Proszę, aby przy rozwiązaniu zadania znalazło się imię i nazwisko autora.

LISTA 1

1. Kinga Długosz

Zad 1. / Lista 1.

$$\langle a, b\rangle = a^T b = b^T a = tr(ab^T)$$

tr - ślad macierzy

$$tr\left(\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\right) = a + e + i$$

$$tr(A+B) = tr(A) + tr(B) \qquad tr(c \cdot A) = c \cdot tr(A)$$

$$a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \qquad \boxed{a \circ b = a_1 \cdot b_1 + a_2 \cdot b_2 + a_3 \cdot b_3}$$

$$a^T = [a_1 \ a_2 \ a_3] \qquad b^T = [b_1 \ b_2 \ b_3]$$

$$* \ a^T b = [a_1 \ a_2 \ a_3] \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$* \ b^T a = [b_1 \ b_2 \ b_3] \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = b_1 a_1 + b_2 a_2 + b_3 a_3$$

$$* \ tr(ab^T)$$

$$ab^T = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} [b_1 \ b_2 \ b_3] = \begin{bmatrix} a_1 b_1 & a_1 b_2 & a_1 b_3 \\ a_2 b_1 & a_2 b_2 & a_2 b_3 \\ a_3 b_1 & a_3 b_2 & a_3 b_3 \end{bmatrix}$$

$$tr(ab^T) = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$a^T b = b^T a = tr(ab^T) = a \circ b$$

2. Piotr Bednarek

LISTA 1 Piotr Bednarek

Zad 2.

Pokazać, że długość wektora $||a|| = \sqrt{a_1^2+a_2^2+a_3^2}$ jest równa $\sqrt{\langle a,a \rangle}$

Wzór ogólny $\langle a,b \rangle = a_1\cdot b_1 + a_2\cdot b_2 + \dots + a_n\cdot b_n$ w $\mathbb{R}^n$

dlatego $\sqrt{\langle a,a \rangle} = \sqrt{a_1\cdot a_1 + a_2\cdot a_2 + a_3\cdot a_3} = \sqrt{a_1^2+a_2^2+a_3^2} = ||a||$

c.k.d

3. Adam Bednorz

LISTA 1 zad. 3 Adam Bednor

$$a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

1. $\langle a, b \rangle \stackrel{?}{=} \langle b, a \rangle$

$$L = \langle a, b \rangle = \sum_{i=1}^{3} a_i b_i = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$P = \langle b, a \rangle = \sum_{i=1}^{3} b_i a_i = b_1 a_1 + b_2 a_2 + b_3 a_3$$

$L = P$ c.n.u

2. $\langle k a, b \rangle \stackrel{?}{=} k \langle a, b \rangle \qquad k \in \mathbb{R}$

$$L = \langle k a, b \rangle = \sum_{i=1}^{3} k a_i b_i = k a_1 b_1 + k a_2 b_2 + k a_3 b_3 =$$

$$= k (a_1 b_1 + a_2 b_2 + a_3 b_3) = k \langle a, b \rangle = P$$

c.n.u

4. Maciej Salamoński

4) 1. $a \times b = -b \times a$ (antyprzemienność)

$$L = a \times b = \det\begin{bmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} = i a_2 b_3 + k a_1 b_2 + j a_3 b_1 - k a_2 b_1 - i a_3 b_2 - j a_1 b_3 = i(a_2 b_3 - a_3 b_2) - j(a_1 b_3 - a_3 b_1) + k(a_1 b_2 - a_2 b_1)$$

$$\begin{matrix} i & j & k \\ a_1 & a_2 & a_3 \end{matrix}$$

$$= \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix} \begin{bmatrix} i \\ j \\ k \end{bmatrix}$$

$$P = -b \times a = \det\begin{bmatrix} i & j & k \\ -b_1 & -b_2 & -b_3 \\ a_1 & a_2 & a_3 \end{bmatrix} = -i a_3 b_2 - k a_2 b_1 - j a_1 b_3 + k a_1 b_2 + i a_2 b_3 + a_3 b_1 j$$

$$\begin{matrix} i & j & k \\ -b_1 & -b_2 & -b_3 \end{matrix}$$

$$= i(a_2 b_3 - a_3 b_2) - j(a_1 b_3 - a_3 b_1) + k(a_1 b_2 - a_2 b_1)$$

$$= \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix} \begin{bmatrix} i \\ j \\ k \end{bmatrix}$$

