

LISTA 4

1. Krzysztof Ragan

Lagun
2ad. 1

LISTA 4

$$L[\omega]R^T = [L\omega]$$

$$R = \text{rot}(x, \alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix}, \quad R^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix}$$

$$[\omega] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

$$L = L[\omega]R^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 \cos \alpha + \sin \alpha \omega_2 & -\sin \alpha \omega_1 & -\omega_1 \cos \alpha \\ \omega_3 \sin \alpha - \omega_2 \cos \alpha & \cos \alpha \omega_1 & -\omega_1 \sin \alpha \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix} =$$

$$\Rightarrow \begin{bmatrix} 0 & -\omega_3 \cos \alpha - \omega_2 \sin \alpha & \omega_3 \sin \alpha + \omega_2 \cos \alpha \\ \omega_3 \cos \alpha + \sin \alpha \omega_2 & 0 & -\sin \alpha \omega_1 - \omega_1 \cos \alpha \\ \omega_3 \sin \alpha - \omega_2 \cos \alpha & \underbrace{\omega_1 \cos^2 \alpha + \omega_1 \sin^2 \alpha}_{\omega_1} & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\omega_3 \cos \alpha - \omega_2 \sin \alpha & \omega_3 \sin \alpha + \omega_2 \cos \alpha \\ \omega_3 \cos \alpha + \sin \alpha \omega_2 & 0 & -\sin \alpha \omega_1 - \omega_1 \cos \alpha \\ \omega_3 \sin \alpha - \omega_2 \cos \alpha & \omega_1 & 0 \end{bmatrix}$$

$$R \cdot \omega = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \begin{bmatrix} \omega_1 \\ \cos \alpha \omega_2 - \sin \alpha \omega_3 \\ \sin \alpha \omega_2 + \cos \alpha \omega_3 \end{bmatrix}$$

$$P = [L \cdot \omega] = \begin{bmatrix} 0 & -\sin \alpha \omega_2 - \cos \alpha \omega_3 & \cos \alpha \omega_2 - \sin \alpha \omega_3 \\ \sin \alpha \omega_2 + \cos \alpha \omega_3 & 0 & -\omega_1 \\ \sin \alpha \omega_3 - \cos \alpha \omega_2 & \omega_1 & 0 \end{bmatrix}$$

$$L = P$$

2. Gabriel Mainka

ROBOTYKA 8w

zad. 2 lista 4

$$\mathcal{L}_s = [\omega_s] = \dot{R}R^T$$

$$\mathcal{L}_B = [\omega_B] = R^T \dot{R}$$

dla $R(x, \alpha)$

$$R^T \mathcal{L}_s R = R^T \dot{R} R^T R = R^T \dot{R} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\alpha & s_\alpha \\ 0 & -s_\alpha & c_\alpha \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\alpha & -s_\alpha \\ 0 & s_\alpha & c_\alpha \end{bmatrix} =$$

$$= R^T \dot{R} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos^2 \alpha + \sin^2 \alpha & -s_\alpha c_\alpha + s_\alpha c_\alpha \\ 0 & -s_\alpha \cos \alpha + s_\alpha \cos \alpha & \cos^2 \alpha + \sin^2 \alpha \end{bmatrix} = R^T \dot{R} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \mathcal{L}_B$$

analogiczne dla $R(y, \beta), R(z, \gamma)$

dla $R(z, \gamma)$

$$R \mathcal{L}_B R^T = R R^T \dot{R} R^T = \begin{bmatrix} c_\gamma & -s_\gamma & 0 \\ s_\gamma & c_\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_\gamma & s_\gamma & 0 \\ -s_\gamma & c_\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} R^T =$$

$$= \begin{bmatrix} c_\gamma^2 + s_\gamma^2 & c_\gamma s_\gamma - c_\gamma s_\gamma & 0 \\ c_\gamma s_\gamma - c_\gamma s_\gamma & c_\gamma^2 + s_\gamma^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} R^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} R^T = \mathcal{L}_s$$

W ostatniej linii zamiast \dot{R}^T powinno być $\dot{R}R^T$.

