

HW 1: Signal Properties

ECE 3220: Signals and Systems

Johnathan Trachte

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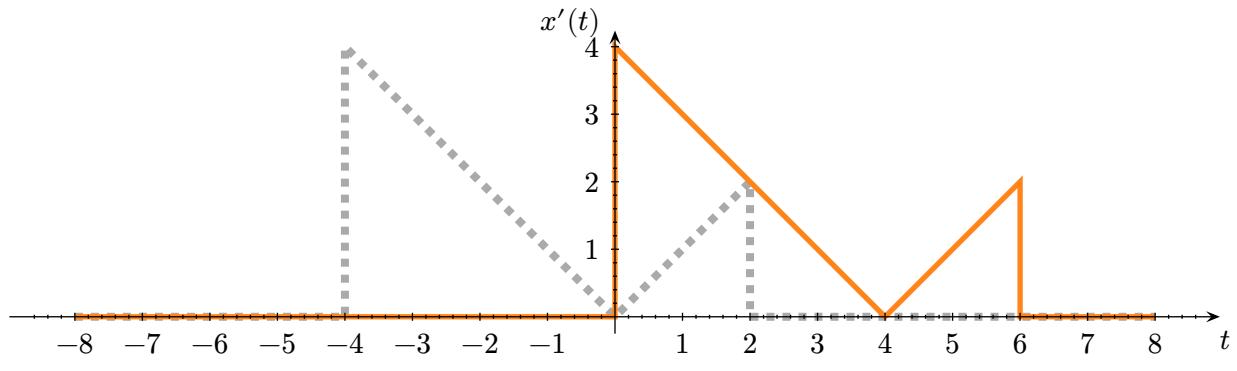
PROBLEM 1

$x(t)$ is shown in gray on graphs 1.1 - 1.5. Graph $x'(t)$.

$$1.1 \quad x'(t) = x(t - 4)$$

$$x(t) = \begin{cases} 0, & t < -4 \\ -t, & -4 < t < 0 \\ t, & 0 < t < 2 \\ 0, & t > 2 \end{cases}$$

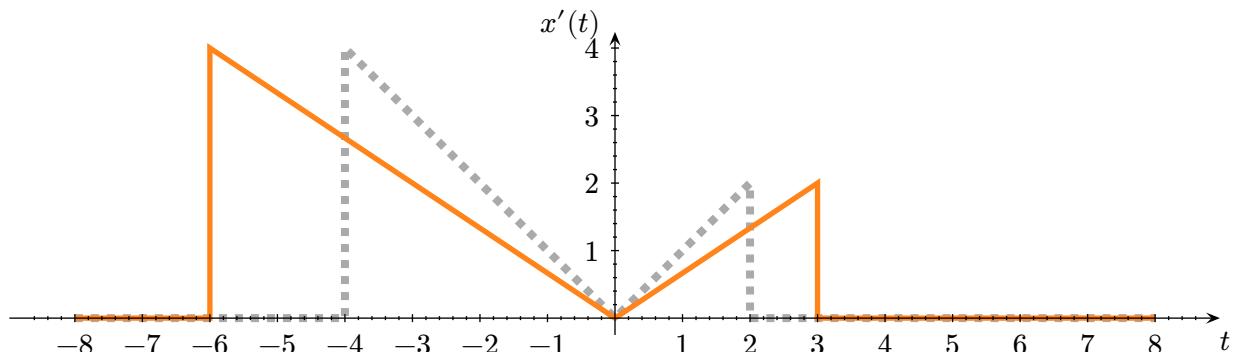
$$x(t - 4) = \begin{cases} 0, & t - 4 < -4 \\ -(t-4), & -4 < t - 4 < 0 \\ t-4, & 0 < t - 4 < 2 \\ 0, & t - 4 > 2 \end{cases} = \begin{cases} 0, & t < 0 \\ -(t-4), & 0 < t < 4 \\ t-4, & 4 < t < 6 \\ 0, & t > 6 \end{cases} = x'(t)$$



