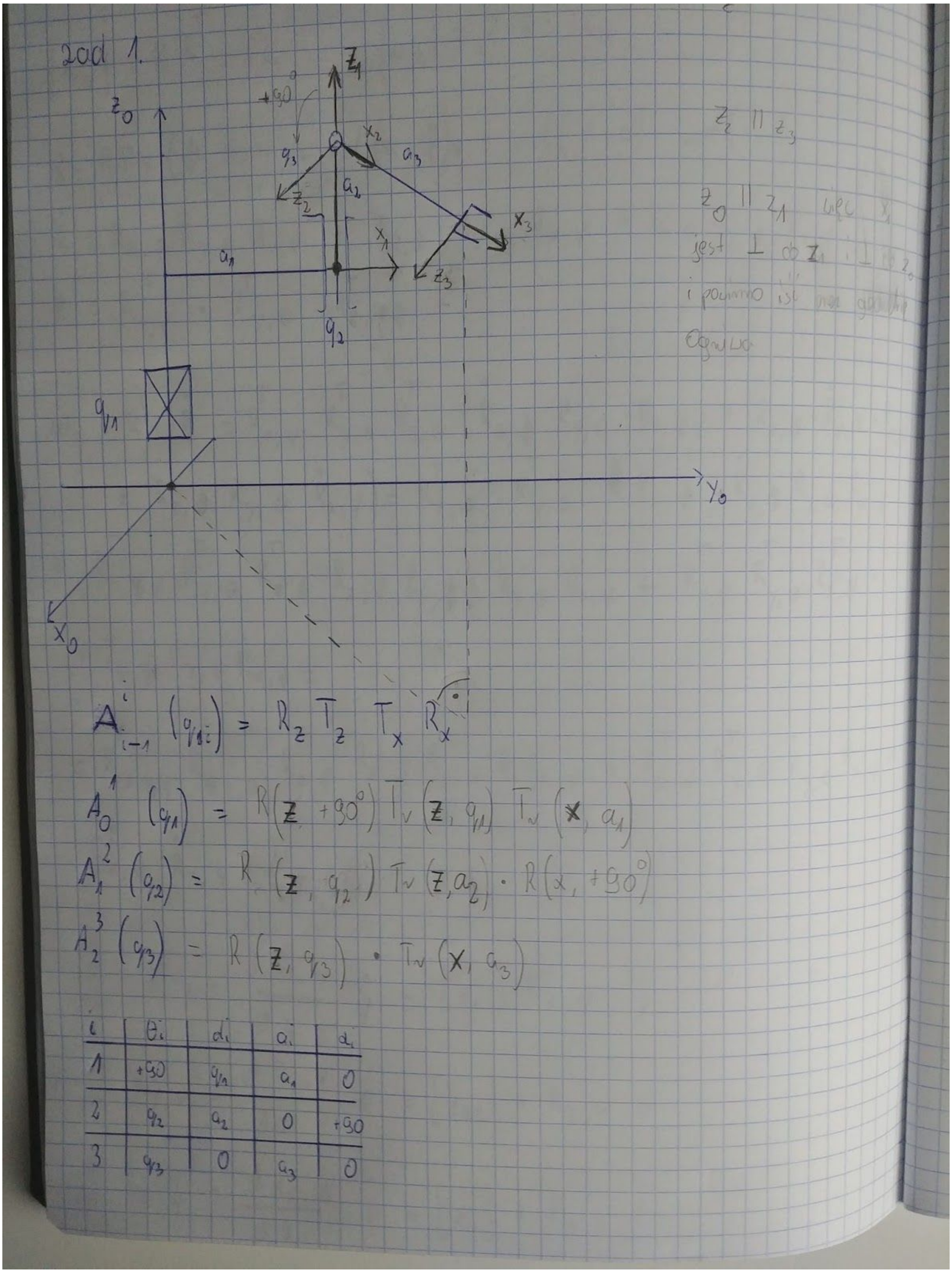
LISTA 5

1.
Igor Dominiak - Oznaczenie współrzędnych, zapisanie transformacji



zad 1.

