LISTA 3

1. Karolina Głuszek

1.

$$R(v,\theta) = I_3 + [v]\sin\theta + (1-\cos\theta)[v]^2$$

$$R(-v,-\theta) = I_3 + [-v]\sin(-\theta) + (1-\cos(-\theta))[-v]^2 =$$

$$= I_3 + [v]\sin\theta + (1-\cos\theta)[v]^2 = R(v,\theta)$$

2. Szymon Tomala

Szymon Tomala

2. $R^{-1} = R^T = R(u, -\theta)$

$$R(u,\theta) = I_3 + [u]\sin\theta + (1-\cos\theta)[u]^2$$

$$U = \begin{bmatrix} 0 & -u_z & u_y \\ u_z & 0 & -u_x \\ -u_y & u_x & 0 \end{bmatrix} \qquad U^T = \begin{bmatrix} 0 & u_z & -u_y \\ -u_z & 0 & u_x \\ u_y & -u_x & 0 \end{bmatrix} = -[U]$$

- $$R^T(u, +\theta) = \left[I_3 + [u]^T \sin\theta + (1-\cos\theta)[u]^2\right]^T$$

$$\left([u]^2\right)^T = \left(\begin{bmatrix} 0 & -u_z & u_y \\ u_z & 0 & -u_x \\ -u_y & u_x & 0 \end{bmatrix}\begin{bmatrix} 0 & -u_z & u_y \\ u_z & 0 & -u_x \\ -u_y & u_x & 0 \end{bmatrix}\right)^T = \begin{bmatrix} -u_z^2-u_y^2 & u_x u_y & u_x u_z \\ u_y u_x & -u_z^2-u_x^2 & u_z u_y \\ u_z u_x & u_z u_y & -u_y^2-u_x^2 \end{bmatrix}^T =$$

$$= \begin{bmatrix} -u_z^2-u_y^2 & u_x u_y & u_x u_z \\ u_y u_x & -u_z^2-u_x^2 & u_z u_y \\ u_z u_x & u_z u_y & -u_y^2-u_x^2 \end{bmatrix}, \text{ czyli } \left[[u]^2\right]^T = [u]^2$$

$$R^T(u, +\theta) = I_3 + [u]^T \sin\theta + (1-\cos\theta)[u]^2 = I_3 - [u]\sin\theta + (1-\cos\theta)[u]^2$$

$$R(u, -\theta) = I_3 + [u]\sin(-\theta) + (1-\cos(-\theta))[u]^2 = I_3 - [u]\sin\theta + (1-\cos\theta)[u]^2$$

$$R^{-1} = R^T \Rightarrow I = RR^T$$

$$RR^T = \left(I_3 + [u]\sin\theta + (1-\cos\theta)[u]^2\right)\left(I_3 - \sin(\theta)[u] + (1-\cos\theta)[u]^2\right) =$$

$$= R(u,\theta)\cdot R(u,-\theta) = I$$, bo gdy obrócimy obiekt o kąt θ, a później o $-\theta$ to wracamy się [illegible]

3. Paweł Troszczyński

Lista 3 zad. 3 Paweł Troszczyński

$$[v] = \frac{R - R^T}{2\sin\theta} \qquad v = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} \qquad \|v\| = 1$$

$$L = [v] = \begin{bmatrix} 0 & -v_z & v_y \\ v_z & 0 & -v_x \\ -v_y & v_x & 0 \end{bmatrix}$$

Niech $\cos\theta = c\theta$ i $\sin\theta = s\theta$

$$R = \begin{bmatrix} v_x^2(1-c\theta)+c\theta & v_xv_y(1-c\theta)-v_zs\theta & v_xv_z(1-c\theta)+v_ys\theta \\ v_xv_y(1-c\theta)+v_zs\theta & v_y^2(1-c\theta)+c\theta & v_yv_z(1-c\theta)-v_xs\theta \\ v_xv_z(1-c\theta)-v_ys\theta & v_yv_z(1-c\theta)+v_xs\theta & v_z^2(1-c\theta)+c\theta \end{bmatrix}$$

$$R^T = \begin{bmatrix} v_x^2(1-c\theta)+c\theta & v_xv_y(1-c\theta)+v_zs\theta & v_xv_z(1-c\theta)-v_ys\theta \\ v_xv_y(1-c\theta)-v_zs\theta & v_y^2(1-c\theta)+c\theta & v_yv_z(1-c\theta)+v_xs\theta \\ v_xv_z(1-c\theta)+v_ys\theta & v_yv_z(1-c\theta)-v_xs\theta & v_z^2(1-c\theta)+c\theta \end{bmatrix}$$

