LISTA 2

1. Weronika Jakubowska

Zad. 1 Lista 2

$$R = \begin{bmatrix} 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

Obliczam macierz transponowaną R^T

$$R^T = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{bmatrix}$$

Sprawdzam czy zachodzi warunek $R^T R = I_3$

$$R^T R = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{bmatrix} \cdot \begin{bmatrix} 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$0 \cdot 0 + 1 \cdot 1 + 0 \cdot 0 = 1$

$0 \cdot \left(-\frac{\sqrt{2}}{2}\right) + 1 \cdot 0 + 0 \cdot \frac{\sqrt{2}}{2} = 0$

$0 \cdot \frac{\sqrt{2}}{2} + 1 \cdot 0 + 0 \cdot \frac{\sqrt{2}}{2} = 0$

$-\frac{\sqrt{2}}{2} \cdot 0 + 0 \cdot 1 + \frac{\sqrt{2}}{2} \cdot 0 = 0$

$-\frac{\sqrt{2}}{2} \cdot \left(-\frac{\sqrt{2}}{2}\right) + 0 \cdot 0 + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{1}{2} + \frac{1}{2} = 1$

$-\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} + 0 \cdot 0 + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = -\frac{1}{2} + \frac{1}{2} = 0$

$\frac{\sqrt{2}}{2} \cdot 0 + 0 \cdot 1 + \frac{\sqrt{2}}{2} \cdot 0 = 0$

$\frac{\sqrt{2}}{2} \cdot \left(-\frac{\sqrt{2}}{2}\right) + 0 \cdot 0 + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = -\frac{1}{2} + \frac{1}{2} = 0$

$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} + 0 \cdot 0 + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{1}{2} + \frac{1}{2} = 1$

Warunek $R^T R = I_3$ zachodzi

Sprawdziemy, czy zachodzi warunek $RR^T = I_3$

$$RR^T = \begin{bmatrix} 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$0 \cdot 0 + \left(-\frac{\sqrt{2}}{2}\right) \cdot \left(-\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{1}{2} + \frac{1}{2} = 1$

$0 \cdot 1 + \left(-\frac{\sqrt{2}}{2}\right) \cdot 0 + \frac{\sqrt{2}}{2} \cdot 0 = 0$

$0 \cdot 0 + \left(-\frac{\sqrt{2}}{2}\right) \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = -\frac{1}{2} + \frac{1}{2} = 0$

$1 \cdot 0 + 0 \cdot \left(-\frac{\sqrt{2}}{2}\right) + 0 \cdot \frac{\sqrt{2}}{2} = 0$

$1 \cdot 1 + 0 \cdot 0 + 0 \cdot 0 = 1$

$1 \cdot 0 + 0 \cdot \frac{\sqrt{2}}{2} + 0 \cdot \frac{\sqrt{2}}{2} = 0$

$0 \cdot 0 + \frac{\sqrt{2}}{2} \cdot \left(-\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = -\frac{1}{2} + \frac{1}{2} = 0$

$0 \cdot 1 + \frac{\sqrt{2}}{2} \cdot 0 + \frac{\sqrt{2}}{2} \cdot 0 = 0$

$0 \cdot 0 + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \frac{1}{2} + \frac{1}{2} = 1$

Zachodzi warunek $RR^T = I_3$

Sprawdzam, czy zachodzi warunek $\det R = 1$

$$\det R = \begin{vmatrix} 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{vmatrix} \begin{matrix} 0 & -\frac{\sqrt{2}}{2} \\ 1 & 0 \\ 0 & \frac{\sqrt{2}}{2} \end{matrix} = \frac{1}{2} - \left(-\frac{1}{2}\right) = 1$$

$0 \quad 0 \quad -\frac{1}{2} \quad 0 \quad 0 \quad \frac{1}{2}$

Zachodzi warunek $\det R = 1$

Macierz R

- jest nieosobliwa (wyznacznik różny od 0)
- jest to macierz ortogonalna bo spełnia warunek ~~RR^T~~
  $R^TR = RR^T = I_3$
- 2 własności → macierz odwrotna do macierzy R jest ~~jest~~ jej macierzą transponowaną, tj $R^{-1} = R^T$. Macierz ta też jest ortogonalna
- macierze ortogonalne służą do przekształceń ortogonalnych,

