

Softmax (Multiclass Logistic) Regression on MNIST

This notebook implements Softmax Regression — a multiclass extension of Logistic Regression — to classify MNIST handwritten digits (0–9). We'll train it manually (using basic PyTorch tensors) and compare the results with the PyTorch built-in implementation (`nn.Linear + CrossEntropyLoss`).

Import Libraries

```
from torchvision import datasets, transforms
import matplotlib.pyplot as plt
import torch
import torch.nn as nn
import numpy as np
from torch.utils.data import DataLoader
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LogisticRegression
from sklearn.metrics import accuracy_score
from torch.utils.data import TensorDataset, DataLoader
from sklearn.metrics import confusion_matrix, ConfusionMatrixDisplay
```

Load and Preprocess Data

```
transform = transforms.Compose([
    transforms.ToTensor() # Convert to tensor and scale to [0,1]
])
```

```
mnist_train = datasets.MNIST(root='data', train=True, transform=transform, download=True)
mnist_test = datasets.MNIST(root='data', train=False, transform=transform, download=True)
```

```
X = torch.stack([img.view(-1) for img, _ in mnist_train])
y = torch.tensor([label for _, label in mnist_train])
```

Train/Validation/Test Split

```
X_train, X_temp, y_train, y_temp = train_test_split(X, y, test_size=0.4, stratify=y)
X_val, X_test, y_val, y_test = train_test_split(X_temp, y_temp, test_size=0.5, stratify=y_temp)
print(f"Train set size: {X_train.shape[0]}")
print(f"Validation set size: {X_val.shape[0]}")
print(f"Test set size: {X_test.shape[0]}")
```

```
Train set size: 36000
Validation set size: 12000
Test set size: 12000
```

Data Loaders

```
batch_size = 64
train_loader = DataLoader(TensorDataset(X_train, y_train), batch_size=batch_size, shuffle=True)
val_loader = DataLoader(TensorDataset(X_val, y_val), batch_size=batch_size)
test_loader = DataLoader(TensorDataset(X_test, y_test), batch_size=batch_size)
```

Helper Functions

```
def softmax(z):
    exp_z = torch.exp(z - torch.max(z, dim=1, keepdim=True).values)
    return exp_z / exp_z.sum(dim=1, keepdim=True)
```

```
def cross_entropy_loss(y_hat, y_true):
    eps = 1e-15
    y_hat = torch.clamp(y_hat, eps, 1 - eps)
    one_hot = torch.nn.functional.one_hot(y_true, num_classes=10).float()
    return -torch.mean(torch.sum(one_hot * torch.log(y_hat), dim=1))
```

Initialize Parameters

```
n_features = 784
n_classes = 10
W = torch.zeros((n_features, n_classes), requires_grad=True)
b = torch.zeros(n_classes, requires_grad=True)
```

```
learning_rate = 0.01
epochs = 100
train_losses = []
val_losses = []
train_accuracies = []
val_accuracies = []
```

