Stat 330 (Spring 2018): Homework 4

Due: February 14, 2018

Show all of your work, and *please* staple your assignment if you use more than one sheet. Write your name, the course number and the section on every sheet. Problems marked with * will be graded and one additional randomly chosen problem will be graded.

1. An urn contains 4 balls numbered 1, 2, 3, 4. Two balls are drawn at random from the urn without replacement. Define a random variable X to be the sum of the two numbers drawn.
   (a) What is $Im(X)$?
   (b) Make a table giving the probability mass function (pmf) for X.
   (c) What is the probability that the sum of the numbers will be less than 5?
   (d) Find the expected value and variance of X.

2. * Let X be a random variable with image $Im(X) = \{0, 1, 2, 3\}$.
   (a) Fill in the blank in the table below to make it a valid probability mass function:

x	0	1	2	3
$p_X(x)$	0.5	0.25	0.1	

   (b) Draw a graph of its cumulative distribution function (cdf).
   (c) Determine the probability that X is neither 0 nor 2.
   (d) Find the expected value and variance of X.
   (e) Let Y be a random variable with $Y = 4X - 3$. Determine the image of Y.
   (f) Using the rules for computing expected values and variances of a linear function of a random variable, find the expected value and variance of Y, using the corresponding values of X.

3. A computer virus is trying to corrupt two files. The first file will be corrupted with probability 0.4. Independently of it, the second file will be corrupted with probability 0.3.
   (a) Find the probability mass function (pmf) of X, the number of corrupted files.
   (b) Draw a graph of its cdf.

4. Let X denote the number of busy servers at the checkout counters in a store at 6 pm. Suppose that the cumulative distribution function (cdf) of X is:

$$F_X(t) = \begin{cases} 0, & \text{if } t < 0; \\ 0.2, & \text{if } 0 \le t < 1; \\ 0.5, & \text{if } 1 \le t < 2; \\ 0.8, & \text{if } 2 \le t < 3; \\ 0.9, & \text{if } 3 \le t < 4; \\ 1, & \text{if } t \ge 4. \end{cases}$$

   (a) Find the probability mass function (pmf) of X. (X is a discrete random variable.)
   (b) Compute $P(X > 2)$ and $P(X = 4 \mid X > 2)$.

5. Every day, the number of network blackouts has a following pmf:

x	0	1	2
$p_X(x)$	0.7	0.2	0.1

   A small internet trading company estimates that each network blackout results in a \$500 loss. Compute expected and variance of this company's daily loss due to blackouts.

6. * A lab network consisting of 20 computers was attacked by a computer virus. This virus enters each computer with probability 0.4, independently of other computers. Find the probability that
   (a) it entered exactly 6 computers
   (b) it entered at least 10 computers
   (c) it entered all 20 computers