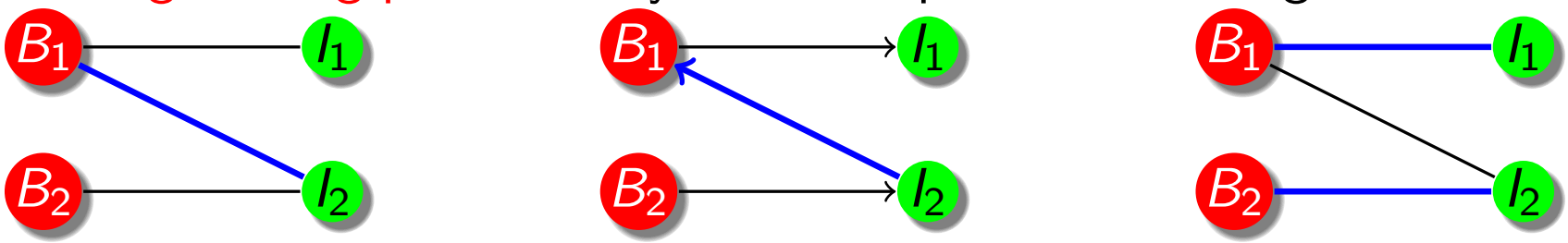

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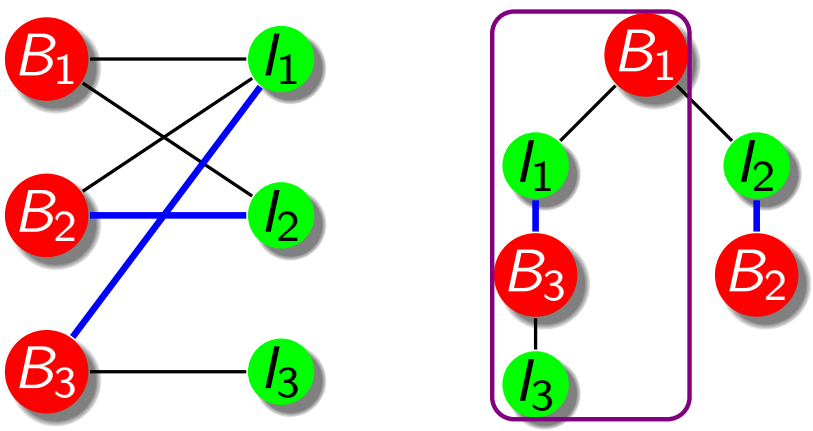

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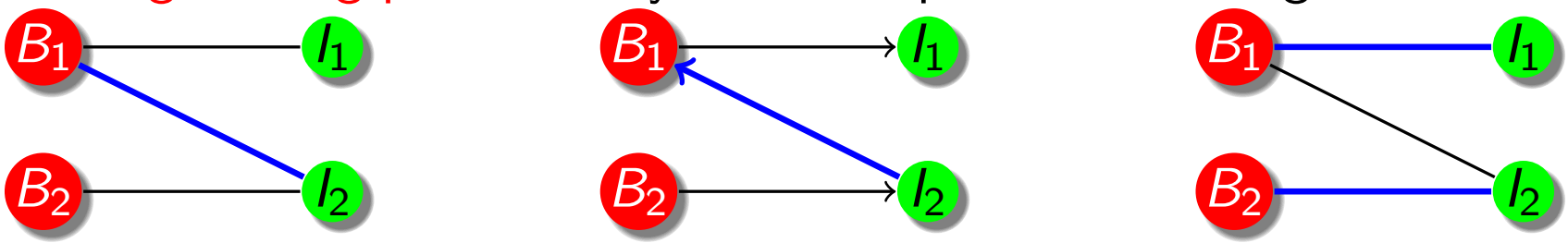

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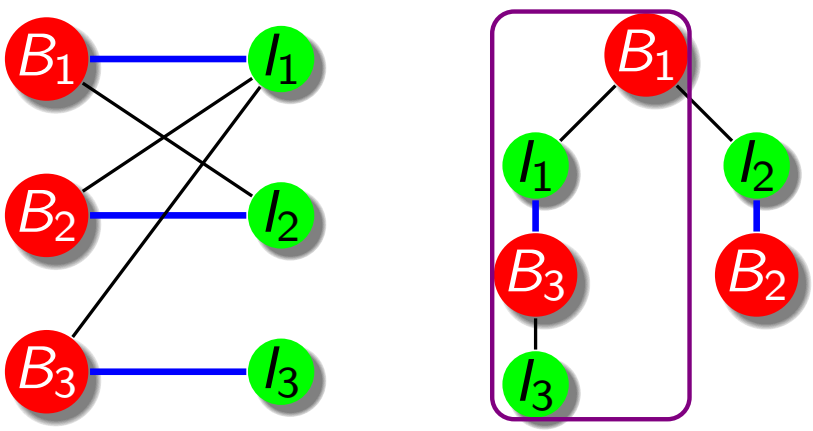