$$L = P \rightarrow a \times b = -b \times a$$

2. $(ka) \times b = k(a \times b)$

zgodność z mnożeniem przez skalar

$$L = (ka) \times b = \det \begin{bmatrix} i & j & k \\ ka_1 & ka_2 & ka_3 \\ b_1 & b_2 & b_3 \end{bmatrix} \begin{matrix} \\ \\ \\ i & j & k \\ ka_1 & ka_2 & ka_3 \end{matrix} = ik a_2 b_3 + k^2 a_1 b_2 + jk a_3 b_1 - k^2 a_2 b_1 - ik a_3 b_2 - jk a_1 b_3$$

$$L = ik(a_2 b_3 - a_3 b_2) + jk(a_3 b_1 - a_1 b_3) + k^2(a_1 b_2 - a_2 b_1)$$

$$P = k(a \times b) = k \cdot \det \begin{bmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}$$

$$= k \cdot [i(a_2 b_3 - a_3 b_2) - j(a_1 b_3 - a_3 b_1) + k(a_1 b_2 - a_2 b_1)]$$

$$= ik(a_2 b_3 - a_3 b_2) + jk(a_3 b_1 - a_1 b_3) + k^2(a_1 b_2 - a_2 b_1)$$

$$L = P \rightarrow (ka) \times b = k(a \times b)$$

5. Piotr Gorzelnik

Lista I z. 5 | Piotr Gorzelnik

iloczyn skalarny

Wektory są ortogonalne ($\perp$) $\iff \langle a, b \rangle = 0$

$$a \times b = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix} \quad // \text{Własność z zadania 4}$$

$$\langle a, a \times b \rangle = a_1(a_2 b_3 - a_3 b_2) + a_2(a_3 b_1 - a_1 b_3) + a_3(a_1 b_2 - a_2 b_1) =$$
$$= a_1 a_2 b_3 - a_1 a_3 b_2 + a_2 a_3 b_1 - a_2 a_1 b_3 + a_3 a_1 b_2 - a_3 a_2 b_1 = 0$$

oraz

$$\langle b, a \times b \rangle = b_1(a_2 b_3 - a_3 b_2) + b_2(a_3 b_1 - a_1 b_3) + b_3(a_1 b_2 - a_2 b_1) =$$
$$= a_2 b_1 b_3 - a_3 b_1 b_2 + a_3 b_1 b_2 - a_1 b_2 b_3 + a_1 b_2 b_3 - a_2 b_1 b_3 = 0$$

Zatem wektory $\vec{a}, \vec{b}$ są ortogonalne.

6. Krzysztof Ragan

ZAD. 6

$$A_{3\times3} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

Warunek macierzy skośnie symetrycznej:

$$A^T = -A$$

$$a_{ji} = -a_{ij}$$

MACIERZ SKOŚNIE SYMETRYCZNA:

$$A_{3\times3} = \begin{bmatrix} 0 & A_{12} & A_{13} \\ -A_{12} & 0 & A_{23} \\ -A_{13} & -A_{23} & 0 \end{bmatrix}$$

$$a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}; \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$a \times b = \begin{bmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} = \begin{bmatrix} \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}; & -\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}; & \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \end{bmatrix} =$$

$$\begin{bmatrix} a_2 b_3 - b_2 a_3 \\ -a_1 b_3 + a_3 b_1 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$$

$$Ab = \begin{bmatrix} 0 & A_{12} & A_{13} \\ -A_{12} & 0 & A_{23} \\ -A_{13} & -A_{23} & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} A_{12} b_2 + A_{13} b_3 \\ -A_{12} b_1 + A_{23} b_3 \\ -A_{13} b_1 - A_{23} b_2 \end{bmatrix}$$

$$a \times b = Ab$$

$$\begin{bmatrix} a_2 b_3 - b_2 a_3 \\ -a_1 b_3 + a_3 b_1 \\ a_1 b_2 - a_2 b_1 \end{bmatrix} = \begin{bmatrix} A_{12} b_2 + A_{13} b_3 \\ -A_{12} b_1 + A_{23} b_3 \\ -A_{13} b_1 - A_{23} b_2 \end{bmatrix}$$

$$a_2 b_3 - b_2 a_3 = A_{12} b_2 + A_{13} b_3 \Rightarrow A_{12} = -a_3; \; A_{13} = a_2$$

$$-a_1 b_3 + a_3 b_1 = -A_{12} b_1 + A_{23} b_3 \Rightarrow A_{12} = -a_3; \; A_{23} = -a_1$$

$$a_1 b_2 - a_2 b_1 = -A_{13} b_1 - A_{23} b_2 \Rightarrow A_{13} = a_2; \; A_{23} = -a_1$$

$$A = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

Czy macierz A spełnia równanie $A + A^T = 0$?

