

LISTA 4

1. Krzysztof Ragan

LISTA 4

Ragan
Zad. 1

$$L[W]R^T = [LW]$$

$$R = \text{rot}(x, \alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\alpha & -s_\alpha \\ 0 & s_\alpha & c_\alpha \end{bmatrix}, \quad R^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\alpha & s_\alpha \\ 0 & -s_\alpha & c_\alpha \end{bmatrix}$$

$$[W] = \begin{bmatrix} 0 & -w_3 & w_2 \\ w_3 & 0 & -w_1 \\ -w_2 & w_1 & 0 \end{bmatrix}$$

$$L = L[W]R^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\alpha & -s_\alpha \\ 0 & s_\alpha & c_\alpha \end{bmatrix} \begin{bmatrix} 0 & -w_3 & w_2 \\ w_3 & 0 & -w_1 \\ -w_2 & w_1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\alpha & s_\alpha \\ 0 & -s_\alpha & c_\alpha \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & -w_3 & w_2 \\ w_3 c_\alpha + s_\alpha w_2 & -s_\alpha w_1 & -w_1 c_\alpha \\ w_3 s_\alpha - w_2 c_\alpha & c_\alpha w_1 & -w_1 s_\alpha \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\alpha & s_\alpha \\ 0 & -s_\alpha & c_\alpha \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & -w_3 c_\alpha - w_2 s_\alpha & -w_3 s_\alpha + w_2 c_\alpha \\ w_3 c_\alpha + s_\alpha w_2 & 0 & -s_\alpha^2 w_1 - w_1 c_\alpha^2 \\ w_3 s_\alpha - w_2 c_\alpha & w_1 c_\alpha^2 + w_1 s_\alpha^2 & 0 \end{bmatrix} \begin{matrix} \\ -w_1 \\ \end{matrix}$$

$$R \cdot W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\alpha & -s_\alpha \\ 0 & s_\alpha & c_\alpha \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} w_1 \\ c_\alpha w_2 - s_\alpha w_3 \\ s_\alpha w_2 + c_\alpha w_3 \end{bmatrix}$$

$$P = [R \cdot W] = \begin{bmatrix} 0 & -s_\alpha w_2 - c_\alpha w_3 & c_\alpha w_2 - s_\alpha w_3 \\ s_\alpha w_2 + c_\alpha w_3 & 0 & -w_1 \\ s_\alpha w_3 - c_\alpha w_2 & w_1 & 0 \end{bmatrix}$$

$$L = P$$

2. Gabriel Mainka

ROBOTYKA 84

zad. 2, lista 4

$$\Omega_S = [\omega_S] = \dot{R}R^T \quad \Omega_B = [\omega_B] = R^T \dot{R}$$

dla $R(x, \alpha)$

$$R^T \Omega_S R = R^T \dot{R} R^T R = R^T \dot{R} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\alpha + s\alpha & 0 \\ 0 & -s\alpha & c\alpha \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\alpha - s\alpha & 0 \\ 0 & s\alpha & c\alpha \end{bmatrix} =$$

$$= R^T \dot{R} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos^2 \alpha + \sin^2 \alpha & -s\alpha c\alpha + s\alpha c\alpha \\ 0 & -\sin\alpha \cos\alpha + \sin\alpha \cos\alpha & \cos^2 \alpha + \sin^2 \alpha \end{bmatrix} = R^T \dot{R} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \Omega_B$$

analogicznie dla $R(y, \beta)$, $R(z, \gamma)$

dla $R(z, \gamma)$

$$R \Omega_B R^T = R R^T \dot{R} R^T = \begin{bmatrix} c\gamma & -s\gamma & 0 \\ s\gamma & c\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\gamma & s\gamma & 0 \\ -s\gamma & c\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \dot{R} R^T =$$

$$= \begin{bmatrix} c^2 \gamma + s^2 \gamma & c\gamma s\gamma - c\gamma s\gamma & 0 \\ c\gamma s\gamma - c\gamma s\gamma & c^2 \gamma + s^2 \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \dot{R} R^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \dot{R} R^T = \Omega_S$$

W ostatniej linii zamiast $\dot{R}T$ powinno być $\dot{R}R^T$.