2nd 3/

$$\begin{cases} \Omega_s = R \Omega_b R^T \\ R[\omega_s] R^T = [R\omega_b] \end{cases}$$

$$\Omega_s = [R\omega_b]$$

$$\omega_s = R\omega_b$$

3.

4. Hubert Górski

zad. 4

Hubert Górski

$$V_s = \begin{bmatrix} R & [T]R \\ 0 & R \end{bmatrix} V_B \quad \begin{bmatrix} R & [T]R \\ 0 & R \end{bmatrix} = Z$$

$$V_B = Z^{-1} V_s \quad Z^{-1} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} R & [T]R \\ 0 & R \end{bmatrix} = \begin{bmatrix} I_3 & 0 \\ 0 & I_3 \end{bmatrix}$$

$$AR + B_0 = I_3 \rightarrow A = R^T$$

$$A[T]R + BR = 0 \rightarrow BR = -A[T]R \rightarrow B = -R^T[T]$$

$$CR + DO = 0 \rightarrow C = 0$$

$$C[T]R + DR = I_3 \rightarrow D = R^T$$

$$Z^{-1} = \begin{bmatrix} R^T & -R^T[T] \\ 0 & R^T \end{bmatrix}$$

$$V_B = \begin{bmatrix} R^T & -R^T[T] \\ 0 & R^T \end{bmatrix} V_s$$

5. a) Przemysław Kudełka

2.5 a)

$$\dot{V}_B = \begin{pmatrix} R^T \dot{T} \\ \omega_B \end{pmatrix} \quad \dot{V}_S = \begin{pmatrix} [T] \omega_S + \dot{T} \\ \omega_S \end{pmatrix}$$

$$\begin{bmatrix} R(t) & T(t) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \text{rot}(2, \kappa(t)) & 0 \\ 0 & 1 \end{bmatrix} \quad \kappa(t) = -3t$$

$$R(t) = \begin{bmatrix} C(-3t) & -S(-3t) & 0 \\ S(-3t) & C(-3t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R^T(t) = \begin{bmatrix} C(-3t) & S(-3t) & 0 \\ -S(-3t) & C(-3t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\dot{R}(t) = \begin{bmatrix} 3S(-3t) & 3C(-3t) & 0 \\ -3C(-3t) & 3S(-3t) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

wzory na te macierze potrzebne do wyrowadzenia metodiki:

$$\dot{\Omega}_S = \dot{R} R^T \quad \dot{\Omega}_B = R^T \dot{R} \quad \text{jedna } \dot{\Omega} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

$$\dot{R} R^T = 3S(-3t)(C(-3t) - 3C(-3t)S(-3t))$$

$$\dot{R} R^T =$$

$$\dot{R}R^T = \begin{bmatrix} 3S(-3t)C(-3t) & -3C(-3t)S(-3t) & 3S^2(-3t) + 3C^2(-3t) & 0 \\ -3C^2(-3t) - 3S^2(-3t) & -3C(-3t)S(-3t) + 3S(-3t)C(-3t) & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

$$\omega_S = \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix} \quad T=0 \quad \dot{T}=0 \Rightarrow V_S = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -3 \end{pmatrix}$$

$$R^T \dot{R} = \begin{bmatrix} 3S(-3t)C(-3t) & -3C(-3t)S(-3t) & 3C^2(-3t) + 3S^2(-3t) & 0 \\ -3S^2(-3t) - 3C^2(-3t) & 3C(-3t)(-S(-3t)) + 3S(-3t)C(-3t) & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \Rightarrow V_B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -3 \end{pmatrix}$$

b) Wiktor Springer

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N.S.

b) lista translacyjna

$$\begin{bmatrix} R(t) & T(t) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} I_3 & [1, 3t, -t^2]^T \\ 0 & 1 \end{bmatrix}$$

$$T = [1, 3t, -t^2]^T \quad \ddot{T} = [0, 3, -2t]^T$$

$$R(t) = I_3 \quad \dot{R}(t) = 0$$

$$[\omega_B] = R^T \cdot \ddot{R} = 0 \quad [\omega_s] = \dot{R} R^T = 0$$

$$V_B = \begin{pmatrix} R^T \ddot{T} \\ \omega_B \end{pmatrix} = \begin{pmatrix} [0, 3, -2t]^T \\ 0 \end{pmatrix}$$

$$V_S = \begin{pmatrix} [T] \omega_s + \ddot{T} \\ \omega_s \end{pmatrix} = \begin{pmatrix} [0, 3, -2t]^T \\ 0 \end{pmatrix}$$

c) Maciej Salamoński

$$\text{zad 5} \quad V_B = \begin{pmatrix} RT \\ W_B \end{pmatrix}, \quad V_s = \begin{pmatrix} [T]W_s + \dot{T} \\ W_s \end{pmatrix}$$

