HW 1: Signal Properties

ECE 3220: Signals and Systems
Johnathan Trachte
2/6/26

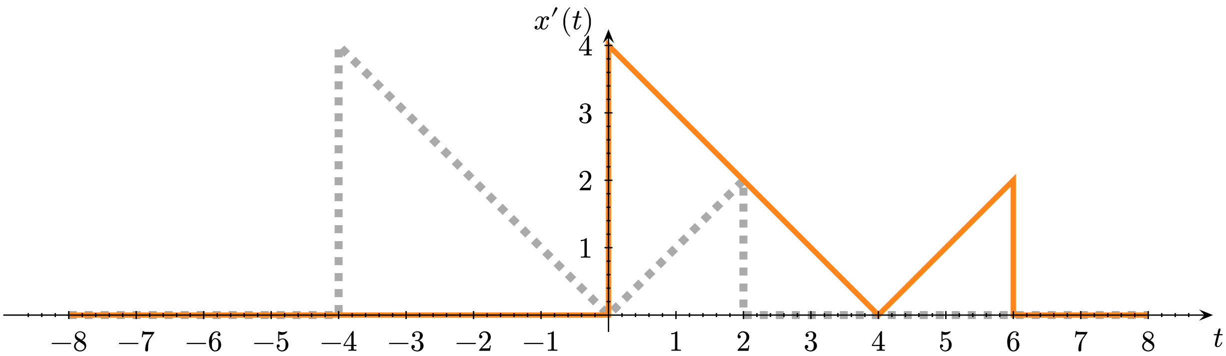
Problem 1

$x(t)$ is shown in gray on graphs 1.1 - 1.5. Graph $x'(t)$.

1.1 $x'(t) = x(t-4)$

$$x(t) = \begin{cases} 0, & t<-4 \\ -t, & -4<t<0 \\ t, & 0<t<2 \\ 0, & t>2 \end{cases}$$

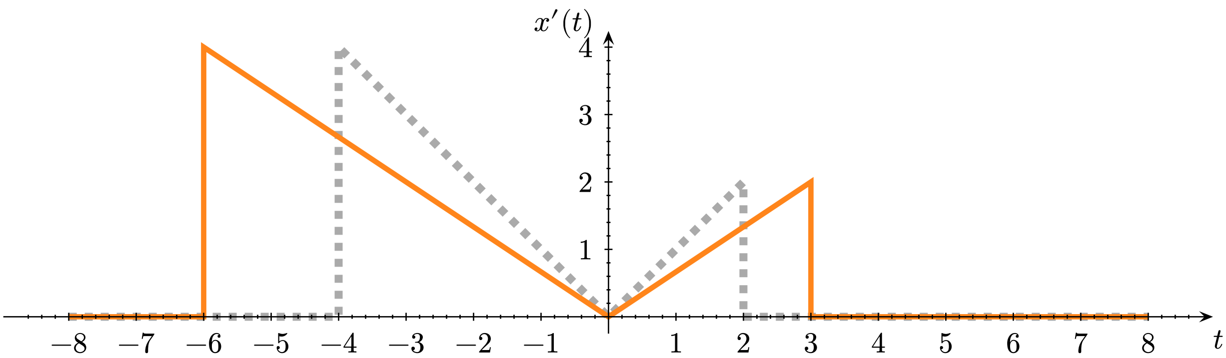
$$x(t-4) = \begin{cases} 0, & t-4<-4 \\ -(t-4), & -4<t-4<0 \\ t-4, & 0<t-4<2 \\ 0, & t-4>2 \end{cases} = \begin{cases} 0, & t<0 \\ -(t-4), & 0<t<4 \\ t-4, & 4<t<6 \\ 0, & t>6 \end{cases} = x'(t)$$



