Changing the Reference Level of a Factor in a GLM Model Does Not Change the Overall Model Results

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The second recommendation of the GFSSC: (https://docs.google.com/document/d/1p891XH3e12wDIZsUdKpWcBtpXqshs9a5/edit):

Other recommendations of the GFSSC on how to improve the method included:

- ...
- *Explore the sensitivity of the abundance index for vermillion rockfish to the reference level against which all the data used in the model are scaled. The logistic model makes an arbitrary choice for the reference level, and depending on the software, the reference levels for the fixed effects are probably associated with the factor levels from the data point that happens to occur first (or last) in the data set. If these reference levels are not supported by appreciable observations from all years in the series, then the resulting annual index values may not be reliable.*

to some degree follows the incomplete explanation (with R code) of Jim Greene (12th Mar 2022) here:

https://www.researchgate.net/post/Does_changing_the_reference_level_in_an_R_model_cause_any_difference_in_results

What is unique for a factor in a GLM are the estimated differences of the pairwise comparisons, regardless whether one level is set to zero or a different contrast treatment is used. That is why a 2 level factor only has a sign change when the other level is set to zero. However, the summary output of a GLM >2 level factor is only a subset of all pairwise factors. Say a factor has four levels (3 degrees of freedom). In R, the combination of 4 items taking 2 at a time is:

```
combn(4, 2)
     [,1] [,2] [,3] [,4] [,5] [,6]
[1,]    1    1    1    2    2    3
[2,]    2    3    4    3    4    4
```

If the first level is set to zero then the GLM summary for the factor with four levels will only show the estimated differences for 3 (equal to the degrees of freedom) of the pairwise

comparisons: (1, 2), (1, 3), and (1, 4) . However, the other three differences are not given. In R, the function stats::relevel() re-orders the levels of a factor so that the reference level specified is first (and hence set to zero with the default treatment contrasts). Two additional relevel() calls, one with a 2nd level reference and the other with a 3rd reference is one way to see all six pairwise differences.

Vermilion, thru 2019, has been caught on 118 of the 121 sites. Site is treated as a factor in the GLM and has 117 degrees of freedom. The number of different pairwise combinations is 6,903:

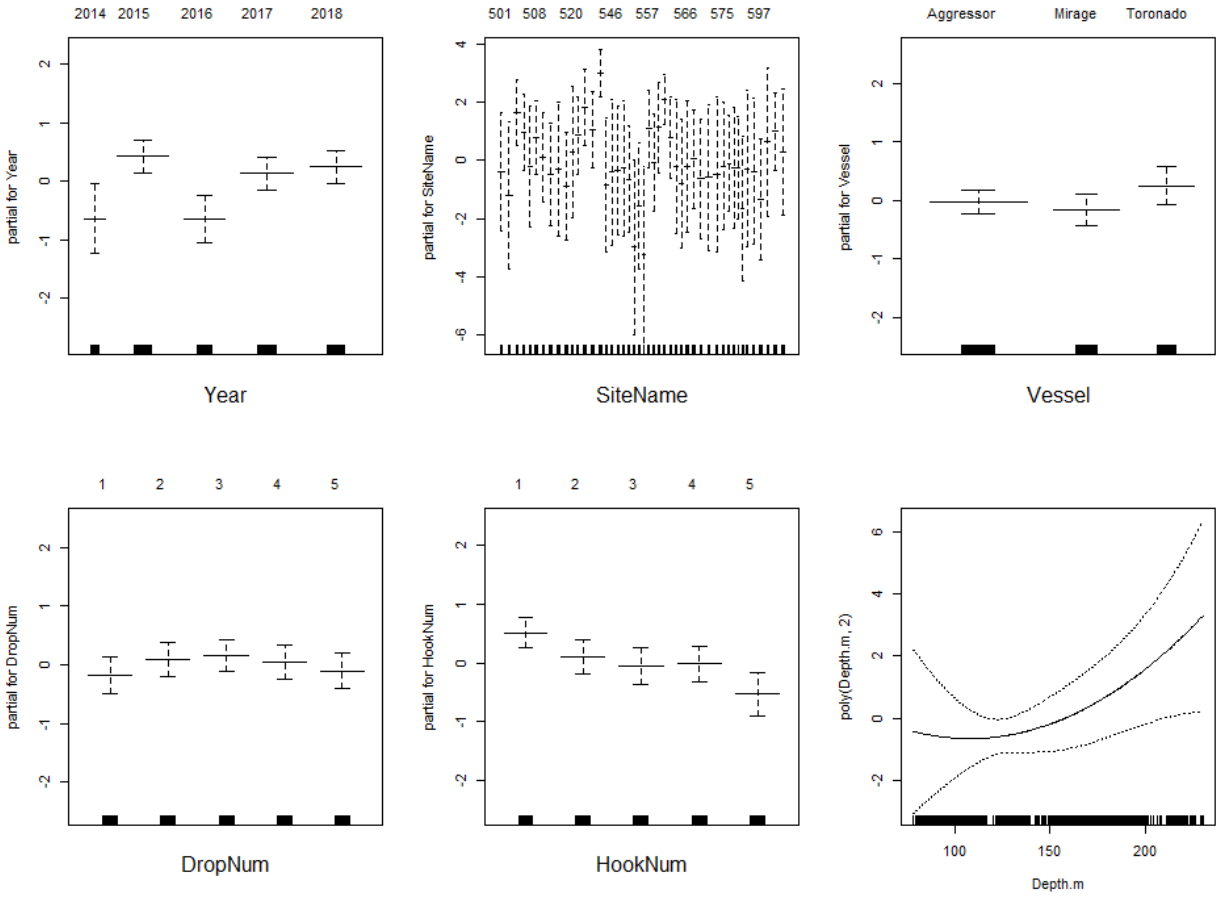
```
dim(combn(118, 2))
 [1]    2 6903
```

