1.
    a. Paweł Troszczyński

Zad. 1a)

$$k(q) = \begin{bmatrix} -s_2 c_3 a_3 \\ c_2 c_3 a_3 + a_1 \\ s_3 a_3 + a_2 + q_1 \\ q_2 + \frac{\pi}{2} \\ -q_3 \\ \frac{\pi}{2} \end{bmatrix}$$

$$J(q) = \begin{bmatrix} 0 & -c_2 c_3 a_3 & s_2 s_3 a_3 \\ 0 & -s_2 c_3 a_3 & -c_2 s_3 a_3 \\ 1 & 0 & c_3 a_3 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Liczba wierszy jest większa niż liczba kolumn, więc wszystkie konfiguracje są osobliwe. Dlatego ograniczono przekształcenie zadaniowe tego manipulatora wyłącznie do położeń efektora. Wtedy Jakobian ma postać

$$J_3(q) = \begin{bmatrix} 0 & -c_2 c_3 a_3 & s_2 s_3 a_3 \\ 0 & -s_2 c_3 a_3 & -c_2 s_3 a_3 \\ 1 & 0 & c_3 a_3 \end{bmatrix}$$

Liczba wierszy jest równa liczbie kolumn, więc konfiguracje osobliwe spełniają zależność:

$\det J_3(q_1) = 0$

rozwinięcie wzdłuż pierwszej kolumny

$$\det J_3(q_1) = (-1)^{1+3} \cdot \begin{vmatrix} -c_2c_3a_3 & s_2s_3a_3 \\ -s_2c_3a_3 & -c_2s_3a_3 \end{vmatrix} =$$

$$= -c_2^2 \cdot c_3s_3 \cdot a_3^2 + s_2^2 c_3s_3a_3^2 =$$

$$= a_3^2 \cdot c_3s_3 (c_2^2 + s_2^2) = a_3^2 \cdot c_3s_3$$

Więc

$$\det J_3(q_1) = a_3^2 \cdot c_3s_3 = 0 \Rightarrow 1^\circ \; c_3 = 0$$

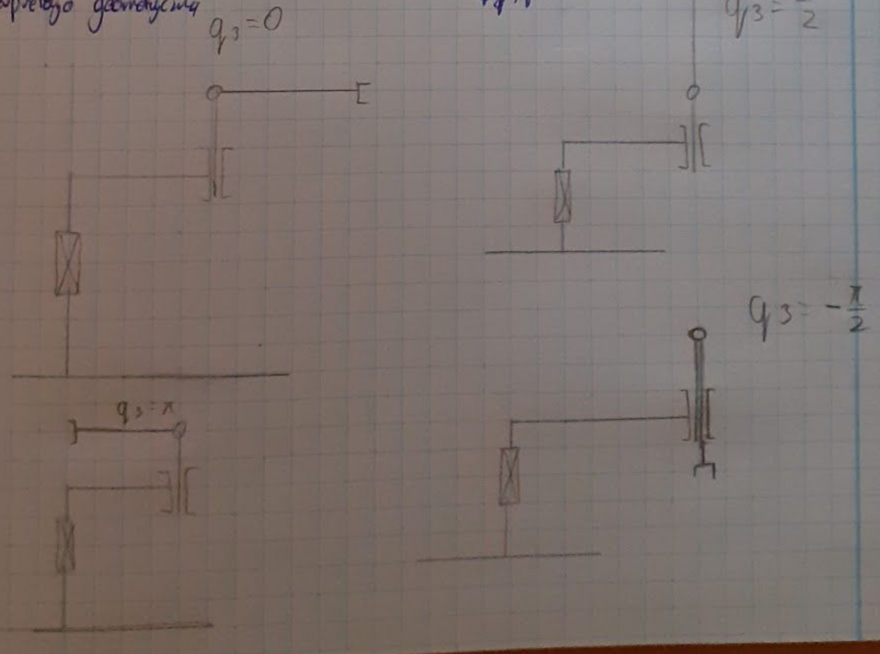
$$2^\circ \; s_3 = 0$$

czyli osobliwość występuje gdy

$q_3 = k \cdot \frac{\pi}{2}, \; k \in \mathbb{Z}$

q_1, q_2 dowolne

Interpretacja geometryczna



b) Kajetan Zdanowicz

$$k(q) = \begin{bmatrix} -s_{12}(c_3a_3+a_2) - s_1a_1 \\ c_{12}(c_3a_3+a_2) - c_1a_1 \\ s_3a_3 + d_1 + d_2 \\ q_1 \\ \frac{\pi}{2} \\ q_3 \end{bmatrix}$$