Train loop

```
# --- Training loop ---
for epoch in range(epochs):
    epoch_loss = 0.0
    correct_train = 0
    total_train = 0

    # ----- TRAINING PHASE -----
    for X_batch, y_batch in train_loader:
        # Forward pass
        y_pred = softmax(X_batch @ W + b)
        loss = cross_entropy_loss(y_pred, y_batch)

        # Backward pass
        loss.backward()

        # Gradient update (manual SGD)
        with torch.no_grad():
            W -= learning_rate * W.grad
            b -= learning_rate * b.grad

        # Reset gradients
        W.grad.zero_()
        b.grad.zero_()

        # Accumulate batch loss
        epoch_loss += loss.item()
        # Compute training accuracy per batch
        preds = y_pred.argmax(dim=1)
        correct_train += (preds == y_batch).sum().item()
        total_train += y_batch.size(0)

    # Compute average training loss for this epoch
    avg_train_loss = epoch_loss / len(train_loader)
    train_acc = correct_train / total_train
    train_losses.append(avg_train_loss)
    train_accuracies.append(train_acc)

    # ----- VALIDATION PHASE -----
    with torch.no_grad():
        val_loss_total = 0.0
        correct = 0
        total = 0

        for X_val_batch, y_val_batch in val_loader:
            y_val_pred = softmax(X_val_batch @ W + b)
            val_loss_total += cross_entropy_loss(y_val_pred, y_val_batch).item()

            # Compute accuracy
            y_val_pred_label = y_val_pred.argmax(dim=1)
            correct += (y_val_pred_label == y_val_batch).sum().item()
            total += y_val_batch.size(0)

        avg_val_loss = val_loss_total / len(val_loader)
        val_acc = correct / total

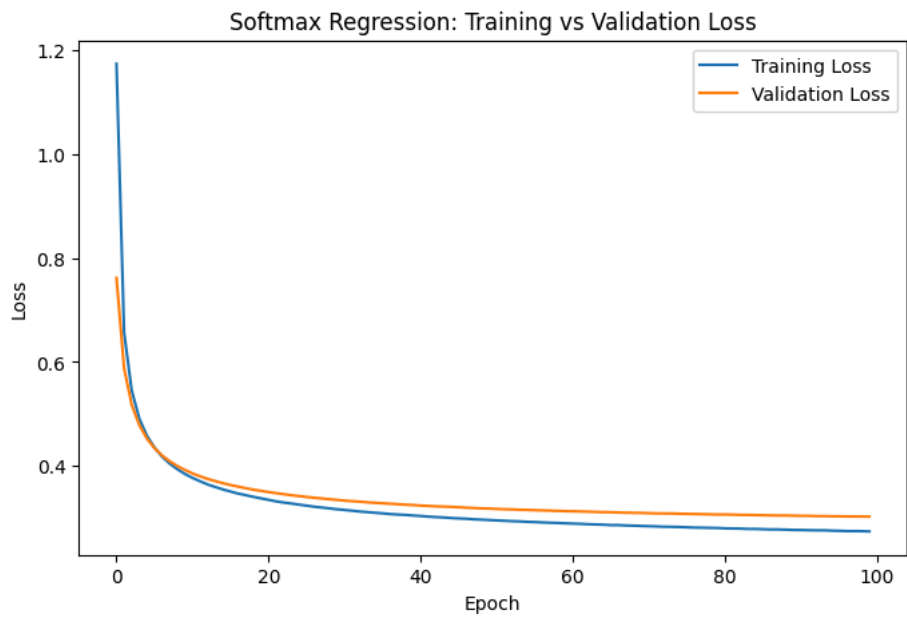
    val_losses.append(avg_val_loss)
    val_accuracies.append(val_acc)
```

```
# Print progress for this epoch
print(f"Epoch {epoch+1}/{epochs} | "
      f"Train Loss: {avg_train_loss:.4f} | "
      f"Val Loss: {avg_val_loss:.4f} | "
      f"Train Acc: {train_accuracies[-1]:.4f} | "
      f"Val Acc: {val_accuracies[-1]:.4f}")
```