$z_2 \parallel z_3$

$z_0 \parallel z_1$ więc x_1 jest $\perp$ do z_1 i $\perp$ do z_0 i pomimo iść [illegible] ogólno

$$A_{i-1}^{i}(q_i) = R_z T_z T_x R_x$$

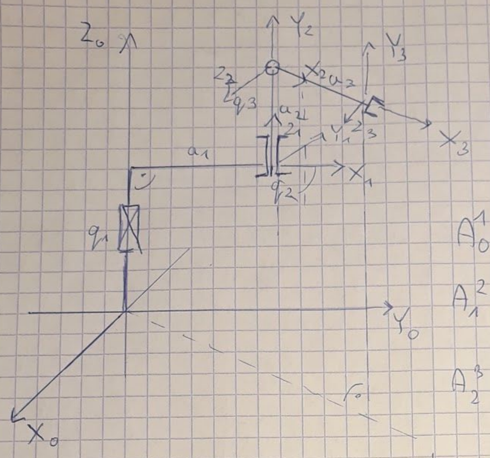
$$A_0^1(q_1) = R(z, +90^\circ)\, T_v(z, q_1)\, T_v(x, a_1)$$

$$A_1^2(q_2) = R(z, q_2)\, T_v(z, a_2) \cdot R(x, +90^\circ)$$

$$A_2^3(q_3) = R(z, q_3) \cdot T_v(x, a_3)$$

i	θ_i	d_i	a_i	α_i
1	+90	q_1	a_1	0
2	q_2	a_2	0	+90
3	q_3	0	a_3	0

Kinematyka - Krzysztof Górski



$\cancel{A_0^1 = Tr(Z, q_1)\, Rot(Z, 90) \cdot Tr(X_1, a_1)}$

$$A_0^1 = Tr(Z, q_1)\, Rot(Z, 90)\, Tr(X, a_1)$$

$$A_1^2 = Rot(Z, q_2)\, Tr(Z, a_2)\, Rot(X, 90)$$

$$A_2^3 = Rot(Z, q_3)\, Tr(X, a_3)$$

i	θ_i	d_i	a_i	α_i
1	$+90°$	q_1	a_1	$\cancel{90}$ 0
2	q_2	a_2	0	$+90$
3	q_3	$\cancel{a_3}$ 0	a_3	0

$$A_0^1 = \underbrace{\begin{bmatrix} c\frac{\pi}{2} & -s\frac{\pi}{2} & 0 & 0 \\ s\frac{\pi}{2} & c\frac{\pi}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{Rot(Z,90)} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{Tr(Z,q)} \underbrace{\begin{bmatrix} 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{Tr(X,a_1)}$$

$$= \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & a_1 \\ 0 & 0 & 1 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1^2 = \overset{\text{Rot}(z,q_2)}{\begin{bmatrix} cq_2 & -sq_2 & 0 & 0 \\ sq_2 & cq_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \overset{\text{Tr}(z,a_2)}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & a_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \overset{\text{Tr}(x,90)}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} =$$

$$A_1^2 = \begin{bmatrix} cq_2 & -sq_2 & 0 & 0 \\ sq_2 & cq_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & a_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} cq_2 & 0 & sq_2 & 0 \\ sq_2 & 0 & -cq_2 & 0 \\ 0 & 1 & 0 & a_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2^3 = \overset{\text{Rot}(z,q_3)}{\begin{bmatrix} cq_3 & -sq_3 & 0 & 0 \\ sq_3 & cq_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \underset{\text{Tr}(x,a_3)}{\begin{bmatrix} 1 & 0 & 0 & a_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} = \begin{bmatrix} cq_3 & -sq_3 & 0 & a_3 cq_3 \\ sq_3 & cq_3 & 0 & a_3 sq_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$K_0^3 = A_0^1 \cdot A_1^2 \cdot A_2^3$$

$$K_0^3 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & a_1 \\ 0 & 0 & 1 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cq_2 & 0 & sq_2 & 0 \\ sq_2 & 0 & -cq_2 & 0 \\ 0 & 1 & 0 & a_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cq_3 & -sq_3 & 0 & a_3 cq_3 \\ sq_3 & cq_3 & 0 & a_3 sq_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$K_0^3 = \begin{bmatrix} -sq_2 & 0 & cq_1 & 0 \\ cq_2 & 0 & sq_2 & a_1 \\ 0 & 1 & 0 & a_2+q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cq_3 & -sq_3 & 0 & a_3 cq_3 \\ sq_3 & cq_3 & 0 & a_3 sq_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$K_0^3 = \begin{bmatrix} -sq_2 cq_3 & sq_2 cq_3 & cq_1 & -sq_2 cq_3 a_3 \\ cq_2 cq_3 & -cq_2 sq_3 & sq_2 & cq_2 cq_3 a_3 + a_1 \\ sq_3 & cq_3 & 0 & sq_3 a_3 + a_2 + q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Maciej Salamoński - Wyrażenie kinematyki we współrzędnych RPY

$$K(q) = \begin{bmatrix} -S_2 C_3 & S_2 C_3 & C_1 & -S_2 C_3 a_3 \\ C_2 C_3 & -C_2 S_3 & S_2 & -C_2 C_3 a_3 + a_1 \\ S_3 & C_3 & 0 & S_3 a_3 + a_2 + q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_{RPY} = R(Z, \varphi) \cdot R(Y, \theta) \cdot R(X, \psi)$$