$$J_{3\times3} = \begin{bmatrix} -c_{12}(c_3a_3+a_2) - c_1a_1 & -c_{12}(c_3a_3+a_2) & s_{12}s_3a_3 \\ -s_{12}(c_3a_3+a_2) + s_1a_1 & -s_{12}(c_3a_3+a_2) & -c_{12}s_3a_3 \\ 0 & 0 & c_3a_3 \end{bmatrix}$$

$$\det J_{3\times3} = \Big(\overbrace{c_{12}(c_3a_3+a_2)}^{x} + c_1a_1\Big)\Big(\overbrace{s_{12}(c_3a_3+a_2)}^{y}\Big)c_3a_3$$

$$- c_3a_3\Big(\underbrace{s_{12}(c_3a_3+a_2)}_{y} + s_1a_1\Big)\Big(\underbrace{c_{12}(c_3a_3+a_2)}_{x}\Big)$$

$$c_3a_3\big[(x + c_1a_1)\cdot y - (y + s_1a_1)x\big] = 0$$

1* $c_3a_3 = 0$ $\vee$ $\cancel{xy} + c_1a_1y - \cancel{xy} - s_1a_1x = 0 \;/:a_1$

$$c_1y - s_1x = 0$$

$$c_1s_{12}(c_3a_3+a_2) - s_1c_{12}(c_3a_3+a_2) = 0$$

2* $c_3a_3 + a_2 = 0$ $\vee$ $c_1s_{12} - s_1c_{12} = 0$

$$C_1 S_{12} - S_1 C_{12} = 0$$

$$\cos q_1 \cdot \sin(q_1 + q_2) - \sin q_1 \cdot \cos(q_1 + q_2) = 0$$

$$\cancel{\cos q_1 \cdot \sin q_1 \cdot \cos q_2} + \cos q_1 \cdot \cos q_1 \cdot \sin q_2 \cancel{- \sin q_1 \cos q_1 \cos q_2}$$
$$+ \sin q_1 \cdot \sin q_1 \cdot \sin q_2 = 0$$