2. Igor Dominiak

zadanie 2 Lista 2

$i, j, k \quad i = \left(\frac{\sqrt{3}}{2}, 0, \frac{1}{2}\right)^T, \quad j = ?, \quad k = (0, 1, 0)^T$

układ prawoskrętny $\quad \|i\| = \frac{3}{4} + \frac{1}{4} = 1 \qquad \|k\| = 1$

$$\begin{cases} i \times j = k \\ j \times k = i \\ k \times i = j \end{cases}$$

$$j = \left|\begin{array}{ccc|cc} i & j & k & i & j \\ 0 & 1 & 0 & 0 & 1 \\ \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \end{array}\right| = \frac{1}{2} i - \frac{\sqrt{3}}{2} k = \underline{\left(\frac{1}{2}, 0, -\frac{\sqrt{3}}{2}\right)}$$

$\|k\| = \frac{1}{4} + \frac{3}{4} = 1$

Sprawdzenie:

$$\det\begin{vmatrix} i_1 & i_2 & i_3 \\ j_1 & j_2 & j_3 \\ k_1 & k_2 & k_3 \end{vmatrix} = 1 \Rightarrow \left|\begin{array}{ccc|cc} \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 & 0 & 1 \end{array}\right| = 1 \cdot \frac{1}{2} \cdot \frac{1}{2} - \left(\frac{\sqrt{3}}{2} \cdot -\frac{\sqrt{3}}{2}\right) \cdot 1$$

$$= \frac{3}{4} + \frac{1}{4} = \underline{\underline{1}}$$

3. Karolina Głuszek

3. Sprawdzamy warunki prawoskrętności:

$i \times j = k$

$j \times k = i$

$k \times i = j$

$$i_0 \times j_0 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \end{vmatrix} = \frac{\sqrt{2}}{2}\vec{i} + 0\vec{j} + \frac{\sqrt{2}}{2}\vec{k} = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \end{bmatrix} = k_0$$

$$j_0 \times k_0 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 0 \\ \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{vmatrix} = \frac{\sqrt{2}}{2}\vec{i} + 0\vec{j} - \frac{\sqrt{2}}{2}\vec{k} = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ -\frac{\sqrt{2}}{2} \end{bmatrix} = i_0$$

$$k_0 \times i_0 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} \end{vmatrix} = 0\vec{i} + 1\vec{j} + 0\vec{k} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = j_0$$

Układ 0 jest prawoskrętny

$$i_1 \times j_1 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \end{vmatrix} = 0\vec{i} + 0\vec{j} + 1\vec{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \neq k_1$$

Układ 1 nie jest prawoskrętny

Wersory po zmianie:

$$i_1 = \begin{bmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \\ 0 \end{bmatrix}, \quad j_1 = \begin{bmatrix} -\frac{\sqrt{3}}{2} \\ \frac{1}{2} \\ 0 \end{bmatrix}, \quad k_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$i_1 \times j_1 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \end{vmatrix} = 0\vec{i} + 0\vec{j} + 1\vec{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = k_1$$

$$j_1 \times k_1 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{vmatrix} = \frac{1}{2}\vec{i} + \frac{\sqrt{3}}{2}\vec{j} + 0\vec{k} = \begin{bmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \\ 0 \end{bmatrix} = i_1$$

$$k_1 \times i_1 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & +1 \\ +\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \end{vmatrix} = -\frac{\sqrt{3}}{2}\vec{i} + \frac{1}{2}\vec{j} + 0\vec{k} = \begin{bmatrix} -\frac{\sqrt{3}}{2} \\ \frac{1}{2} \\ 0 \end{bmatrix} = j^1$$