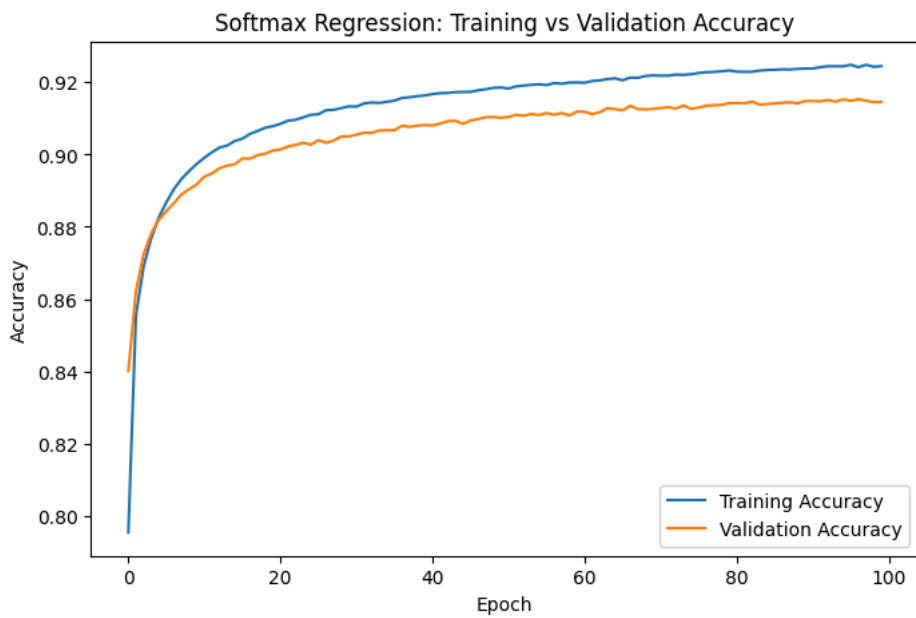
Epoch 1/100	Train Loss: 1.1733	Val Loss: 0.7617	Train Acc: 0.7954	Val Acc: 0.8401
Epoch 2/100	Train Loss: 0.6570	Val Loss: 0.5879	Train Acc: 0.8556	Val Acc: 0.8622
Epoch 3/100	Train Loss: 0.5452	Val Loss: 0.5179	Train Acc: 0.8689	Val Acc: 0.8723
Epoch 4/100	Train Loss: 0.4913	Val Loss: 0.4788	Train Acc: 0.8768	Val Acc: 0.8782
Epoch 5/100	Train Loss: 0.4582	Val Loss: 0.4529	Train Acc: 0.8826	Val Acc: 0.8819
Epoch 6/100	Train Loss: 0.4355	Val Loss: 0.4344	Train Acc: 0.8868	Val Acc: 0.8843
Epoch 7/100	Train Loss: 0.4181	Val Loss: 0.4202	Train Acc: 0.8905	Val Acc: 0.8866
Epoch 8/100	Train Loss: 0.4046	Val Loss: 0.4091	Train Acc: 0.8933	Val Acc: 0.8890
Epoch 9/100	Train Loss: 0.3940	Val Loss: 0.4001	Train Acc: 0.8955	Val Acc: 0.8904
Epoch 10/100	Train Loss: 0.3848	Val Loss: 0.3924	Train Acc: 0.8974	Val Acc: 0.8918
Epoch 11/100	Train Loss: 0.3772	Val Loss: 0.3858	Train Acc: 0.8991	Val Acc: 0.8939
Epoch 12/100	Train Loss: 0.3706	Val Loss: 0.3802	Train Acc: 0.9006	Val Acc: 0.8948
Epoch 13/100	Train Loss: 0.3645	Val Loss: 0.3752	Train Acc: 0.9019	Val Acc: 0.8962
Epoch 14/100	Train Loss: 0.3596	Val Loss: 0.3710	Train Acc: 0.9025	Val Acc: 0.8969
Epoch 15/100	Train Loss: 0.3550	Val Loss: 0.3670	Train Acc: 0.9038	Val Acc: 0.8973
Epoch 16/100	Train Loss: 0.3510	Val Loss: 0.3635	Train Acc: 0.9044	Val Acc: 0.8989
Epoch 17/100	Train Loss: 0.3471	Val Loss: 0.3605	Train Acc: 0.9057	Val Acc: 0.8988
Epoch 18/100	Train Loss: 0.3440	Val Loss: 0.3574	Train Acc: 0.9066	Val Acc: 0.8998
Epoch 19/100	Train Loss: 0.3408	Val Loss: 0.3546	Train Acc: 0.9074	Val Acc: 0.9002
Epoch 20/100	Train Loss: 0.3379	Val Loss: 0.3524	Train Acc: 0.9079	Val Acc: 0.9011
Epoch 21/100	Train Loss: 0.3350	Val Loss: 0.3499	Train Acc: 0.9085	Val Acc: 0.9014
Epoch 22/100	Train Loss: 0.3322	Val Loss: 0.3479	Train Acc: 0.9094	Val Acc: 0.9022
Epoch 23/100	Train Loss: 0.3298	Val Loss: 0.3460	Train Acc: 0.9096	Val Acc: 0.9027
Epoch 24/100	Train Loss: 0.3279	Val Loss: 0.3439	Train Acc: 0.9103	Val Acc: 0.9032
Epoch 25/100	Train Loss: 0.3258	Val Loss: 0.3423	Train Acc: 0.9110	Val Acc: 0.9028
Epoch 26/100	Train Loss: 0.3239	Val Loss: 0.3406	Train Acc: 0.9112	Val Acc: 0.9040
Epoch 27/100	Train Loss: 0.3218	Val Loss: 0.3391	Train Acc: 0.9123	Val Acc: 0.9032
Epoch 28/100	Train Loss: 0.3203	Val Loss: 0.3377	Train Acc: 0.9124	Val Acc: 0.9039
Epoch 29/100	Train Loss: 0.3186	Val Loss: 0.3362	Train Acc: 0.9129	Val Acc: 0.9050