$$\cos^2 q_1 \sin q_2 + \sin^2 q_1 \cdot \sin q_2 = 0$$

$$\sin q_2 (\cos^2 q_1 + \sin^2 q_1) = 0$$

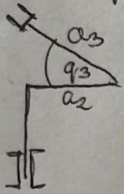
3^*
$$\sin q_2 = 0$$

$1^*)$ $\cos q_3 = 0$

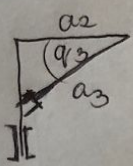
$q_3 = \frac{\pi}{2}$

$q_3 = \frac{3\pi}{2}$

$2^*)$ $\cos q_3 = \frac{-a_2}{a_3}$



$a_3 > a_2$



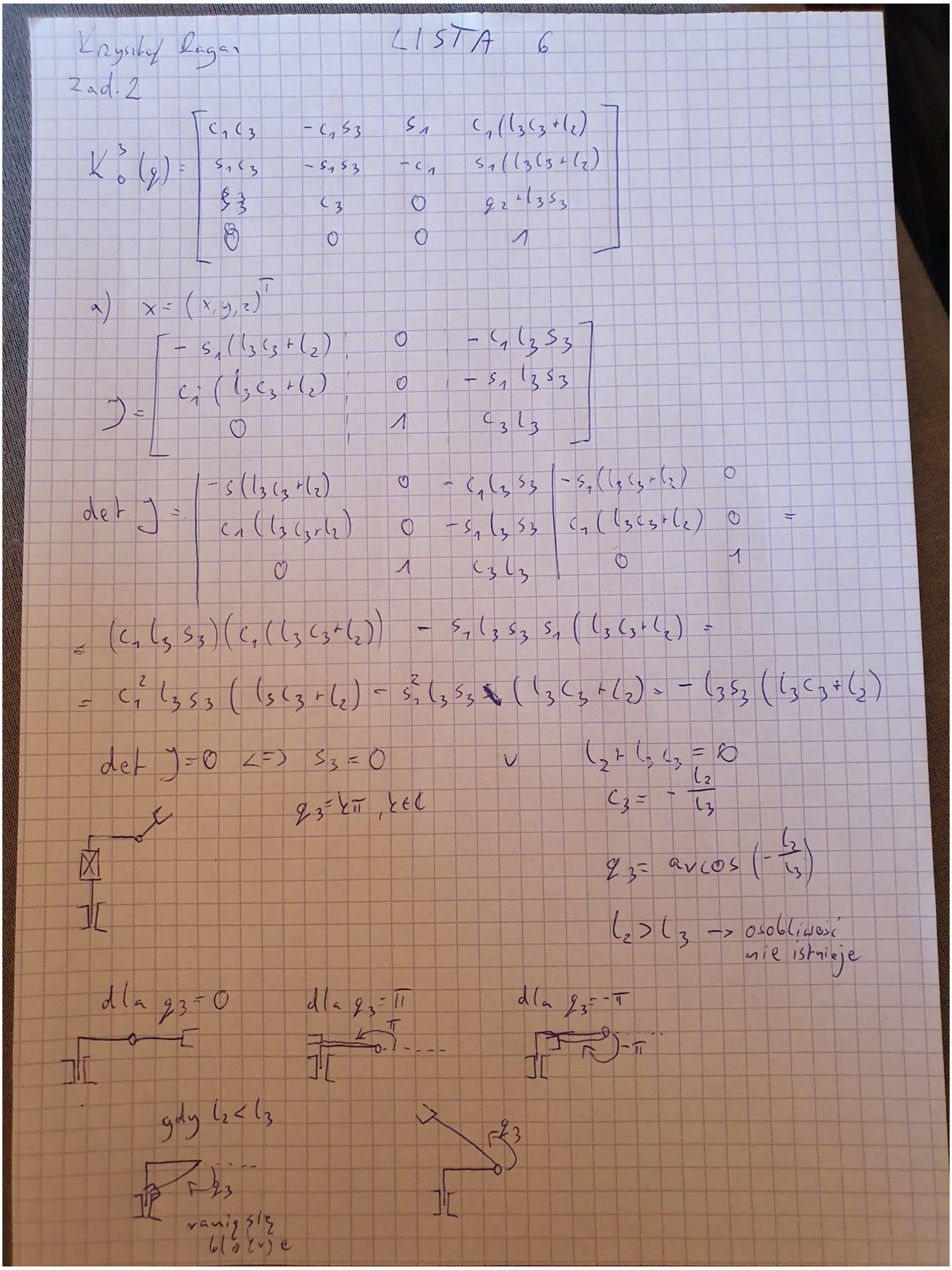
$3^*)$ $\sin q_2 = 0$

$q_2 = \{-\pi, 0, \pi, [illegible]\}$

c.

2.

a) Krzysztof Ragan



Krzysztof Ragan LISTA 6

Zad. 2

$$K_0^3(q) = \begin{bmatrix} c_1c_3 & -c_1s_3 & s_1 & c_1(l_3c_3+l_2) \\ s_1c_3 & -s_1s_3 & -c_1 & s_1(l_3c_3+l_2) \\ s_3 & c_3 & 0 & q_2+l_3s_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

a) $x = (x, y, z)^T$

$$J = \begin{bmatrix} -s_1(l_3c_3+l_2) & 0 & -c_1l_3s_3 \\ c_1(l_3c_3+l_2) & 0 & -s_1l_3s_3 \\ 0 & 1 & c_3l_3 \end{bmatrix}$$

$$\det J = \begin{vmatrix} -s(l_3c_3+l_2) & 0 & -c_1l_3s_3 \\ c_1(l_3c_3+l_2) & 0 & -s_1l_3s_3 \\ 0 & 1 & c_3l_3 \end{vmatrix} \begin{matrix} -s_1(l_3c_3+l_2) & 0 \\ c_1(l_3c_3+l_2) & 0 \\ 0 & 1 \end{matrix} =$$