$$R - R^T = \begin{bmatrix} 0 & -2v_zs\theta & 2v_ys\theta \\ 2v_zs\theta & 0 & -2v_xs\theta \\ -2v_ys\theta & 2v_xs\theta & 0 \end{bmatrix}$$

$$P = \frac{R-R^T}{2\sin\theta} = \frac{1}{2\sin\theta}\begin{bmatrix} 0 & -2v_z\sin\theta & 2v_y\sin\theta \\ 2v_z\sin\theta & 0 & -2v_x\sin\theta \\ -2v_y\sin\theta & 2v_x\sin\theta & 0 \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & -v_z & v_y \\ v_z & 0 & -v_x \\ -v_y & v_x & 0 \end{bmatrix}$$

$L = P$ c.n.d.

4. Adam Bednorz

Adam Bednorz

Lista 3 zad. 4 Korzystając ze wzoru podanego na wykładzie wylicz macierz obrotu wokół wektora $\omega = (1,1,1)^T$ o kąt $\frac{\pi}{3}$.

Dane:

$$\omega = \begin{pmatrix}1\\1\\1\end{pmatrix}, \quad \theta = \frac{\pi}{3} \qquad \sin\theta = \frac{\sqrt{3}}{2} \qquad \cos\theta = \frac{1}{2}$$

$$\|\omega\| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$$

$$u = \frac{\omega}{\|\omega\|} = \begin{pmatrix}\frac{1}{\sqrt{3}}\\ \frac{1}{\sqrt{3}}\\ \frac{1}{\sqrt{3}}\end{pmatrix} = \begin{pmatrix}\frac{\sqrt{3}}{3}\\ \frac{\sqrt{3}}{3}\\ \frac{\sqrt{3}}{3}\end{pmatrix}$$

Wzór:

$$rot(u,\theta) = \begin{bmatrix} u_x^2(1-C\theta)+C\theta & u_xu_y(1-C\theta)-u_zS\theta & u_xu_z(1-C\theta)+u_yS\theta \\ u_xu_y(1-C\theta)+u_zS\theta & u_y^2(1-C\theta)+C\theta & u_yu_z(1-C\theta)-u_xS\theta \\ u_yu_z(1-C\theta)-u_yS\theta & u_yu_z(1-C\theta)+u_xS\theta & u_z^2(1-C\theta)+C\theta \end{bmatrix}$$

$$= \begin{bmatrix} \left(\frac{\sqrt{3}}{2}\right)^2\left(1-\frac{1}{2}\right)+\frac{1}{2} & \left(\frac{\sqrt{3}}{3}\right)^2\left(1-\frac{1}{2}\right)-\frac{\sqrt{3}}{3}\cdot\frac{\sqrt{3}}{2} & \left(\frac{\sqrt{3}}{3}\right)^2\left(1-\frac{1}{2}\right)+\frac{\sqrt{3}}{3}\cdot\frac{\sqrt{3}}{2} \\ \left(\frac{\sqrt{3}}{3}\right)^2\left(1-\frac{1}{2}\right)+\frac{\sqrt{3}}{3}\cdot\frac{\sqrt{3}}{2} & \left(\frac{\sqrt{3}}{3}\right)^2\left(1-\frac{1}{2}\right)+\frac{1}{2} & \left(\frac{\sqrt{3}}{3}\right)^2\left(1-\frac{1}{2}\right)-\frac{\sqrt{3}}{3}\cdot\frac{\sqrt{3}}{2} \\ \left(\frac{\sqrt{3}}{3}\right)^2\left(1-\frac{1}{2}\right)-\frac{\sqrt{3}}{3}\cdot\frac{\sqrt{3}}{2} & \left(\frac{\sqrt{3}}{3}\right)^2\left(1-\frac{1}{2}\right)+\frac{\sqrt{3}}{3}\cdot\frac{\sqrt{3}}{2} & \left(\frac{\sqrt{3}}{3}\right)^2\left(1-\frac{1}{2}\right)+\frac{1}{2} \end{bmatrix} =$$

$$\begin{bmatrix} \frac{3}{9}\cdot\frac{1}{2}+\frac{1}{2} & \frac{3}{9}\cdot\frac{1}{2}-\frac{1}{2} & \frac{3}{9}\cdot\frac{1}{2}+\frac{1}{2} \\ \frac{3}{9}\cdot\frac{1}{2}+\frac{1}{2} & \frac{3}{9}\cdot\frac{1}{2}+\frac{1}{2} & \frac{3}{9}\cdot\frac{1}{2}-\frac{1}{2} \\ \frac{3}{9}\cdot\frac{1}{2}-\frac{1}{2} & \frac{3}{9}\cdot\frac{1}{2}+\frac{1}{2} & \frac{3}{9}\cdot\frac{1}{2}+\frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{bmatrix}$$