$$1.2 \quad x'\left(\frac{t}{1.5}\right) = x\left(\frac{t}{1.5} - 4\right)$$

Using $x(t)$ defined in problem 1.1,

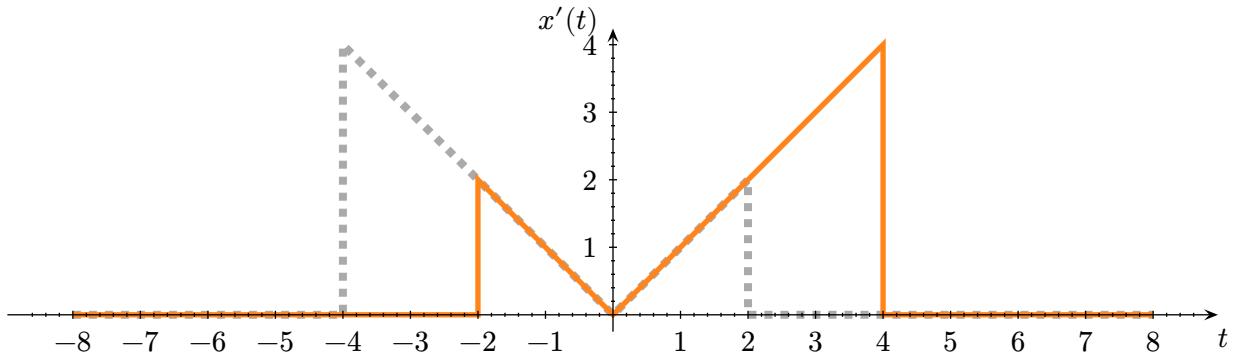
$$x\left(\frac{t}{1.5}\right) = \begin{cases} 0, & \frac{t}{1.5} < -4 \\ -\frac{t}{1.5}, & -4 < \frac{t}{1.5} < 0 \\ \frac{t}{1.5}, & 0 < \frac{t}{1.5} < 2 \\ 0, & \frac{t}{1.5} > 2 \end{cases} = \begin{cases} 0, & t < -6 \\ -\left(\frac{t}{1.5}\right), & -6 < t < 0 \\ \frac{t}{1.5}, & 0 < t < 3 \\ 0, & t > 3 \end{cases} = x'\left(\frac{t}{1.5}\right) = x'(t)$$



$$1.3 \quad x'(t) = x(-t)$$

Using $x(t)$ defined in problem 1.1,

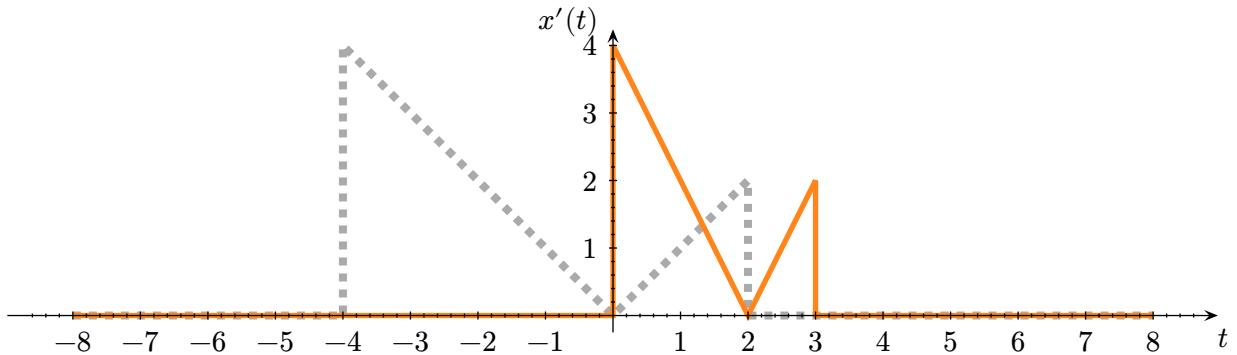
$$x(-t) = \begin{cases} 0, & -t < -4 \\ -(-t), & -4 < -t < 0 \\ -t, & 0 < -t < 2 \\ 0, & -t > 2 \end{cases} = \begin{cases} 0, & t < -2 \\ -t, & -2 < t < 0 \\ t, & 0 < t < 4 \\ 0, & t > 4 \end{cases} = x'(t)$$