2013/

$$\begin{cases} \Omega_s = R \Omega_b R^T \\ R[\omega_b] R^T = [R\omega_b] \end{cases}$$

$$\Omega_s = [R\omega_b]$$

$$\omega_s = R\omega_b$$

3.

4. Hubert Górski

zad. 4

Hubert Górski

$$V_s = \begin{bmatrix} R & [T]R \\ 0 & R \end{bmatrix} V_B$$

$$\begin{bmatrix} R & [T]R \\ 0 & R \end{bmatrix} = Z$$

$$V_B = Z^{-1} V_s$$

$$Z^{-1} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} R & [T]R \\ 0 & R \end{bmatrix} = \begin{bmatrix} I_3 & 0 \\ 0 & I_3 \end{bmatrix}$$

$$AR + B_0 = I_3 \rightarrow A = R^{-1}$$

$$A[T]R + BR = 0 \rightarrow BR = -A[T]R \rightarrow B = -R^{-1}[T]$$

$$CR + DR = 0 \rightarrow C = 0$$

$$C[T]R + DR = I_3 \rightarrow D = R^{-1}$$

$$Z^{-1} = \begin{bmatrix} R^{-1} & -R^{-1}[T] \\ 0 & R^{-1} \end{bmatrix}$$

$$V_B = \begin{bmatrix} R^{-1} & -R^{-1}[T] \\ 0 & R^{-1} \end{bmatrix} V_s$$

5. a) Przemysław Kudelka

2.5 a)

$$V_B = \begin{pmatrix} R^T \dot{T} \\ \omega_B \end{pmatrix} \quad V_S = \begin{pmatrix} [T] \omega_S + \dot{T} \\ \omega_S \end{pmatrix}$$

$$\begin{bmatrix} R(t) & T(t) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(2, \alpha(t)) & 0 \\ 0 & 1 \end{bmatrix}$$

$$\alpha(t) = -3t$$

$$R(t) = \begin{bmatrix} \cos(-3t) & -\sin(-3t) & 0 \\ \sin(-3t) & \cos(-3t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R^T(t) = \begin{bmatrix} \cos(-3t) & \sin(-3t) & 0 \\ -\sin(-3t) & \cos(-3t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\dot{R}(t) = \begin{bmatrix} 3\sin(-3t) & 3\cos(-3t) & 0 \\ -3\cos(-3t) & 3\sin(-3t) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Wzory na te macierze potrzebne są do wypracowanie
prędkości:

$$\Omega_S = \dot{R} R^T$$

$$\Omega_B = R^T \dot{R}$$

$$\text{gdzie } \Omega = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

$$\dot{R} R^T = 3\sin(-3t)\cos(-3t) - 3\cos(-3t)\sin(-3t)$$

$$\dot{R} R^T = \begin{bmatrix} 3S(-3t)C(-3t) - 3C(-3t)S(-3t) & 3S^2(-3t) + 3C^2(-3t) & 0 \\ -3C^2(-3t) - 3S^2(-3t) & -3C(-3t)S(-3t) + 3S(-3t)C(-3t) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 3 & 0 \\ -3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ 0 & \omega_2 & \omega_1 & 0 \end{bmatrix}$$

$$\omega_S = \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix} \quad T=0 \quad \dot{T}=0 \Rightarrow V_S = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -3 \end{pmatrix}$$

$$R^T \dot{R} = \begin{bmatrix} 3S(-3t)C(-3t) - 3C(-3t)S(-3t) & 3C^2(-3t) + 3S^2(-3t) & 0 \\ -3S^2(-3t) - 3C^2(-3t) & 3C(-3t)S(-3t) + 3S(-3t)C(-3t) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 3 & 0 \\ -3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \Rightarrow V_B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -3 \end{pmatrix}$$

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N.S.

