HW 2: System Properties

ECE 3220: Signals and Systems
Johnathan Trachte
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Problem 1. An LTI system is specified by the equation:

$$(D^2 + 5D + 6)y(t) = (D + 1)x(t)$$

A. Find the characteristic polynomial, equation, roots, and modes of the system.

By observation,

Characteristic polynomial: $Q(\lambda) = \lambda^2 + 5\lambda + 6$

Characteristic equation: $\lambda^2 + 5\lambda + 6 = (\lambda + 2)(\lambda + 3) = 0$

Characteristic roots: $\lambda_1 = -2, \lambda_2 = -3$

Characteristic modes: $y(t) = c_1 e^{-2t} + c_2 e^{-3t}$

B. Find the zero input components of the response $y_0(t)$ if the initial conditions are $y_0(0^-) = 2$ and $y_0'(0^-) = -1$

$y(t) = c_1 e^{-2t} + c_2 e^{-3t}$

$y(0) = c_1 e^{-2\cdot 0} + c_2 e^{-3\cdot 0}$

$2 = c_1 + c_2$

$y'(t) = -2c_1 e^{-2t} - 3c_2 e^{-3t}$

$y'(0) = -2c_1 e^{-2\cdot 0} - 3c_2 e^{-3\cdot 0}$

$-1 = -2c_1 - 3c_2$

Equations $2 = c_1 + c_2$ and $-1 = -2c_1 - 3c_2$ can now be solved as a system of equations:

$c_1 = 2 - c_2 \rightarrow -1 = -2(2 - c_2) - 3c_2$

$-1 = -4 + 2c_2 - 3c_2$

$3 = -1c_2$

$c_2 = -3$

$c_1 = 2 - (-3)$

$c_1 = 5$

$\boxed{y_0(t) = (5e^{-2t} - 3e^{-3t})u(t)}$

Problem 2. Find the impulse response of an LTIC system specified by the equation:

$$(D^2 + 6D + 9)y(t) = (2D + 9)x(t)$$

$h(t) = b_0\delta(t) + [P(D)y_n(t)]u(t)$

By observation, $P(D) = 2D + 9$; because $P(D)$ contains no D^2 term, $b_0 = 0$.

Solve for $y_n(t)$:

$\lambda^2 + 6\lambda + 9 = (\lambda + 3)^2 = 0 \to \lambda_1 = \lambda_2 = \lambda = -3$

$y_n(0) = 0$, $y_n'(0) = 1$

$y_n(t) = c_1 e^{-3t} + c_2 t e^{-3t}$

$0 = c_1 e^{-3 \cdot 0} + c_2 e^{-3 \cdot 0}$

$0 = c_1 + c_2$

$y_n'(t) = -3c_1 e^{-3t} + c_2 e^{-3t} - 3c_2 t e^{-3t}$

$1 = -3c_1 e^{-3 \cdot 0} + c_2 e^{-3 \cdot 0} - 3c_2 \cdot 0 \cdot e^{-3 \cdot 0}$

$1 = -3c_1 + c_2$

Equations $0 = c_1 + c_2$ and $1 = -3c_1 + c_2$ can now be solved as a system of equations:

$c_1 = -c_2 \to 1 = -3(-c_2) + c_2$

$1 = 3c_2 + c_2$

$1 = 4c_2$

$c_2 = \frac{1}{4}$

$c_1 = -\frac{1}{4}$

$$
\begin{aligned}
y_n(t) &= -\tfrac{1}{4}e^{-3t} + \tfrac{1}{4}te^{-3t} \\
h(t) &= 0 \cdot \delta(t) + \left[(2D + 9)\left(\tfrac{-e^{-3t}}{4} + \tfrac{te^{-3t}}{4}\right)\right]u(t) \\
&= \left[2\frac{d\left(-\frac{e^{-3t}}{4}\right)}{dt} + 2\frac{d\left(\frac{te^{-3t}}{4}\right)}{dt} - \frac{9e^{-3t}}{4} + \frac{9te^{-3t}}{4}\right]u(t) \\
&= \left[2\left(\tfrac{3e^{-3t}}{4}\right) + 2\left(\tfrac{e^{-3t} - 3te^{-3t}}{4}\right) - \tfrac{9e^{-3t}}{4} + \tfrac{9te^{-3t}}{4}\right]u(t) \\
&= \left[\tfrac{6e^{-3t} + 2e^{-3t} - 6te^{-3t} - 9e^{-3t} + 9te^{-3t}}{4}\right]u(t) \\
&= \left[\tfrac{-e^{-3t} + 3te^{-3t}}{4}\right]u(t)
\end{aligned}
$$

$$\boxed{h(t) = \left[\tfrac{-e^{-3t} + 3te^{-3t}}{4}\right]u(t)}$$

Problem 3. The unit impulse of an LTIC system is given by $h(t) = e^{-t}u(t)$. Determine the zero-state system response if the input $x(t)$ is:

A. $u(t)$

For two causal systems (which both $h(t)$ and $x(t)$ are, based on the $u(t)$ term in each function), $y_{\text{ZSR}}(t) = x(t) * h(t) = \int_0^t x(\tau)h(t-\tau)d\tau$. Therefore,

$$\begin{aligned} y_{\text{ZSR}}(t) &= u(t) * e^{-t}u(t) \\ &= \int_0^t u(\tau)e^{-(t-\tau)}u(t-\tau)d\tau \\ &= \int_0^t e^{-t} \cdot e^{-\tau}d\tau \\ &= e^{-t}\int_0^t e^{-\tau}d\tau \\ &= e^{-t} \cdot [-e^{-\tau}]_0^t \\ &= e^{-t} \cdot [-e^{-t} - (-1)] \\ &= e^{-t} - e^{-2t} \end{aligned}$$

$$\boxed{y_{\text{ZSR}}(t) = (e^{-t} - e^{-2t})u(t)}$$

B. $e^{-t}u(t)$

$$\begin{aligned} y_{\text{ZSR}}(t) &= e^{-t}u(t) * e^{-t}u(t) \\ &= \int_0^t e^{-\tau}u(\tau)e^{-(t-\tau)}u(t-\tau)d\tau \\ &= \int_0^t e^{-\tau}e^{\tau}e^{-t}d\tau \\ &= e^{-t}\int_0^t e^0 d\tau \\ &= e^{-t} \cdot [\tau]_0^t \end{aligned}$$

$$\boxed{y_{\text{ZSR}}(t) = (te^{-t})u(t)}$$

C. $e^{-2t}u(t)$

$$\begin{aligned} y_{\text{ZSR}}(t) &= e^{-2t}u(t) * e^{-t}u(t) \\ &= \int_0^t e^{-2\tau}u(\tau)e^{-(t-\tau)}u(t-\tau)d\tau \\ &= \int_0^t e^{-2\tau}e^{\tau}e^{-t}d\tau \\ &= e^{-t}\int_0^t e^{-\tau}d\tau \\ &= e^{-t}[-e^{-\tau}]_0^t \\ &= e^{-t}[-e^{-t} - (-1)] \\ &= e^{-t} - e^{-2t} \end{aligned}$$

$$\boxed{y_{\text{ZSR}}(t) = (e^{-t} - e^{-2t})u(t)}$$

D. $\sin(3t)u(t)$

$$
\begin{aligned}
y_{\text{ZSR}}(t) &= \sin(3t)u(t) * e^{-t}u(t) \\
&= \int_0^t \sin(3\tau)u(\tau)e^{-(t-\tau)}u(t-\tau)d\tau \\
&= \int_0^t \sin(3\tau)e^{-t}e^{\tau}d\tau \\
&= e^{-t}\int_0^t \sin(3\tau)e^{\tau}d\tau \\
&= \left[e^{-t}e^{\tau}\sin(3\tau)\right]_{\tau=0}^{\tau=t} - 3e^{-t}\int_0^t \cos(3\tau)e^{\tau}d\tau
\end{aligned}
$$

Let $u_1 = \sin(3\tau)$ and $v_1 = e^{\tau}$; then $du_1 = 3\cos(3\tau)d\tau$ and $dv_1 = e^{\tau}d\tau$.

$$
= \left[e^{-t}\left(e^{\tau}\sin(3\tau) - 3e^{\tau}\cos(3\tau)\right)\right]_{\tau=0}^{\tau=t} - 9e^{-t}\int_0^t \sin(3\tau)e^{\tau}d\tau
$$

Let $u_2 = \cos(3\tau)$ and $v_2 = e^{\tau}$; then $du_2 = -3\sin(3\tau)d\tau$ and $dv_2 = e^{\tau}d\tau$.