5. Marcin Bober

zadanie 5 Marcin Bober

$$\begin{bmatrix} rot(x,\alpha) & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I & T \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} rot(x,\alpha) & \boxed{rot(x,\alpha)\cdot T} \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} I & T \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} rot(x,\alpha) & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} rot(x,\alpha) & \boxed{T} \\ 0 & 1 \end{bmatrix}$$

$rot(x,\alpha)\cdot T = T$ tylko gdy dotyczą tych samych osi

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\alpha & -s_\alpha \\ 0 & s_\alpha & c_\alpha \end{bmatrix} \cdot \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}$$

$rot(x,\alpha)\cdot T \neq T$ gdy dotyczą innych osi

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\alpha & -s_\alpha \\ 0 & s_\alpha & c_\alpha \end{bmatrix} \cdot \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ b\,c_\alpha \\ b\,s_\alpha \end{bmatrix} \neq \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}$$

6. Piotr Gorzelnik

z.6 lista 3 | Piotr Gorzelnik

$L = Rot(x,\alpha)\,Trans(x,a)$

$P = Trans(x,a)\,Rot(x,\alpha)$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_\alpha & S_\alpha & 0 \\ 0 & -S_\alpha & C_\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & C_\alpha & S_\alpha & 0 \\ 0 & -S_\alpha & C_\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

$$P = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_\alpha & S_\alpha & 0 \\ 0 & -S_\alpha & C_\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & C_\alpha & S_\alpha & 0 \\ 0 & -S_\alpha & C_\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

$(1) = (2) \qquad L = P$

Dla sytuacji, gdy składamy translacje z obrotem wobec tej samej osi o ten sam kąt.

Złożenie translacji z rotacją TYLKO względem tej samej osi jest przemienne.

7. Patryk Szydlik

1 Zadanie 7 - Patryk Szydlik

Macierz jednorodna rotacji ma postać:

$$K = \begin{bmatrix} R_{3x3} & T_{3x1} \\ 0_{1x3} & 1 \end{bmatrix}$$

Szukamy macierzy do niej odwrotnej, którą można przedstawić w postaci blokowej jako:

$$K^{-1} = \begin{bmatrix} A_{3x3} & B_{3x1} \\ C_{1x3} & D_{1x1} \end{bmatrix}$$

Korzystając z definicji macierzy odwrotnej możemy zapisać równość:

$$K \cdot K^{-1} = I \qquad dlatego \qquad \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} RA + TC & RB + TD \\ C & D \end{bmatrix} = I_{4x4} = \begin{bmatrix} I_{3x3} & 0_{3x1} \\ 0_{1x3} & 1 \end{bmatrix}$$

Możemy rozwiązać powstały układ równań:

$$\begin{cases} RA + TC = I \\ RB + TD = 0 \\ C = 0 \\ D = 1 \end{cases} \qquad \begin{cases} RA = I \\ RB + T = 0 \\ C = 0 \\ D = 1 \end{cases} \qquad \begin{cases} R^{-1}RA = R^{-1}I \\ RB = -T \\ C = 0 \\ D = 1 \end{cases} \qquad \begin{cases} A = R^{-1} \\ R^{-1}RB = -R^{-1}T \\ C = 0 \\ D = 1 \end{cases}$$

Otrzymujemy rozwiązanie:

$$\begin{cases} A = R^{-1} \\ B = -R^{-1}T \\ C = 0 \\ D = 1 \end{cases} \qquad K^{-1} = \begin{bmatrix} R^{-1} & -R^{-1}T \\ 0 & 1 \end{bmatrix}$$

Korzystając z ortogonalności macierzy obrotu R wiemy, że macierz do niej transponowana jest równa jej odwrotności, a stąd:

$$R^T = R^{-1} \qquad dlatego \qquad K^{-1} = \begin{bmatrix} R^T & -R^T T \\ 0 & 1 \end{bmatrix}$$