b) gsta translacija

$$\begin{bmatrix} R(t) & T(t) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbb{I}_3 & [1, 3t, -t^2]^T \\ 0 & 1 \end{bmatrix}$$

$$T = [1, 3t, -t^2]^T \quad \dot{T} = [0, 3, -2t]^T$$

$$R(t) = \mathbb{I}_3 \quad \dot{R}(t) = 0$$

$$[\omega_B] = R^T \cdot \dot{R} = 0 \quad [\omega_S] = \dot{R} R^T = 0$$

$$V_B = \begin{pmatrix} R^T \dot{T} \\ \omega_B \end{pmatrix} = \begin{pmatrix} [0, 3, -2t]^T \\ 0 \end{pmatrix}$$

$$V_S = \begin{pmatrix} [T] \omega_S + \dot{T} \\ \omega_S \end{pmatrix} = \begin{pmatrix} [0, 3, -2t]^T \\ 0 \end{pmatrix}$$

c) Maciej Salamoński

zad 5 $V_B = \begin{pmatrix} R^T \dot{T} \\ \omega_B \end{pmatrix}, V_s = \begin{pmatrix} [T] \omega_s + \dot{T} \\ \omega_s \end{pmatrix}$

* połączenia powyższych równań

$$\begin{bmatrix} R(t) & T(t) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \text{rot}(z, \alpha(t)) & [1, 3t, -t^2]^T \\ 0 & 1 \end{bmatrix}, \alpha(t) = -3t$$

$$R = \text{rot}(z, \alpha(t)) = \begin{bmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c(-3t) & -s(-3t) & 0 \\ s(-3t) & c(-3t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c(3t) & s(3t) & 0 \\ -s(3t) & c(3t) & 0 \\ 0 & 0 & 1 \end{bmatrix}, T = \begin{pmatrix} 1 \\ 3t \\ -t^2 \end{pmatrix} \rightarrow \dot{T} = \begin{pmatrix} 0 \\ 3 \\ -2t \end{pmatrix}$$

$$\underline{\underline{[W]}} = \underline{\underline{\Omega_B}} = \underline{\underline{R^T \cdot \dot{R}}} = \begin{bmatrix} c(3t) & -s(3t) & 0 \\ s(3t) & c(3t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3s(3t) & 3c(3t) & 0 \\ -3c(3t) & -3s(3t) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -3cs(3t) + 3cs(3t) & 3c^2(3t) + 3s^2(3t) & 0 \\ -3s^2(3t) - 3c^2(3t) & 3cs(3t) - 3cs(3t) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 3 & 0 \\ -3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \underline{\underline{\omega_B}} = \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix}, \underline{\underline{R^T \cdot \dot{T}}} = \begin{bmatrix} c(3t) & -s(3t) & 0 \\ s(3t) & c(3t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 0 \\ 3 \\ -2t \end{pmatrix}$$

$$\underline{\underline{V_B}} = \begin{pmatrix} -3s(3t) \\ 3c(3t) \\ -2t \\ 0 \\ 0 \\ -3 \end{pmatrix} = \begin{pmatrix} -3s(3t) \\ 3c(3t) \\ -2t \end{pmatrix}$$

$$V_s = \begin{pmatrix} [T] w_s + \dot{T} \\ w_s \end{pmatrix}, \quad \dot{T} = \begin{pmatrix} 0 \\ 3 \\ -2t \end{pmatrix} \quad (-T = \begin{pmatrix} 1 \\ 3t \\ -t^2 \end{pmatrix})$$

$$[w] = \underline{R_s} = \dot{R} R^T$$

$$[w] = \begin{bmatrix} -3s(3t) & 3c(3t) & 0 \\ -3c(3t) & -3s(3t) & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} c(3t) & -s(3t) & 0 \\ s(3t) & c(3t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -3cs(3t) + 3cs(3t) & 3s^2(3t) + 3c^2(3t) & 0 \\ -3c^2(3t) - 3s^2(3t) & 3cs(3t) - 3cs(3t) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 3 & 0 \\ -3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow w_s = \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix}$$

$$[T] = \begin{bmatrix} 0 & t^2 & 3t \\ -t^2 & 0 & -1 \\ 3t & 1 & 0 \end{bmatrix} \rightarrow [T] w_s = \begin{bmatrix} 0 & t^2 & 3t \\ -t^2 & 0 & -1 \\ 3t & 1 & 0 \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix}$$

$$\underline{[T] w_s} = \begin{pmatrix} -9t \\ 3 \\ 0 \end{pmatrix}$$

$$V_s = \begin{bmatrix} [T] w_s + \dot{T} \\ w_s \end{bmatrix} = \begin{pmatrix} -9t \\ 6 \\ -2t \\ 0 \\ 0 \\ -3 \end{pmatrix}$$