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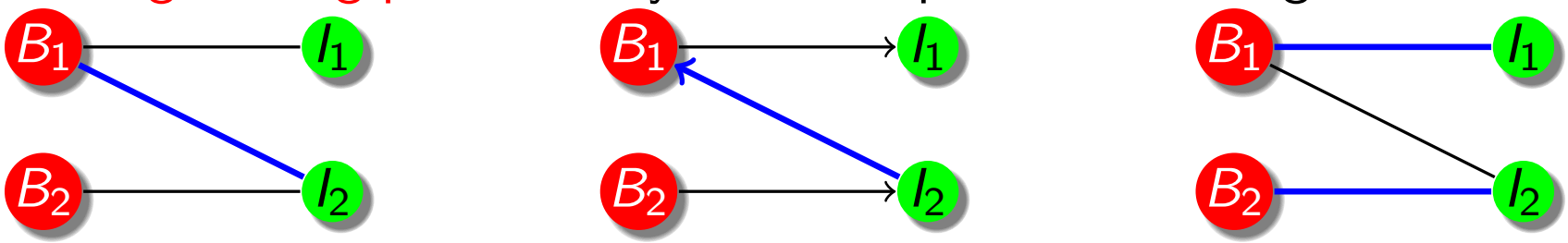

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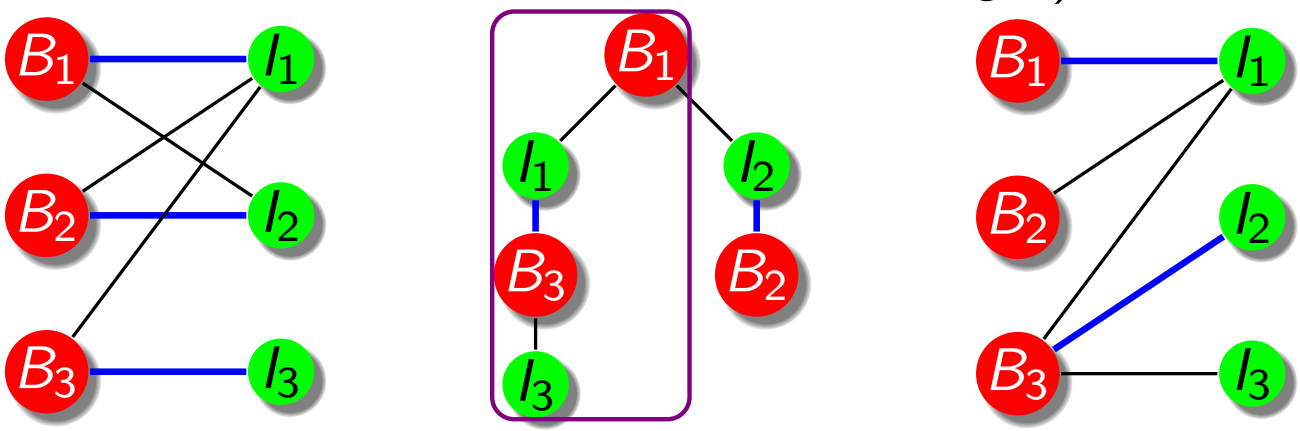

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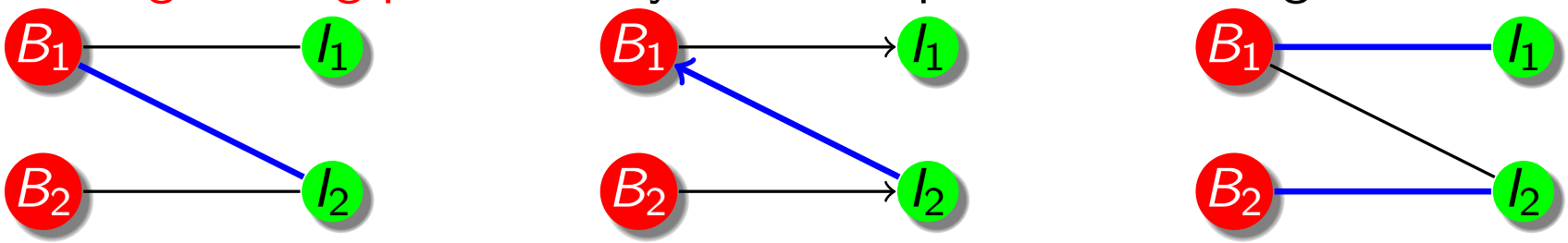