The 117 pairwise differences shown in the model summary are only a fraction of the total (127/6903= 0.0169).

Using the Cowcod data inside the CCA, a GLM model with default Site '501' set to zero gives the following figure, summary GLM output, and yearly index:

```
dim(combn(46, 2)) # 46 sites inside the CCA have caught Cowcod (out of 79 sites as of 2019).
[1]    2 1035

dev.new(); par(mfrow = c(2, 3))
plot.Gam(Glm <- glm(NumCow ~ Year + SiteName + Vessel + DropNum + HookNum + poly(Depth.m, 2),
data = CowCod.2018.CC.In, family = binomial), scale = 5, se = TRUE)
```


2014 2015 2016 2017 2018
partial for Year
Year
501 508 520 546 557 566 575 597
partial for SiteName
SiteName
Aggressor Mirage Toronado
partial for Vessel
Vessel
1 2 3 4 5
partial for DropNum
DropNum
1 2 3 4 5
partial for HookNum
HookNum
poly(Depth.m, 2)
100 150 200
Depth.m

```
summary(Glm)

Call:
glm(formula = NumCow ~ Year + SiteName + Vessel + DropNum + HookNum +
    poly(Depth.m, 2), family = binomial, data = CowCod.2018.CC.In)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-0.6527  -0.1734  -0.1089  -0.0777   3.6709

Coefficients:
                  Estimate Std. Error z value Pr(>|z|)
(Intercept)       -5.634174   1.122711  -5.018 5.21e-07 ***
Year2015           1.056229   0.343729   3.073  0.00212 **
Year2016          -0.009766   0.394643  -0.025  0.98026
Year2017           0.763814   0.359699   2.123  0.03371 *
Year2018           0.874550   0.361456   2.420  0.01554 *
SiteName502       -0.814525   1.858448  -0.438  0.66118
SiteName503        2.036290   1.211274   1.681  0.09274 .
SiteName506        1.349011   1.232385   1.095  0.27368
SiteName507        0.172081   1.435932   0.120  0.90461
SiteName508        1.169394   1.163709   1.005  0.31495
SiteName509        0.493452   1.269564   0.389  0.69751
SiteName512       -0.085415   1.411946  -0.060  0.95176
SiteName514        0.092868   1.495193   0.062  0.95047
SiteName518       -0.499450   1.530247  -0.326  0.74413
SiteName520        0.679355   1.464380   0.464  0.64270
SiteName525        1.249222   1.254567   0.996  0.31938
SiteName526        2.218295   1.300433   1.706  0.08804 .
SiteName527        1.447481   1.145896   1.263  0.20652
SiteName531        3.386144   1.068468   3.169  0.00153 **
SiteName543       -0.450674   1.604677  -0.281  0.77883
SiteName546       -0.020164   1.570024  -0.013  0.98975
SiteName547        0.026433   1.461238   0.018  0.98557
SiteName548        0.112273   1.499170   0.075  0.94030
SiteName549       -0.255500   1.465097  -0.174  0.86156
SiteName551       -2.586888   2.044829  -1.265  0.20584
SiteName553       -1.198341   1.652456  -0.725  0.46834
SiteName554       -2.871422   2.033039  -1.412  0.15784
SiteName557        1.473574   1.273923   1.157  0.24739
SiteName558        0.315725   1.449405   0.218  0.82756
SiteName559        1.521580   1.426693   1.067  0.28619
SiteName560        2.488208   1.051524   2.366  0.01797 *
