EMTH171 Case Study 1 Assignment:

Task One:

Function $g(F_t)$ which a root could be found for with Newton's method was derived from equation (1).

$$F_t = F_{t-1} + \Psi\Delta\Theta \cdot \ln\left[\frac{\Psi\Delta\Theta + F_t}{\Psi\Delta\Theta + F_{t-1}}\right] + K \cdot \Delta t \tag{1}$$

This is a nonlinear equation used to determine F_t. It takes the values for the previous cumulative infiltration (F_{t-1}), soil suction (Ψ), change in moisture content ($\Delta\Theta$), hydraulic conductivity (K), and change in time (Δt). This equation was rearranged with all terms on the right-hand side. Therefore, $g(F_t)$ is given as equation (2).

$$\boldsymbol{g(F_t) = F_t - F_{t-1} - \Psi\Delta\Theta \cdot \ln\left[\frac{\Psi\Delta\Theta + F_t}{\Psi\Delta\Theta + F_{t-1}}\right] - K \cdot \Delta t} \tag{2}$$

By differentiating this function, a derivative $g'(F_t)$ was found which was needed to use Newton's Method:

$$\boldsymbol{g'(F_t) = \frac{F_t}{\Psi\Delta\Theta + F_t}} \tag{3}$$

The code in the appendix for task one used the values for Loamy Sand soil in (Table 2) with an initial previous cumulative infiltration value (F_{t-1}) of 1 cm to find an approximation of (F_t). Using 4 iterations for Newton's Method the value was found to be $\mathbf{2.37\ cm}$.

Task Two:

Below are the unique input parameters used for both soil types.

Table 1: Input parameters for sandy clay loam soil.

Sandy Clay Loam Soil Parameters:	***Symbol***	***Value***	***Unit***
Time Interval	Δt	0.25	h
Soil Suction	Ψ	21.85	cm
Hydraulic Conductivity	K	0.15	cm/h
Effective Porosity	Θ_e	0.330	Dimensionless
Effective Saturation	S_e	0.45	Dimensionless
Change in Moisture Content	$\Delta\Theta$	0.1815	Dimensionless

Table 2: Input parameters for loamy sand soil.

Loamy Sand Soil Parameters:	***Symbol***	***Value***	***Unit***
Time Interval	Δt	0.25	h
Soil Suction	Ψ	6.13	cm
Hydraulic Conductivity	K	2.99	cm/h
Effective Porosity	Θ_e	0.401	Dimensionless
Effective Saturation	S_e	0.45	Dimensionless
Change in Moisture Content	$\Delta\Theta$	0.22055	Dimensionless

Below are the MATLAB generated graphs made using the values found from Newton's Method and the values in the table appendix.

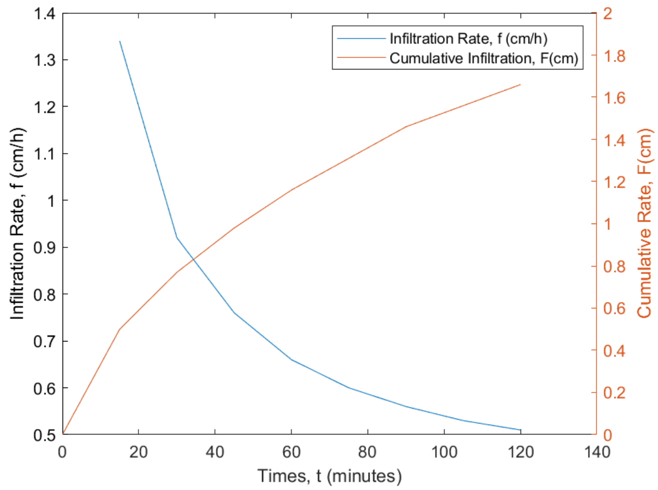

Figure 1: Infiltration rate and cumulative infiltration over time for Sandy Clay Loam soil.