$$\varphi \in \langle 0, 2\pi \rangle, \quad \theta \in \langle -\tfrac{\pi}{2}, \tfrac{\pi}{2} \rangle, \quad \psi \in \langle 0, 2\pi \rangle$$

$$= \begin{bmatrix} C\varphi & -S\varphi & 0 \\ S\varphi & C\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} C\theta & 0 & S\theta \\ 0 & 1 & 0 \\ -S\theta & 0 & C\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\psi & -S\psi \\ 0 & S\psi & C\psi \end{bmatrix}$$

$$= \begin{bmatrix} C\varphi C\theta & C\varphi S\theta S\psi - S\varphi C\psi & C\varphi S\theta C\psi + S\varphi S\psi \\ S\varphi C\theta & S\varphi S\theta S\psi + C\varphi C\psi & S\varphi S\theta C\psi - C\varphi S\psi \\ -S\theta & C\theta S\psi & C\theta C\psi \end{bmatrix}$$

1) $-S\theta = S_3$

$S\theta = -S_3 \quad | f^{-1}$

$\underline{\theta = -q_3}$

2) $C\theta S\psi = C_3$

$C3 = C(-3)$ bo $\cos(-x) = \cos(x)$

$\cancel{C3} S\psi = \cancel{C3}$

$S\psi = 1 \quad | f^{-1}$

$\underline{\psi = \frac{\pi}{2}}$

3) $C\varphi C\theta = -S_2 C_3$

$C\varphi \cancel{C3} = -S_2 \cancel{C3}$

$C\varphi = -S_2 \quad | f^{-1}$

$* \sin\left(\frac{\pi}{2} - \alpha\right) = \sin(-\alpha)$

$\frac{\pi}{2} - \varphi = -q_2 \rightarrow \underline{\varphi = q_2 + \frac{\pi}{2}}$

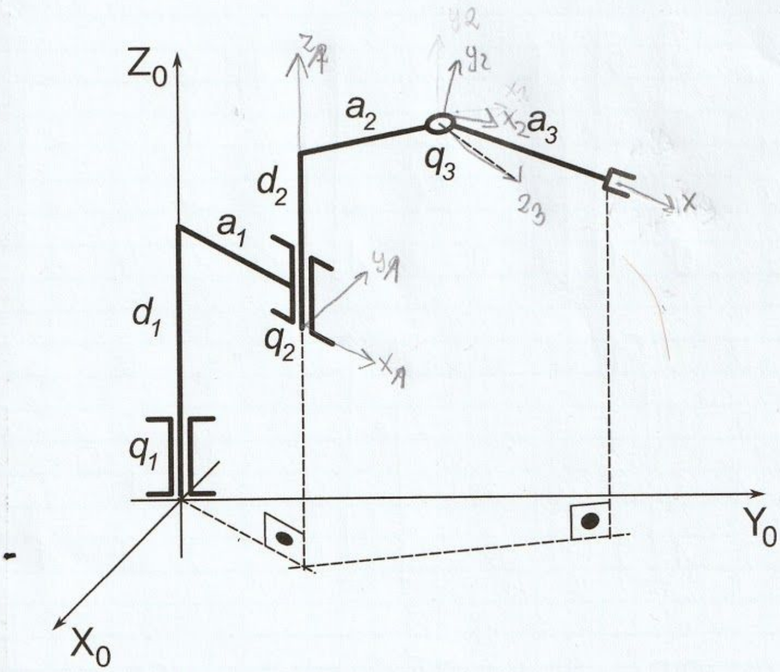
$$\underline{(\varphi, \theta, \psi) = \left(q_2 + \frac{\pi}{2}, -q_3, \frac{\pi}{2}\right)}$$

$$K(q) = \left(-S_2 C_3 a_3, -C_2 C_3 a_3 + a_1, S_3 a_3 + a_2 + q_1, q_2 + \frac{\pi}{2}, -q_3, \frac{\pi}{2}\right)^T$$

2. Weronika Jakubowska - osie lokalnych układów współrzędnych zgodnie z algorytmem Denavita-Hartenberga

Zad 2

2 Należy umieścić osie lokalnych układów współrzędnych zgodnie z algorytmem Denavita-Hartenberga. Wyliczyć kinematykę i wyrazić ją we współrzędnych (kartezjańskie, kąty Eulera ZXZ). W tym celu należy wyliczyć macierz reprezentacji kątów ZXZ.