6. Konrad Białek

Konrad Białek
Zad. 6 / LISTA 4

Przebiegi statyczne układu X_0, Y_0, Z_0 w układach X_1, Y_1, Z_1

$$\begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{q_1 q_2} & -s_{q_1 q_2} & 0 & l_1 c_{q_1} + l_2 c_{q_1 q_2} \\ s_{q_1 q_2} & c_{q_1 q_2} & 0 & l_1 s_{q_1} + l_2 s_{q_1 q_2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{ gdzie } \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} \cos^{-1} \left(\frac{l_1}{l_1 + l_2 c_{q_1 q_2}} \right) \\ \cos^{-1} \left(\frac{l_1 s_{q_1} + l_2 s_{q_1 q_2}}{l_1 + l_2 c_{q_1 q_2}} \right) \end{pmatrix}$$

$$\cos(q_1 + q_2) = c_{q_1 q_2}, \sin(q_1 + q_2) = s_{q_1 q_2}, \cos(q_1) = c_{q_1}, \sin(q_1) = s_{q_1}$$

$$R^T = \begin{bmatrix} c_{q_1 q_2} & s_{q_1 q_2} & 0 \\ -s_{q_1 q_2} & c_{q_1 q_2} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \dot{R} = \begin{bmatrix} -\dot{q}_2 s_{q_1 q_2} & -\dot{q}_2 c_{q_1 q_2} & 0 \\ \dot{q}_2 c_{q_1 q_2} & -\dot{q}_2 s_{q_1 q_2} & 0 \\ 0 & 0 & 0 \end{bmatrix}, R = \begin{bmatrix} c_{q_1 q_2} & -s_{q_1 q_2} & 0 \\ s_{q_1 q_2} & c_{q_1 q_2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} l_1 c_{q_1} + l_2 c_{q_1 q_2} \\ l_1 s_{q_1} + l_2 s_{q_1 q_2} \\ 0 \end{bmatrix} \dot{T} = \begin{bmatrix} -l_2 \dot{q}_2 s_{q_1 q_2} \\ l_2 \dot{q}_2 c_{q_1 q_2} \\ 0 \end{bmatrix}, [T] = \begin{bmatrix} 0 & 0 & l_1 s_{q_1} + l_2 s_{q_1 q_2} \\ 0 & 0 & -l_1 c_{q_1} - l_2 c_{q_1 q_2} \\ -l_1 s_{q_1} - l_2 s_{q_1 q_2} & l_1 c_{q_1} + l_2 c_{q_1 q_2} & 0 \end{bmatrix}$$

$$[w_B] = \Omega_B = \begin{bmatrix} c_{q_1 q_2} & s_{q_1 q_2} & 0 \\ -s_{q_1 q_2} & c_{q_1 q_2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \dot{R} = \begin{bmatrix} -\dot{q}_2 s_{q_1 q_2} & -\dot{q}_2 c_{q_1 q_2} & 0 \\ \dot{q}_2 c_{q_1 q_2} & -\dot{q}_2 s_{q_1 q_2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[w_B] = \begin{bmatrix} -\dot{q}_2^2 c_{q_1 q_2}^2 s_{q_1 q_2}^2 + \dot{q}_2^2 s_{q_1 q_2}^2 c_{q_1 q_2}^2 & -\dot{q}_2^2 c_{q_1 q_2}^2 s_{q_1 q_2}^2 & -\dot{q}_2^2 s_{q_1 q_2}^2 c_{q_1 q_2}^2 \\ \dot{q}_2^2 s_{q_1 q_2}^2 + \dot{q}_2^2 c_{q_1 q_2}^2 & \dot{q}_2^2 s_{q_1 q_2}^2 c_{q_1 q_2}^2 & -\dot{q}_2^2 c_{q_1 q_2}^2 s_{q_1 q_2}^2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[w_B] = \begin{bmatrix} 0 & -\dot{q}_2^2 (c_{q_1 q_2}^2 + s_{q_1 q_2}^2) & 0 \\ \dot{q}_2^2 (s_{q_1 q_2}^2 + c_{q_1 q_2}^2) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\dot{q}_2^2 & 0 \\ \dot{q}_2^2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$w_B = \begin{pmatrix} 0 \\ 0 \\ \dot{q}_2 \end{pmatrix}, w_S = R w_B = \begin{bmatrix} c_{q_1 q_2} & -s_{q_1 q_2} & 0 \\ s_{q_1 q_2} & c_{q_1 q_2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ \dot{q}_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \dot{q}_2 \end{pmatrix}$$