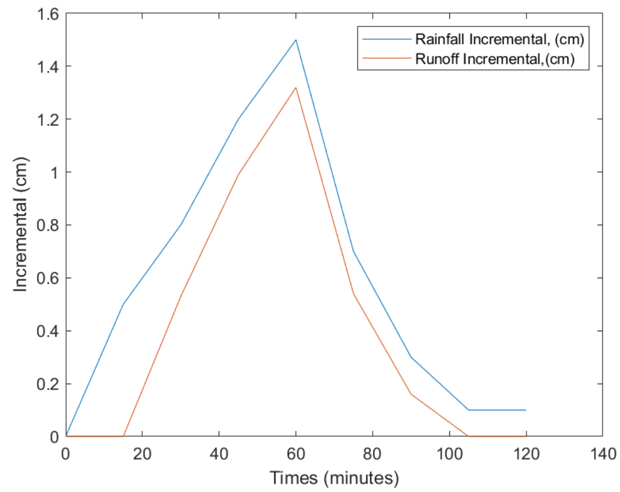

Figure 2: Incremental rainfall and runoff over time for Sandy Clay Loam soil.

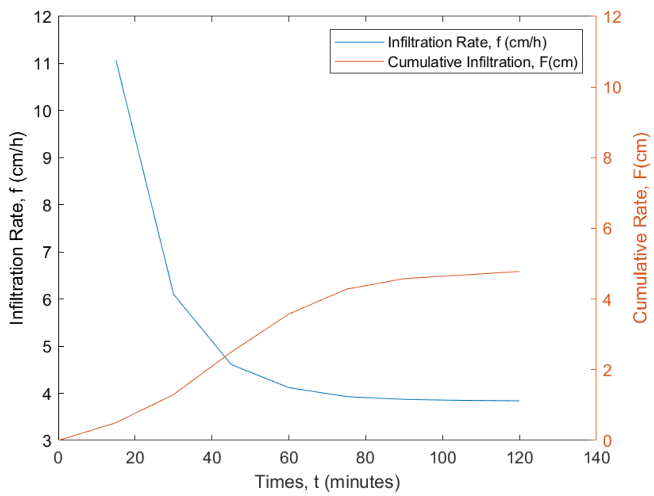

Figure 3: Infiltration rate and cumulative infiltration over time for Loamy Sand soil.

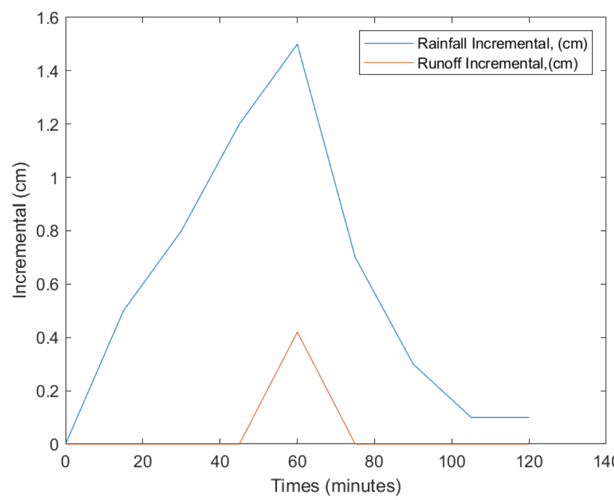

Figure 4: Incremental rainfall and runoff over time for Loamy Sand soil.

Task Three:

In task two the cumulative runoff for both soils was found during an average rainfall event. The catchments total land area is 100,000 m^2 and the land is comprised of 50% Loamy sand and 50% sandy clay loam soil. This information was used to produce the table below.

Table 3: Relevant values for volume calculations.

	Sandy Clay Loam	*Loamy Sand*	*Units*
Cumulative Runoff Height	0.0354	0.0042	m
Land Area	50,000	50,000	m^2
Runoff Volume	1770	210	m^3

The volume in (Table 3) can be found by the following calculations,

$$Sandy\ Clay\ Loam\ Volume = 50{,}000\ \mathrm{m}^2 * 0.0354\ \mathrm{m} = 1770\ m^3$$

$$Loamy\ Sand\ Volume = 50{,}000\ \mathrm{m}^2 * 0.0042\mathrm{m} = 210\ m^3$$

$$Total\ Volume = 1770\ m^3 + 210\ m^3 = 1980\ m^3$$

Therefore, the total volume for an average rainfall event is 1980 m^3 as there are 26 of these events within a year the yearly average rainfall runoff can be determined.

$$Yearly\ Volume = 1980\ m^3 * 26 = 51{,}480\ m^3$$

As the reservoir is being designed to store a fourth of the total yearly runoff,

$$\boldsymbol{Reservoir\ Volume} = \frac{\mathbf{51,480}\ \boldsymbol{m^3}}{\mathbf{4}} = \mathbf{12,870}\ \boldsymbol{m^3}$$

The average rainfall event does not accurately represent the fluctuations in rainfall due to wet and dry seasons that occur throughout the year. This creates an uncertainty with the yearly rainfall volume and therefore the reservoir volume needed. A large percent of the rainfall may occur only during the wet season which means the reservoir itself could be nearly empty until the wet season occurs.