SiteName561        1.180996   1.174898   1.005  0.31481
SiteName563        0.165243   1.491343   0.111  0.91177
SiteName564       -0.416478   1.546238  -0.269  0.78766
SiteName566        0.196739   1.473313   0.134  0.89377
SiteName568        0.434835   1.484229   0.293  0.76955
SiteName569       -0.251057   1.436903  -0.175  0.86130
SiteName573       -0.204481   1.566236  -0.131  0.89613
SiteName574       -0.104149   1.631159  -0.064  0.94909
SiteName575        0.198947   1.453351   0.137  0.89112
SiteName581        0.275513   1.398039   0.197  0.84377
SiteName582        0.139130   1.431287   0.097  0.92256
SiteName584        0.116750   1.502684   0.078  0.93807
SiteName586       -1.253449   1.750082  -0.716  0.47385
SiteName590        0.095607   1.636829   0.058  0.95342
SiteName592        0.012994   1.568782   0.008  0.99339
SiteName597       -0.946427   1.607126  -0.589  0.55593
SiteName598        1.016238   1.562395   0.650  0.51541
SiteName609        1.380375   1.149784   1.201  0.22993
SiteName611        0.675488   1.422831   0.475  0.63497
VesselMirage      -0.140488   0.212154  -0.662  0.50784
VesselToronado     0.282219   0.232909   1.212  0.22562
DropNum2           0.264774   0.243441   1.088  0.27676
DropNum3           0.335791   0.239661   1.401  0.16118
DropNum4           0.224934   0.246469   0.913  0.36144
DropNum5           0.077354   0.254926   0.303  0.76156
HookNum2          -0.408147   0.211154  -1.933  0.05324 .
HookNum3          -0.565099   0.219472  -2.575  0.01003 *
HookNum4          -0.529912   0.217654  -2.435  0.01491 *
```

```
HookNum5          -1.045472   0.255090  -4.098 4.16e-05 ***
poly(Depth.m, 2)1 93.620079  67.926761   1.378  0.16813
poly(Depth.m, 2)2 42.514555  36.643643   1.160  0.24596
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 2000.6  on 13313  degrees of freedom
Residual deviance: 1734.1  on 13252  degrees of freedom
AIC: 1858.1

Number of Fisher Scoring iterations: 8


Index <- rbind(cbind(Estimate = 0, 'Std. Error' = 0), summary(Glm)$coeff[2:5,1:2])
row.names(Index)[1] <- "Year2014"
Index
             Estimate Std. Error
Year2014  0.000000000  0.0000000
Year2015  1.056229189  0.3437288
Year2016 -0.009765786  0.3946432
Year2017  0.763813537  0.3596992
Year2018  0.874550269  0.3614564


anova(Glm, test ='Cp')
Analysis of Deviance Table

Model: binomial, link: logit

Response: NumCow

Terms added sequentially (first to last)

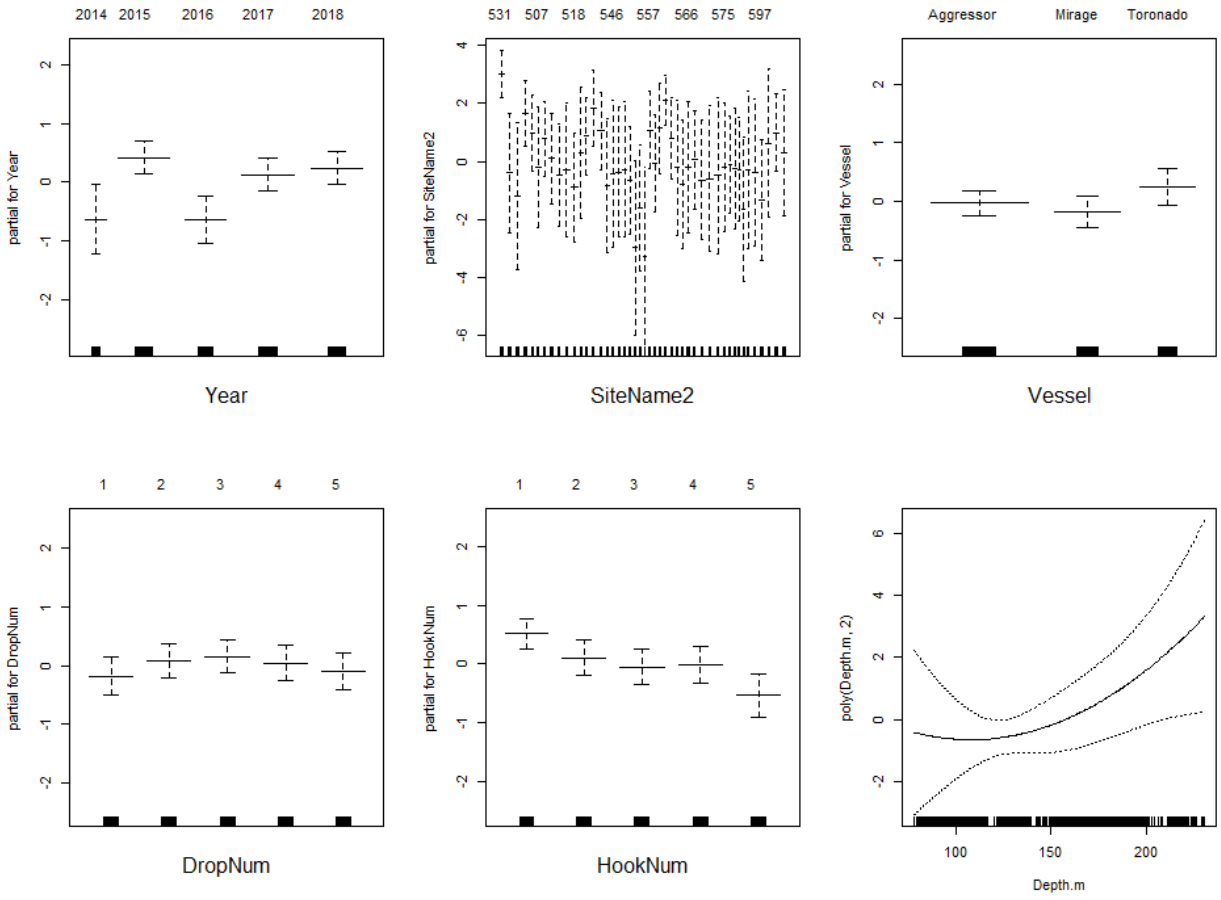

                 Df Deviance Resid. Df Resid. Dev     Cp
NULL                             13313     2000.6 2002.6
Year              4   13.870     13309     1986.7 1996.7
SiteName2        45  222.647     13264     1764.0 1864.1
Vessel            2    2.722     13262     1761.3 1865.3
DropNum           4    3.179     13258     1758.2 1870.2
HookNum           4   19.565     13254     1738.6 1858.6
poly(Depth.m, 2)  2    4.476     13252     1734.1 1858.1
```