$$V_B = \begin{pmatrix} R^T \dot{T} \\ \omega_B \end{pmatrix}, V_S = \begin{pmatrix} [T] \omega_S + \dot{T} \\ \omega_S \end{pmatrix}$$

$$R^T \dot{T} = \begin{bmatrix} c_{q_1 q_2} s_{q_1 q_2} & 0 \\ -s_{q_1 q_2} c_{q_1 q_2} & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} -l_2 \dot{q}_2 s_{q_1 q_2} \\ l_2 \dot{q}_2 c_{q_1 q_2} \\ 0 \end{pmatrix} = \begin{pmatrix} -l_2 \dot{q}_2 s_{q_1 q_2} s_{q_1 q_2} + l_2 \dot{q}_2 s_{q_1 q_2} c_{q_1 q_2} \\ l_2 \dot{q}_2 s_{q_1 q_2}^2 - l_2 \dot{q}_2 c_{q_1 q_2}^2 \\ 0 \end{pmatrix}$$

$$R^T \dot{T} = \begin{pmatrix} 0 \\ l_2 \dot{q}_2 (s_{q_1 q_2}^2 + c_{q_1 q_2}^2) \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ l_2 \dot{q}_2 \\ 0 \end{pmatrix}$$

$$[T] \omega_S + \dot{T} = \begin{bmatrix} 0 & 0 & l_1 \dot{q}_1 + l_2 s_{q_1 q_2} \\ 0 & 0 & -l_1 c_{q_1} - l_2 c_{q_1 q_2} \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ \dot{q}_2 \end{pmatrix} + \begin{pmatrix} -l_2 \dot{q}_2 s_{q_1 q_2} \\ l_2 \dot{q}_2 c_{q_1 q_2} \\ 0 \end{pmatrix}$$

$$[T] \omega_S + \dot{T} = \begin{pmatrix} l_1 \dot{q}_2 s_{q_1} + l_2 \dot{q}_2 s_{q_1 q_2} \\ -l_1 \dot{q}_2 c_{q_1} - l_2 \dot{q}_2 c_{q_1 q_2} \\ 0 \end{pmatrix} + \begin{pmatrix} -l_2 \dot{q}_2 s_{q_1 q_2} \\ l_2 \dot{q}_2 c_{q_1 q_2} \\ 0 \end{pmatrix} = \begin{pmatrix} l_1 \dot{q}_2 s_{q_1} \\ -l_1 \dot{q}_2 c_{q_1} \\ 0 \end{pmatrix}$$

$$V_B = \begin{pmatrix} 0 \\ l_2 \dot{q}_2 \\ 0 \\ 0 \\ 0 \\ \dot{q}_2 \end{pmatrix} = \begin{pmatrix} R^T \dot{T} \\ \omega_B \end{pmatrix}$$

$$V_S = \begin{pmatrix} l_1 \dot{q}_2 s_{q_1} \\ -l_1 \dot{q}_2 c_{q_1} \\ 0 \\ 0 \\ 0 \\ \dot{q}_2 \end{pmatrix} = \begin{pmatrix} [T] \omega_S + \dot{T} \\ \omega_S \end{pmatrix}$$

$$\alpha = q_1 + q_2$$

$$\begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{q_1+q_2} & -s_{q_1+q_2} & 0 & l_1 c_{q_1} + l_2 c_{q_1+q_2} \\ s_{q_1+q_2} & c_{q_1+q_2} & 0 & l_1 s_{q_1} + l_2 s_{q_1+q_2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} c_\alpha & -s_\alpha & 0 \\ s_\alpha & c_\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\dot{R} = \begin{bmatrix} -\dot{\alpha} s_\alpha & -\dot{\alpha} c_\alpha & 0 \\ \dot{\alpha} c_\alpha & -\dot{\alpha} s_\alpha & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$T = \begin{bmatrix} l_1 c_{q_1} + l_2 c_\alpha \\ l_1 s_{q_1} + l_2 s_\alpha \\ 0 \end{bmatrix}$$

$$\dot{T} = \begin{bmatrix} -\dot{q}_1 l_1 s_{q_1} - \dot{\alpha} l_2 s_\alpha \\ \dot{q}_1 l_1 c_{q_1} + \dot{\alpha} l_2 c_\alpha \\ 0 \end{bmatrix}$$

$$R^T = \begin{bmatrix} c_\alpha & s_\alpha & 0 \\ -s_\alpha & c_\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

