

1.

Assume that L is regular, then $\exists m \in \mathbb{N}, \forall x \in L, |x| > m, \exists u, v, w \in \{0, 1\}^*, |uv| \leq m, |v| > 0, \forall k \in \mathbb{N}, uv^k w \in L$. Consider the String $0^{\frac{m}{2}} 1^{\frac{m}{2}} 0^{\frac{m}{2}} 1^{\frac{m}{2}}$, then $u = 0^{\frac{m}{2}-i} 1^{\frac{m}{2}-j}, v = 0^i 1^j, w = 0^{\frac{m}{2}} 1^{\frac{m}{2}} : i > 0 \implies j = \frac{m}{2}$. since $|v| > 0$ that means $j \geq 0$. When $k = 0$ the first half of the string will contain $0^{\frac{m}{2}-i} 1^{\frac{m}{2}-j} 0^{\frac{m}{2}} 1^{\frac{m}{2}}$. if $j < \frac{m}{2}$ then the first half of the string will contain a run of 0's a run of 1's and then another run of 0's, while the second half only contains a run of 0's followed by a run of 1's so $k = 0, j < \frac{m}{2} \implies uvw \notin L$. When $j = \frac{m}{2}, i < \frac{m}{2}$ the first half of the string will only contain 0's while the second half will contain both 0's and 1's, so $k = 0, j = \frac{m}{2}, i < \frac{m}{2} \implies uvw \notin L$. When $j = \frac{m}{2}, i = \frac{m}{2}$ the first half of the string will contain only 0's and the second half will contain only 1's, so $k = 0, j = \frac{m}{2}, i = \frac{m}{2} \implies uvw \notin L$. Since for all values for i, j cause $uvw \notin L$ when $k = 0$, this valid string does not pump $\forall k \in \mathbb{N}$, which means that L can not be regular.

2.

Assume that L is regular, then $\exists m \in \mathbb{N}, \forall x \in L, |x| > m, \exists u, v, w \in \{0, 1\}^*, |uv| \leq m, |v| > 0, \forall k \in \mathbb{N}, uv^k w \in L$. Consider the String $a^m b^{m+m!}$ then $u = a^{m-i}, v = a^i, w = b^m b^{m!}, |uv| \neq |w|$. Now consider $uv^k w$ when $k = \frac{m!}{i} + 1$.

$$\begin{aligned} |uv^k| &\neq |w| \\ |a^{m-i}(a^i)^{\frac{m!}{i}+1}| &\neq |b^m b^{m!}| \\ |a^{m-i} a^{m!+i}| &\neq |b^m b^{m!}| \\ |a^m a^{m!}| &= |b^m b^{m!}| \end{aligned}$$

So this string is not in the language $\forall i, m, k \in \mathbb{N} : i \leq m, k = \frac{m!}{i} + 1$ which is a contradiction proving that L is not regular.