Po zmianie wersorów układ 1 jest prawoskrętny

Wyznaczamy macierz obrotu R_0^1

$$p_0 = R_0^1 \cdot p_1$$

$$R_0^1 = \begin{bmatrix} \begin{bmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \\ 0 \end{bmatrix} \circ \begin{bmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ -\frac{\sqrt{2}}{2} \end{bmatrix} & \begin{bmatrix} -\frac{\sqrt{3}}{2} \\ \frac{1}{2} \\ 0 \end{bmatrix} \circ \begin{bmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ -\frac{\sqrt{2}}{2} \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \circ \begin{bmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ -\frac{\sqrt{2}}{2} \end{bmatrix} \\ \begin{bmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \\ 0 \end{bmatrix} \circ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} & \begin{bmatrix} -\frac{\sqrt{3}}{2} \\ \frac{1}{2} \\ 0 \end{bmatrix} \circ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \circ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \\ \begin{bmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \\ 0 \end{bmatrix} \circ \begin{bmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \end{bmatrix} & \begin{bmatrix} -\frac{\sqrt{3}}{2} \\ \frac{1}{2} \\ 0 \end{bmatrix} \circ \begin{bmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \circ \begin{bmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \end{bmatrix} \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{\sqrt{2}}{4} & -\frac{\sqrt{6}}{4} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{\sqrt{2}}{4} & -\frac{\sqrt{6}}{4} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

$$p_1 = \left(R_0^1\right)^{-1} \cdot p_0$$

Wiedząc, że macierz obrotu jest macierzą ortogonalną

$$\left(R_0^1\right)^{-1} = \left(R_0^1\right)^T$$

$$p_1 = \begin{bmatrix} \frac{\sqrt{2}}{4} & \frac{\sqrt{3}}{2} & \frac{\sqrt{2}}{4} \\ -\frac{\sqrt{6}}{4} & \frac{1}{2} & -\frac{\sqrt{6}}{4} \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}+\sqrt{3}}{2} \\ \frac{1-\sqrt{6}}{2} \\ 0 \end{bmatrix}$$

4. Jan Bronicki

4. Macierze bazowe dla grupy obrotów $SO(3)$ to elementarne obroty względem osi głównych, czyli

$$rot(x,\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\phi & -s_\phi \\ 0 & s_\phi & c_\phi \end{bmatrix}, \qquad rot(y,\theta) = \begin{bmatrix} c_\theta & 0 & s_\theta \\ 0 & 1 & 0 \\ -s_\theta & 0 & c_\theta \end{bmatrix}, \qquad rot(z,\psi) = \begin{bmatrix} c_\psi & -s_\psi & 0 \\ s_\psi & c_\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Pokazać na konkretnym przykładzie (wykorzystując ogólną postać macierzy bazowych), że mnożenie macierzy obrotu względem różnych osi jest nieprzemienne, zaś względem tej samej osi – jest przemienne.

$$rot(y,\alpha)\cdot rot(y,\beta) = rot(y,\beta)\cdot rot(y,\alpha)$$

$$rot(y,\alpha)\cdot rot(z,\beta) \neq rot(z,\beta)\cdot rot(y,\beta)$$

$$R_1 = rot(y,\alpha)\cdot rot(z,\beta) = \begin{bmatrix} C_\alpha & 0 & S_\alpha \\ 0 & 1 & 0 \\ -S_\alpha & 0 & C_\alpha \end{bmatrix}\cdot\begin{bmatrix} C_\beta & -S_\beta & 0 \\ S_\beta & C_\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_\beta C_\alpha & -S_\beta C_\alpha & S_\alpha \\ S_\beta & C_\beta & 0 \\ -S_\alpha C_\beta & S_\alpha S_\beta & C_\alpha \end{bmatrix}$$

$$R_2 = rot(z,\beta)\, rot(y,\alpha) = \begin{bmatrix} C_\alpha & -S_\beta & 0 \\ S_\beta & C_\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} C_\alpha & 0 & S_\alpha \\ 0 & 1 & 0 \\ -S_\alpha & 0 & C_\alpha \end{bmatrix} = \begin{bmatrix} C_\beta C_\alpha & -S_\beta & C_\beta S_\alpha \\ S_\beta C_\alpha & C_\beta & S_\beta S_\alpha \\ -S_\alpha & 0 & C_\alpha \end{bmatrix}$$

$$R_1 \neq R_2 \,!$$

$$R_3 = rot(y,\alpha)\cdot rot(y,\beta) = \begin{bmatrix} C_\alpha & 0 & S_\alpha \\ 0 & 1 & 0 \\ -S_\alpha & 0 & C_\alpha \end{bmatrix}\cdot\begin{bmatrix} C_\beta & 0 & S_\beta \\ 0 & 1 & 0 \\ -S_\beta & 0 & C_\beta \end{bmatrix} =$$