There is also an uncertainty with the number of rainfall events that occur, 26 standard rainfalls were used to predict the required volume for the reservoir but there may be more or less within a year and therefore the reservoir may not be able to hold a quarter the total runoff produced.

By adding additional volume to the reservoir, if more rainfall events occur within a year there would be additional space that could hold the extra runoff produced. Therefore, this reduces the uncertainty on the reservoir's ability to store a quarter the total runoff produced.

Appendix of Tables (Task Two):

Table 4: Infiltration for sandy clay loam soil during an average rainfall.

Time	Rainfall			Infiltration		Runoff	
(min)	Incremental (cm)	Cumulative (cm)	Intensity (cm/h)	f (cm/h)	F (cm)	Cumulative (cm)	Incremental (cm)
0	0.0	0.0	2.0	0.00	0.00	0	0
15	0.5	0.5	3.2	1.34	0.50	0	0
30	0.8	1.3	4.8	**0.92**	**0.77**	**0.53**	**0.53**
45	1.2	2.5	6.0	**0.76**	**0.98**	**1.52**	**0.99**
60	1.5	4.0	2.8	**0.66**	**1.16**	**2.84**	**1.32**
75	0.7	4.7	1.2	**0.60**	**1.31**	**3.39**	**0.54**
90	0.3	5.0	0.4	**0.56**	**1.46**	**3.54**	**0.16**
105	0.1	5.1	0.4	0.53	1.56	0	0
120	0.1	5.2	0.0	0.51	1.66	0	0

Table 5: Infiltration for loamy sand soil during an average rainfall.

Time	Rainfall			Infiltration		Runoff	
(min)	Incremental (cm)	Cumulative (cm)	Intensity (cm/h)	f (cm/h)	F (cm)	Cumulative (cm)	Incremental (cm)
0	0.0	0.0	2.0	0.00	0.00	0	0
15	0.5	0.5	3.2	11.07	0.50	0	0
30	0.8	1.3	4.8	6.10	1.30	0	0
45	1.2	2.5	6.0	4.61	2.50	0	0
60	1.5	4.0	2.8	**4.12**	**3.58**	**0.42**	**0.42**
75	0.7	4.7	1.2	3.93	4.28	0	0
90	0.3	5.0	0.4	3.87	4.58	0	0
105	0.1	5.1	0.4	3.85	4.68	0	0
120	0.1	5.2	0.0	3.84	4.78	0	0

Code Appendix:

Task One Newton's Method (Sandy Clay Loam):

```
% EMTH-171 Case Study One
% John Elliott and Samuel Vallance
% Using Newtons-Method to find the infiltration of water and therefore the
% runoff of water given a rain event.


% Reset virtual environment
clear
clc
close all

% ======================| Variables Initialization |======================

% Soil properties and time interval:
%   These values change depending on the soil type.
%   The soil being used is(Sandy Clay Loam).
soilSuction = 6.13; % cm
moistureChange = 0.22055;
con = soilSuction * moistureChange; % Constant value (cm)
conductivity = 2.99; % cm/h
time = 15/60; % h

% g is the soil infiltration rate equation and gd is its derivative
% g takes the inputs x and z which are F(t) and F(t-1) respectively.
g = @(x, z) x - z - con*log((con + x) / (con + z)) - conductivity*time;
gd = @(x) (x / (con + x));

% currentVal is the value of F(t-1) used by the function g
% trialVal is the guess used to started Newtons-Method
currentVal = 1;
trialVal = 3;
N = 4;

% =====================| Newtons-Method Calculations |=====================

% Iterates through newtons method (N) times to find an approximation of (F(t)).
for jj = 1:N
    trialVal = trialVal - g(trialVal, currentVal)/gd(trialVal);
end

% =======================| Display Calculations |=======================

disp(trialVal)
```