1.2 $x'(t) = x\left(\frac{t}{1.5}\right)$

Using $x(t)$ defined in problem 1.1,

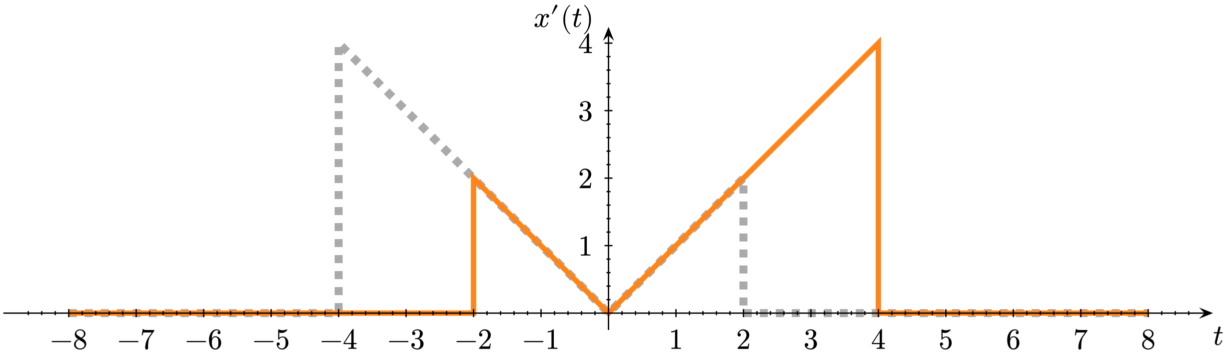
$$x\left(\tfrac{t}{1.5}\right) = \begin{cases} 0, & \frac{t}{1.5}<-4 \\ -\frac{t}{1.5}, & -4<\frac{t}{1.5}<0 \\ \frac{t}{1.5}, & 0<\frac{t}{1.5}<2 \\ 0, & \frac{t}{1.5}>2 \end{cases} = \begin{cases} 0, & t<-6 \\ -\left(\frac{t}{1.5}\right), & -6<t<0 \\ \frac{t}{1.5}, & 0<t<3 \\ 0, & t>3 \end{cases} = x'(t)$$



1.3 $x'(t) = x(-t)$

Using $x(t)$ defined in problem 1.1,

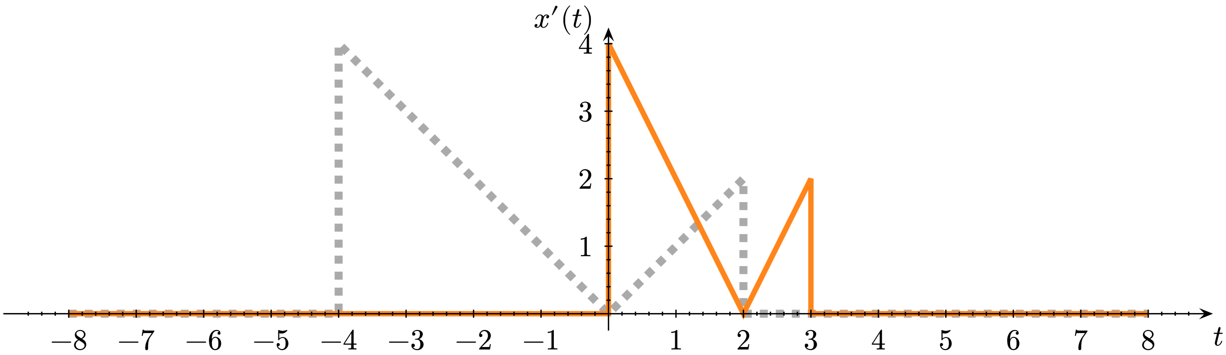
$$x(-t) = \begin{cases} 0, & -t<-4 \\ -(-t), & -4<-t<0 \\ -t, & 0<-t<2 \\ 0, & -t>2 \end{cases} = \begin{cases} 0, & t<-2 \\ -t, & -2<t<0 \\ t, & 0<t<4 \\ 0, & t>4 \end{cases} = x'(t)$$



1.4 $x'(t) = x(2t-4)$

Using $x(t-4)$ defined in problem 1.1,

$$x(2t-4) = \begin{cases} 0, & 2t<0 \\ -(2t-4), & 0<2t<4 \\ 2t-4, & 4<2t<6 \\ 0, & 2t>6 \end{cases} = \begin{cases} 0, & t<0 \\ -(2t-4), & 0<t<2 \\ 2t-4, & 2<t<3 \\ 0, & t>3 \end{cases} = x'(t)$$

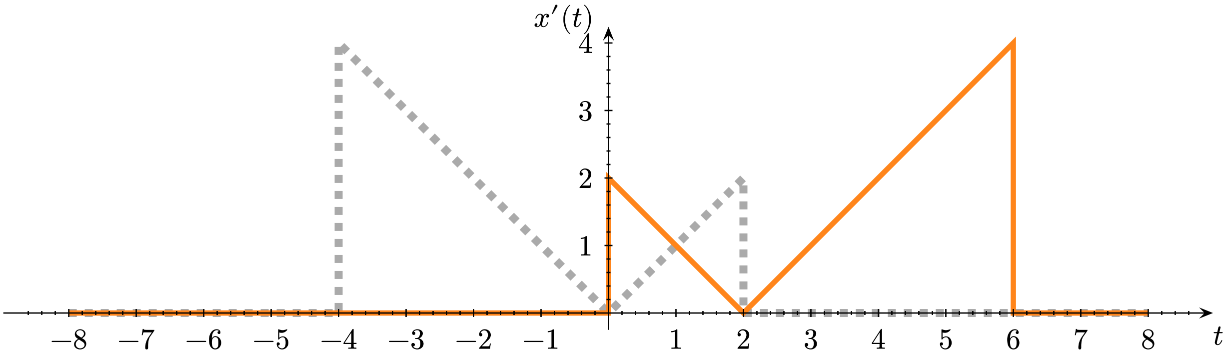


1.5 $x'(t) = x(2-t)$

Using $x(t)$ defined in problem 1.1,

$$x(t+2) = \begin{cases} 0, & t+2<-4 \\ -(t+2), & -4<t+2<0 \\ t+2, & 0<t+2<2 \\ 0, & t+2>2 \end{cases} = \begin{cases} 0, & t<-6 \\ -(t+2), & -6<t<-2 \\ t+2, & -2<t<0 \\ 0, & t>0 \end{cases}$$