* $V_B = \begin{pmatrix} R^T \dot{T} \\ \omega_B \end{pmatrix}$ - Skretmih ω
preestozeni

$$R^T \dot{T} = \begin{bmatrix} C_\alpha & S_\alpha & 0 \\ -S_\alpha & C_\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\dot{q}_1 l_1 S_{q_1} - \dot{\alpha} l_2 S_\alpha \\ \dot{q}_1 l_2 C_{q_1} + \dot{\alpha} l_2 C_\alpha \\ 0 \end{bmatrix} = \begin{bmatrix} l_1 \dot{q}_1 S_{q_2} \\ l_1 \dot{q}_1 C_{q_2} \cdot l_2 (\dot{q}_1 + \dot{q}_2) \\ 0 \end{bmatrix}$$

$$[\omega_B] = R^T \dot{R} = \begin{bmatrix} C_\alpha & S_\alpha & 0 \\ -S_\alpha & C_\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\dot{\alpha} S_\alpha & -\dot{\alpha} C_\alpha & 0 \\ \dot{\alpha} C_\alpha & -\dot{\alpha} S_\alpha & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\dot{\alpha} & 0 \\ \dot{\alpha} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$V_B = \begin{bmatrix} l_1 \dot{q}_1 S_{q_2} \\ l_1 \dot{q}_1 C_{q_2} \cdot l_2 (\dot{q}_1 + \dot{q}_2) \\ 0 \\ 0 \\ 0 \\ \dot{\alpha} \end{bmatrix} \quad \omega_B = \begin{pmatrix} 0 \\ 0 \\ \dot{\alpha} \end{pmatrix}$$

* $V_S = \begin{pmatrix} [T] \omega_S + \dot{T} \\ \omega_S \end{pmatrix}$ - Skretmih ω
like $\omega_S = \begin{pmatrix} 0 \\ 0 \\ \dot{\alpha} \end{pmatrix}$

$$[T] \omega_S = \begin{bmatrix} 0 & 0 & l_1 S_{q_1} + l_2 S_\alpha \\ 0 & 0 & -l_1 C_{q_1} - l_2 C_\alpha \\ -l_1 S_{q_1} - l_2 S_\alpha & l_1 C_{q_1} + l_2 C_\alpha & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} \dot{\alpha} l_1 S_{q_1} + \dot{\alpha} l_2 S_\alpha \\ -\dot{\alpha} l_1 C_{q_1} - \dot{\alpha} l_2 C_\alpha \\ 0 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} \dot{\alpha} l_1 S_{q_1} + \dot{\alpha} l_2 S_\alpha \\ -\dot{\alpha} l_1 C_{q_1} - \dot{\alpha} l_2 C_\alpha \\ 0 \end{bmatrix}}_B + \underbrace{\begin{bmatrix} -\dot{q}_1 l_1 S_{q_1} - \dot{\alpha} l_2 S_\alpha \\ \dot{q}_1 l_1 C_{q_1} + \dot{\alpha} l_2 C_\alpha \\ 0 \end{bmatrix}}_{\dot{T}} = \begin{bmatrix} l_1 S_{q_1} (\dot{\alpha} - \dot{q}_1) \\ l_1 C_{q_1} (\dot{q}_1 - \dot{\alpha}) \\ 0 \end{bmatrix} \quad V_S = \begin{bmatrix} l_1 S_{q_1} (\dot{\alpha} - \dot{q}_1) \\ l_1 C_{q_1} (\dot{q}_1 - \dot{\alpha}) \\ 0 \\ 0 \\ 0 \\ \dot{\alpha} \end{bmatrix}$$

8. Marcin Bober