Task Two Newton's Method (Sandy Clay Loam):

```
% EMTH-171 Case Study One
% John Elliott and Samuel Vallance
% Using Newtons-Method to find the infiltration of water and therefore the
% runoff of water given a rain event.

% Reset virtual environment
clear
clc
close all

% ======================| Variables Initialization |======================

% Rainfall dependent values
cumulative = [1.3, 2.5, 4.0, 4.7, 5.0, 5.1, 5.2];
intensity = [4.8, 6.0, 2.8, 1.2, 0.4, 0.4, 0.0];

% Soil properties and time interval:
%   These changes depending on the soil type.
%   The soil being used is (Sandy Clay Loam)
soilSuction = 21.85; %cm
moistureChange = 0.1815;
con = soilSuction * moistureChange;
conductivity = 0.15; %cm/h
time = 15/60; %h

% g is the soil infiltration rate equation and gd is its derivative
% g takes the inputs x and z which are F(t) and F(t-1) respectively.
g = @(x, z) x - z - con*log((con + x) / (con + z)) - conductivity*time;
gd = @(x) (x / (con + x));

% Initialization arrays that store all caculated values to advoid rezising
% a zero array is made to the maxium possible size (endSize).
endSize = length(intensity);
fArray = zeros(1, endSize);
FArray = zeros(1, endSize);
runoffArray = zeros(2, endSize);

% currentVal is is the value of F(t-1) used by the function g
% trialVal is the guess used to started Newtons-Method
currentVal = 0.5;
trialVal = 1;
N = 4;

% =====================| Newtons-Method Calculations |=====================

% Reapeat finding F(t) untill it again becomes less than rainfall intensity
for ii = 1:endSize
    % Iterates through newtons method (N) times to find an approximation of
    % (F(t)).
    for jj = 1:N
        trialVal = trialVal - g(trialVal, currentVal)/gd(trialVal);
    end

    currentVal = trialVal; % Updates now F(t)
    FArray(ii) = trialVal;
    % Runoff calculations
```

```
    runoffArray(1, ii + 1) = cumulative(ii) - FArray(ii);

    fArray(ii) = conductivity *((con / trialVal) + 1);
    runoffArray(2, ii + 1) = runoffArray(1, ii + 1) - runoffArray(1, ii);

    if fArray(ii) > intensity(ii) % Check to see if infiltration is greater again
        break
    end
end
% ========================| Display Calculations |========================

fprintf("==== F (cm/h) ====\n")
fprintf("%.2f\n",FArray(1, 1:ii))
fprintf("\n==== f (cm/h) ====\n")
fprintf("%.2f\n",fArray(1, 1:ii))
fprintf("\n==== Runoff (cm) ====\n")
disp("Cumulative: Incremental:")
fprintf("%.2f %11.2f\n", runoffArray(1:2, 2:ii + 1))
```

Task Two Plotting (Sandy Clay Loam):

```
% EMTH-171 Case Study One
% John Elliott and Samuel Vallance
% Plots values for the infiltration arrays and rainfall and runoff arrays.

% Reset virtual environment
clear
clc
close all


% ======================| Variables Initialization |======================

% Array of values being plotted for (Sandy Clay Loam)
timearray = [0, 15, 30, 45, 60, 75, 90, 105, 120];
infiltration = [1.34, 0.92, 0.76, 0.66, 0.60, 0.56, 0.53, 0.51];
cumulativeInfiltration = [0.00, 0.50, 0.77, 0.98, 1.16, 1.31, 1.46, 1.56, 1.66];
runoff = [0, 0, 0.53, 0.99, 1.32, 0.54, 0.16, 0, 0];
incremental = [0.0, 0.5, 0.8, 1.2, 1.5, 0.7, 0.3, 0.1, 0.1];

% ==============================| Plotting |==============================

% Plot for Infiltration Rate and Cumulative Infiltration.
figure(1)
plot(timearray(2:end), infiltration)
ylabel('Infiltration Rate, f (cm/h)')
xlabel('Times, t (minutes)')
yyaxis right % Second y-axis scale

hold on
plot(timearray, cumulativeInfiltration)
ylabel('Cumulative Rate, F(cm)')
legend('Infiltration Rate, f (cm/h)','Cumulative Infiltration, F(cm)')
hold off
% Limits graph produce to better centre it.
ylim([0,2])
xlim([0,140])

% Plot for Rainfall Incremental and Runoff Incremental.
figure(2)
plot(timearray, incremental)
ylabel('Incremental (cm)')
xlabel('Times (minutes)')

hold on
plot(timearray, runoff)
legend('Rainfall Incremental, (cm)','Runoff Incremental,(cm)')
hold off
% Limits graph produce to better centre it.
ylim([0,1.6])
xlim([0,140])
```