Epoch 30/100	Train Loss: 0.3169	Val Loss: 0.3349	Train Acc: 0.9133	Val Acc: 0.9050
Epoch 31/100	Train Loss: 0.3156	Val Loss: 0.3336	Train Acc: 0.9133	Val Acc: 0.9055
Epoch 32/100	Train Loss: 0.3141	Val Loss: 0.3325	Train Acc: 0.9141	Val Acc: 0.9061
Epoch 33/100	Train Loss: 0.3126	Val Loss: 0.3316	Train Acc: 0.9144	Val Acc: 0.9060
Epoch 34/100	Train Loss: 0.3114	Val Loss: 0.3302	Train Acc: 0.9143	Val Acc: 0.9067
Epoch 35/100	Train Loss: 0.3102	Val Loss: 0.3293	Train Acc: 0.9146	Val Acc: 0.9067
Epoch 36/100	Train Loss: 0.3091	Val Loss: 0.3284	Train Acc: 0.9149	Val Acc: 0.9067
Epoch 37/100	Train Loss: 0.3077	Val Loss: 0.3275	Train Acc: 0.9157	Val Acc: 0.9080
Epoch 38/100	Train Loss: 0.3068	Val Loss: 0.3267	Train Acc: 0.9159	Val Acc: 0.9077
Epoch 39/100	Train Loss: 0.3061	Val Loss: 0.3257	Train Acc: 0.9162	Val Acc: 0.9080
Epoch 40/100	Train Loss: 0.3049	Val Loss: 0.3251	Train Acc: 0.9164	Val Acc: 0.9082
Epoch 41/100	Train Loss: 0.3040	Val Loss: 0.3240	Train Acc: 0.9167	Val Acc: 0.9081
Epoch 42/100	Train Loss: 0.3028	Val Loss: 0.3233	Train Acc: 0.9170	Val Acc: 0.9086
Epoch 43/100	Train Loss: 0.3019	Val Loss: 0.3226	Train Acc: 0.9171	Val Acc: 0.9093
Epoch 44/100	Train Loss: 0.3012	Val Loss: 0.3220	Train Acc: 0.9173	Val Acc: 0.9093
Epoch 45/100	Train Loss: 0.3001	Val Loss: 0.3215	Train Acc: 0.9173	Val Acc: 0.9085
Epoch 46/100	Train Loss: 0.2993	Val Loss: 0.3208	Train Acc: 0.9174	Val Acc: 0.9094
Epoch 47/100	Train Loss: 0.2987	Val Loss: 0.3200	Train Acc: 0.9178	Val Acc: 0.9098
Epoch 48/100	Train Loss: 0.2976	Val Loss: 0.3193	Train Acc: 0.9181	Val Acc: 0.9103
Epoch 49/100	Train Loss: 0.2970	Val Loss: 0.3186	Train Acc: 0.9185	Val Acc: 0.9104
Epoch 50/100	Train Loss: 0.2962	Val Loss: 0.3183	Train Acc: 0.9186	Val Acc: 0.9102
Epoch 51/100	Train Loss: 0.2955	Val Loss: 0.3176	Train Acc: 0.9183	Val Acc: 0.9104
Epoch 52/100	Train Loss: 0.2948	Val Loss: 0.3170	Train Acc: 0.9189	Val Acc: 0.9110
Epoch 53/100	Train Loss: 0.2941	Val Loss: 0.3166	Train Acc: 0.9191	Val Acc: 0.9107
Epoch 54/100	Train Loss: 0.2936	Val Loss: 0.3161	Train Acc: 0.9193	Val Acc: 0.9113
Epoch 55/100	Train Loss: 0.2928	Val Loss: 0.3157	Train Acc: 0.9194	Val Acc: 0.9109
Epoch 56/100	Train Loss: 0.2923	Val Loss: 0.3151	Train Acc: 0.9193	Val Acc: 0.9115
Epoch 57/100	Train Loss: 0.2916	Val Loss: 0.3148	Train Acc: 0.9198	Val Acc: 0.9110
Epoch 58/100	Train Loss: 0.2910	Val Loss: 0.3143	Train Acc: 0.9196	Val Acc: 0.9115