8. Kinga Długosz

Kinga Długosz ROBOTYKA 23.10.2020r

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$$* \; R = rot(z,\varphi) = \begin{bmatrix} c\varphi & -s\varphi & 0 \\ s\varphi & c\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\frac{\partial R}{\partial \varphi} = \begin{bmatrix} -\sin\varphi & -\cos\varphi & 0 \\ \cos\varphi & -\sin\varphi & 0 \\ 0 & 0 & 0 \end{bmatrix} = A \cdot R = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} c\varphi & -s\varphi & 0 \\ s\varphi & c\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$* \; R = rot(x,\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\alpha & -s\alpha \\ 0 & s\alpha & c\alpha \end{bmatrix}$$

$$\frac{\partial R}{\partial \alpha} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sin\alpha & -\cos\alpha \\ 0 & \cos\alpha & -\sin\alpha \end{bmatrix} = A \cdot R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\alpha & -s\alpha \\ 0 & s\alpha & c\alpha \end{bmatrix}$$

$$* \; R = rot(y,\beta) = \begin{bmatrix} c\beta & 0 & s\beta \\ 0 & 1 & 0 \\ -s\beta & 0 & c\beta \end{bmatrix}$$

$$\frac{\partial R}{\partial \beta} = \begin{bmatrix} -s\beta & 0 & c\beta \\ 0 & 0 & 0 \\ -c\beta & 0 & -s\beta \end{bmatrix} = A \cdot R = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} c\beta & 0 & s\beta \\ 0 & 1 & 0 \\ -s\beta & 0 & c\beta \end{bmatrix}$$

Czyli $\frac{\partial Rot(z,\phi)}{\partial \phi} = [(0,0,1)^T]Rot(z,\phi)$, $\frac{\partial Rot(x,\alpha)}{\partial \alpha} = [(1,0,0)^T]Rot(x,\alpha)$, $\frac{\partial Rot(y,\beta)}{\partial \beta} = [(0,1,0)^T]Rot(y,\beta)$

9. Szymon Zajda

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Zad. 9 Szymon Zajda

$[\omega_S] = \dot{R}R^T = \Omega_S \quad [\omega_B] = R^T\dot{R} = \Omega_B$

$R = RPY(\phi, \theta, \psi) = rot(Z, \phi) \cdot rot(Y, \theta) \cdot rot(X, \psi)$

$\dot{R} = \dot{R}_{Z,\phi} R_{Y,\theta} R_{X,\psi} + R_{Z,\phi} \dot{R}_{Y,\theta} R_{X,\psi} + R_{Z,\phi} R_{Y,\theta} \dot{R}_{X,\psi}$

$R^T = R_{X,\psi}^T \cdot R_{Y,\theta}^T \cdot R_{Z,\phi}^T$

$\omega_B = R^T \cdot \dot{R} = R_{X,\psi}^T R_{Y,\theta}^T R_{Z,\phi}^T (\dot{R}_{Z,\phi} R_{Y,\theta} R_{X,\psi} + R_{Z,\phi} \dot{R}_{Y,\theta} R_{X,\psi} + R_{Z,\phi} R_{Y,\theta} \dot{R}_{X,\psi}) = R_{X,\psi}^T R_{Y,\theta}^T R_{Z,\phi}^T \dot{R}_{Z,\phi} R_{Y,\theta} R_{X,\psi}$
$+ R_{X,\psi}^T R_{Y,\theta}^T R_{Z,\phi}^T R_{Z,\phi} \dot{R}_{Y,\theta} R_{X,\psi} + R_{X,\psi}^T R_{Y,\theta}^T R_{Z,\phi}^T R_{Z,\phi} R_{Y,\theta} \dot{R}_{X,\psi}$
$= R_{X,\psi}^T R_{Y,\theta}^T R_{Z,\phi}^T \dot{R}_{Z,\phi} R_{Y,\theta} R_{X,\psi} + R_{X,\psi}^T R_{Y,\theta}^T \dot{R}_{Y,\theta} R_{X,\psi} + R_{X,\psi}^T \dot{R}_{X,\psi}$

$$\frac{d}{dt} R_{X,\psi} = \frac{\partial R_{X,\psi}}{\partial \psi} \dot{\psi} = [e_1] R_{X,\psi} \cdot \dot{\psi}$$

$$\frac{d}{dt} R_{Y,\theta} = \frac{\partial R_{Y,\theta}}{\partial \theta} \dot{\theta} = [e_2] R_{Y,\theta} \dot{\theta}$$

$$\frac{d}{dt} R_{Z,\phi} = \frac{\partial R_{Z,\phi}}{\partial \phi} \dot{\phi} = [e_3] R_{Z,\phi} \dot{\phi}$$