$$
\begin{aligned}
e^{-t}\int_0^t \sin(3\tau)e^{\tau}d\tau &= \left[e^{-t}\left(e^{\tau}\sin(3\tau) - 3e^{\tau}\cos(3\tau)\right)\right]_{\tau=0}^{\tau=t} - 9e^{-t}\int_0^t \sin(3\tau)e^{\tau}d\tau \\
10e^{-t}\int_0^t \sin(3\tau)e^{\tau}d\tau &= \left[e^{-t}\left(e^{\tau}\sin(3\tau) - 3e^{\tau}\cos(3\tau)\right)\right]_{\tau=0}^{\tau=t} \\
\int_0^t \sin(3\tau)e^{\tau}d\tau &= \frac{\left[e^{\tau}\sin(3\tau) - 3e^{\tau}\cos(3\tau)\right]_{\tau=0}^{\tau=t}}{10} \\
&= \frac{\left(e^{t}\sin(3t) - 3e^{t}\cos(3t)\right) - \left(e^{0}\sin(3\cdot 0) - 3e^{0}\cos(3\cdot 0)\right)}{10} \\
&= \frac{e^{t}\sin(3t) - 3e^{t}\cos(3t) + 3}{10} \\
&= \frac{\sin(3t) - 3\cos(3t) + 3e^{-t}}{10}
\end{aligned}
$$

The numerator is of the form $a\cos(\omega t) + b\sin(\omega t) + e^{-t}$; therefore it can be rewritten in the form $c\cos(\omega t + \theta) + e^{-t}$.

Let $a = c\cos(\theta)$; then $\frac{a}{c} = \cos(\theta)$.

$a = -3$

Let $b = -c\sin(\theta)$; then $-\frac{b}{c} = \sin(\theta)$.

$b = 1$

$$
\begin{aligned}
c &= \sqrt{a^2 + b^2} \\
&= \sqrt{(-3)^2 + 1^2} \\
&= \sqrt{10}
\end{aligned}
$$

$$
\frac{a}{c} = \cos(\theta) \to \theta = \arccos\left(\frac{a}{c}\right)
$$

$$
\begin{aligned}
\theta &= \arccos\left(-\frac{3}{\sqrt{10}}\right) \\
&= 18.43°
\end{aligned}
$$

Therefore, $\frac{\sin(3t) - 3\cos(3t) + 3e^{-t}}{10}$ can be rewritten as $\frac{\sqrt{10}\cos(3t + 18.43°) + 3e^{-t}}{10}$.

$$
\boxed{y_{\text{ZSR}}(t) = \frac{\sqrt{10}\cos(3t + 18.43°) + 3e^{-t}}{10}.}
$$

PROBLEM 4. Consider an integrator system given by $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$.

A. Determine the unit impulse response $h_i(t)$ of the system.

The unit impulse response of a system is the system output given $\delta(t)$ as an input. Therefore, $h_i(t) = \int_{-\infty}^{t} \delta(\tau)d\tau = u(t)$.

$$\boxed{h_i(t) = u(t)}$$

B. Determine the impulse response $h_p(t)$ of two such integrators in parallel.

For parallel systems, $y_p(t) = x(t) * (h_1(t) + h_2(t))$.

To find the impulse response, $x(t) = \delta(t)$. Using two of the same integrator, $h_1(t) = h_2(t) = h_i(t) = u(t)$. Therefore,

$$\begin{aligned} h_p(t) &= \delta(t) * (u(t) + u(t)) \\ &= \delta(t) * 2u(t) \\ &= \int_{-\infty}^{\infty} \delta(\tau)2u(t-\tau)d\tau \\ &= 2u(t-0)\int_{-\infty}^{\infty} \delta(0)d\tau && \delta(\tau) \text{ has a value only when } \tau = 0 \\ &= 2u(t) \cdot 1 \end{aligned}$$

$$\boxed{h_p(t) = 2u(t) \quad ^{1}}$$

C. Determine the impulse response $h_s(t)$ of two such integrators in series.

For series systems, $y_s(t) = x(t) * (h_1(t) * h_2(t))$.

