Stat 330 (Spring 2018): Homework 8 Due: March 30, 2018

Show all of your work, and *please* staple your assignment if you use more than one sheet. Write your name, the course number and the section on every sheet. Problems marked with * will be graded and one additional randomly chosen problem will be graded.

1. During any minute, a computer device is either in a busy mode or in an idle mode. A busy mode is followed by an idle mode with probability 0.1. An idle mode is followed by a busy mode with probability 0.6.
   (a) What is the 1-step transition matrix?
   (b) Suppose that the device was in idle mode at 10:00 a.m. What is the probability that it will be busy at 10:02 a.m.?
   (c) Suppose that the device was in idle mode at 10:00 a.m. today. Approximate the probability that it will be busy at noon tomorrow via steady state.

2. * A Markov chain has 3 possible states: A, B, and C. Every hour, it makes a transition to a *different* state. From state A, transitions to states B and C are equally likely. From state B, transitions to states A and C are equally likely. From state C, it always makes a transition to state A.
   (a) Write down the transition probability matrix.
   (b) If the initial distribution for states A, B, and C is $P_0 = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, find the distribution of state after 2 transitions, i.e., the distribution of X_2 (or $X(2)$).
   (c) Show that this is a regular Markov Chain.
   (d) Find the steady-state distribution of states.

3. * Customers comes to a restaurant at a rate of 5 customers per hour. We assume customer arrivals is a homogenous Poisson process.
   (a) What is the probability of more than 2 customer arrivals in a period of one hour?
   (b) What is the probability of more than 4 customer arrivals in a period of 2 hours?
   (c) What is the expected value and the variance of inter-arrival times?
   (d) Compute the probability that the next customer does not arrive during the next 30 minutes?
   (e) Compute the probability that the time till the third customer arrives exceeds 40 minutes?

4. Suppose that the telephone calls arriving at the switchboard of a small corporation follows a Poisson process. Let $X(t)$ be the number of calls received by time t that day, with $X(0) = 0$. Suppose that the (intensity) parameter is $\lambda = 0.5$ calls per minute. Assume that the number of calls arriving during any period has a Poisson distribution.
   (a) What is the probability that no calls will arrive in a 5-minute period?
   (b) What is the expected amount of time between the 10th and 11th callers?
   (c) On average, how much time is needed to get their 100th caller of the day?

5. Every day, Eric takes the same street from his home to the university. There are 4 street lights along his way, and Eric has noticed the following Markov dependence. If he sees a green light at an intersection, then 60% of time the next light is also green, and 40% of time the next light is red. However, if he sees a red light, then 70% of time the next light is also red, and 30% of time the next light is green. Let 1 = "green light" and 2 = "red light" with the state space $\{1, 2\}$.
   (a) Construct the transition probability matrix for the street lights.
   (b) If the first light is green, what is the probability that the third light is red?
   (c) Eric's classmate Jacob has *many* street lights between his home and the university. If the *first* street light is green, what is the probability that the *last* street light is red? (Use the steady-state distribution.)

6. An internet service provider offers special discounts to every third connecting customer. Its customers connect to the internet according to a Poisson process with the rate of 5 customers per minute.

   (a) Compute the probability that no offer is made during the first 2 minutes.

   (b) Compute the expectation and variance of the time of the first offer.