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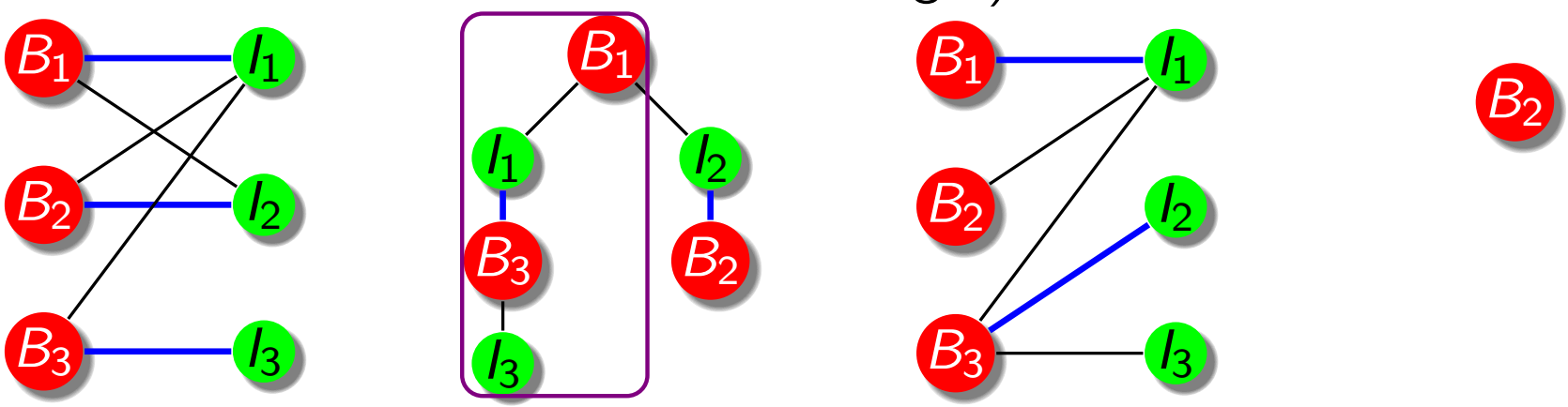

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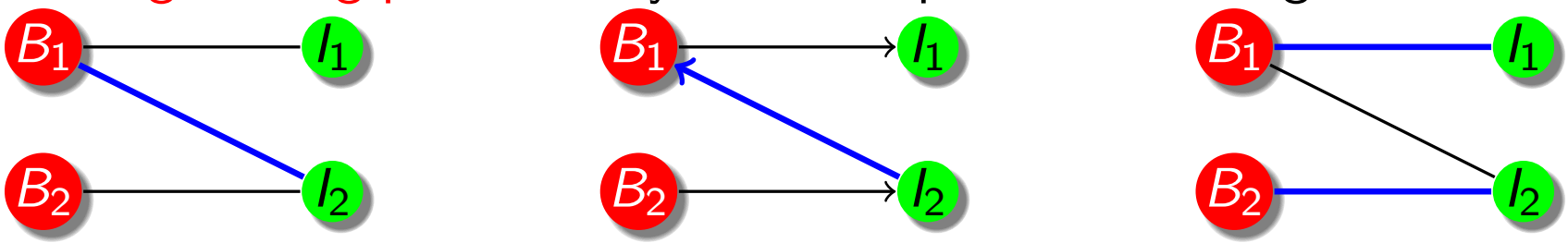

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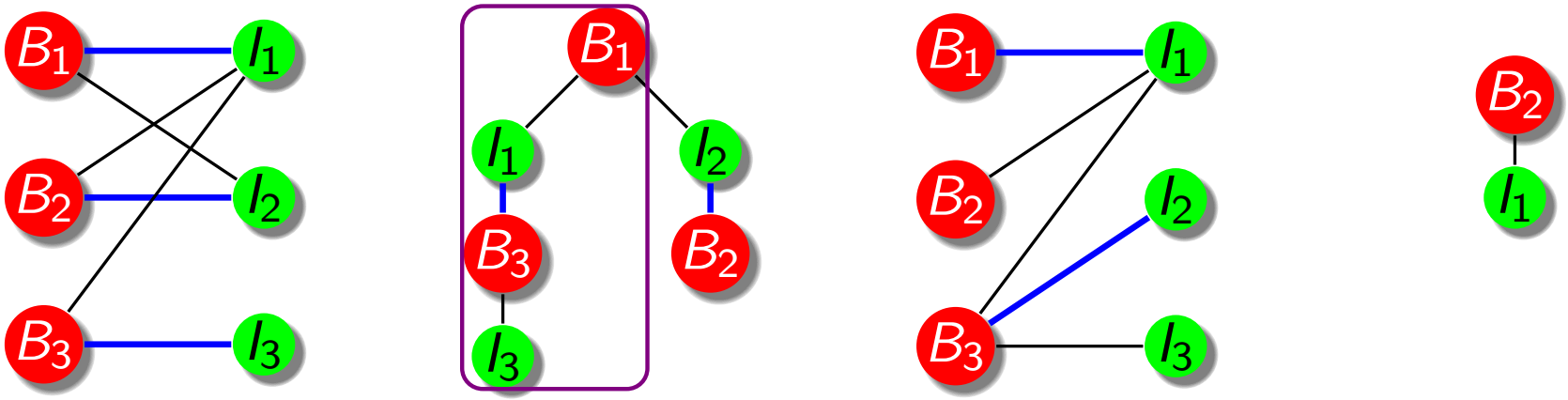