$$A_{i-1}^{i}(q_i) = R_z T_z T_x R_x$$

$$A_0^1 = Rot(z, q_1 + 90^\circ) \cdot Tv(x, a_1)$$

$$A_1^2 = Rot(z, q_2)\ Tv(z, d_1 + d_2) \cdot Tv(x, a_2) \cdot Rot(x, 90^\circ)$$

$$A_2^3 = Rot(z, q_3) \cdot Tv(x, a_3)$$

$$A_{i-1}^{i}(q_i) = R(z,\theta_i)\cdot Tr(z,d_i)\cdot Tr(x,a_i)\cdot Rot(x,\alpha_i)$$

i	θ_i	d_i	a_i	α_i
1	q_1+90°	0	a_1	0
2	q_2	d_2+d_1	a_2	90°
3	q_3	0	a_3	0

2b) Tomasz Gniazdowski - kinematyka

TOMASZ GNIAZDOWSKI

2B.

	i	θ_i	d_i	a_i	α_i
A_0^1	1	$q_1+\frac{\pi}{2}$	0	a_1	0
A_1^2	2	q_2	d_1+d_2	a_2	$\frac{\pi}{2}$
A_2^3	3	q_3	0	a_3	0

$\sin(0)=0$ $\quad \cos(\frac{\pi}{2}+\alpha)=-\sin(\alpha)$

$\cos(0)=1$ $\quad \sin(\alpha+\frac{\pi}{2})=\cos(\alpha)$

$\cos(\frac{\pi}{2})=0$

$\sin(\frac{\pi}{2})=1$

$$A_0^3 = A_0^1 A_1^2 A_2^3$$

$$A_0^1 = \begin{bmatrix} -s_1 & -c_1 & 0 & 0 \\ c_1 & -s_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} -s_1 & -c_1 & 0 & -a_1 s_1 \\ c_1 & -s_1 & 0 & a_1 c_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1^2 = \begin{bmatrix} C_2 & -S_2 & 0 & 0 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 1 & d_1+d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} C_2 & 0 & S_2 & a_2C_2 \\ S_2 & 0 & -C_2 & a_2S_2 \\ 0 & 1 & 0 & d_1+d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2^3 = \begin{bmatrix} C_3 & -S_3 & 0 & 0 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} C_3 & -S_3 & 0 & C_3a_3 \\ S_3 & C_3 & 0 & S_3a_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_0^2 = A_0^1 \cdot A_1^2 = \begin{bmatrix} -s_1 & -c_1 & 0 & -s_1 a_1 \\ c_1 & -s_1 & 0 & c_1 a_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & 0 & s_2 & c_2 a_2 \\ s_2 & 0 & -c_2 & s_2 a_2 \\ 0 & 1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$-a_2(s_1 c_2 + c_1 s_2) - s_1 a_1$$

$$/\!/$$

$$\begin{bmatrix} -s_1 c_2 - c_1 s_2 & 0 & -s_1 s_2 + c_1 c_2 & -s_1 c_2 a_2 - c_1 s_2 a_2 - s_1 a_1 \\ c_1 c_2 - s_1 s_2 & 0 & c_1 s_2 + s_1 c_2 & c_1 c_2 a_2 - s_1 s_2 a_2 + c_1 a_1 \\ 0 & 1 & 0 & d_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$a_2(c_1 c_2 - s_1 s_2) + c_1 a_1 \; =$$

$$A_0^2 = \begin{bmatrix} -s_{1+2} & 0 & c_{1+2} & -a_2 s_{1+2} - s_1 a_1 \\ c_{1+2} & 0 & s_{1+2} & a_2 c_{1+2} + c_1 a_1 \\ 0 & 1 & 0 & d_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_0^2 \cdot A_2^3 = K =$$

$$\begin{cases} -s_{1+2} c_3 a_3 - a_2 s_{1+2} - s_1 a_1 \\ c_{1+2} c_3 a_3 + a_2 c_{1+2} + c_1 a_1 \end{cases}$$

$$\begin{bmatrix} -s_{1+2} & 0 & c_{1+2} & -a_2 s_{1+2} - s_1 a_1 \\ c_{1+2} & 0 & s_{1+2} & a_2 c_{1+2} + c_1 a_1 \\ 0 & 1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & -s_3 & 0 & c_3 a_3 \\ s_3 & c_3 & 0 & s_3 a_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -s_{1+2} c_3 & s_{1+2} s_3 & c_{1+2} & \\ c_{1+2} c_3 & -s_3 c_{1+2} & s_{1+2} & \\ s_3 & c_3 & 0 & s a_3 + d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c)Wiktor Springer, współrzędne kartezjańskie,ZXZ

$$K(q) = \begin{bmatrix} -s_{12}c_3 & s_3 s_{12} & c_{12} & -s_{12}c_3 a_3 - a_2 s_{12} - s_1 a_1 \\ c_3 s_{12} & -c_{12} s_3 & s_{12} & c_{12} c_3 a_3 + a_2 c_{12} - c_1 a_1 \\ s_3 & c_3 & 0 & d_1 + d_2 + s_3 a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$ZXZ = \begin{bmatrix} c_1 c_3 - c_2 s_1 s_3 & -c_1 s_3 - c_2 c_3 s_1 & s_1 s_2 \\ c_3 s_1 + c_1 c_2 s_3 & c_1 c_2 c_3 - s_1 s_3 & -c_1 s_2 \\ s_2 s_3 & c_3 s_2 & c_2 \end{bmatrix}$$