$$1.4 \quad x'(t) = x(2t - 4)$$

Using $x(t - 4)$ defined in problem 1.1,

$$x(2t - 4) = \begin{cases} 0, & 2t < 0 \\ -(2t-4), & 0 < 2t < 4 \\ 2t-4, & 4 < 2t < 6 \\ 0, & 2t > 6 \end{cases} = \begin{cases} 0, & t < 0 \\ -(2t-4), & 0 < t < 2 \\ 2t-4, & 2 < t < 3 \\ 0, & t > 3 \end{cases} = x'(t)$$

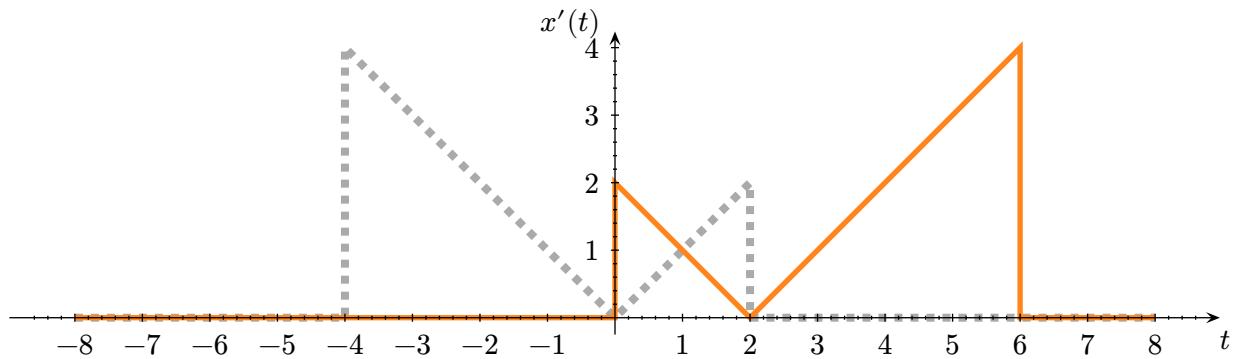


$$1.5 \quad x'(t) = x(2-t)$$

Using $x(t)$ defined in problem 1.1,

$$x(t+2) = \begin{cases} 0, & t+2 < -4 \\ -(t+2), & -4 < t+2 < 0 \\ t+2, & 0 < t+2 < 2 \\ 0, & t+2 > 2 \end{cases} = \begin{cases} 0, & t < -6 \\ -(t+2), & -6 < t < -2 \\ t+2, & -2 < t < 0 \\ 0, & t > 0 \end{cases}$$

$$x(-t+2) = \begin{cases} 0, & -t < -6 \\ -(-t+2), & -6 < -t < -2 \\ -t+2, & -2 < -t < 0 \\ 0, & -t > 0 \end{cases} = \begin{cases} 0, & t < 0 \\ -t+2, & 0 < t < 2 \\ t+2, & 2 < t < 6 \\ 0, & t > 6 \end{cases}$$



PROBLEM 2

Evaluate each integral:

$$2.1 \quad \int_{-\infty}^{\infty} \delta(\tau)x(t - \tau)d\tau$$

$\delta(\tau)$ has a value only when $\tau = 0$.

$$\begin{aligned} & \int_{-\infty}^{\infty} \delta(\tau)x(t - \tau)d\tau \\ &= \int_{-\infty}^{\infty} \delta(\tau)x(t - 0)d\tau \\ &= x(t) \int_{-\infty}^{\infty} \delta(\tau) \\ &= x(t) \cdot 1 \end{aligned}$$

$$\int_{-\infty}^{\infty} \delta(\tau)x(t - \tau)d\tau = x(t)$$

$$2.2 \quad \int_{-\infty}^{\infty} x(\tau)\delta(t - \tau)d\tau$$

$\delta(t - \tau)$ has a value only when $\tau = t$.