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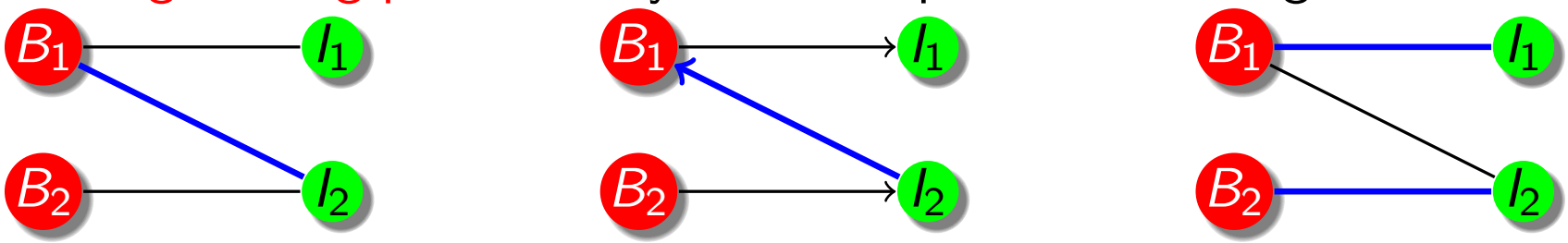

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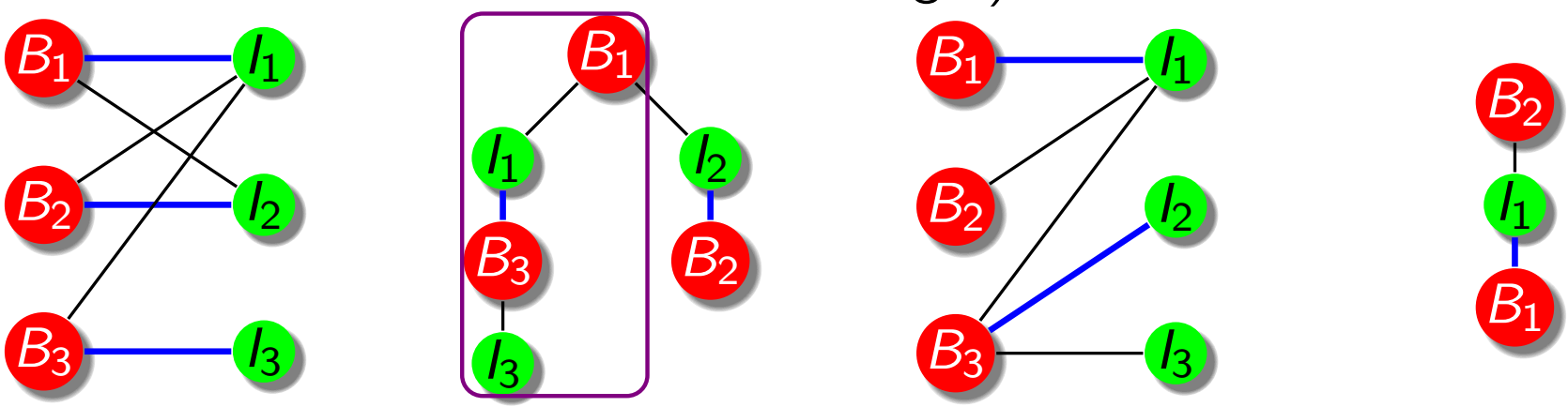