Visualize Learning Curves

```
plt.figure(figsize=(8,5))
plt.plot(train_losses, label='Training Loss')
plt.plot(val_losses, label='Validation Loss')
plt.xlabel('Epoch')
plt.ylabel('Loss')
plt.title('Softmax Regression: Training vs Validation Loss')
plt.legend()
plt.show()
```



```
plt.figure(figsize=(8,5))
plt.plot(train_accuracies, label='Training Accuracy')
plt.plot(val_accuracies, label='Validation Accuracy')
plt.xlabel('Epoch')
plt.ylabel('Accuracy')
plt.title('Softmax Regression: Training vs Validation Accuracy')
plt.legend()
plt.show()
```



Evaluate on Test Set

```

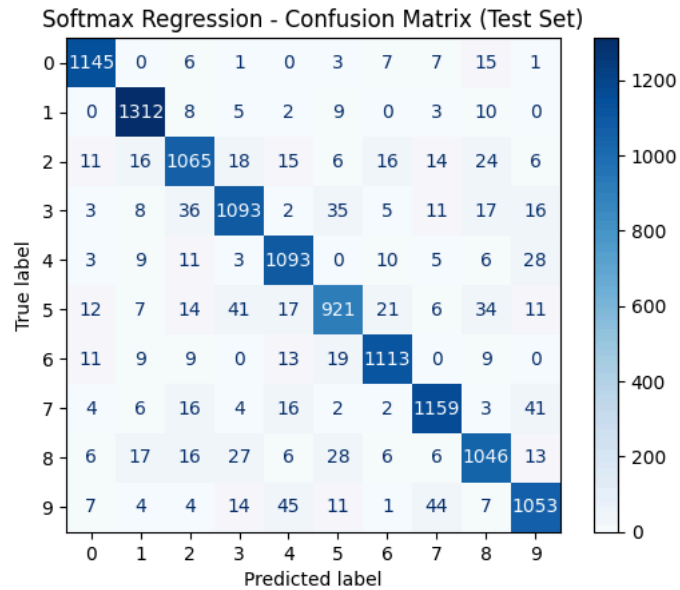
with torch.no_grad():
    logits_test = X_test @ W + b
    y_test_pred = softmax(logits_test)
    y_test_pred_label = y_test_pred.argmax(dim=1)

test_acc = (y_test_pred_label == y_test).float().mean().item()
print(f"\nFinal Test Accuracy: {test_acc:.4f}")

cm = confusion_matrix(y_test, y_test_pred_label)
ConfusionMatrixDisplay(cm).plot(cmap='Blues')
plt.title("Softmax Regression - Confusion Matrix (Test Set)")
plt.show()

```

Final Test Accuracy: 0.9167



Per-Class Accuracy

```

classes = list(range(10))
per_class_acc = []
for c in classes:
    mask = (y_test == c)
    acc_c = (y_test_pred_label[mask] == y_test[mask]).float().mean().item()
    per_class_acc.append(acc_c)
    print(f"Accuracy for digit {c}: {acc_c:.4f}")

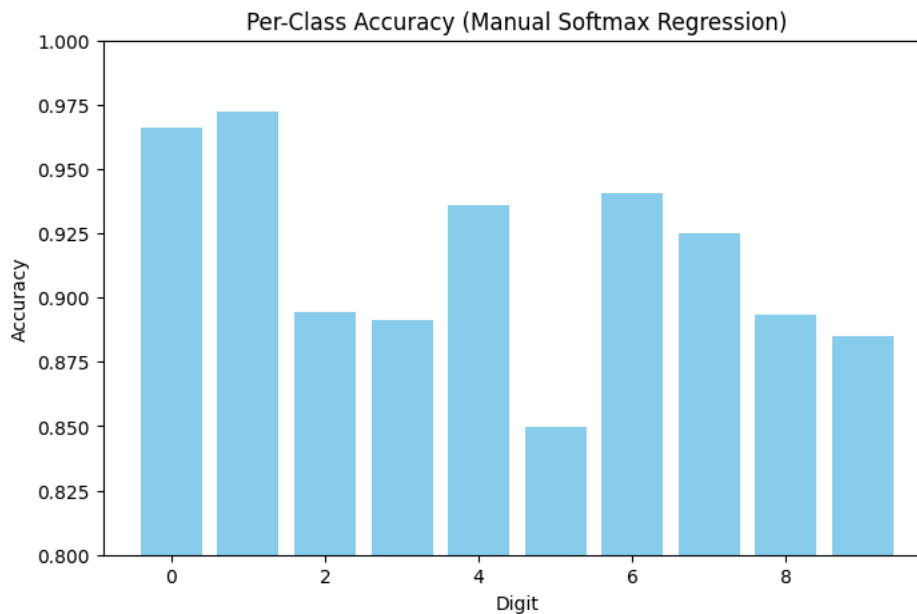
plt.figure(figsize=(8,5))
plt.bar(classes, per_class_acc, color='skyblue')
plt.xlabel('Digit')
plt.ylabel('Accuracy')
plt.title('Per-Class Accuracy (Manual Softmax Regression)')
plt.ylim(0.8, 1.0)
plt.show()

