Stat 330 (Spring 2018): Homework 9

Due: April 11th, 2018

Show all of your work, and *please* staple your assignment if you use more than one sheet. Write your name, the course number and the section on every sheet. Problems marked with * will be graded and one additional randomly chosen problem will be graded.

1. A secure digital communications system has a receiving station with a CPU that can decode messages at a rate of 1 per minute, and a buffer that will hold up to 3 of these messages. Suppose that messages arrive according to a Poisson process with rate 0.7 per minute. Making the assumption that decode times are exponential random variables, answer the following questions about the large t behavior of the receiving station. (Note: the number messages in the system is the number in the buffer plus the one being processed). **Solve this as a Birth and Death Process question. Then you can go back and verify your answers using the appropriate queueing system**

   (a) Draw a state diagram of the number of messages in the system.

   (b) What fraction of the time is the CPU idle?

   (c) Let X be the number of messages in the system (for large t). What is the probability mass function (pmf) of X?

   (d) What fraction of incoming messages are lost (i.e. arrive when the buffer is full)?

   (e) What is the mean number of messages in the system? What is the mean number of messages in the buffer?

2. A library has only one librarian to help borrowers check out their books. Borrowers arrive according to a Poisson process at a rate of 5 per hour. The service time for each customer follows an Exponential distribution with expectation 0.1 hour. If the librarian is serving someone, arriving borrowers will stay in a line.

   (a) What is this queueing system?

   (b) What is the traffic intensity?

   (c) What is the (steady state) probability that there is no one in the *queue*?

   (d) What is the (steady state) expected waiting time to check out (time in the queue plus time in service)?

3. * There is only one restroom in a restaurant, where customers go to this restroom according to a Possion process with average rate of 16 people every hour. The time that a customer uses the restroom follows an Exponential distribution with expectation 3 minutes. If the restroom is being used, newly arrived customers stay in the queue. Find (large t behavior of) the following quantities.

   (a) The expected number of customers in the restroom and waiting at any time.

   (b) The proportion of time when no one is using the restroom.

   (c) The probability that there are less than 3 customers either waiting or using the restroom.

   (d) The average time that an arrival stands in line.

   (e) The probability that a customer is in the queue for longer than 15 min.

4. * A network router operates as an M/M/1/10 queue, with data packets arriving as a Poisson process, being stored in a buffer, and then, in the order of their arrival, sent onwards by a processor. The arrival rate is $\lambda = 1$/millisecond, and the processor operates at a rate of $\mu = 1.2$/millisecond. Any packets arriving at times when the buffer is already full are discarded.

   (a) Draw the transition diagram for this system.

   (b) Find the expected time until the next change of system state if the *system* is currently

      i. empty;

      ii. full;

      iii. neither empty nor full.

(c) Find the steady-state distribution (pmf) of the system (report probabilities with 4 decimal places).

(d) Find the probability that the processer is busy.

(e) Find the average number of packets in the system.

(f) Find the rate that arriving packets are discarded.

(g) Find the expected length of time spent by each packet in the system.

5. Jobs are sent to a printer at random times, according to a Poisson process of arrivals, with a rate of 12 jobs per hour. The time it takes to print a job is an Exponential random variable, independent of the arrival time, with the average of 2 minutes per job.

   (a) A job is sent to a printer at noon. When is it expected to be printed?

   (b) How often does the total number of jobs in the queue and currently being printed exceed 2?

6. Trucks arrive at a weigh station according to a Poisson process with the average rate of 1 truck every 10 minutes. Inspection time is Exponential with the average of 3 minutes. When a truck is on the scale, the other trucks stay in a line waiting for their turn. Compute:

   (a) the expected number of trucks in the line at any time

   (b) the proportion of time when the weigh station is empty

   (c) the expected time each truck spends at the station, from arrival till departure.