$$x(-t+2) = \begin{cases} 0, & -t<-6 \\ -(-t+2), & -6<-t<-2 \\ -t+2, & -2<-t<0 \\ 0, & -t>0 \end{cases} = \begin{cases} 0, & t<0 \\ -t+2, & 0<t<2 \\ t+2, & 2<t<6 \\ 0, & t>6 \end{cases}$$



Problem 2

Evaluate each integral:

2.1 $\int_{-\infty}^{\infty} \delta(\tau)x(t-\tau)d\tau$

$\delta(\tau)$ has a value only when $\tau = 0$.

$\int_{-\infty}^{\infty} \delta(\tau)x(t-\tau)d\tau$

$= \int_{-\infty}^{\infty} \delta(\tau)x(t-0)d\tau$

$= x(t)\int_{-\infty}^{\infty} \delta(\tau)$

$= x(t) \cdot 1$

$$\boxed{\int_{-\infty}^{\infty} \delta(\tau)x(t-\tau)d\tau = x(t)}$$

2.2 $\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$

$\delta(t-\tau)$ has a value only when $\tau = t$.

$\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$

$= x(t)\int_{-\infty}^{\infty} \delta(t-\tau)d\tau$

$= x(t) \cdot 1$

$$\boxed{\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau = x(t)}$$

2.3 $\int_{-\infty}^{\infty} \delta(t)e^{-j\omega t}dt$

$\delta(t)$ has a value only when $t = 0$.

$\int_{-\infty}^{\infty} \delta(t)e^{-j\omega t}dt$

$= \int_{-\infty}^{\infty} \delta(t)e^{-j\omega \cdot 0}dt$

$= e^{-j\omega \cdot 0}\int_{-\infty}^{\infty} \delta(t)dt$

$= 1 \cdot 1$

$$\boxed{\int_{-\infty}^{\infty} \delta(t)e^{-j\omega t}dt = 1}$$

2.4 $\int_{-\infty}^{\infty} \delta(2t-3)\sin(\pi t)dt$

$\delta(2t-3)$ has a value only when $2t = 3$, or $t = 1.5$.

$\int_{-\infty}^{\infty} \delta(2t-3)\sin(\pi t)dt$

$= \int_{-\infty}^{\infty} \delta(2t-3)\sin(\pi \cdot 1.5)dt$

$= \sin(\pi \cdot 1.5) \int_{-\infty}^{\infty} \delta(2t-3)dt$

$= \sin(\pi \cdot 1.5) \cdot 1$

$= \sin(\pi \cdot 1.5)$

$$\boxed{\int_{-\infty}^{\infty} \delta(2t-3)\sin(\pi t)dt = 0.0822 = -1 \text{ rad}}$$

2.5 $\int_{-\infty}^{\infty} \delta(t+3)e^{-t}dt$

$\delta(t+3)$ has a value only when $t = -3$.

$\int_{-\infty}^{\infty} \delta(t+3)e^{-t}dt$

$= e^{-(-3)} \int_{-\infty}^{\infty} \delta(t+3)dt$

$= e^3 \cdot 1$

$= e^3$

$$\boxed{\int_{-\infty}^{\infty} \delta(t+3)e^{-t}dt = 20.086}$$

2.6 $\int_{-\infty}^{\infty} (t^3+4)\delta(1-t)dt$

$\delta(1-t)$ has a value only when $t = 1$.

$\int_{-\infty}^{\infty} (t^3+4)\delta(1-t)dt$

$= \int_{-\infty}^{\infty} (1^3+4)\delta(1-t)dt$

$= (1^3+4) \int_{-\infty}^{\infty} \delta(1-t)dt$

$= (1^3+4) \cdot 1$

$= (1^3+4)$

$$\boxed{\int_{-\infty}^{\infty} (t^3+4)\delta(1-t)dt = 5}$$

2.7 $\int_{-\infty}^{\infty} x(2-t)\delta(3-t)dt$

$\delta(3-t)$ has a value only when $t = 3$.