To find the impulse response, $x(t) = \delta(t)$. Using two of the same integrator, $h_1(t) = h_2(t) = h_i(t) = u(t)$. Therefore,

$$\begin{aligned} h_s(t) &= \delta(t) * (u(t) * u(t)) \\ &= \delta(t) * \int_{-\infty}^{\infty} u(\tau)u(t-\tau)d\tau \\ &= \delta(t) * \int_{0}^{t} u(\tau)u(t-\tau)d\tau \\ &= \delta(t) * \int_{0}^{t} 1 \cdot 1 d\tau \\ &= \delta(t) * 1 \\ &= \int_{-\infty}^{\infty} \delta(\tau) \cdot 1 d\tau \\ &= 1 \end{aligned}$$

$$\boxed{h_s(t) = 1 \quad ^{2}}$$

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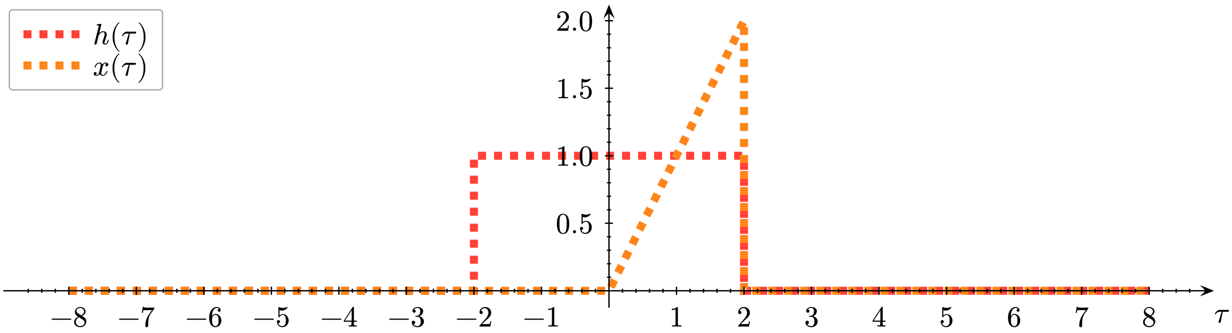
[1] Note that the final solved convolution is equal to $h_1(t) + h_2(t)$

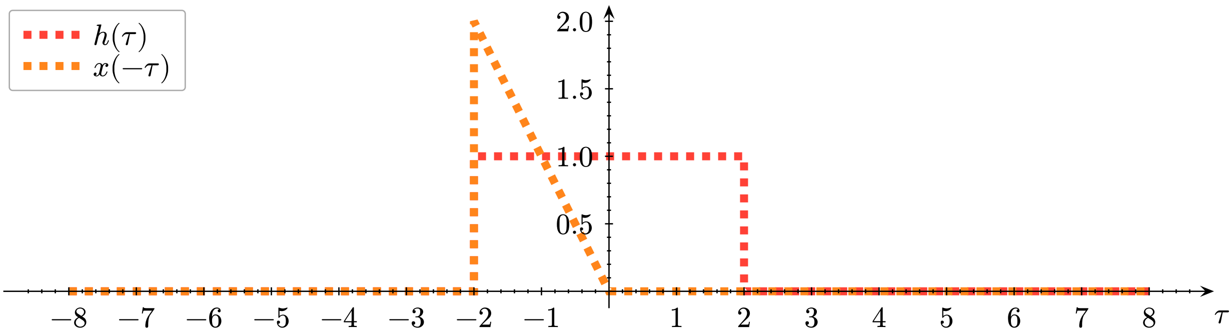
[2] Note that the final solved convolution is equal to $h_1(t) * h_2(t)$

Problem 5. An analog LTIC system with impulse response function $h(t) = u(t+2) - u(t-2)$ is presented with an input $x(t) = t(u(t) - u(t-2))$.



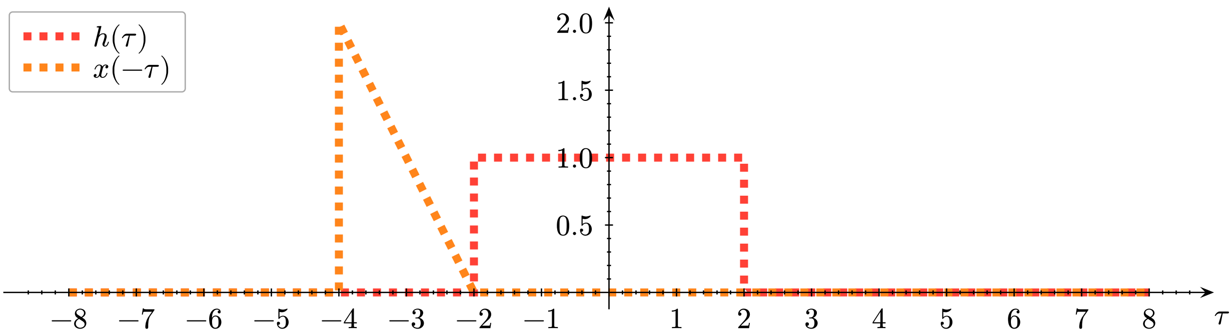
A. Determine and plot the system output $y(t) = x(t) \star h(t)$.