$$\begin{aligned} & \int_{-\infty}^{\infty} x(\tau)\delta(t - \tau)d\tau \\ &= x(t) \int_{-\infty}^{\infty} \delta(t - \tau)d\tau \\ &= x(t) \cdot 1 \end{aligned}$$

$$\int_{-\infty}^{\infty} x(\tau)\delta(t - \tau)d\tau = x(t)$$

$$2.3 \quad \int_{-\infty}^{\infty} \delta(t)e^{-j\omega t}dt$$

$\delta(t)$ has a value only when $t = 0$.

$$\begin{aligned} & \int_{-\infty}^{\infty} \delta(t)e^{-j\omega t}dt \\ &= \int_{-\infty}^{\infty} \delta(t)e^{-j\omega \cdot 0}dt \\ &= e^{-j\omega \cdot 0} \int_{-\infty}^{\infty} \delta(t)dt \\ &= 1 \cdot 1 \end{aligned}$$

$$\int_{-\infty}^{\infty} \delta(t)e^{-j\omega t}dt = 1$$

$$2.4 \quad \int_{-\infty}^{\infty} \delta(2t - 3) \sin(\pi t) dt$$

$\delta(2t - 3)$ has a value only when $2t = 3$, or $t = 1.5$.

$$\begin{aligned} & \int_{-\infty}^{\infty} \delta(2t - 3) \sin(\pi t) dt \\ &= \int_{-\infty}^{\infty} \delta(2t - 3) \sin(\pi \cdot 1.5) dt \\ &= \sin(\pi \cdot 1.5) \int_{-\infty}^{\infty} \delta(2t - 3) dt \\ &= \sin(\pi \cdot 1.5) \cdot 1 \\ &= \sin(\pi \cdot 1.5) \end{aligned}$$

$$\boxed{\int_{-\infty}^{\infty} \delta(2t - 3) \sin(\pi t) dt = 0.0822 = -1 \text{ rad}}$$

$$2.5 \quad \int_{-\infty}^{\infty} \delta(t + 3) e^{-t} dt$$

$\delta(t + 3)$ has a value only when $t = -3$.

$$\begin{aligned} & \int_{-\infty}^{\infty} \delta(t + 3) e^{-t} dt \\ &= e^{-(-3)} \int_{-\infty}^{\infty} \delta(t + 3) dt \\ &= e^3 \cdot 1 \\ &= e^3 \end{aligned}$$

$$\boxed{\int_{-\infty}^{\infty} \delta(t + 3) e^{-t} dt = 20.086}$$

$$2.6 \quad \int_{-\infty}^{\infty} (t^3 + 4) \delta(1 - t) dt$$

$\delta(1 - t)$ has a value only when $t = 1$.

$$\begin{aligned} & \int_{-\infty}^{\infty} (t^3 + 4) \delta(1 - t) dt \\ &= \int_{-\infty}^{\infty} (1^3 + 4) \delta(1 - t) dt \\ &= (1^3 + 4) \int_{-\infty}^{\infty} \delta(1 - t) dt \\ &= (1^3 + 4) \cdot 1 \\ &= (1^3 + 4) \end{aligned}$$

$$\boxed{\int_{-\infty}^{\infty} (t^3 + 4) \delta(1 - t) dt = 5}$$

$$2.7 \quad \int_{-\infty}^{\infty} x(2-t)\delta(3-t)dt$$

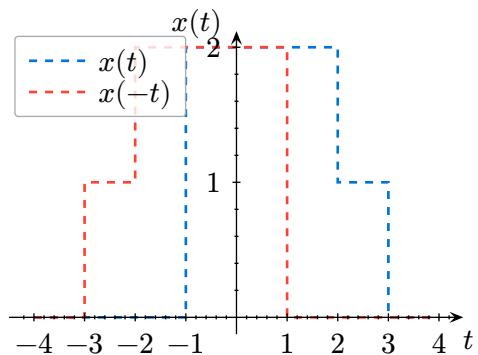
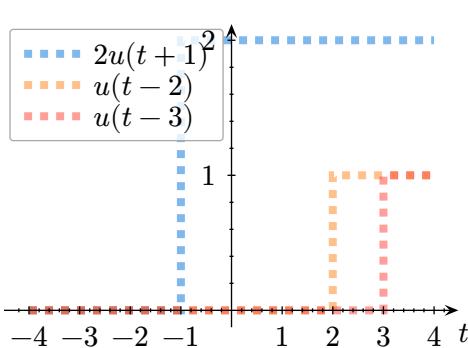
$\delta(3-t)$ has a value only when $t = 3$.