$1 = \varphi$
$2 = \Theta$
$3 = \psi$

(1) $c_\Theta = 0 \Rightarrow \Theta = \frac{\pi}{2} \vee -\frac{\pi}{2}$

$s_\Theta = 1 = \frac{\pi}{2}$ więc

(2) $c_\psi = c_3 \Rightarrow \psi = q_3$

(3) $c_\varphi c_{q_3} = -s_{12} c_3$

$c_\varphi = -s(q_1 + q_2)$

$c(\alpha + \frac{\pi}{2}) = -s(q_1 + q_2)$

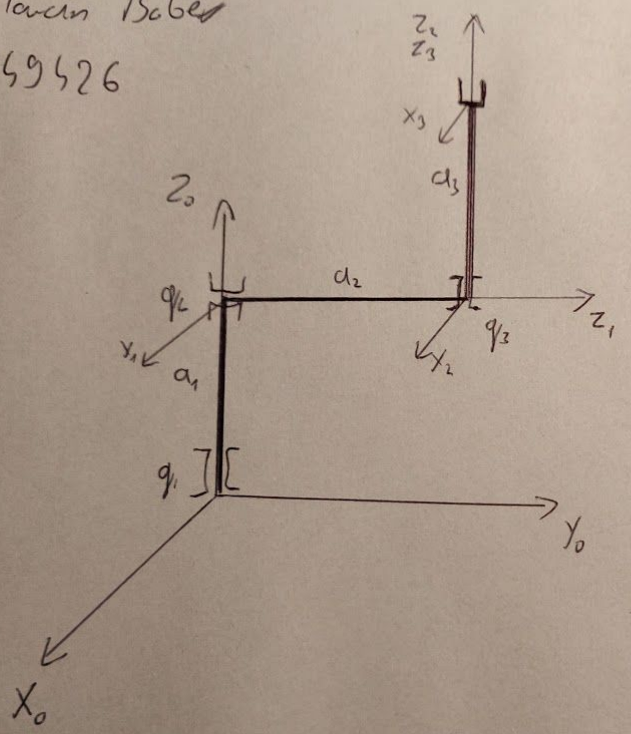
$\Theta = \frac{\pi}{2}$
$\psi = q_3$
$\varphi = q_1 + q_2 + \frac{\pi}{2}$

$$K(q) = \begin{bmatrix} -s_{12}(c_3 a_3 + a_2) - s_1 a_1 \\ c_{12}(c_3 a_3 + a_2) - c_1 a_1 \\ d_1 + d_2 + s_3 a_3 \\ q_1 + q_2 + \frac{\pi}{2} \\ \frac{\pi}{2} \\ q_3 \end{bmatrix}$$

3. Marcin Bober - Oznaczenie współrzędnych, zapisanie transformacji

Marcin Bober

249426



$$A_0^1 = Rot(z, q_1)\, T(z, a_1)\, Rot\left(x, -\tfrac{\pi}{2}\right)$$

$$A_1^2 = R(z, q_2)\, T(z, a_2)\, Rot\left(x, \tfrac{\pi}{2}\right)$$

$$A_2^3 = \cancel{T}\, Rot(z, q_3)\, T(z, a_3)$$

i	θ_i	d_i	a_i	α_i
1	q_1	a_1	0	$-\frac{\pi}{2}$
2	q_2	a_2	0	$\frac{\pi}{2}$
3	q_3	a_3	0	0

