The Convex Hull

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Started project with a focus on trying to understand intuitively convex hull. After the explanation in class plus reading a bunch of different articles, I gained a better understanding of convex hull and its uses.

The convex hull is used for many things. It has been used in automotive engineering to help define the shape of a vehicle by determining a less complex model than a fully, accurately rendered car.

Like many things a models reduce complexity and ease computation. This is why complex hull calculations have been made for video games, or any 3D modeling for things like hit-boxes, and 3D collision meshes.

A convex hull is the smallest convex shape which contains a given set of points. The common example is a bunch of nails, which represent data points, surrounded by a rubber band. The rubber band connects the data points to form a convex hull. Typically, convex hulls are regular polygons, but can be irregularly shaped. The convex hull can look similar to gift-wrapping a toy in an oddly shaped box; the shape of the present does not reflect the true shape of the item, or set-points, inside the box.

The gift wrapping algorithm, also known as the Jarvis March, is an algorithm for computing the convex hull of a given set of points.

There are many different algorithms that are used. They are Jarvis March, Graham Scan, Chans Algorithm, Quick-Hull, Divide and Conquer, Monotone Chain. For purposes of this discussion, I successfully implemented three models:

- Gift wrapping, a.k.a. Jarvis March O(nh) - One of the simplest (although not the most time efficient in the worst case) planar algorithms. It has O(nh) time complexity where n is the number of points in the set, and h is the number of points in the hull. In the worst case, the complexity is (n^2) An advantage of the Jarvis March is that it is easy to program, to quickly identify a set of convex points in a subset to declare the convex hull.[1]

- Graham Scan O(n log n) - Graham Scan is a slightly more sophisticated, but much more efficient algorithm, published in 1972. If the points are already sorted by one of the coordinates or by the angle to a fixed vector, then the algorithm takes O(n) time A bit more complex than Jarvis March, but the trade-off is worth it if working with convex hulls a lot. The reduction from n^2 to n log n is massive for time savings. [1]

- Chans Algorithm O(n log h) - A simpler, optimal output-sensitive algorithm created in 1966. It combines gift wrapping with the execution of an O(n log n) algorithm (such as Graham scan) on a small subsets of the input. Chans Algorithm is very quick and is the fastest algorithm, but it works by combining two other algorithms (ex: Jarvis March and Graham Scan), which make Chans Algorithm complex and requires detailed examination of the 2 combined algorithms. A Chans Scan can be used when calculating convex hulls frequently, or if the data set is large. [1]

While the results of these three algorithms produce the same outcome, the implementation difference is important. Scientists are interested in them because they vary by speed. This is important when dealing with large data sets, time matters. A more efficient algorithm means spending less time on computations.

References

[1] Convex hull algorithms (2022/April/30) in Wikipedia https://en.wikipedia.org/wiki/Convex_hull_algorithms