$$\begin{aligned} & \int_{-\infty}^{\infty} x(2-t)\delta(3-t)dt \\ &= \int_{-\infty}^{\infty} x(2-3)\delta(3-t)dt \\ &= (x(2-3)) \int_{-\infty}^{\infty} \delta(3-t)dt \\ &= (x(2-3)) \cdot 1 \end{aligned}$$

$$\boxed{\int_{-\infty}^{\infty} x(2-t)\delta(3-t)dt = x(-1)}$$

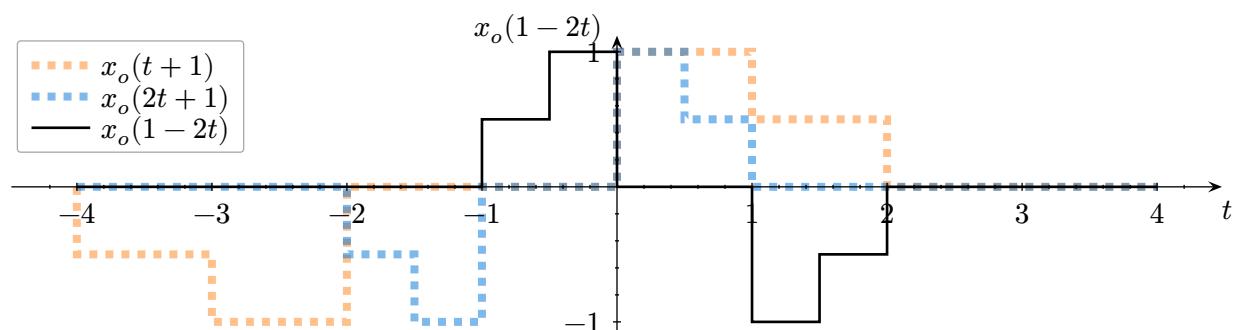
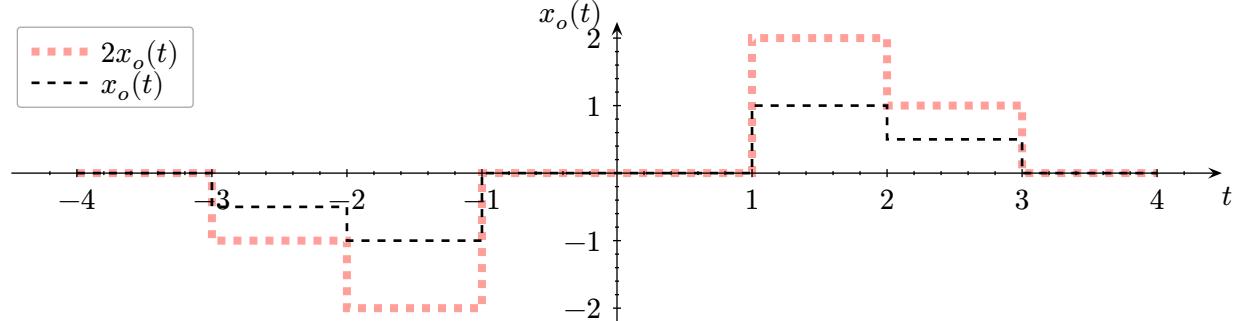
PROBLEM 3

$$x(t) = 2u(t+1) - u(t-2) - u(t-3).$$



3.1 Letting $x_o(t)$ designate the odd portion of $x(t)$, graph $x_o(1-2t)$.

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$



3.2 Letting $x_e(t)$ designate the even portion of $x(t)$, graph $x_e(2 + \frac{t}{3})$

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