$$\cos(\alpha+\beta) = C_\alpha C_\beta - S_\alpha S_\beta$$
$$\sin(\alpha+\beta) = S_\alpha C_\beta + S_\beta C_\alpha$$

$$= \begin{bmatrix} C_\alpha C_\beta - S_\alpha S_\beta & 0 & C_\alpha S_\beta + S_\alpha C_\beta \\ 0 & 1 & 0 \\ -S_\alpha C_\beta - S_\beta C_\alpha & 0 & -S_\alpha S_\beta + C_\alpha C_\beta \end{bmatrix}$$

$$R_4 = rot(y,\beta)\cdot rot(y,\alpha) = \begin{bmatrix} C_\beta & 0 & S_\beta \\ 0 & 1 & 0 \\ -S_\beta & 0 & C_\beta \end{bmatrix}\cdot\begin{bmatrix} C_\alpha & 0 & S_\alpha \\ 0 & 1 & 0 \\ -S_\alpha & 0 & C_\alpha \end{bmatrix} = \begin{bmatrix} C_\alpha C_\beta - S_\alpha S_\beta & 0 & C_\beta S_\alpha + C_\alpha S_\beta \\ 0 & 1 & 0 \\ -S_\beta C_\alpha - S_\alpha C_\beta & 0 & -S_\alpha S_\beta + C_\alpha C_\beta \end{bmatrix}$$

$$R_3 = R_4 \,!$$

5. Wiktor Springer

$$2\cdot\sin\Theta$$

$$RPY(\alpha,\beta,\varphi)=rot(z,\alpha)\circ rot(y,\beta)\circ rot(x,\varphi)$$

$$\begin{bmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} c\beta & 0 & s\beta \\ 0 & 1 & 0 \\ -s\beta & 0 & c\beta \end{bmatrix}\circ\begin{bmatrix} 1 & 0 & 0 \\ 0 & c\varphi & -s\varphi \\ 0 & s\varphi & c\varphi \end{bmatrix}$$

$$=\begin{bmatrix} c\alpha c\beta & -s\alpha & c\alpha s\beta \\ s\alpha c\beta & c\alpha & s\alpha s\beta \\ -s\beta & 0 & c\beta \end{bmatrix}\cdot\begin{bmatrix} 1 & 0 & 0 \\ 0 & c\varphi & -s\varphi \\ 0 & s\varphi & c\varphi \end{bmatrix}$$

$$\cancel{=\begin{bmatrix} c\alpha c\beta & -s\alpha c\varphi & c\alpha s\beta s\varphi \\ & & \\ & & \end{bmatrix}}$$

$$=\begin{bmatrix} c\alpha c\beta & -s\alpha c\varphi + c\alpha s\beta s\varphi & s\alpha s\varphi + c\alpha s\beta c\varphi \\ s\alpha c\beta & c\alpha c\varphi + s\alpha s\beta s\varphi & -c\alpha s\varphi + s\alpha s\beta c\varphi \\ -s\beta & c\beta s\varphi & c\beta c\varphi \end{bmatrix}$$

6. Kajetan Zdanowicz

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Zad. 6

$$R = RPY\left(\overset{\alpha}{\frac{\pi}{2}}, \overset{\beta}{\frac{\pi}{3}}, \overset{\gamma}{0}\right)$$

$$RPY(\alpha, \beta, \gamma) = rot(z, \alpha) \cdot rot(y, \beta) \cdot rot(x, \gamma)$$

$$RPY(\alpha, \beta, \gamma) = \begin{bmatrix} C\alpha & -S\alpha & 0 \\ S\alpha & C\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} C\beta & 0 & S\beta \\ 0 & 1 & 0 \\ -S\beta & 0 & C\beta \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\gamma & -S\gamma \\ 0 & S\gamma & C\gamma \end{bmatrix}$$

$$= \begin{bmatrix} C\alpha C\beta & -S\alpha & C\alpha S\beta \\ S\alpha C\beta & C\alpha & S\alpha S\beta \\ -S\beta & 0 & C\beta \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\gamma & -S\gamma \\ 0 & S\gamma & C\gamma \end{bmatrix} = \begin{bmatrix} C\alpha C\beta & -S\alpha C\gamma + C\alpha S\beta S\gamma & S\alpha S\gamma + C\alpha S\beta C\gamma \\ S\alpha C\beta & C\alpha C\gamma + S\alpha S\beta S\gamma & -C\alpha S\gamma + S\alpha S\beta C\gamma \\ -S\beta & C\beta S\gamma & C\beta C\gamma \end{bmatrix}$$