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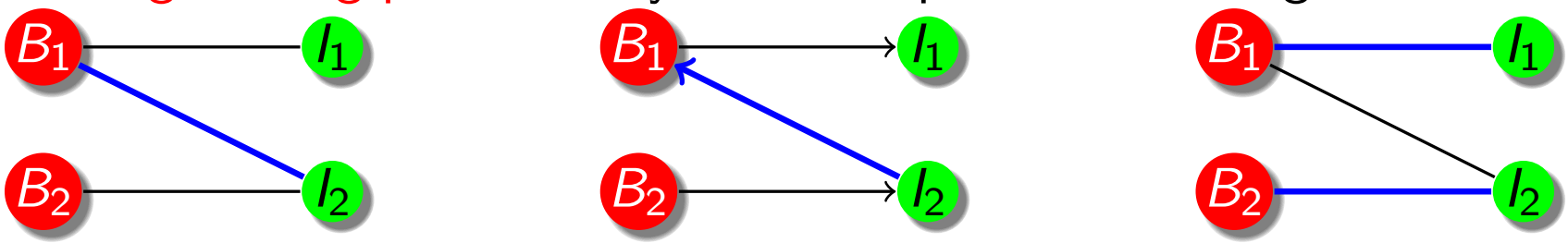

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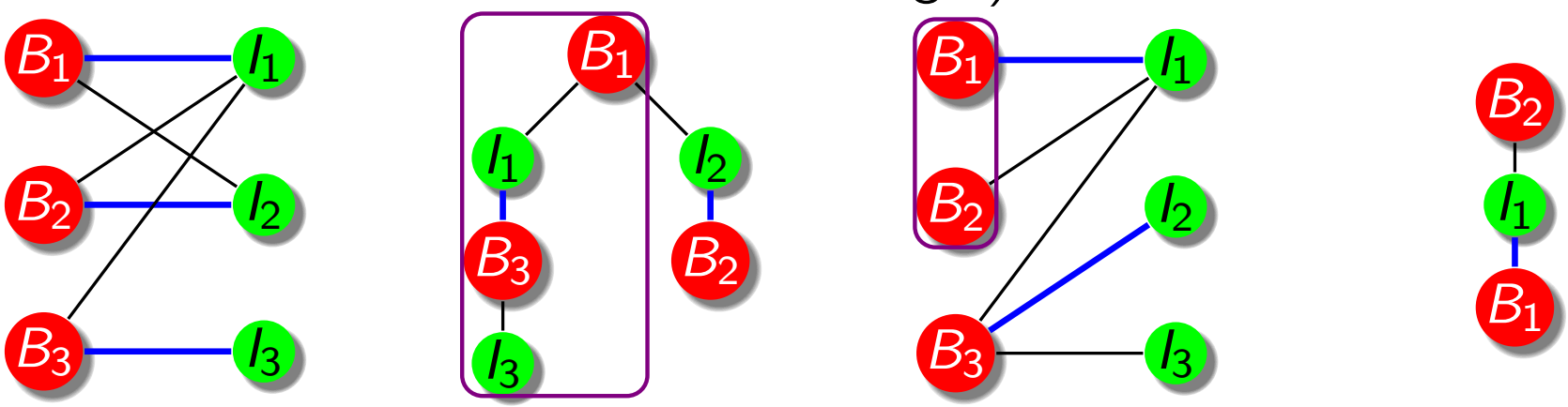

Advanced Material

1. Set price 0 to each item
2. Until the preferred-item graph has not a perfect matching:
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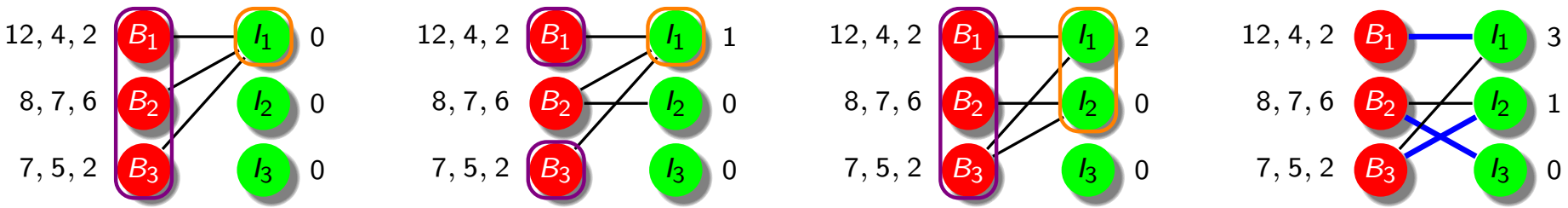

Does the algorithm end?

