Homework 3
Com S 311
Due: Sep 28, 11:59PM

Late Submission Due: Sep 29, 11:59PM (25% penalty)

Outcomes.

- Gain an in-depth understanding of BSTs.

Please see guidelines for submission instructions.

1. Let A be an array consisting of elements in the following order

$$11, 40, 20, 13, 21, 50, 60, 9, 2, 10$$

(75 Points)

   (a) Draw the binary search tree that is obtained by adding/inserting the elements (without balancing) in the order they appear in the array.

   (b) If the resulting tree is not balanced, perform balancing as per AVL tree discussed in lectures. Specify the steps needed to balance (left rotation/right rotation etc) and draw the resulting balanced tree.

   (c) Draw the hash table (using chain hashing) obtained by by adding the above elements using the hash function $(2x + 3)\%5$.

2. A BST is *perfectly balanced* if every for every node v number of nodes in the left subtree of v equals number of nodes in the right subtree of v. Given a BST, *balance factor of a node* V is

$$\text{Height of left subtree of } V - \text{Height of right subtree of } V$$

(60 Points)

   (a) Let T be a perfectly balanced BST holding $n = 2^\ell - 1$ distinct integers. Give an algorithm to find an element that is smaller than $2^{\ell-2} - 1$ many elements. What is the run time of your algorithm? Justify the correctness of your algorithm.

   (b) Let T be the following BST: the root node is A which has a right subtree G and a left subtree rooted a node B. The node B has a left subtree C and a right subtree rooted at D. The node D has left subtree E and a right subtree F. Suppose that balance factor of A is 2, balance factor of B is -1, balance factor of D is 0.

   Draw the tree obtained by balancing T. Draw the intermediate trees (if any) obtained while balancing. Once you balance the tree, what are the balance factors of the nodes A, B and D ?

3. Let $A = [a_1, a_2, \cdots a_n]$ be an integer array consisting of n integers. (80 Points)

   (a) Design an algorithm that constructs a Binary Tree T such that the leaves of T from left to right are $a_1, a_2, \cdots, a_n$. I.e, left most leaf of T is a_1, second from left is a_2, and so on, and the rightmost leaf is a_n. Derive the asymptotic run time of your algorithm in terms of n.

   (b) Design a data structure that supports the following operations in $O(\log n)$ time.

   - `increment(i, val)`: Updates the value of a_i to $a_i + val$.
   - `SS(`ℓ`)`: Returns $\sum_{i=\ell}^{n} a_i$

   Describe the data structure, algorithms to perform above operations, and the derive the time bounds.

4. Design a data structure D that supports the following operations in $O(\log n)$ time, where n is the number of integers stored in D. (80 Points)

   - `add(x):` Adds/inserts integer x into D.
   - `remove(k):` Removes the kth largest integer from D.
   - `search(x):` Returns true if x is in D.

GUIDE LINES:

- It is important to know whether your really know! For each problem, if you write the statement "I do not know how to solve this problem" (and nothing else), you will receive 20% credit for that problem. If you do write a solution, then your grade could be anywhere between 0% to 100%. To receive this 20% credit, you must explicitly state that you do not know how to solve the problem.
- You must work on the homework problems on your own. You should write the final solutions alone, without consulting any one. Your writing should demonstrate that you understand the proofs completely.
- When proofs are required, you should make them both clear and rigorous. Do not hand waive.
- If you hand writing is not legible, then your homework will not be graded.
- Any concerns about grading should be made within one week of returning the homework.
- **Please submit your HW via blackboard. If you type your solutions, then please submit pdf version. If you hand-write your solutions, then please scan your solutions and submit a pdf version. Please make sure that the quality of the scan is good, and your hand writing is legible. Name your file must be** *YourNetID-HW3.pdf*. **For example, if your net id is** `bondj`**, then the file name should be** *bondj-HW3.pdf*. **HW's submitted in incorrect format (non pdf, incorrect file name etc) will incur a penalty of 30%**