$S\alpha = 1$ $\qquad C\alpha = 0$

$S\beta = \frac{\sqrt{3}}{2}$ $\qquad C\beta = \frac{1}{2}$

$S\gamma = 0$ $\qquad C\gamma = 1$

$$RPY\left(\frac{\pi}{2}, \frac{\pi}{3}, 0\right) = \begin{bmatrix} 0 & -1 & 0 \\ \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

7. Izabella Cwojdzińska

zad 7 Izabella Cwojdzińska 243001

$$R = \begin{bmatrix} 0 & -\sqrt{2}/2 & \sqrt{2}/2 \\ 1 & 0 & 0 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} =$$

$$= E(\alpha, \beta, \gamma)$$

$c_\beta = \sqrt{2}/2 \to 1^\circ \quad \beta = \frac{\pi}{4} \quad s_\beta = \frac{\sqrt{2}}{2}$

$$E(\alpha, \tfrac{\pi}{4}, \gamma) = \begin{bmatrix} c_\alpha \frac{\sqrt{2}}{2} \cdot c_\gamma - s_\alpha s_\gamma & -s_\gamma \cdot c_\alpha \cdot \frac{\sqrt{2}}{2} - s_\alpha c_\gamma & \frac{\sqrt{2}}{2} \cdot c_\alpha \\ s_\alpha \cdot \frac{\sqrt{2}}{2} \cdot c_\gamma + s_\gamma c_\alpha & -s_\alpha s_\gamma \frac{\sqrt{2}}{2} + c_\alpha c_\gamma & \frac{\sqrt{2}}{2} \cdot s_\alpha \\ -\frac{\sqrt{2}}{2} \cdot \cos\gamma & \frac{\sqrt{2}}{2} \cdot s_\gamma & \frac{\sqrt{2}}{2} \end{bmatrix} = *$$

$-\frac{\sqrt{2}}{2} \cdot c_\gamma = 0 \Rightarrow c_\gamma = 0 \quad \Rightarrow \gamma = \frac{\pi}{2}$

$\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} \cdot s_\gamma \Rightarrow s_\gamma = 1$

$$* = E(\alpha, \tfrac{\pi}{4}, \tfrac{\pi}{2}) = \begin{bmatrix} -s_\alpha & -c_\alpha \frac{\sqrt{2}}{2} & \sqrt{2}/2\, c_\alpha \\ c_\alpha & -s_\alpha \sqrt{2}/2 & \sqrt{2}/2\, s_\alpha \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$$

$\begin{cases} -s_\alpha = 0 \\ c_\alpha = 1 \\ -\sqrt{2}/2 = -\cos\alpha \frac{\sqrt{2}}{2} \end{cases} \qquad \begin{aligned} 0 &= -s_\alpha \cdot \tfrac{\sqrt{2}}{2} \\ \sqrt{2}/2 &= \sqrt{2}/2\, c_\alpha \\ 0 &= \sqrt{2}/2 \cdot s_\alpha \end{aligned}$

$\begin{cases} -s_\alpha = 0 \\ c_\alpha = 1 \\ c_\alpha = 1 \end{cases} \qquad \begin{aligned} -s_\alpha &= 0 \\ c_\alpha &= 1 \\ s_\alpha &= 0 \end{aligned}$