* połączenia powyższych równań

$$\begin{bmatrix} R(t) & T(t) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \omega_0 + (\zeta, \alpha(t)) & [1, 3t, -t^2]^T \\ 0 & 1 \end{bmatrix}, \quad \alpha(t) = -3t$$

$$R = \omega_0 t (\zeta, \alpha(t)) = \begin{bmatrix} c_\omega & -s_\omega & 0 \\ s_\omega & c_\omega & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c(-3t) & -s(-3t) & 0 \\ s(-3t) & c(-3t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c(3t) & s(3t) & 0 \\ -s(3t) & c(3t) & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad T = \begin{pmatrix} 1 \\ 3t \\ -t^2 \end{pmatrix} \rightarrow \dot{T} = \begin{pmatrix} 0 \\ 3 \\ -2t \end{pmatrix}$$

$$\underline{\underline{[w]}} = \underline{\underline{Q_B}} = \underline{\underline{R^T \cdot \dot{R}}} = \begin{bmatrix} c(3t) & -s(3t) & 0 \\ s(3t) & c(3t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3s(3t) & 3c(3t) & 0 \\ -3c(3t) & -3s(3t) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -3cs(3t) + 3cs(3t) & 3c^2(3t) + 3s^2(3t) & 0 \\ -3s^2(3t) - 3c^2(3t) & 3cs(3t) - 3cs(3t) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 3 & 0 \\ -3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \underline{\underline{W_B}} = \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix}, \quad \underline{\underline{R^T \cdot \dot{T}}} = \begin{bmatrix} c(3t) & -s(3t) & 0 \\ s(3t) & c(3t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 0 \\ 3 \\ -2t \end{pmatrix}$$

$$\underline{\underline{V_B}} = \begin{pmatrix} -3s(3t) \\ 3c(3t) \\ -2t \\ 0 \\ 0 \\ -3 \end{pmatrix}$$

$$V_s = \begin{pmatrix} [T]w_s + \dot{T} \\ w_s \end{pmatrix}, \quad \dot{T} = \begin{pmatrix} 0 \\ 3 \\ -2t \end{pmatrix} \quad \leftarrow T = \begin{pmatrix} 1 \\ 3t \\ -t^2 \end{pmatrix}$$

$$[W] = \underline{\underline{J}}_s = \underline{\underline{R}}^T \underline{\underline{R}}$$

$$[W] = \begin{bmatrix} -3s(3t) & 3c(3t) & 0 \\ -3c(3t) & -3s(3t) & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} c(3t) & s(3t) & 0 \\ s(3t) & c(3t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -3cs(3t) + 3cs(3t) & 3s^2(3t) + 3c^2(3t) & 0 \\ -3c^2(3t) - 3s^2(3t) & 3cs(3t) - 3cs(3t) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 3 & 0 \\ -3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow w_s = \begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix}$$

$$[T] = \begin{bmatrix} 0 & t^2 & 3t \\ -t^2 & 0 & -1 \\ 3t & 1 & 0 \end{bmatrix} \rightarrow [T]w_s = \begin{bmatrix} 0 & t^2 & 3t \\ -t^2 & 0 & -1 \\ 3t & 1 & 0 \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$[\bar{T}]w_s = \begin{pmatrix} -9t \\ 3 \\ 0 \end{pmatrix}$$

$$V_s = \begin{bmatrix} [\bar{T}]w_s + \dot{T} \\ w_s \end{bmatrix} = \begin{pmatrix} -9t \\ 6 \\ -2t \\ 0 \\ -3 \end{pmatrix}$$