$\Omega_B = R_{X,\psi}^T R_{Y,\theta}^T R_{Z,\phi}^T [e_3] R_{Z,\phi} R_{X,\psi} \dot{\phi} + R_{X,\psi}^T R_{Y,\theta}^T [e_2] R_{Y,\theta} R_{X,\psi} \dot{\theta}$
$+ R_{X,\psi}^T [e_1] R_{X,\psi} = R_{X,\psi}^T R_{Y,\theta}^T [R_{Z,\phi}^T e_3] R_{Y,\theta} R_{X,\psi} \dot{\phi} + R_{X,\psi}^T [R_{Y,\theta}^T e_2]$
$R_{X,\psi} \dot{\theta} + [R_{X,\psi} e_1] \dot{\psi} = [R_{X,\psi}^T R_{Y,\theta}^T R_{Z,\phi}^T e_3] \dot{\phi} + [R_{X,\psi}^T R_{Y,\theta}^T e_2] \dot{\theta}$
$+ [R_{X,\psi}^T e_1] \dot{\psi}$

$$\dot\psi\left[R_x^T e_1\right] = \dot\psi\left[\begin{bmatrix}1 & 0 & 0\\ 0 & c_\psi & s_\psi\\ 0 & -s_\psi & c_\psi\end{bmatrix}\cdot\begin{pmatrix}1\\0\\0\end{pmatrix}\right] = \dot\psi\begin{bmatrix}1\\0\\0\end{bmatrix} = \dot\psi\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{bmatrix}$$

$$\dot\theta\left[R_x^T R_y^T e_2\right] = \dot\theta\left[\begin{bmatrix}1 & 0 & 0\\ 0 & c_\psi & s_\psi\\ 0 & -s_\psi & c_\psi\end{bmatrix}\begin{bmatrix}c_\theta & 0 & -s_\theta\\ 0 & 1 & 0\\ s_\theta & 0 & c_\theta\end{bmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}\right] =$$

$$= \dot\theta\left[\begin{bmatrix}1 & 0 & 0\\ 0 & c_\psi & s_\psi\\ 0 & -s_\psi & c_\psi\end{bmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}\right] = \dot\theta\begin{bmatrix}0\\ c_\psi\\ -s_\psi\end{bmatrix} = \dot\theta\begin{bmatrix}0 & s_\psi & c_\psi\\ -s_\psi & 0 & 0\\ -c_\psi & 0 & 0\end{bmatrix}$$

$$\dot\phi\left[R_x^T R_y^T R_z^T e_3\right] = \dot\phi\left[\begin{bmatrix}1 & 0 & 0\\ 0 & c_\psi & s_\psi\\ 0 & -s_\psi & c_\psi\end{bmatrix}\begin{bmatrix}c_\theta & 0 & -s_\theta\\ 0 & 1 & 0\\ s_\theta & 0 & c_\theta\end{bmatrix}\begin{bmatrix}c_\phi & s_\phi & 0\\ -s_\phi & c_\phi & 0\\ 0 & 0 & 1\end{bmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}\right] =$$

$$= \dot\phi\left[\begin{bmatrix}1 & 0 & 0\\ 0 & c_\psi & s_\psi\\ 0 & -s_\psi & c_\psi\end{bmatrix}\begin{pmatrix}-s_\theta\\ 0\\ -c_\theta\end{pmatrix}\right] = \dot\phi\begin{bmatrix}-s_\theta\\ s_\psi c_\theta\\ c_\psi c_\theta\end{bmatrix} = \dot\phi\begin{bmatrix}0 & -c_\psi c_\theta & s_\psi c_\theta\\ c_\psi c_\theta & 0 & s_\theta\\ -s_\psi c_\theta & -s_\theta & 0\end{bmatrix}$$

$$\Omega_B = \begin{bmatrix}0 & \dot\theta s_\psi - \dot\phi c_\psi c_\theta & \dot\theta c_\psi + \dot\phi s_\psi c_\theta\\ -\dot\theta s_\psi + \dot\phi c_\psi c_\theta & 0 & -\dot\psi + \dot\phi s_\theta\\ -\dot\theta c_\psi - \dot\phi s_\psi c_\theta & \dot\psi - \dot\phi s_\theta & 0\end{bmatrix}$$

$$\omega_B = \begin{pmatrix}\dot\psi - \dot\phi s_\theta\\ \dot\theta c_\psi + \dot\phi s_\psi c_\theta\\ -\dot\theta s_\psi + \dot\phi c_\psi c_\theta\end{pmatrix}$$

$$\omega_B = \begin{bmatrix}-s_\theta & 0 & 1\\ s_\psi c_\theta & c_\psi & 0\\ c_\psi c_\theta & -s_\psi & 0\end{bmatrix}\begin{bmatrix}\dot\phi\\ \dot\theta\\ \dot\psi\end{bmatrix}, \quad \omega_s = R\,\omega_B$$