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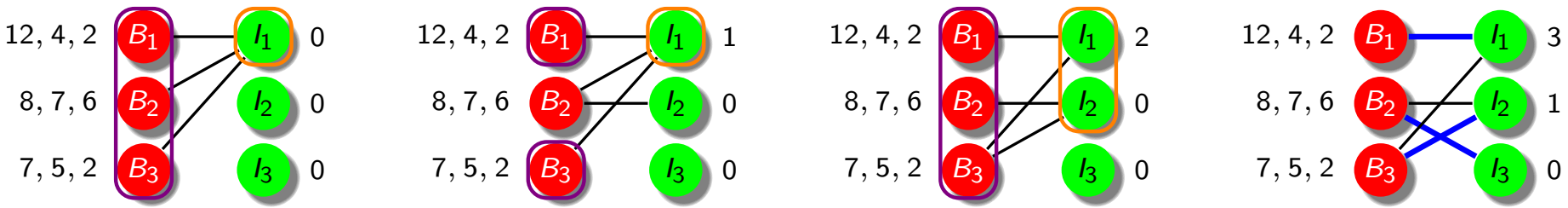

Does the algorithm end?

- Potential function: sum of price + sum of buyers' utilities

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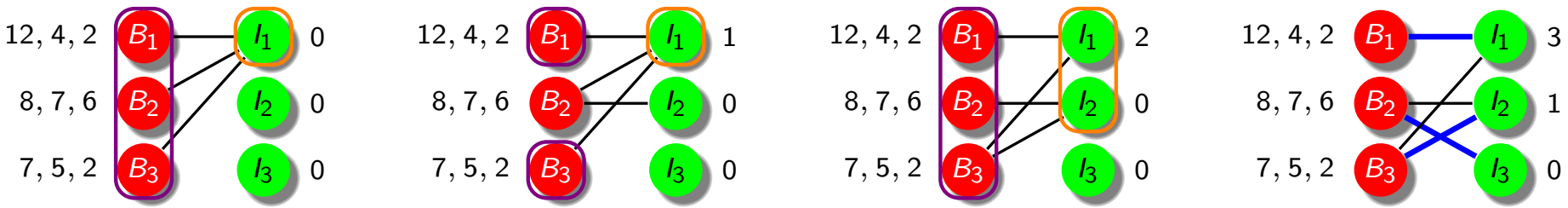

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- Potential function: sum of price + sum of buyers' utilities
- Potential function is always at least 0 (price reduction)

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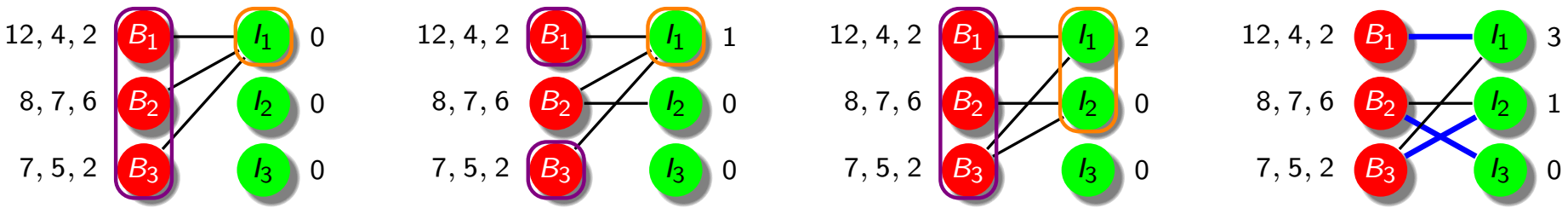

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- Potential function: sum of price + sum of buyers' utilities
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- At each step, we raise the price of $N(S)$ items

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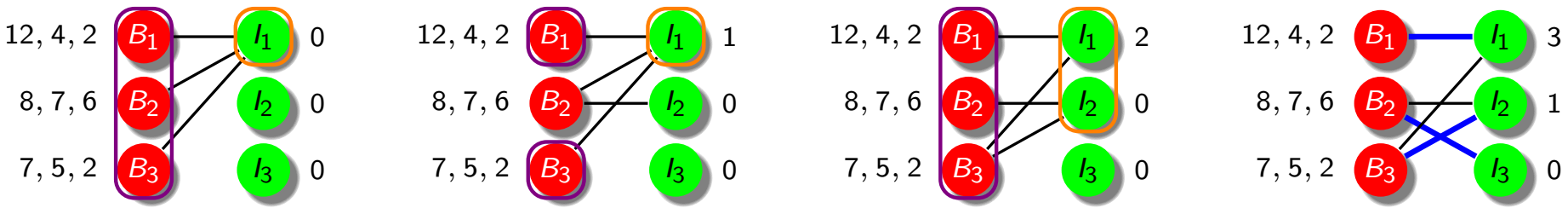