Graph $x(-\tau)$:



The graph of $x(-\tau)$ has two points of interest, $\tau_1 = 0$ and $\tau_2 = -2$. These points will be tracked for each case below.

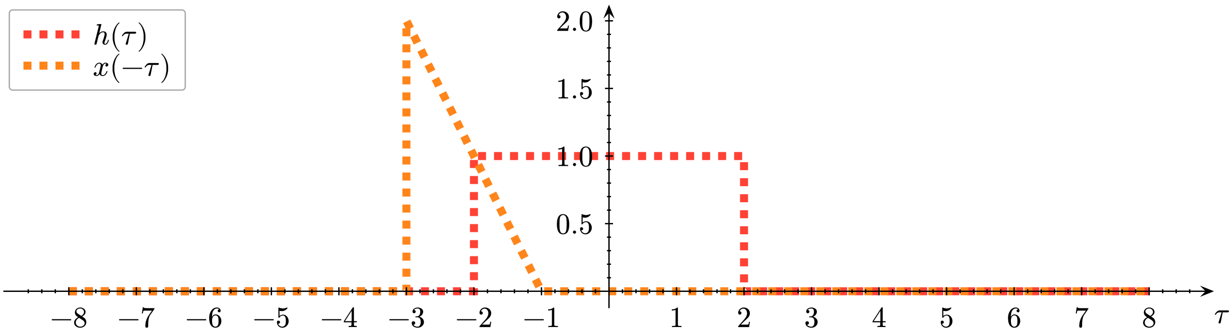
Case 1: $t < -2$



$\tau_1 = 0 + t$, $\tau_2 = -2 + t$

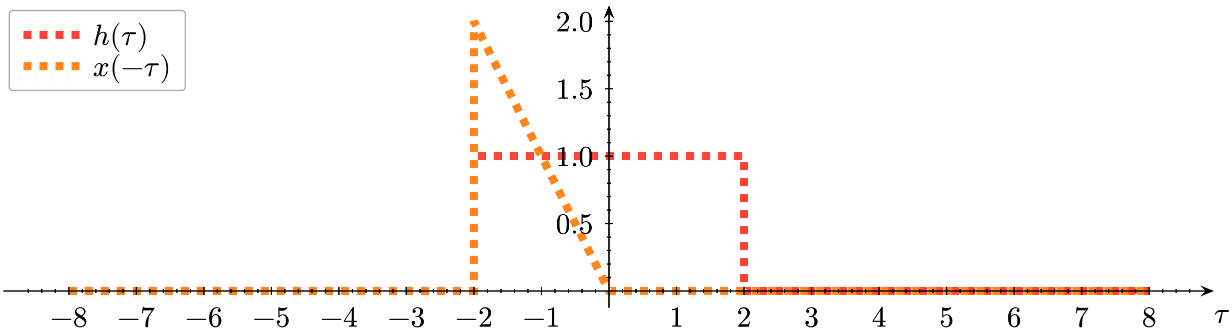
Since $h(t)$ and $x(t)$ do not overlap where either function is greater than 0, $\int x(\tau)h(t-\tau)d\tau = 0$.