Looking at the number of Cowcod by site, ‘531’ has the most Cowcod over all years. Each year had at least one Cowcod at this site.

Site ‘531’ is changed to the first level reference site using relevel():

```
# *** When using plot.Gam(), relevel() cannot be inside the formula call ***

dev.new(); par(mfrow = c(2, 3))
CowCod.2018.CC.In$SiteName2 <- relevel(SiteName, '531', '1')
plot.Gam(Glm2 <- glm(NumCow ~ Year + SiteName2 + Vessel + DropNum + HookNum + poly(Depth.m, 2),
data = CowCod.2018.CC.In, family = binomial), scale = 5, se = TRUE)
```




```
summary(Glm2)

Call:
glm(formula = NumCow ~ Year + SiteName2 + Vessel + DropNum +
    HookNum + poly(Depth.m, 2), family = binomial, data = CowCod.2018.CC.In)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-0.6527  -0.1734  -0.1089  -0.0777   3.6709

Coefficients:
                 Estimate Std. Error z value Pr(>|z|)
(Intercept)      -2.248030   0.537575  -4.182 2.89e-05 ***
Year2015          1.056229   0.343729   3.073 0.002120 **
Year2016         -0.009766   0.394643  -0.025 0.980258
Year2017          0.763814   0.359699   2.123 0.033714 *
Year2018          0.874550   0.361456   2.420 0.015541 *
SiteName2501     -3.386144   1.068468  -3.169 0.001529 **
SiteName2502     -4.200669   1.476923  -2.844 0.004452 **
SiteName2503     -1.349854   0.474461  -2.845 0.004441 **
SiteName2506     -2.037133   0.555720  -3.666 0.000247 ***
SiteName2507     -3.214063   1.028032  -3.126 0.001769 **
SiteName2508     -2.216750   0.653624  -3.391 0.000695 ***
SiteName2509     -2.892692   0.742032  -3.898 9.69e-05 ***
SiteName2512     -3.471559   0.840874  -4.129 3.65e-05 ***
SiteName2514     -3.293276   1.287006  -2.559 0.010501 *
SiteName2518     -3.885594   0.991926  -3.917 8.96e-05 ***
SiteName2520     -2.706789   1.226175  -2.208 0.027279 *
SiteName2525     -2.136922   0.582438  -3.669 0.000244 ***
SiteName2526     -1.167849   0.604267  -1.933 0.053277 .
SiteName2527     -1.938663   0.769830  -2.518 0.011792 *
```

```
SiteName2543      -3.836818   1.125865  -3.408 0.000655 ***
SiteName2546      -3.406308   1.415816  -2.406 0.016133 *
SiteName2547      -3.359712   1.213213  -2.769 0.005618 **
SiteName2548      -3.273871   1.293681  -2.531 0.011385 *
SiteName2549      -3.641644   0.910577  -3.999 6.35e-05 ***
SiteName2551      -5.973032   1.711711  -3.490 0.000484 ***
SiteName2553      -4.584485   1.172196  -3.911 9.19e-05 ***
SiteName2554      -6.257566   1.683728  -3.716 0.000202 ***
SiteName2557      -1.912570   0.586824  -3.259 0.001117 **
SiteName2558      -3.070419   0.864131  -3.553 0.000381 ***
SiteName2559      -1.864564   0.827379  -2.254 0.024223 *
SiteName2560      -0.897936   0.458058  -1.960 0.049960 *