3.b) Hubert Górski-obliczenie kinematyki zad. 3 b Hubert Gębski

Obliczenie Kinematyki

$$A_0^1 = Rot(z, q_1) \cdot Tr(z, a_1) \cdot Rot(x, \tfrac{\pi}{2}) =$$

$$= \begin{bmatrix} c(q_1) & -\sin(q_1) & 0 & 0 \\ s(q_1) & c(q_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c(q_1) & -s(q_1) & 0 & 0 \\ s(q_1) & c(q_1) & 0 & 0 \\ 0 & 0 & 1 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c(q_1) & 0 & s(q_1) & 0 \\ s(q_1) & 0 & -c(q_1) & 0 \\ 0 & 1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1^2 = Rot(z, q_2) \cdot Tr(z, a_2) \cdot Rot(x, \tfrac{\pi}{2}) = \begin{bmatrix} c(q_2) & 0 & s(q_2) & 0 \\ s(q_2) & 0 & -c(q_2) & 0 \\ 0 & 1 & 0 & a_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2^3 = Rot(z, q_3) \cdot Tr(z, a_3) = \begin{bmatrix} c(q_3) & -s(q_3) & 0 & 0 \\ s(q_3) & c(q_3) & 0 & 0 \\ 0 & 0 & 1 & a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$K = A_0^1 A_1^2 A_2^3 = \begin{bmatrix} c(q_1) & 0 & s(q_1) & 0 \\ s(q_1) & 0 & -c(q_1) & 0 \\ 0 & 1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} c(q_2) & 0 & s(q_2) & 0 \\ s(q_2) & 0 & -c(q_2) & 0 \\ 0 & 1 & 0 & a_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} c(q_3) & -s(q_3) & 0 & 0 \\ s(q_3) & c(q_3) & 0 & 0 \\ 0 & 0 & 1 & a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} c(q_1)c(q_2) & s(q_1) & c(q_1)s(q_2) & a_2 s(q_1) \\ s(q_1)c(q_2) & -c(q_1) & s(q_1)s(q_2) & -a_2 c(q_1) \\ s(q_2) & 0 & -c(q_2) & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c(q_3) & -s(q_3) & 0 & 0 \\ s(q_3) & c(q_3) & 0 & 0 \\ 0 & 0 & 1 & a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} * \end{bmatrix}$$

$$\begin{bmatrix} * \end{bmatrix} = \begin{bmatrix} c(q_1)c(q_2)c(q_3) + s(q_1)s(q_3) & -c(q_1)c(q_2)s(q_3) + s(q_1)c(q_3) & c(q_1)s(q_2) & a_2 s(q_1) + a_3 c(q_1)s(q_2) \\ s(q_1)c(q_2)c(q_3) - c(q_1)s(q_3) & -s(q_1)c(q_2)s(q_3) - c(q_1)c(q_3) & s(q_1)s(q_2) & -a_2 c(q_1) + a_3 s(q_1)s(q_2) \\ s(q_2)c(q_3) & -s(q_2)s(q_3) & -c(q_2) & -a_3 c(q_2) + a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c) Patryk Szydlik - współrzędne kartezjańskie, ZYZ

NOWE PRAWIDŁOWE ROZWIĄZANIE:

$$K = \begin{bmatrix} C_{\theta_1}C_{\theta_2}C_{\theta_3} - S_{\theta_1}S_{\theta_3} & -S_{\theta_1}C_{\theta_3} - C_{\theta_1}C_{\theta_2}S_{\theta_3} & C_{\theta_1}S_{\theta_2} & a_3C_{\theta_1}S_{\theta_2} - a_2S_{\theta_1} \\ S_{\theta_1}C_{\theta_2}C_{\theta_3} + C_{\theta_1}S_{\theta_3} & -S_{\theta_1}C_{\theta_2}S_{\theta_3} + C_{\theta_1}C_{\theta_3} & S_{\theta_1}S_{\theta_2} & a_3S_{\theta_1}S_{\theta_2} + a_2C_{\theta_1} \\ -S_{\theta_2}C_{\theta_3} & S_{\theta_2}S_{\theta_3} & C_{\theta_2} & a_3C_{\theta_2} + a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Z_\alpha Y_\beta Z_\gamma = \begin{bmatrix} C_\alpha C_\beta C_\gamma - S_\alpha S_\beta & -S_\alpha C_\gamma - C_\alpha C_\beta S_\gamma & C_\alpha S_\beta \\ C_\alpha S_\gamma + S_\alpha C_\beta C_\gamma & -S_\alpha C_\beta S_\gamma + C_\alpha C_\gamma & S_\alpha S_\beta \\ -S_\beta C_\gamma & S_\beta S_\gamma & C_\beta \end{bmatrix}$$

$\alpha = \Theta_1 \qquad \beta = \Theta_2 \qquad \gamma = \Theta_3$

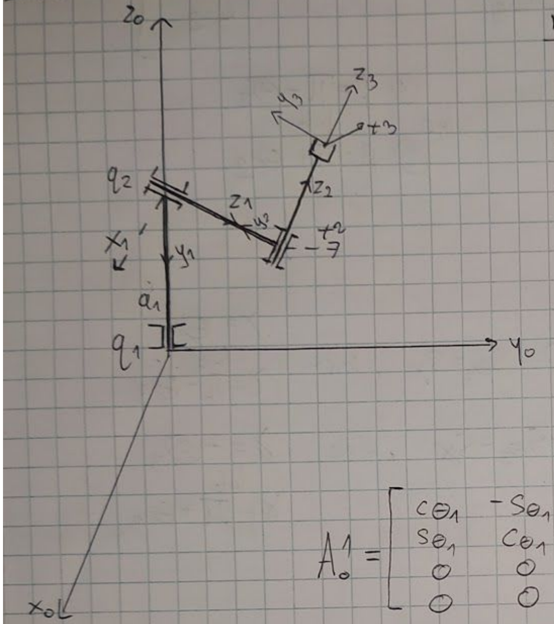
Kinematyka we współrzędnych (kartezjańskie, Eulera ZYZ)

