

1.

$$\begin{aligned}
S &\rightarrow QE \\
Q &\rightarrow 0QZ \mid 1QN \mid F \\
FN &\rightarrow FR1 \\
FZ &\rightarrow FR0 \\
F &\rightarrow \epsilon \\
R0N &\rightarrow NR0 \\
R0Z &\rightarrow ZR0 \\
R0E &\rightarrow 0 \\
R1N &\rightarrow NR1 \\
R1Z &\rightarrow ZR1 \\
R1E &\rightarrow 1 \\
R11 &\rightarrow 11 \\
R10 &\rightarrow 10 \\
R01 &\rightarrow 01 \\
R00 &\rightarrow 00
\end{aligned}$$

basically it generates  $ww^R$  which a normal grammar can do and then reverses  $w^R$  with the power of an unrestricted grammar to form  $ww$ .

2.

the assignment didn't ask for proofs so I'm hoping that these intuitions are enough of an answer.

a)

This language is acceptable because  $M$  will use a finite number of cells when it halts or when it gets caught in a loop over the same section of tape which can be detected by tracking the machine's configurations. In both of these cases we can accept  $\rho(M)\rho(w)$ , however for us to be able to decide that it will not use a finite number of cells we would have to be able to decide that  $M$  will not halt on  $w$  and if we could do that we could solve the halting problem.

b)

This can be decided because if  $M$  halts using  $n$  or less cells we can accept  $\rho(M)\rho(w)01^n0$ , and if we track the configurations of  $M$  and detect that a configuration is repeated after using less than  $n$  cells we know that it will continue to loop over those  $n$  cells, allowing us to accept  $\rho(M)\rho(w)$  even if  $M$  doesn't halt. If however  $M$  ever goes past  $n$  cells we can halt and reject. Allowing us to decide  $L$ .