```

```

Accuracy for digit 0: 0.9662
Accuracy for digit 1: 0.9726
Accuracy for digit 2: 0.8942
Accuracy for digit 3: 0.8915
Accuracy for digit 4: 0.9358
Accuracy for digit 5: 0.8496
Accuracy for digit 6: 0.9408
Accuracy for digit 7: 0.9250
Accuracy for digit 8: 0.8933
Accuracy for digit 9: 0.8849

```



PyTorch Built-in Model

```

model_torch = nn.Sequential(
    nn.Linear(784, 10)
)

criterion = nn.CrossEntropyLoss()
optimizer = torch.optim.SGD(model_torch.parameters(), lr=0.01)

```

```

epochs_builtin = 100
for epoch in range(epochs_builtin):
    total_loss = 0
    correct = 0
    total = 0
    for X_batch, y_batch in train_loader:
        outputs = model_torch(X_batch)
        loss = criterion(outputs, y_batch)

        optimizer.zero_grad()
        loss.backward()
        optimizer.step()

        total_loss += loss.item()
        preds = outputs.argmax(dim=1)
        correct += (preds == y_batch).sum().item()
        total += y_batch.size(0)

    print(f"[PyTorch Model] Epoch {epoch+1}/{epochs_builtin} | "
          f"Loss: {total_loss/len(train_loader):.4f} | Train Acc: {correct/total:.4f}")

```

```

[PyTorch Model] Epoch 1/100 | Loss: 1.1756 | Train Acc: 0.7678
[PyTorch Model] Epoch 2/100 | Loss: 0.6592 | Train Acc: 0.8540
[PyTorch Model] Epoch 3/100 | Loss: 0.5466 | Train Acc: 0.8679
[PyTorch Model] Epoch 4/100 | Loss: 0.4924 | Train Acc: 0.8762
[PyTorch Model] Epoch 5/100 | Loss: 0.4590 | Train Acc: 0.8819
[PyTorch Model] Epoch 6/100 | Loss: 0.4360 | Train Acc: 0.8863
[PyTorch Model] Epoch 7/100 | Loss: 0.4188 | Train Acc: 0.8902
[PyTorch Model] Epoch 8/100 | Loss: 0.4054 | Train Acc: 0.8927
[PyTorch Model] Epoch 9/100 | Loss: 0.3947 | Train Acc: 0.8952
[PyTorch Model] Epoch 10/100 | Loss: 0.3854 | Train Acc: 0.8972