SiteName2561      -2.205148   0.792357  -2.783 0.005385 **
SiteName2563      -3.220901   1.278174  -2.520 0.011738 *
SiteName2564      -3.802622   1.072149  -3.547 0.000390 ***
SiteName2566      -3.189405   1.243223  -2.565 0.010305 *
SiteName2568      -2.951309   0.915121  -3.225 0.001260 **
SiteName2569      -3.637201   1.030794  -3.529 0.000418 ***
SiteName2573      -3.590625   1.408715  -2.549 0.010807 *
SiteName2574      -3.490294   1.511444  -2.309 0.020930 *
SiteName2575      -3.187197   1.199238  -2.658 0.007868 **
SiteName2581      -3.110631   0.798406  -3.896 9.78e-05 ***
SiteName2582      -3.247014   1.034696  -3.138 0.001700 **
SiteName2584      -3.269394   0.949490  -3.443 0.000575 ***
SiteName2586      -4.639593   1.318601  -3.519 0.000434 ***
SiteName2590      -3.290537   1.515412  -2.171 0.029902 *
SiteName2592      -3.373150   1.411609  -2.390 0.016868 *
SiteName2597      -4.332572   1.108170  -3.910 9.24e-05 ***
SiteName2598      -2.369906   1.481759  -1.599 0.109735
SiteName2609      -2.005769   0.778629  -2.576 0.009994 **
SiteName2611      -2.710656   1.261599  -2.149 0.031667 *
VesselMirage      -0.140488   0.212154  -0.662 0.507843
VesselToronado     0.282219   0.232909   1.212 0.225622
DropNum2           0.264774   0.243441   1.088 0.276758
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DropNum5           0.077354   0.254926   0.303 0.761556
HookNum2          -0.408147   0.211154  -1.933 0.053245 .
HookNum3          -0.565099   0.219472  -2.575 0.010029 *
HookNum4          -0.529912   0.217654  -2.435 0.014906 *
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```

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poly(Depth.m, 2)  2    4.476     13252     1734.1 1858.1
```

Note the when setting the site with the most Cowcod as the reference, which also happens to be extreme compared to the norm, many more significant pairwise differences (out of the possible 1,035) are shown in the GLM summary compared with the summary when the default site of ‘501’ is set to zero. Also in the figures, observe that plot.Gam() does calculate a value for the levels which have been set to zero and Site ‘531’ can be seen to be moved to the first position in the second figure.

Even though the second model’s summary shows more significant pairwise differences, observe that the AIC, analysis of deviance table, yearly index estimates, and the standard error of those estimates are exactly the same for both models.

Lastly, note that for the binomial model, where the dispersion is assumed to be 1, Mallows' *Cp* statistic “which is closely related to AIC (and multiple of it if the dispersion is known).” Is equal to the AIC [see the R help for anova.glm()].