$\int_{-\infty}^{\infty} x(2-t)\delta(3-t)dt$

$= \int_{-\infty}^{\infty} x(2-3)\delta(3-t)dt$

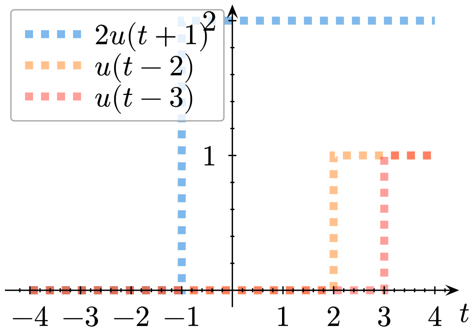
$= (x(2-3)) \int_{-\infty}^{\infty} \delta(3-t)dt$

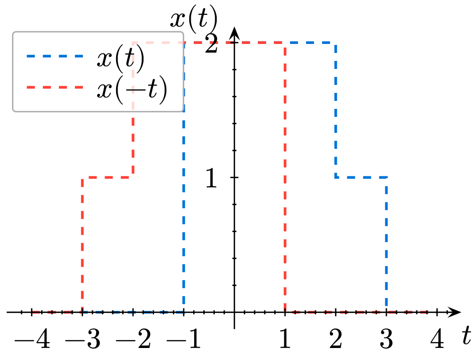
$= (x(2-3)) \cdot 1$

$$\boxed{\int_{-\infty}^{\infty} x(2-t)\delta(3-t)dt = x(-1)}$$

Problem 3

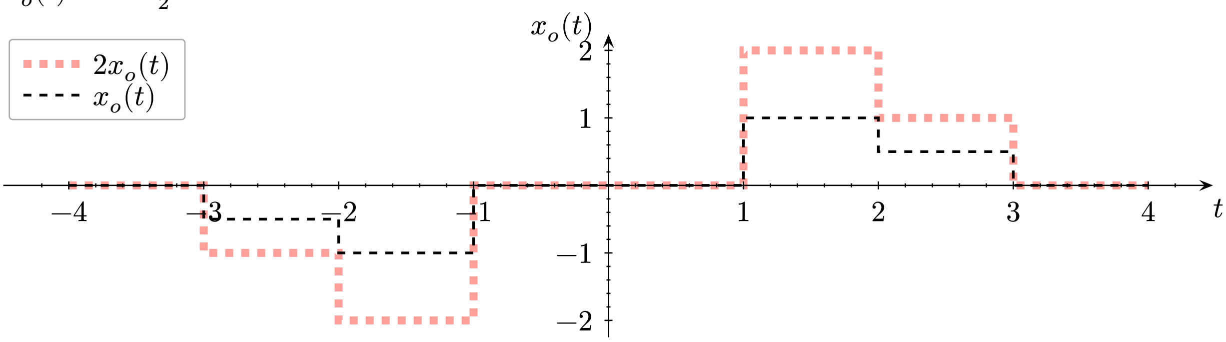
$x(t) = 2u(t+1) - u(t-2) - u(t-3)$.

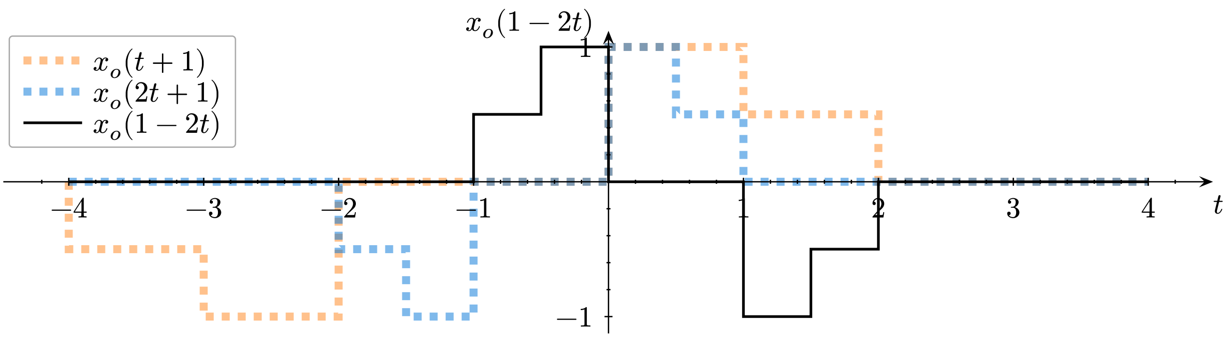




3.1 Letting $x_o(t)$ designate the odd portion of $x(t)$, graph $x_o(1-2t)$.

$x_o(t) = \frac{x(t)-x(-t)}{2}$





3.2 Letting $x_e(t)$ designate the even portion of $x(t)$, graph $x_e\left(2+\frac{t}{3}\right)$

$x_e(t) = \frac{x(t)+x(-t)}{2}$