6. Konrad Białek

Konrad Białek
Zad. 6 / LISTA 4

Przelatstowanie układu $X_0Y_0Z_0$ w układ $X_1Y_1Z_1$

$$\begin{bmatrix} R & T \end{bmatrix} = \begin{bmatrix} c_{q_1q_2} & -s_{q_1q_2} & 0 & l_1c_{q_1} + l_2s_{q_1q_2} \\ s_{q_1q_2} & c_{q_1q_2} & 0 & l_1s_{q_1} + l_2c_{q_1q_2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{ gdzie } \begin{cases} q_1 = \cos \alpha \\ q_2 = \frac{\pi}{2} - \beta \end{cases}$$

$$\cos(q_1+q_2) = c_{q_1q_2}, \sin(q_1+q_2) = s_{q_1q_2}, \cos(q_1) = c_{q_1}, \sin(q_1) = s_{q_1}$$

$$R^T = \begin{bmatrix} c_{q_1q_2} & -s_{q_1q_2} & 0 \\ -s_{q_1q_2} & c_{q_1q_2} & 0 \\ 0 & 0 & 1 \end{bmatrix}, R = \begin{bmatrix} -\dot{q}_2s_{q_1q_2} & -\dot{q}_2c_{q_1q_2} & 0 \\ \dot{q}_2c_{q_1q_2} & -\dot{q}_2s_{q_1q_2} & 0 \\ 0 & 0 & 1 \end{bmatrix}, R = \begin{bmatrix} c_{q_1q_2} & -s_{q_1q_2} & 0 \\ s_{q_1q_2} & c_{q_1q_2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F(l_1, l_2, s_{q_1q_2}) \dot{T} = \begin{bmatrix} -l_2\dot{q}_2s_{q_1q_2} \\ l_2\dot{q}_2c_{q_1q_2} \\ 0 \end{bmatrix}, \dot{T} = \begin{bmatrix} 0 & 0 & l_1s_{q_1} + l_2s_{q_1q_2} \\ 0 & 0 & -l_1c_{q_1} - l_2c_{q_1q_2} \\ 0 & 0 & 0 \end{bmatrix}$$

$$[w_B] = \omega_B = R^T \dot{R} = \begin{bmatrix} c_{q_1q_2} & -s_{q_1q_2} & 0 \\ -s_{q_1q_2} & c_{q_1q_2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\dot{q}_2s_{q_1q_2} & -\dot{q}_2c_{q_1q_2} & 0 \\ \dot{q}_2c_{q_1q_2} & -\dot{q}_2s_{q_1q_2} & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -\dot{q}_2^2c_{q_1q_2}^2 + \dot{q}_2^2s_{q_1q_2}^2 & -\dot{q}_2^2c_{q_1q_2}s_{q_1q_2} & 0 \\ \dot{q}_2^2s_{q_1q_2}^2 + \dot{q}_2^2c_{q_1q_2}^2 & \dot{q}_2^2s_{q_1q_2}c_{q_1q_2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[w_B] = \begin{bmatrix} -\dot{q}_2^2c_{q_1q_2}^2 + \dot{q}_2^2s_{q_1q_2}^2 & -\dot{q}_2^2c_{q_1q_2}s_{q_1q_2} & 0 \\ \dot{q}_2^2s_{q_1q_2}^2 + \dot{q}_2^2c_{q_1q_2}^2 & \dot{q}_2^2s_{q_1q_2}c_{q_1q_2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[w_B] = \begin{bmatrix} 0 & -\dot{q}_2 & 0 \\ \dot{q}_2(c_{q_1q_2}^2 + s_{q_1q_2}^2) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\dot{q}_2 & 0 \\ \dot{q}_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\omega_B = \begin{pmatrix} 0 \\ 0 \\ \dot{q}_2 \end{pmatrix}, \omega_S = R \omega_B = \begin{bmatrix} c_{q_1q_2} & -s_{q_1q_2} & 0 \\ s_{q_1q_2} & c_{q_1q_2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ \dot{q}_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \dot{q}_2 \end{pmatrix}$$