$\begin{cases} s_\alpha = 0 \\ c_\alpha = 1 \end{cases} \Rightarrow \alpha = 0$

$c_\beta = \frac{\sqrt{2}}{2} \to 2^\circ \quad \beta = -\frac{\pi}{4} \Rightarrow s_\beta = -\frac{\sqrt{2}}{2}$

$$E(\alpha, -\tfrac{\pi}{4}, \gamma) = \begin{bmatrix} c_\alpha \cdot \frac{\sqrt{2}}{2} \cdot c_\gamma - s_\alpha c_\gamma & -s_\gamma c_\alpha \cdot \frac{\sqrt{2}}{2} - s_\alpha c_\gamma & -\frac{\sqrt{2}}{2} \cdot c_\alpha \\ s_\alpha \cdot \frac{\sqrt{2}}{2} \cdot c_\gamma + s_\gamma c_\alpha & -s_\alpha s_\gamma \cdot \frac{\sqrt{2}}{2} + c_\alpha c_\gamma & -\frac{\sqrt{2}}{2} \cdot s_\alpha \\ \sqrt{2}/2\, c_\gamma & -\frac{\sqrt{2}}{2} \cdot s_\gamma & \sqrt{2}/2 \end{bmatrix}$$

$\left.\begin{aligned} \frac{\sqrt{2}}{2} \cdot c_\gamma = 0 &\Rightarrow c_\gamma = 0 \\ -\frac{\sqrt{2}}{2} \cdot s_\gamma = \frac{\sqrt{2}}{2} &\Rightarrow s_\gamma = -1 \end{aligned}\right\} \gamma = -\frac{\pi}{2}$

$$E(\alpha, -\frac{\pi}{4}, -\frac{\pi}{2}) = \begin{bmatrix} s_\alpha & \sqrt{2}/2\, c_\alpha & -\sqrt{2}/2\, c_\alpha \\ -c_\alpha & s_\alpha \frac{\sqrt{2}}{2} & -\sqrt{2}/2\, s_\alpha \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$$

$\begin{cases} s_\alpha = 0 \\ -c_\alpha = 1 \end{cases}$ $\quad \frac{\sqrt{2}}{2} c_\alpha = -\frac{\sqrt{2}}{2}$ $\quad s_\alpha \frac{\sqrt{2}}{2} = 0$ $\quad -\frac{\sqrt{2}}{2} c_\alpha = \frac{\sqrt{2}}{2}$ $\quad -\sqrt{2}/2\, s_\alpha = 0$

$\begin{cases} s_\alpha = 0 \\ c_\alpha = -1 \end{cases}$ $\quad c_\alpha = -1$ $\quad s_\alpha = 0$

$\begin{cases} s_\alpha = 0 \\ c_\alpha = -1 \end{cases} > \alpha = \pi$

$E(\pi, -\frac{\pi}{4}, -\frac{\pi}{2})$

RPY

$$RPY(\alpha, \beta, \gamma) = \begin{bmatrix} c_\alpha c_\beta & -s_\alpha c_\gamma + c_\alpha s_\beta s_\gamma & s_\alpha s_\gamma + c_\alpha s_\beta c_\gamma \\ s_\alpha c_\beta & c_\alpha c_\gamma + s_\alpha s_\beta s_\gamma & -c_\alpha s_\gamma + s_\alpha s_\beta c_\gamma \\ -s_\beta & c_\beta s_\gamma & c_\beta c_\gamma \end{bmatrix}$$

$-s_\beta = 0$

$s_\beta = 0$

1° $\beta = 0$

$$\begin{bmatrix} c_\alpha & -s_\alpha c_\gamma & s_\alpha s_\gamma \\ s_\alpha & c_\alpha c_\gamma & -c_\alpha s_\gamma \\ 0 & s_\gamma & c_\gamma \end{bmatrix} = RPY(\alpha, 0, \gamma)$$

$\begin{matrix} s_\gamma = \frac{\sqrt{2}}{2} \\ c_\gamma = \frac{\sqrt{2}}{2} \end{matrix} > \gamma = \frac{\pi}{4}$

$$\begin{bmatrix} c_\alpha & -s_\alpha \frac{\sqrt{2}}{2} & s_\alpha \frac{\sqrt{2}}{2} \\ s_\alpha & c_\alpha \frac{\sqrt{2}}{2} & -c_\alpha \frac{\sqrt{2}}{2} \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} = RPY(\alpha, 0, \frac{\pi}{4})$$

