Stat 330 (Spring 2018): Homework 1

Due: January 17, 2018

Show all of your work, and *please* staple your assignment if you use more than one sheet. Write your name, the course number and the section on every sheet. Problems marked with * will be graded and one additional randomly chosen problem will be graded.

1. Three people (A,B,C) toss a coin in order and the sequence of heads and tails is recorded. The person who flipped a head first is the winner. (e.g. if the result was tails, heads, heads, person B would win)

    (a) Determine the sample space, Ω.

    (b) List the elements that make up the following events: i. $A =$ Person A is the winner, ii. $B =$ Person B is the winner, iii. $C =$ No winner

    (c) List the elements of the following events: i. $\overline{A}$, ii. $\overline{(A \cup B)}$, iii. $A \cap \overline{B}$, iv. $A \cap C$

2. Consider a situation where cars entering an intersection each could turn right, turn left, or go straight. An experiment consists of observing two vehicles moving through the intersection.

    (a) How many elementary outcomes are there in the sample space? List them all. (*Denote your outcomes as: LR means car 1 turned left and car 2 turned R etc*)

    (b) Assuming that all outcomes are equally likely, what is the probability that at least one car turns left?

    (c) Again assuming equally likely outcomes, what is the probability that at most one vehicle turns?

3. * Two fair dice are tossed and the number on each die is recorded, e.g. (3,2) indicates the first die had a 3 and the second die had a 2.

    (a) Determine the sample space (Hint: there are 36 outcomes.).
    Assume all outcomes in (a) are equally likely for the next problems

    (b) What is the probability that the sum of the two dice is 8?

    (c) What is the probability of getting an even number on the first die or a total of 8?

    (d) What is the probability that the minimum of the two numbers is 3?

    (e) Assume that you know the first die shows number 3. Write down what the new sample space would be. Using your new sample space, what is the probability that the sum of the two numbers is 8?

4. Let three sets be $P = \{a, e, d, j, k\}$, $Q = \{e, k, l, m, n\}$, and $L = \{a, d, m, n\}$. The universe is $U = \{a, c, d, e, j, k, l, m, n\}$. Using Demorgan's Law, show that $P \cap \overline{(Q \cup L)} = P \cap (\overline{Q} \cap \overline{L})$
    Hint: *Do not prove. explicitly show the steps by using the defined sets.*

5. * Among employees of a certain firm, 70% know C/C++, 60% know Fortran, and 50% know both languages. What portion of programmers

    (a) does not know Fortran?

    (b) does not know Fortran and does not know C/C++?

    (c) knows C/C++ but not Fortran?

6. Among a large group of patients recovering from shoulder injuries, it is found that 22% visit both a physical therapist and a chiropractor, whereas 12% visit neither of these. The probability that a patient visits a chiropractor exceeds by 0.14 the probability that a patient visits a physical therapist.

    Calculate the probability that a randomly chosen member of this group visits a physical therapist.