

Exercice 4

2.

$$\begin{aligned}
D(n) &= 2D\left(\frac{n}{5}\right) + n && \langle \text{par substitution} \rangle \\
&= 2\left(2D\left(\frac{n}{5^2}\right) + \frac{n}{5}\right) + n && \langle \text{arithmétique} \rangle \\
&= 2^2 D\left(\frac{n}{5^2}\right) + \frac{2n}{5} + n && \langle \text{arithmétique} \rangle \\
&= 2^2 \left(2D\left(\frac{n}{5^3}\right) + \frac{2}{5^2}\right) + \frac{2n}{5} + n && \langle \text{arithmétique} \rangle \\
&= 2^3 D\left(\frac{n}{5^3}\right) + \frac{2^2 n}{5^2} + \frac{2n}{5} + n && \langle \text{arithmétique} \rangle \\
&\vdots \\
&= 2^3 D\left(\frac{n}{5^3}\right) + n \left(\frac{2}{5}\right)^2 + n \left(\frac{2}{5}\right)^1 + n \left(\frac{2}{5}\right)^0 && \langle \text{arithmétique} \rangle \\
&= 2^i D\left(\frac{n}{5^i}\right) + n \left(\frac{2}{5}\right)^{i-1} + \dots + n \left(\frac{2}{5}\right)^1 + n \left(\frac{2}{5}\right)^0 && \langle \text{arithmétique} \rangle \\
&= 2^i D\left(\frac{n}{5^i}\right) + n \sum_{j=0}^{i-1} \left(\frac{2}{5}\right)^j && \langle \text{arithmétique} \rangle \\
&= 2^{\log_5(n)} D(1) + n \left(\frac{1 - \left(\frac{2}{5}\right)^{(i+1)-1}}{1 - \frac{2}{5}}\right) && \langle \text{arithmétique} \rangle \\
&= 2^{\log_5(n)} D(1) + n \left(\frac{1 - \left(\frac{2}{5}\right)^i}{\frac{3}{5}}\right) && \langle \text{arithmétique} \rangle \\
&= 2^{\log_5(n)} D(1) + \frac{5}{3} n \left(1 - \left(\frac{2}{5}\right)^{\log_5(n)}\right) && \langle \text{arithmétique} \rangle \\
&= 7n^{\log_5(2)} + \frac{5}{3} n \left(1 - \left(\frac{n^{\log_5(2)}}{n}\right)\right) && \langle \text{arithmétique} \rangle \\
&= 7n^{\log_5(2)} + \frac{5}{3} n \left(\frac{n - (n^{\log_5(2)})}{n}\right) && \langle \text{arithmétique} \rangle \\
&= 7n^{\log_5(2)} + \frac{5}{3} (n - n^{\log_5(2)}) && \langle \text{arithmétique} \rangle \\
&= \frac{16}{3} n^{\log_5(2)} + \frac{5}{3} n && \langle \text{arithmétique} \rangle
\end{aligned}$$