$$k(\theta) = \begin{pmatrix} a_3C_{\theta_1}S_{\theta_2} - a_2S_{\theta_1} \\ a_3S_{\theta_1}S_{\theta_2} + a_2C_{\theta_1} \\ a_3C_{\theta_2} + a_1 \\ \Theta_1 \\ \Theta_2 \\ \Theta_3 \end{pmatrix}$$

POPRZEDNIE ROZWIĄZANIE:

Patryk Szydlik 248949

Lista 5 Zad 3



	Rotz	Tvz	Tvx	Rotx
n	Θ	a	d	α
1	Θ_1	a_1	0	$-\frac{\pi}{2}$
2	Θ_2	a_2	0	$-\frac{\pi}{2}$
3	Θ_3	a_3	0	0

$$A_0^1 = \begin{bmatrix} C_{\Theta_1} & -S_{\Theta_1} & 0 & 0 \\ S_{\Theta_1} & C_{\Theta_1} & 0 & 0 \\ 0 & 0 & 1 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_{\Theta_1} & 0 & -S_{\Theta_1} & 0 \\ S_{\Theta_1} & 0 & C_{\Theta_1} & 0 \\ 0 & -1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1^2 = \begin{bmatrix} C_{\Theta_2} & -S_{\Theta_2} & 0 & 0 \\ S_{\Theta_2} & C_{\Theta_2} & 0 & 0 \\ 0 & 0 & 1 & a_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_{\Theta_2} & 0 & -S_{\Theta_2} & 0 \\ S_{\Theta_2} & 0 & C_{\Theta_2} & 0 \\ 0 & -1 & 0 & a_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\cancel{A_2^3 =}$

$$A_2^3 = \begin{bmatrix} C_{\Theta_3} & -S_{\Theta_3} & 0 & 0 \\ S_{\Theta_3} & C_{\Theta_3} & 0 & 0 \\ 0 & 0 & 1 & a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_0^2 = A_0^1 \cdot A_1^2 = \begin{bmatrix} C_{\Theta_1} C_{\Theta_2} & S_{\Theta_1} & -C_{\Theta_1} S_{\Theta_2} & -a_2 S_{\Theta_1} \\ S_{\Theta_1} C_{\Theta_2} & -C_{\Theta_1} & -S_{\Theta_1} S_{\Theta_2} & a_2 C_{\Theta_1} \\ -S_{\Theta_2} & 0 & -C_{\Theta_2} & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_0^3 = A_0^2 \cdot A_2^3 = \begin{bmatrix} C_{\Theta_1} C_{\Theta_2} C_{\Theta_3} + S_{\Theta_1} S_{\Theta_3} & S_{\Theta_1} C_{\Theta_3} - C_{\Theta_1} C_{\Theta_2} S_{\Theta_3} & -C_{\Theta_1} S_{\Theta_2} & \left(\begin{matrix} -a_3 C_{\Theta_1} S_{\Theta \ldots} \\ -a_2 S_{\Theta \ldots} \end{matrix}\right. \\ S_{\Theta_1} C_{\Theta_2} C_{\Theta_3} - C_{\Theta_1} S_{\Theta_3} & -S_{\Theta_1} C_{\Theta_2} S_{\Theta_3} - C_{\Theta_1} C_{\Theta_3} & -S_{\Theta_1} S_{\Theta_2} & \left(\begin{matrix} -a_3 S_{\Theta_1} S_{\ldots} \\ +a_2 C_{\Theta_1} \end{matrix}\right. \\ -S_{\Theta_2} C_{\Theta_3} & S_{\Theta_2} S_{\Theta_3} & -C_{\Theta_2} & \left(\begin{matrix} -a_3 C_{\Theta_2} \\ +a_1 \end{matrix}\right. \\ 0 & 0 & 0 & \ldots \end{bmatrix}$$

$$\overset{\overset{1}{\wedge}}{Z_\alpha}\,\overset{\overset{2}{\wedge}}{Y_\beta}\,\overset{\overset{3}{\wedge}}{Z_\gamma} = \begin{bmatrix} C_1C_2C_3 - S_1S_3 & -S_1C_3 - C_1C_2S_3 & C_1S_2 \\ C_1S_3 + S_1C_2C_3 & -S_1C_2S_3 + C_1C_3 & S_1S_2 \\ -S_2C_3 & S_2S_3 & C_2 \end{bmatrix}$$