$$V_B = \begin{pmatrix} R^T \cdot \dot{T} \\ w_B \end{pmatrix}, V_S = \begin{pmatrix} [T]w_S + \dot{T} \\ w_S \end{pmatrix}$$

$$R^T \cdot \dot{T} = \begin{pmatrix} c_{q_1 q_2} s_{q_1 q_2} & 0 \\ -s_{q_1 q_2} c_{q_1 q_2} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -l_2 \dot{q}_2 s_{q_1 q_2} \\ l_2 \dot{q}_2 c_{q_1 q_2} \\ 0 \end{pmatrix} = \begin{pmatrix} l_2 \dot{q}_2 c_{q_1 q_2} s_{q_1 q_2} + l_2 \dot{q}_2 s_{q_1 q_2} c_{q_1 q_2} \\ l_2 \dot{q}_2 s_{q_1 q_2}^2 + l_2 \dot{q}_2 c_{q_1 q_2}^2 \\ 0 \end{pmatrix}$$

$$R^T \cdot \dot{T} = \begin{pmatrix} 0 \\ l_2 \dot{q}_2 (s_{q_1 q_2}^2 + c_{q_1 q_2}^2) \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ l_2 \dot{q}_2 \\ 0 \end{pmatrix}$$

$$[T]w_S + \dot{T} = \begin{pmatrix} 0 & 0 & l_1 s_{q_1} + l_2 s_{q_1 q_2} \\ 0 & 0 & -l_1 c_{q_1} + l_2 c_{q_1 q_2} \\ -l_1 s_{q_1} - l_2 s_{q_1 q_2} & l_1 c_{q_1} + l_2 c_{q_1 q_2} & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \dot{q}_2 \\ 0 \end{pmatrix} = \begin{pmatrix} l_1 \dot{q}_2 s_{q_1} + l_2 \dot{q}_2 s_{q_1 q_2} \\ -l_1 \dot{q}_2 c_{q_1} + l_2 \dot{q}_2 c_{q_1 q_2} \\ 0 \end{pmatrix}$$

$$[T]w_S + \dot{T} = \begin{pmatrix} l_1 \dot{q}_2 s_{q_1} + l_2 \dot{q}_2 s_{q_1 q_2} \\ -l_1 \dot{q}_2 c_{q_1} + l_2 \dot{q}_2 c_{q_1 q_2} \\ 0 \end{pmatrix} + \begin{pmatrix} -l_2 \dot{q}_2 s_{q_1 q_2} \\ l_2 \dot{q}_2 c_{q_1 q_2} \\ 0 \end{pmatrix} = \begin{pmatrix} -l_1 \dot{q}_2 c_{q_1} \\ 0 \\ 0 \end{pmatrix}$$

$$V_B = \begin{pmatrix} 0 \\ l_2 \dot{q}_2 \\ 0 \\ 0 \\ 0 \\ \dot{q}_2 \end{pmatrix} = \begin{pmatrix} R^T \cdot \dot{T} \\ w_B \end{pmatrix}, V_S = \begin{pmatrix} l_1 \dot{q}_2 s_{q_1} \\ -l_1 \dot{q}_2 c_{q_1} \\ 0 \\ 0 \\ 0 \\ \dot{q}_2 \end{pmatrix} = \begin{pmatrix} [T]w_S + \dot{T} \\ w_S \end{pmatrix}$$

7. Jan Bronicki

$$\alpha = q_1 + q_2$$

$$\begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{q_1+q_2} & -s_{q_1+q_2} & 0 & l_1 c_{q_1} + l_2 c_{q_1+q_2} \\ s_{q_1+q_2} & c_{q_1+q_2} & 0 & l_1 s_{q_1} + l_2 s_{q_1+q_2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} c_\alpha & -s_\alpha & 0 \\ s_\alpha & c_\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \dot{R} = \begin{bmatrix} -\dot{\alpha} s_\alpha & \dot{\alpha} c_\alpha & 0 \\ \dot{\alpha} c_\alpha & -\dot{\alpha} s_\alpha & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$T = \begin{bmatrix} l_1 c_{q_1} + l_2 c_\alpha \\ l_1 s_{q_1} + l_2 s_\alpha \\ 0 \end{bmatrix} \quad \dot{T} = \begin{bmatrix} -\dot{q}_1 l_1 s_{q_1} - \dot{\alpha} l_2 s_\alpha \\ \dot{q}_1 l_1 c_{q_1} + \dot{\alpha} l_2 c_\alpha \\ 0 \end{bmatrix}$$

