

**HW 12 Due 1 dec 2017**

1. You are employed as a programmer, and you are asked to write a program that:
  - (a) receives in input a generic C program  $x$ , and counts the number of statements in  $x$ .
  - (b) receives in input a generic C program  $x$  and an input string  $w$ , and counts the number of statements executed at least once when  $x$  runs on input  $w$ .
  - (c) receives in input a generic C program  $x$  and an input string  $w$ , and counts the number of statements never executed when  $x$  runs on input  $w$ .
  - (d) receives in input a generic C program  $x$  and decides whether  $x$  is syntactically correct.
  - (e) receives in input two natural numbers and computes a specific function  $f : \mathbb{N}^2 \rightarrow \mathbb{N}$ .
  - (f) receives in input a generic arithmetic expression  $e$  composed of integers and the four arithmetic operators, and computes its value.
  - (g) halts on the empty string.
  - (h) receives in input a generic C program  $x$  and decides whether  $x$  halts only on the empty string.
  - (i) receives in input two generic regular expressions and decides whether they are equivalent.
  - (j) receives in input a generic C program  $x$  and the name of one of its functions,  $f$ , and decides whether  $x$  can ever call  $f$ .
  - (k) receives in input a generic C program  $x$ , an input string  $w$ , and the name of one of its functions,  $f$ , and decides whether  $x$  calls  $f$  when running on input  $w$ .
  - (l) receives in input two generic C programs  $x_1$  and  $x_2$  and an input string  $w$ , and decides whether  $x_1$  and  $x_2$  produce the same output when running on input  $w$ .
  - (m) receives in input two generic C programs  $x_1$  and  $x_2$ , and decides whether  $x_1$  and  $x_2$  produce the same output when running on every possible input.
  - (n) receives in input two generic C programs  $x_1$  and  $x_2$ , and decides whether  $x_1$  and  $x_2$  produce the same output when running on at least one input.
  - (o) receives in input a generic C program  $x$ , an input string  $w$ , and a natural number  $n$ , and decides whether  $x$  uses less than  $n$  bytes of memory when running on input  $w$ . \*
  - (p) receives in input a generic C program  $x$ , an input string  $w$ , and decides whether there is  $n \in \mathbb{N}$  such that  $x$  uses less than  $n$  bytes of memory running on input  $w$ . \*

For each case, write “Y” close to the question if the program can be written, i.e., if the corresponding problem is solvable in general (semi-solvable is not enough), “N” otherwise. Do not concern yourself with memory limitations, that is, assume that the computer used to run your program has a large, effectively unbounded, amount of memory. Cases marked with a “\*” are particularly challenging.

160

2. Use reduction to prove that the language

$$L = \{\rho(M)\rho(w) : \text{the TM } M \text{ never enters its initial state again when running on } w\}$$

is undecidable.

100