$\boxed{\text{I}}$ Element 3x3 $\quad -C_{\theta_2} = C_\beta$, więc ① $\beta = \pi - \theta_2$ lub ② $\beta = \pi + \theta_2$

$\boxed{\text{II}}$ Element 3x2 $\quad S_{\theta_2}S_{\theta_3} = S_\beta S_\gamma$

dla ① $\beta = \pi - \theta_2 \Rightarrow S_\beta = S_{\theta_2} \Rightarrow S_{\theta_3} = S_\gamma$, więc $\gamma = \theta_3$ lub $\gamma = \pi - \theta_3$

dla ② $\beta = \pi + \theta_2 \Rightarrow S_\beta = -S_{\theta_2} \Rightarrow -S_{\theta_3} = S_\gamma$, więc $\gamma = -\theta_3$ lub $\gamma = \pi + \theta_3$

$\boxed{\text{III}}$ Element 3x1 $\quad -S_{\theta_2}C_{\theta_3} = -S_\beta C_\gamma$

dla ① $\beta = \pi - \theta_2 \Rightarrow C_{\theta_3} = C_\gamma$, więc $\gamma = \theta_3$ lub $\gamma = -\theta_3$

dla ② $\beta = \pi + \theta_2 \Rightarrow -C_{\theta_3} = C_\gamma$, więc $\gamma = \pi - \theta_3$ lub $\pi + \theta_3$

Stąd łącząc II i III mamy
① $\beta = \pi - \theta_2 \quad \gamma = \theta_3$
② $\beta = \pi + \theta_2 \quad \gamma = \pi + \theta_3$

$\boxed{\text{IV}}$ Element 2x3 $\quad -S_{\theta_1}S_{\theta_2} = S_\alpha S_\beta$

dla ① $\quad -S_{\theta_1} = S_\alpha$ więc $\alpha = -\theta_1$ lub $\alpha = \pi + \theta_1$

dla ② $\quad S_{\theta_1} = S_\alpha$ więc $\alpha = \theta_1$ lub $\alpha = \pi - \theta_1$

$\boxed{\text{V}}$ Element 1x3 $\quad -C_{\theta_1}S_{\theta_2} = C_\alpha S_\beta$

dla ① $\quad -C_{\theta_1} = C_\alpha$ więc $\alpha = \pi - \theta_1$ lub $\alpha = \pi + \theta_1$

dla ② $\quad C_{\theta_1} = C_\alpha$ więc $\alpha = \theta_1$ lub $\alpha = -\theta_1$

Stąd łącząc IV i V mamy
① $\alpha = \pi + \theta_1 \quad \beta = \pi - \theta_2 \quad \gamma = \theta_3$
② $\alpha = \theta_1 \quad \beta = \pi + \theta_2 \quad \gamma = \pi + \theta_3$

dla sprawdzenie porównam element 1x1

$$L = C_{\theta_1} C_{\theta_2} C_{\theta_3} + S_{\theta_1} S_{\theta_3} = C_\alpha C_\beta C_\gamma - S_\alpha S_\gamma = P$$

① $\alpha = \pi + \theta_1 \quad \beta = \pi - \theta_2 \quad \gamma = \theta_3$

$$P = (-C_{\theta_1}) \cdot (-C_{\theta_2}) C_{\theta_3} - (-S_{\theta_1}) \cdot S_{\theta_3} = C_{\theta_1} C_{\theta_2} C_{\theta_3} + S_{\theta_1} S_{\theta_3}$$

② $\alpha = \theta_1 \quad \beta = \pi + \theta_2 \quad \gamma = \pi + \theta_3$

$$P = C_{\theta_1}(-C_{\theta_2}) \cdot (-C_{\theta_3}) - S_{\theta_1} \cdot (-S_{\theta_3}) = C_{\theta_1} C_{\theta_2} C_{\theta_3} + S_{\theta_1} S_{\theta_3} = L$$

Kinematyka we współrzędnych (kartezjańskie, Eulera ZYZ)

$$k(\theta) = \begin{pmatrix} -a_3 C_{\theta_1} S_{\theta_2} - a_2 S_{\theta_1} \\ -a_3 S_{\theta_1} S_{\theta_2} + a_2 C_{\theta_1} \\ -a_3 C_{\theta_2} + a_1 \\ \pi + \theta_1 \\ \pi - \theta_2 \\ \theta_3 \end{pmatrix} \quad \text{lub} \quad \begin{matrix} \theta_1 \\ \pi + \theta_2 \\ \pi + \theta_3 \end{matrix}$$