Case 2: $-2 < t < -1$



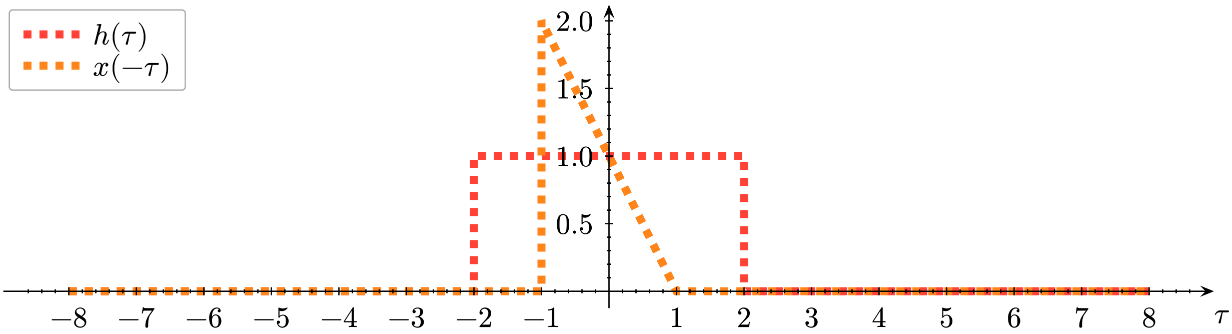
$$
\begin{aligned}
&\tau_1 = 0 + t, \tau_2 = -2 + t \\
&\textstyle\int_{-2}^{t} x(\tau)h(t-\tau)d\tau = \int_{-2}^{t} 1 \cdot (t-\tau)d\tau \\
&\qquad = \textstyle\int_{-2}^{t} t d\tau - \int_{-2}^{t} \tau d\tau \\
&\qquad = [t\tau]_{-2}^{t} - \left[\frac{\tau^2}{2}\right]_{-2}^{t} \\
&\qquad = t^2 + 2t - \left(\frac{t^2}{2} - 2\right) \\
&\qquad = \frac{t^2}{2} + 2t + 2
\end{aligned}
$$

Case 3: $-1 < t < 0$



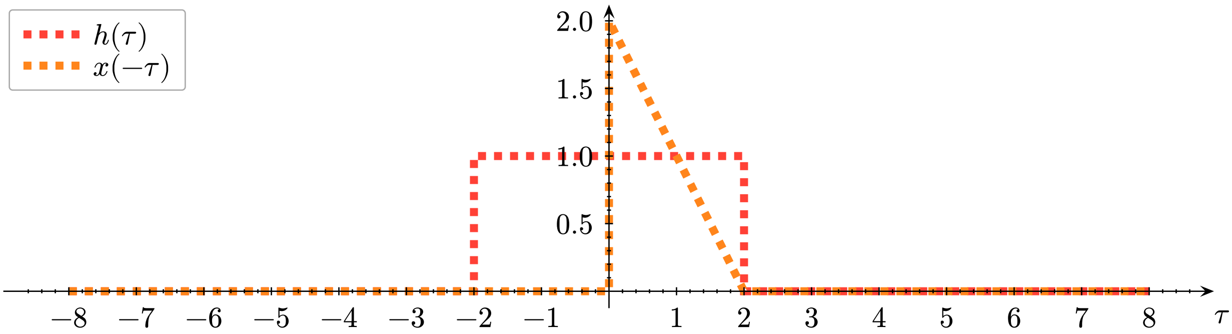
$$
\begin{aligned}
&\tau_1 = 0 + t, \tau_2 = -2 + t \\
&\textstyle\int_{-2}^{t} x(\tau)h(t-\tau)d\tau = \int_{-2}^{t} 1 \cdot (t-\tau)d\tau \\
&\qquad = \textstyle\int_{-2}^{t} t d\tau - \int_{-2}^{t} \tau d\tau \\
&\qquad = [t\tau]_{-2}^{t} - \left[\frac{\tau^2}{2}\right]_{-2}^{t} \\
&\qquad = t^2 + 2t - \left(\frac{t^2}{2} - 2\right) \\
&\qquad = \frac{t^2}{2} + 2t + 2
\end{aligned}
$$

Case 4: $0 < t < 1$



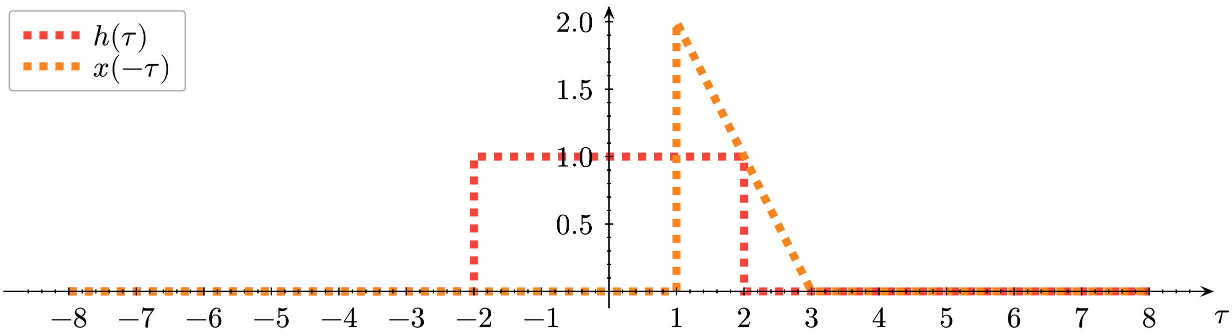
$$
\begin{aligned}
&\tau_1 = 0 + t, \tau_2 = -2 + t \\
&\int_{-2+t}^{t} x(\tau)h(t-\tau)d\tau = \int_{-2+t}^{t} 1 \cdot (t-\tau)d\tau \\
&\qquad = \int_{-2+t}^{t} t d\tau - \int_{-2+t}^{t} \tau d\tau \\
&\qquad = [t\tau]_{-2+t}^{t} - \left[\frac{\tau^2}{2}\right]_{-2+t}^{t} \\
&\qquad = t^2 - (-2t + t^2) - \left(\frac{t^2}{2} - \frac{(-2+t)^2}{2}\right) \\
&\qquad = t^2 + 2t - t^2 - \frac{t^2}{2} + \frac{t^2}{2} - \frac{4t}{2} + \frac{4}{2} \\
&\qquad = 2t - 2t + 2 \\
&\qquad = 2
\end{aligned}
$$