$$A^T = \begin{bmatrix} 0 & a_3 & -a_2 \\ -a_3 & 0 & a_1 \\ a_2 & -a_1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & a_3 & -a_2 \\ -a_3 & 0 & a_1 \\ a_2 & -a_1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Macierz A spełnia powyższe równanie

7.

a. Krzysztof Górski

Krzysztof Górski

$$f(q_1 q_2) = 11 \sin^2(q_2^2 + q_1 q_2) - \ln q_1 \cos(3q_2 + 1)$$

$$\frac{df(q_1 q_2)}{dq_1} = 11 \cdot 2 \sin(q_2^2 + q_1 q_2) \cos(q_2^2 + q_1 q_2) \cdot q_2 -$$

$$- \frac{1}{q_1} \cdot (-1) \sin(3q_2 + 1)$$

$$\frac{df(q_1, q_2)}{dq_1} = 22 \sin\cos(q_2^2 + q_1 q_2) q_2 + \frac{\sin(3q_2 + 1)}{q_1}$$

$$\frac{df(q_1 q_2)}{dq_2} = 22 \sin\cos(q_2^2 + q_1 q_2) \cdot [2q_2 + q_1]$$

$$- \ln q_1 (-1) \sin(3q_2 + 1) \cdot 3$$

$$\frac{df(q_1 q_2)}{dq_2} = 22 \sin\cos(q_2^2 + q_1 q_2) \cdot [2q_2 + q_1] + 3 \ln q_1 \sin(3q_2 +$$

b. Hubert Górski

Hubert Górski

$$f(q_1, q_2) = \begin{bmatrix} tg\, q_1 + q_2^4 q_1^2 \\ arctg(q_1 + 2q_2) + 5q_1 \end{bmatrix}$$

$$\frac{\partial f}{\partial q} = \left[\begin{array}{c|c} \frac{1}{\cos^2 q_1} + 2 q_1 q_2^4 & 4 q_2^3 q_1^2 \\ \hline \frac{1}{1+(q_1+2q_2)^2} + 5 & \frac{2}{1+(q_1+2q_2)^2} \end{array}\right]$$

c. Szymon Tomala

7c)

$$f(q_1, q_2, q_3) = \begin{bmatrix} tg^2 q_3 + q_2^4 q_1^2 \\ arctg(q_1 + 2q_2 + q_3) + 5q_1 \end{bmatrix}$$

$$\frac{\partial f}{\partial q_1} = \begin{bmatrix} 2 q_2^4 q_1 \\ \frac{1}{(q_1+2q_2+q_3)^2+1} + 5 \end{bmatrix}$$

$$\frac{\partial f}{\partial q_2} = \begin{bmatrix} 4 q_1^2 q_2^3 \\ \frac{2}{(q_1+2q_2+q_3)^2+1} \end{bmatrix}$$

Szymon Tomala

$$\frac{\partial f}{\partial q_3} = \begin{bmatrix} 2\, tg\, q_3 \cdot \frac{1}{\cos^2 q_3} \\ \frac{1}{(q_1+2q_2+q_3)^2+1} \end{bmatrix}$$

$$\frac{\partial f}{\partial q} = \left[\frac{\frac{2q_2^4 q_1}{1}}{(q_1+2q_2+q_3)^2+1} + 5 \;\vdots\; \frac{\frac{4q_1^2 q_2^3}{2}}{(q_1+2q_2+q_3)^2+1} \;\vdots\; \frac{\frac{2\,\mathrm{tg}\,q_3 \cdot \frac{1}{\cos^2 q_3}}{1}}{(q_1+2q_2+q_3)^2+1} \right]$$

8. Tomasz Gniazdowski

TOMASZ GNIAZDOWSKI

$$f(q_1(t), q_2(t)) = 7\sin^2 q_1 + q_1 q_2^2$$

$$\frac{df}{dt} = 14\sin(q_1) \cdot \cos(q_1) \cdot \dot{q}_1 + \dot{q}_1 \cdot q_2^2 + q_1 \cdot 2 \cdot q_2 \cdot \dot{q}_2$$