$$\dot{R}^T = \begin{bmatrix} c_\alpha & s_\alpha & 0 \\ -s_\alpha & c_\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$* V_B = \begin{pmatrix} R^T \dot{T} \\ \omega_B \end{pmatrix} - \text{Skewsymm } \omega$$

$$R^T \dot{T} = \begin{bmatrix} c_\alpha & s_\alpha & 0 \\ -s_\alpha & c_\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\dot{q}_1 l_1 s_{q_1} - \dot{\alpha} l_2 s_{q_2} \\ \dot{q}_1 l_2 c_{q_1} + \dot{\alpha} l_1 c_{q_2} \\ 0 \end{bmatrix} = \begin{bmatrix} l_1 \dot{q}_1 s_{q_2} \\ l_1 \dot{q}_1 c_{q_2} + l_2 (\dot{q}_1 + \dot{q}_2) \\ 0 \end{bmatrix}$$

$$[\omega_B] = R^T \dot{R} = \begin{bmatrix} c_\alpha & s_\alpha & 0 \\ -s_\alpha & c_\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\dot{\alpha} s_\alpha & -\dot{\alpha} c_\alpha & 0 \\ \dot{\alpha} c_\alpha & -\dot{\alpha} s_\alpha & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\dot{\alpha} & 0 \\ \dot{\alpha} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$V_B = \begin{bmatrix} l_1 \dot{q}_1 s_{q_2} \\ l_1 \dot{q}_1 c_{q_2} + l_2 (\dot{q}_1 + \dot{q}_2) \\ 0 \\ 0 \\ \vdots \end{bmatrix} \quad \omega_B = \begin{pmatrix} 0 \\ 0 \\ \dot{\alpha} \end{pmatrix}$$

$$* V_S = \begin{pmatrix} T \omega_S + \dot{T} \\ \omega_S \end{pmatrix} - \text{Skewsymm } \omega$$

$$\omega_S = \begin{pmatrix} 0 \\ 0 \\ \dot{\alpha} \end{pmatrix}$$

$$[T] \omega_S = \begin{bmatrix} 0 & 0 & l_1 s_{q_1} + l_2 s_{q_2} \\ 0 & 0 & -l_1 c_{q_1} - l_2 c_{q_2} \\ -l_1 s_{q_1} - l_2 s_{q_2} & l_1 c_{q_1} + l_2 c_{q_2} & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\alpha} \end{bmatrix} = \underbrace{\begin{bmatrix} \dot{\alpha} l_1 s_{q_1} + \dot{\alpha} l_2 s_{q_2} \\ -\dot{\alpha} l_1 c_{q_1} - \dot{\alpha} l_2 c_{q_2} \\ 0 \end{bmatrix}}_{B}$$

$$\underbrace{\begin{bmatrix} \dot{\alpha} l_1 s_{q_1} + \dot{\alpha} l_2 s_{q_2} \\ -\dot{\alpha} l_1 c_{q_1} - \dot{\alpha} l_2 c_{q_2} \\ 0 \end{bmatrix}}_B + \underbrace{\begin{bmatrix} -\dot{q}_1 l_1 s_{q_1} - \dot{\alpha} l_2 s_{q_2} \\ \dot{q}_1 l_1 c_{q_1} + \dot{\alpha} l_2 c_{q_2} \\ 0 \end{bmatrix}}_T = \begin{bmatrix} l_1 s_{q_1} (\dot{\alpha} - \dot{q}_1) \\ l_1 c_{q_1} (\dot{q}_1 - \dot{\alpha}) \\ 0 \end{bmatrix} \quad V_S = \begin{bmatrix} l_1 s_{q_1} (\dot{\alpha} - \dot{q}_1) \\ l_1 c_{q_1} (\dot{q}_1 - \dot{\alpha}) \\ 0 \\ 0 \\ \dot{\alpha} \end{bmatrix}$$

8. Marcin Bober