Case 5: $1 < t < 2$



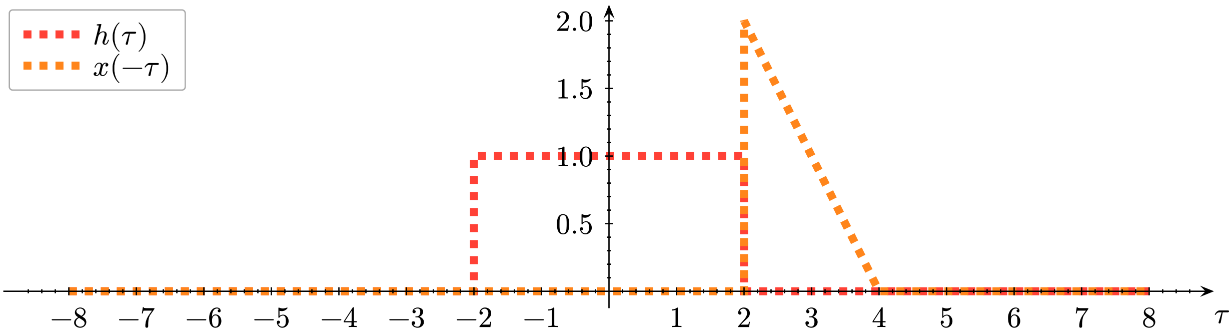
$$
\begin{aligned}
&\tau_1 = 0 + t, \tau_2 = -2 + t \\
&\int_{-2+t}^{t} x(\tau)h(t-\tau)d\tau = \int_{-2+t}^{t} 1 \cdot (t-\tau)d\tau \\
&\qquad = \int_{-2+t}^{t} t d\tau - \int_{-2+t}^{t} \tau d\tau \\
&\qquad = [t\tau]_{-2+t}^{t} - \left[\frac{\tau^2}{2}\right]_{-2+t}^{t} \\
&\qquad = t^2 - (-2t + t^2) - \left(\frac{t^2}{2} - \frac{(-2+t)^2}{2}\right) \\
&\qquad = t^2 + 2t - t^2 - \frac{t^2}{2} + \frac{t^2}{2} - \frac{4t}{2} + \frac{4}{2} \\
&\qquad = 2t - 2t + 2 \\
&\qquad = 2
\end{aligned}
$$

Case 6: $2 < t < 3$



$$
\begin{aligned}
&\tau_1 = 0 + t, \tau_2 = -2 + t \\
&\int_{-2+t}^{2} x(\tau)h(t-\tau)d\tau = \int_{-2+t}^{2} 1 \cdot (t-\tau)d\tau \\
&\qquad = \int_{-2+t}^{2} t d\tau - \int_{-2+t}^{2} \tau d\tau \\
&\qquad = [t\tau]_{-2+t}^{2} - \left[\frac{\tau^2}{2}\right]_{-2+t}^{2} \\
&\qquad = 2t - \left(-2t + t^2\right) - \left(2 - \frac{(-2+t)^2}{2}\right) \\
&\qquad = 2t + 2t - t^2 - 2 + \frac{t^2}{2} - \frac{4t}{2} + \frac{4}{2} \\
&\qquad = -t^2 + \frac{t^2}{2} - 2t + 2t + 2t - 2 + 2 \\
&\qquad = -\frac{t^2}{2} + 2t
\end{aligned}
$$