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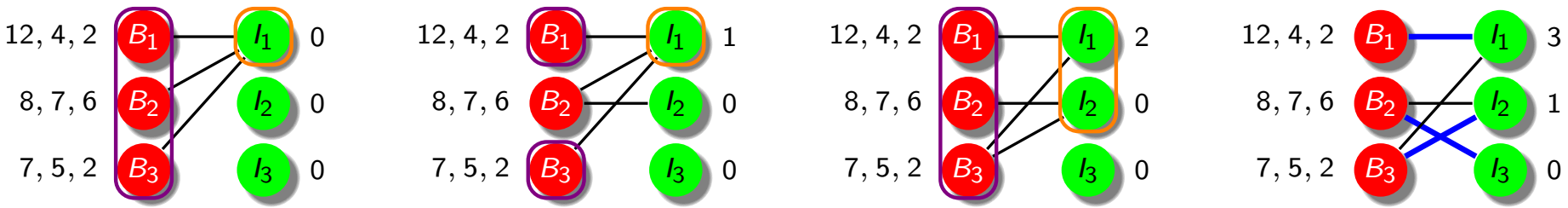

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  - At each step, the potential function decreases

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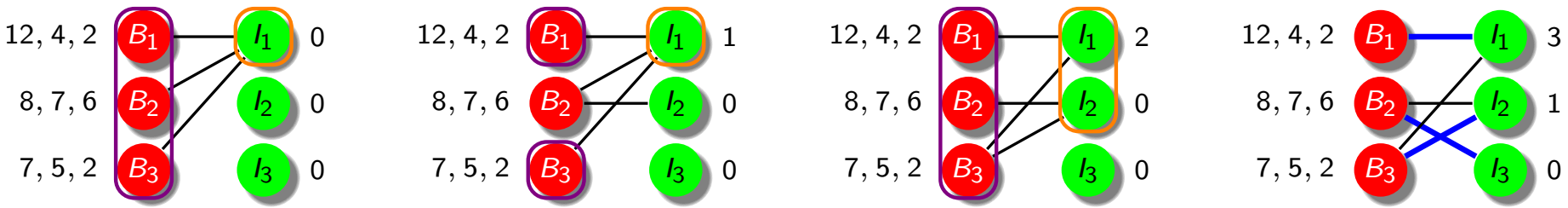

27

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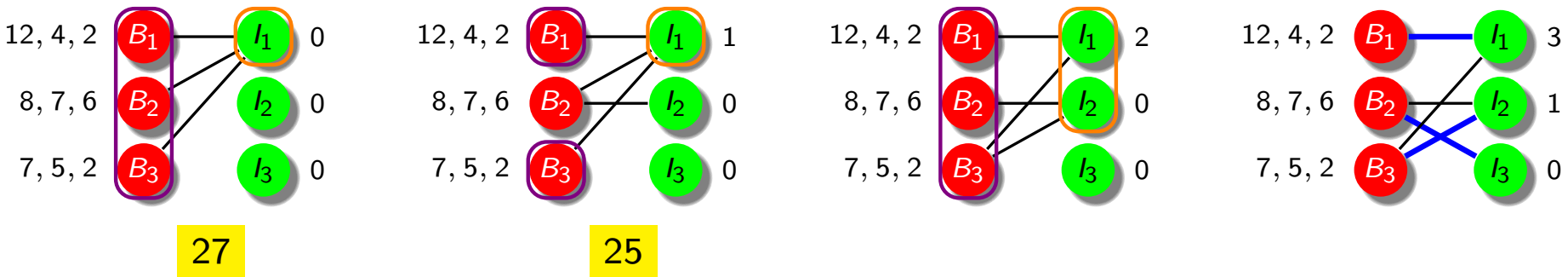

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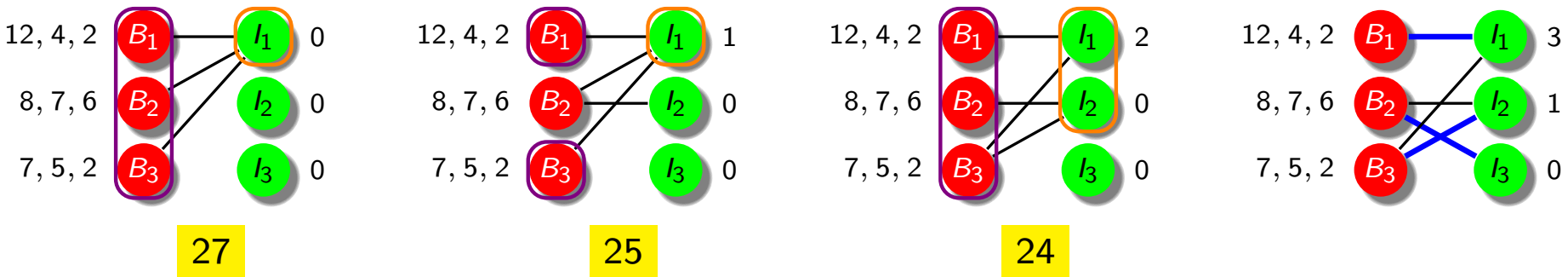