```

[PyTorch Model]	Epoch 11/100	Loss: 0.3777	Train Acc: 0.8987
[PyTorch Model]	Epoch 12/100	Loss: 0.3712	Train Acc: 0.9003
[PyTorch Model]	Epoch 13/100	Loss: 0.3652	Train Acc: 0.9018
[PyTorch Model]	Epoch 14/100	Loss: 0.3601	Train Acc: 0.9030
[PyTorch Model]	Epoch 15/100	Loss: 0.3558	Train Acc: 0.9040
[PyTorch Model]	Epoch 16/100	Loss: 0.3513	Train Acc: 0.9049
[PyTorch Model]	Epoch 17/100	Loss: 0.3476	Train Acc: 0.9058
[PyTorch Model]	Epoch 18/100	Loss: 0.3442	Train Acc: 0.9066
[PyTorch Model]	Epoch 19/100	Loss: 0.3409	Train Acc: 0.9066
[PyTorch Model]	Epoch 20/100	Loss: 0.3384	Train Acc: 0.9077
[PyTorch Model]	Epoch 21/100	Loss: 0.3352	Train Acc: 0.9084
[PyTorch Model]	Epoch 22/100	Loss: 0.3328	Train Acc: 0.9090
[PyTorch Model]	Epoch 23/100	Loss: 0.3305	Train Acc: 0.9091
[PyTorch Model]	Epoch 24/100	Loss: 0.3286	Train Acc: 0.9100
[PyTorch Model]	Epoch 25/100	Loss: 0.3261	Train Acc: 0.9107
[PyTorch Model]	Epoch 26/100	Loss: 0.3242	Train Acc: 0.9112
[PyTorch Model]	Epoch 27/100	Loss: 0.3225	Train Acc: 0.9113
[PyTorch Model]	Epoch 28/100	Loss: 0.3207	Train Acc: 0.9118
[PyTorch Model]	Epoch 29/100	Loss: 0.3191	Train Acc: 0.9124
[PyTorch Model]	Epoch 30/100	Loss: 0.3175	Train Acc: 0.9127
[PyTorch Model]	Epoch 31/100	Loss: 0.3158	Train Acc: 0.9130
[PyTorch Model]	Epoch 32/100	Loss: 0.3146	Train Acc: 0.9134
[PyTorch Model]	Epoch 33/100	Loss: 0.3130	Train Acc: 0.9132
[PyTorch Model]	Epoch 34/100	Loss: 0.3118	Train Acc: 0.9139
[PyTorch Model]	Epoch 35/100	Loss: 0.3105	Train Acc: 0.9141
[PyTorch Model]	Epoch 36/100	Loss: 0.3094	Train Acc: 0.9149
[PyTorch Model]	Epoch 37/100	Loss: 0.3083	Train Acc: 0.9150
[PyTorch Model]	Epoch 38/100	Loss: 0.3072	Train Acc: 0.9154
[PyTorch Model]	Epoch 39/100	Loss: 0.3059	Train Acc: 0.9161
[PyTorch Model]	Epoch 40/100	Loss: 0.3051	Train Acc: 0.9158
[PyTorch Model]	Epoch 41/100	Loss: 0.3040	Train Acc: 0.9166
[PyTorch Model]	Epoch 42/100	Loss: 0.3030	Train Acc: 0.9165
[PyTorch Model]	Epoch 43/100	Loss: 0.3021	Train Acc: 0.9168
[PyTorch Model]	Epoch 44/100	Loss: 0.3012	Train Acc: 0.9170
[PyTorch Model]	Epoch 45/100	Loss: 0.3003	Train Acc: 0.9172
[PyTorch Model]	Epoch 46/100	Loss: 0.2995	Train Acc: 0.9174
[PyTorch Model]	Epoch 47/100	Loss: 0.2990	Train Acc: 0.9176
[PyTorch Model]	Epoch 48/100	Loss: 0.2979	Train Acc: 0.9178
[PyTorch Model]	Epoch 49/100	Loss: 0.2973	Train Acc: 0.9179
[PyTorch Model]	Epoch 50/100	Loss: 0.2964	Train Acc: 0.9183
[PyTorch Model]	Epoch 51/100	Loss: 0.2957	Train Acc: 0.9184
[PyTorch Model]	Epoch 52/100	Loss: 0.2950	Train Acc: 0.9184
[PyTorch Model]	Epoch 53/100	Loss: 0.2942	Train Acc: 0.9186
[PyTorch Model]	Epoch 54/100	Loss: 0.2939	Train Acc: 0.9186
[PyTorch Model]	Epoch 55/100	Loss: 0.2930	Train Acc: 0.9192
[PyTorch Model]	Epoch 56/100	Loss: 0.2923	Train Acc: 0.9193
[PyTorch Model]	Epoch 57/100	Loss: 0.2917	Train Acc: 0.9195

```

with torch.no_grad():
    outputs_test = model_torch(X_test)
    preds_test = outputs_test.argmax(dim=1)
    acc_torch_test = (preds_test == y_test).float().mean().item()

print(f"\nPyTorch Built-in Softmax Regression - Test Accuracy: {acc_torch_test:.4f}")