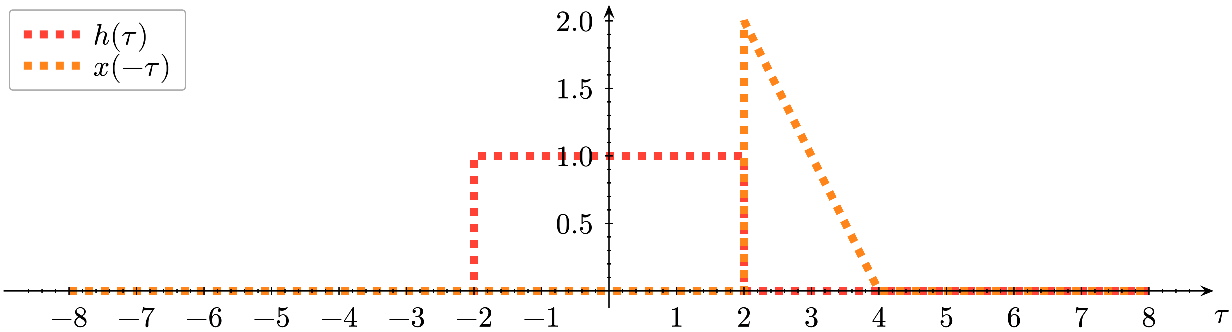
Case 7: $3 < t < 4$



$$\tau_1 = 0 + t, \tau_2 = -2 + t$$

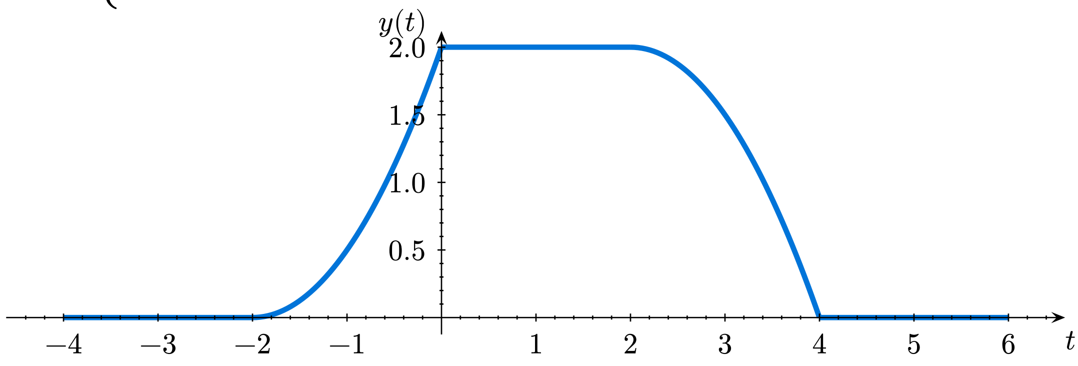
$$\begin{aligned}
\int_{-2+t}^{2} x(\tau)h(t-\tau)d\tau &= \int_{-2+t}^{2} 1 \cdot (t-\tau)d\tau \\
&= \int_{-2+t}^{2} t d\tau - \int_{-2+t}^{2} \tau d\tau \\
&= [t\tau]_{-2+t}^{2} - \left[\tfrac{\tau^2}{2}\right]_{-2+t}^{2} \\
&= 2t - \left(-2t + t^2\right) - \left(2 - \tfrac{(-2+t)^2}{2}\right) \\
&= 2t + 2t - t^2 - 2 + \tfrac{t^2}{2} - \tfrac{4t}{2} + \tfrac{4}{2} \\
&= -t^2 + \tfrac{t^2}{2} - 2t + 2t + 2t - 2 + 2 \\
&= -\tfrac{t^2}{2} + 2t
\end{aligned}$$

Case 8: $4 < t$



Since $h(t)$ and $x(t)$ do not overlap where either function is greater than 0, $\int x(\tau)h(t-\tau)d\tau = 0$.

$$y(t) = \begin{cases} 0 & t<-2 \\ \frac{t^2}{2}+2t+2 & -2<t<0 \\ 2 & 0<t<2 \\ -\frac{t^2}{2}+2t & 2<t<4 \\ 0 & 4<t \end{cases}$$



B. Determine if the system is stable and causal. Justify your reasoning.

Based on the graph of $y(t)$ above, the system is not causal since $y(t) \neq 0 \; \forall \; t < 0$. The system is stable since $\lim_{t\to\infty} y(t) = 0$.