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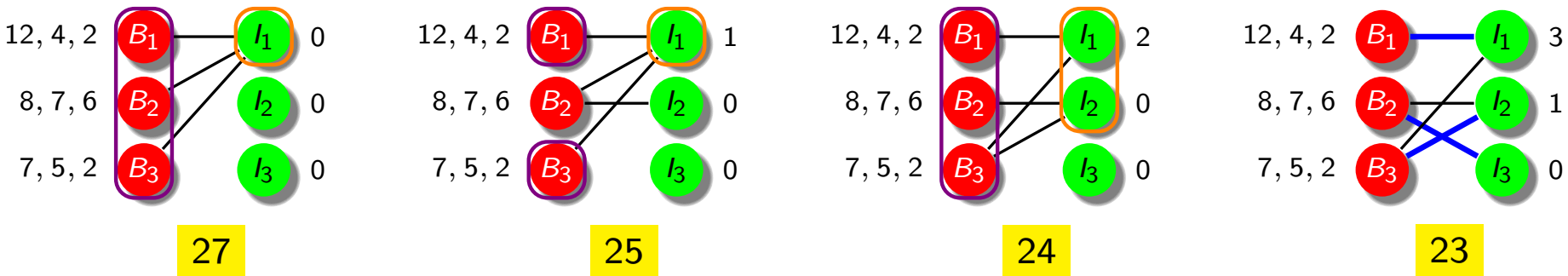

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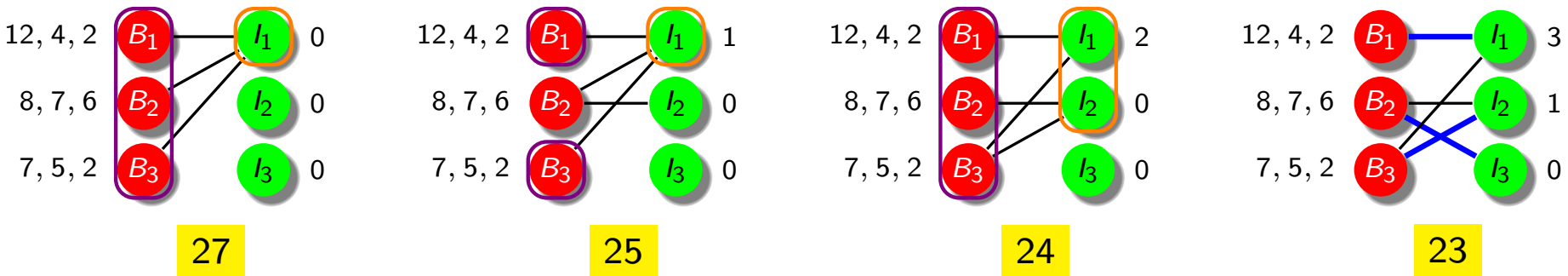

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- At each step, we raise the price of $N(S)$ items
  - We are reducing the utility of S buyers
  - At each step, the potential function decreases
  - Number of steps are finite

Advanced Material

VCG Auctions: Truthfulness

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- If the lie does not affect the assigned slot, payoff unchanged

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- If the lie does not affect the assigned slot, payoff unchanged
- By lying, i takes item k in place of item j

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$$v_{ij} - p_{ij} \overset{?}{\geq} v_{ik} - p_{ik}$$

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- If the lie does not affect the assigned slot, payoff unchanged
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$$v_{ij} - p_{ij} \overset{?}{\geq} v_{ik} - p_{ik}$$

$$v_{ij} - \left[SW(A_{-i}) - SW(A_{-i}^{-j})\right] \overset{?}{\geq} v_{ik} - \left[SW(A_{-i}) - SW(A_{-i}^{-k})\right]$$

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$$v_{ij} + SW(A_{-i}^{-j}) \stackrel{?}{\geq} v_{ik} + SW(A_{-i}^{-k})$$

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$$v_{ij} + SW(A_{-i}^{-j}) \stackrel{?}{\geq} v_{ik} + SW(A_{-i}^{-k})$$

$$SW(A) \stackrel{?}{\geq} v_{ik} + SW(A_{-i}^{-k})$$

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$$v_{ij} + SW(A_{-i}^{-j}) \stackrel{?}{\geq} v_{ik} + SW(A_{-i}^{-k})$$

$$SW(A) \stackrel{?}{\geq} v_{ik} + SW(A_{-i}^{-k}) \qquad \textit{YES}$$