```

PyTorch Built-in Softmax Regression - Test Accuracy: 0.9163

```

classes = list(range(10))
per_class_acc_torch = []

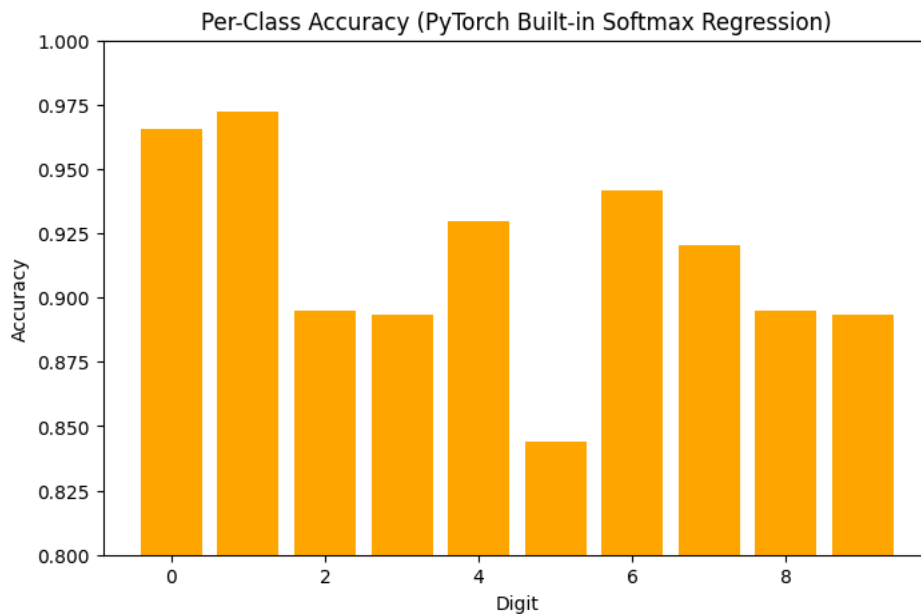
for c in classes:
    mask = (y_test == c)
    acc_c = (preds_test[mask] == y_test[mask]).float().mean().item()
    per_class_acc_torch.append(acc_c)
    print(f"Digit {c}: {acc_c:.4f}")

# Optional: visualize as bar chart
import matplotlib.pyplot as plt

plt.figure(figsize=(8,5))
plt.bar(classes, per_class_acc_torch, color='orange')
plt.xlabel('Digit')
plt.ylabel('Accuracy')
plt.title('Per-Class Accuracy (PyTorch Built-in Softmax Regression)')
plt.ylim(0.8, 1.0)
plt.show()

```

```
Digit 0: 0.9654
Digit 1: 0.9726
Digit 2: 0.8950
Digit 3: 0.8931
Digit 4: 0.9298
Digit 5: 0.8441
Digit 6: 0.9417
Digit 7: 0.9202
Digit 8: 0.8950
Digit 9: 0.8933
```



Compare Manual vs Built-in

```
manual_acc = np.array(per_class_acc)
torch_acc = np.array(per_class_acc_torch)

# --- Plot comparison ---
plt.figure(figsize=(10,6))
bar_width = 0.35
x = np.arange(len(classes))

plt.bar(x - bar_width/2, manual_acc, width=bar_width, label='Manual Softmax', color='skyblue')
plt.bar(x + bar_width/2, torch_acc, width=bar_width, label='PyTorch Built-in', color='orange')

plt.xlabel('Digit Class')
plt.ylabel('Accuracy')
plt.title('Comparison of Per-Class Accuracy: Manual vs PyTorch Softmax Regression')
plt.xticks(x, classes)
plt.ylim(0.8, 1.0)
plt.legend()
plt.grid(axis='y', linestyle='--', alpha=0.6)
plt.show()

# --- Print numerical summary ---
print("Digit | Manual Acc | PyTorch Acc | Difference")
print("-----")
for c in range(10):
    diff = abs(manual_acc[c] - torch_acc[c])
    print(f" {c:>2} | {manual_acc[c]:.4f} | {torch_acc[c]:.4f} | {diff:.4f}")
```


Comparison of Per-Class Accuracy: Manual vs PyTorch Softmax Regression

