

1:

This is an inline equation:  $x + y = 3$ .

This is a displayed equation:

$$x + \frac{y}{z - \sqrt{3}} = 2.$$

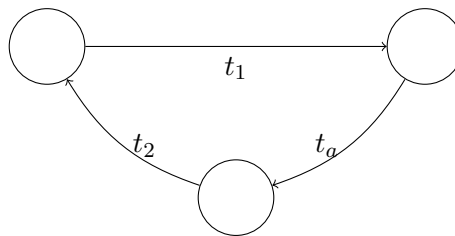
This is how you can define a piece-wise linear function:

$$f(x) = \begin{cases} 3x + 2 & \text{if } x < 0 \\ 7x + 2 & \text{if } x \geq 0 \text{ and } x < 10 \\ 5x + 22 & \text{otherwise.} \end{cases}$$

This is a matrix:

9	9	9	9
6	6	6	
3		3	3

This is a figure incorporated in a LaTeX file



**2:** consider the function  $f(x) = \begin{cases} x/2 & \text{if } x \text{ is even} \\ -\frac{x+1}{2} & \text{if } x \text{ is odd} \\ 0 & \text{if } x = 0 \end{cases}$  where  $x \in \mathbb{N}$  and  $f(x) \in \mathbb{Z}$ . This function is a

bijection between  $\mathbb{N}$  and  $\mathbb{Z}$ , and since there is a bijection between them they must be equinumerous.

**3:** assume that if a set  $S$  is infinite that there can not exist a function  $f$  such that  $f : S \rightarrow S$  and  $f$  is surjective but not injective, or injective but not surjective. consider the function  $g(x) = x^2$  over the infinite set  $f : \mathbb{N} \rightarrow \mathbb{N}$ . the element 3 in the set  $\mathbb{N}$  is a counter example to  $g(x)$  being onto since  $\sqrt{3}$  is not in  $\mathbb{N}$ . assume that there is some  $x \neq y$  where  $g(x) = g(y)$ , that would mean that  $x^2 = y^2$ , but there are not values in  $\mathbb{N}$  for which this is true since they are all positive proving that  $g$  is one-to-one by contradiction. This makes  $g$  an example of a function from an infinite set to itself that is one-to-one yet not onto. Now consider the function  $h = x + 2\lfloor x/2 \rfloor$  for  $h : \mathbb{R} \rightarrow \mathbb{R}$  this function is not one-to-one and  $x = 1$  and  $x = 2$  are a counter example. It is onto though because every value in the range has two values in the domain that equal it. This makes  $h$  an example of a

function that is onto, but not one-to-to. proving that an infinite set  $S$  can have a function  $f : S \rightarrow S$  that is injective but not surjective, and surjective but not injective by contradiction.

**4:**  $R$  is reflexive since if  $m = n$  then  $m - n = 0$  and  $0 \bmod 3$  will always be 0.

$R$  can be shown to be symmetric by direct proof.

assume that for some random values  $m, n \in \mathbb{N}$  where  $(m - n) \bmod 3 = 0$  then  $(n - m) \bmod 3$  must also equal 0 since  $n - m = -(m - n)$  so  $(n, m) \in R$  if  $(m, n) \in R$

$R$  can be shown to be transitive by direct proof

assume that from some random values  $a, b, c \in \mathbb{N}$   $(a, b) \in R$  and  $(b, c) \in R$ . this means that  $a - b = 3n$  and  $b - c = 3m$  where  $m$  and  $n$  are unknown constants. this means that  $a - c = 3n + 3m = 3(n + m)$ . which means that  $(a - c) \bmod 3 = 0$  and  $(a, c) \in R$ .

since  $R$  is reflexive symmetric and transitive it is an Equivalence relation, and its Equivalence classes are  $\{0, 3, 6, 9, \dots, 3n\}$ ,  $\{1, 4, 7, \dots, 3n + 1\}$ ,  $\{2, 5, 8, \dots, 3n + 2\}$  where  $n \in \mathbb{N}$

**5:** Proof by induction on  $n$ .

*Basis:* when  $n = 1$   $\sum_{i=1}^1 i^2 = 1$  and  $(2(1) + 1)(1 + 1)(1)/6 = 1$

*Induction hypothesis:* Assume that this holds true for all  $n$  up to some value  $m$

*Inductive step:*

$$\sum_{i=1}^{n+1} i^2 = \frac{(2(n+1) + 1)(n+1+1)(n+1)}{6} \quad (1)$$

$$(n+1)^2 + \sum_{i=1}^n i^2 = \frac{(2n+3)(n+2)(n+1)}{6} \quad (2)$$

$$(6n^2 + 12n + 6) + (2n+1)(n+1)n = (2n+3)(n+2)(n+1) \quad (3)$$

$$2n^3 + 9n^2 + 13n + 6 = 2n^3 + 9n^2 + 13n + 6 \quad (4)$$

which shows the property holds for  $n + 1$  proving  $\sum_{i=1}^n i^2 = \frac{(2n+1)(n+1)n}{6}$  by induction