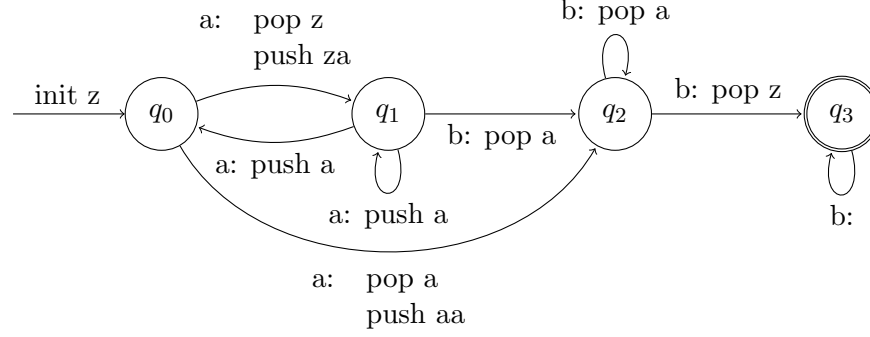


1.



2.

consider the string $w = b^m a^{m+1} b^m$ when applying the pumping lemma to this string there are really three distinct ways of decomposing w into uv^kxy^kz , $vy \neq \epsilon$, $|vxy| \leq m$. One way is where both v and y are comprised of a run of b's, which would look like, $b^{m-i-j}b^{ik}b^{jk}a^{m+1}b^m$ or $b^m a^{m+1}b^{m-i-j}b^{ik}b^{jk}$ or $b^{m-i}b^{ik}a^{m+1}b^{m-j}b^{jk}$ where $i, j \geq 1$, $i \leq j$, all of these fail to pump because when $k = 2$ one of the run's of b is at least $m + i$ and $m + i \geq m + 1$. The second way to split w would be to have $u = b^i, y = a^j$ which would result in $b^{m-i}b^{ik}a^{m+1-j}a^{jk}b^m$ and when $k = 0$, $|a|_w = m + 1 - j$ and one set of b's is of length m still and $m + 1 - j \leq m$; this version is symmetric to if $y = a^j, u = b^i$ and u was in the second run of b's. The third option is that $u = a^i, y^j$ which would result in $b^m a^{m+1-j-i}a^{jk}a^{ik}b^m$ when $k = 0$ the run of a's is at most $m + 1 - j - i$ which is less than both run's of b which is m showing that this valid string $w \in L$ does not pump so L is not context free.

3.

by contradiction if L was a CFL then there must be a way to partition $\forall w \in L$ so that $uvxyz$, $|v||y| \geq 1$, $|vxy| \leq m$ pumps. Consider $w = 1^m 0^m 2 1^{m+1} 0^{m-1}$, we know that v, y cannot contain 2 because then when $k = 0$ the string does not contain a two and is not in the language. If both u, y are on the left side then pumping 2 times will result in α having a 1 in a higher place value making $[\alpha]_2 > [\beta]_2$. Then same would be true if both v and y are on the right side of the 2 and $k = 0$ since at least two characters will be removed from the right string making $[\alpha]_2 > [\beta]_2$. If v is on the right and y is on the left then when $|v| < |y|$ pumping 0 times will result in $|\alpha| > |\beta| \implies [\alpha]_2 > [\beta]_2$, when $|v| > |y|$ pumping 2 times will result in the same, and when $|v| = |y| = i \geq 1$ pumping 0 times will result in $1^m 0^{m-i} 2 1^{m+1-i} 0^{m-1}$ which results in $[\alpha]_2 = [\beta]_2$ when $i = 1$ and $[\alpha]_2 > [\beta]_2$ when $i > 1$. So in no partition does w pump and since $w \in L$, L is not context free