$\begin{cases} s_\alpha = 1 \\ c_\alpha = 0 \\ -s_\alpha \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2} \\ c_\alpha \frac{\sqrt{2}}{2} = 0 \\ s_\alpha \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} \\ -c_\alpha \frac{\sqrt{2}}{2} = 0 \end{cases}$ $\quad \begin{cases} s_\alpha = 1 \\ c_\alpha = 0 \\ s_\alpha = 1 \\ c_\alpha = 0 \\ s_\alpha = 1 \\ c_\alpha = 0 \end{cases} \implies \alpha = \frac{\pi}{2}$

1° $RPY(\alpha, \beta, \gamma) = RPY(\pi/2, 0, \pi/4)$

$2°$ $s\beta = 0$ $\quad \beta = \pi$

$$RPY(\alpha, \pi, \gamma) = \begin{bmatrix} -c_\alpha & -s_\alpha c_\gamma & s_\alpha s_\gamma \\ -s_\alpha & c_\alpha c_\gamma & -c_\alpha s_\gamma \\ 0 & -s_\gamma & -c_\gamma \end{bmatrix}$$

$$\begin{matrix} -s_\gamma = \frac{\sqrt{2}}{2} \\ -c_\gamma = \sqrt{2}/2 \end{matrix} \Rightarrow \begin{matrix} s_\gamma = -\frac{\sqrt{2}}{2} \\ c_\gamma = -\sqrt{2}/2 \end{matrix} > \gamma = -\frac{3\pi}{4}$$

$$RPY(\alpha, \pi, \tfrac{3\pi}{4}) = \begin{bmatrix} -c_\alpha & s_\alpha \frac{\sqrt{2}}{2} & -s_\alpha \sqrt{2}/2 \\ -s_\alpha & -c_\alpha \sqrt{2}/2 & c_\alpha \sqrt{2}/2 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$$

$$\begin{cases} -c_\alpha = 0 & c_\alpha = 0 \\ -s_\alpha = 1 & s_\alpha = -1 \\ s_\alpha \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2} & s_\alpha = -1 \\ -c_\alpha \frac{\sqrt{2}}{2} = 0 & c_\alpha = 0 \\ -s_\alpha \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} & s_\alpha = -1 \\ c_\alpha \frac{\sqrt{2}}{2} = 0 & c_\alpha = 0 \end{cases} \Longrightarrow \alpha = -\frac{\pi}{2}$$

$$2° \quad RPY(\alpha, \beta, \gamma) = \left(-\frac{\pi}{2}, \pi, -\frac{3\pi}{4}\right)$$

$3°$ $s\beta = 0 \Rightarrow \beta = -\pi$

$$RPY(\alpha, -\pi, \gamma) = \begin{bmatrix} -c_\alpha & -s_\alpha c_\gamma & s_\alpha s_\gamma \\ -s_\alpha & c_\alpha c_\gamma & -c_\alpha s_\gamma \\ 0 & -s_\gamma & -c_\gamma \end{bmatrix} =$$

$$\begin{matrix} -s_\gamma = \frac{\sqrt{2}}{2} \\ -c_\gamma = \frac{\sqrt{2}}{2} \end{matrix} \Longrightarrow \gamma = -\frac{3}{4}\pi$$

$$RPY(\alpha, -\pi, \tfrac{\pi}{4}) = \begin{bmatrix} -c_\alpha & s_\alpha \sqrt{2}/2 & -s_\alpha \sqrt{2}/2 \\ -s_\alpha & -c_\alpha \sqrt{2}/2 & c_\alpha \sqrt{2}/2 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$$

$$\begin{cases} -c_\alpha = 0 \\ -s_\alpha = 1 \\ s_\alpha \cdot \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2} \\ -c_\alpha \cdot \frac{\sqrt{2}}{2} = 0 \\ s_\alpha \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} \\ c_\alpha \cdot \frac{\sqrt{2}}{2} = 0 \end{cases} \quad \begin{cases} c_\alpha = 0 \\ s_\alpha = -1 \\ s_\alpha = -1 \\ c_\alpha = 0 \\ s_\alpha = -1 \\ c_\alpha = 0 \end{cases} \Rightarrow \alpha = -\frac{\pi}{2}$$

$3°$

$RPY(\alpha, \beta, \gamma) =$

$RPY(-\frac{\pi}{2}, -\pi, -\frac{3}{4}\pi)$