$$= (c_1l_3s_3)(c_1(l_3c_3+l_2)) - s_1l_3s_3s_1(l_3c_3+l_2) =$$

$$= c_1^2l_3s_3(l_3c_3+l_2) - s_1^2l_3s_3(l_3c_3+l_2) = -l_3s_3(l_3c_3+l_2)$$

$\det J = 0 \iff s_3 = 0 \quad \vee \quad l_2 + l_3c_3 = 0$

$q_3 = k\pi,\ k \in C$

$c_3 = -\frac{l_2}{l_3}$

$q_3 = \arccos\left(-\frac{l_2}{l_3}\right)$

$l_2 > l_3 \rightarrow$ osobliwość nie istnieje

dla $q_3 = 0$ dla $q_3 = \pi$ dla $q_3 = -\pi$

π $-\pi$

gdy $l_2 < l_3$

q_3

ramię się blokuje

q_3

b) Kinga Długosz

Powinno być x=(x,**y**)^T

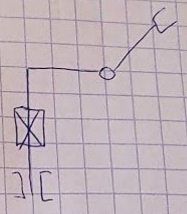
Kinga Długosz

LISTA 6 / zad 2

$$K_0^3(q) = \begin{bmatrix} c_1 c_3 & -c_1 s_3 & s_1 & c_1(l_3 c_3 + l_2) \\ s_1 c_3 & -s_1 s_3 & -c_1 & s_1(l_3 c_3 + l_2) \\ s_3 & c_3 & 0 & q_2 + l_3 s_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) $x = (x, z)^T$

$$J = \begin{bmatrix} -s_1(l_2 + l_3 c_3) & 0 & -c_1 s_3 l_3 \\ c_1(l_2 + l_3 c_3) & 0 & -s_1 s_3 l_3 \end{bmatrix}$$



$$\det(J_1) = \begin{vmatrix} -s_1(l_2 + l_3 c_3) & 0 \\ c_1(l_2 + l_3 c_3) & 0 \end{vmatrix} = \underline{0}$$

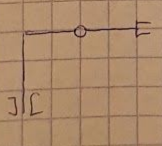
$$\det(J_2) = \begin{vmatrix} 0 & -c_1 s_3 l_3 \\ 0 & -s_1 s_3 l_3 \end{vmatrix} = \underline{0}$$

$$\det(J_3) = \begin{vmatrix} -s_1(l_2 + l_3 c_3) & -c_1 s_3 l_3 \\ c_1(l_2 + l_3 c_3) & -s_1 s_3 l_3 \end{vmatrix} = (+s_1(l_2 + l_3 c_3))(+s_1 s_3 l_3) + c_1 s_3 l_3 \cdot c_1 \cdot (l_2 + l_3 c_3) =$$

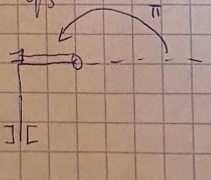
$$= s_1^2 s_3 l_2 l_3 + s_1^2 s_3 l_3^2 c_3 + c_1^2 l_2 l_3 s_3 + c_1^2 c_3 l_3^2 s_3 = \underline{s_3 \cdot l_3 (l_2 + l_3 c_3)}$$

$s_3 l_3 (l_2 + l_3 c_3) = 0 \qquad s_3 = 0 \quad \vee \quad c_3 = -\frac{l_2}{l_3} \quad \Rightarrow \quad \underline{q_3 = k\pi}$

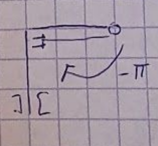
$q_3 = 0$



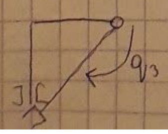
$q_3 = \pi$



$q_3 = -\pi$



$l_2 < l_3$



– RAMIĘ SIĘ BLOKUJE

3. Konrad Białek

Zad. 3 / LISTA 6 Konrad Białek

Tabela parametrów D-H:

i	θ_i	d_i	a_i	α_i
1	$q_1+\frac{\pi}{2}$	d_1	0	$\frac{\pi}{2}$
2	0	q_2	a_2	0
3	q_3	0	a_3	$-\frac{\pi}{2}$

Kinematyki cząstkowe:

$$A_0^1=\begin{bmatrix}-s_1 & 0 & c_1 & 0\\ c_1 & 0 & s_1 & 0\\ 0 & 1 & 0 & d_1\\ 0 & 0 & 0 & 1\end{bmatrix}\quad A_0^2=\begin{bmatrix}-s_1 & 0 & c_1 & -a_2s_1+q_2c_1\\ c_1 & 0 & s_1 & a_2c_1+q_2s_1\\ 0 & 1 & 0 & d_1\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Przeguby:

1. - obrotowy
2. - przesuwny
3. - obrotowy

$$A_0^3=\begin{bmatrix}-s_1c_3 & -c_1 & s_1s_3 & -s_1(a_3c_3+a_2)+q_2c_1\\ c_1c_3 & -s_1 & -c_1s_3 & c_1(a_3c_3+a_2)+q_2s_1\\ s_3 & 0 & c_3 & d_1+a_3s_3\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$J_m(q)=\left[J_{m1}(q)\ J_{m2}(q)\ J_{m3}(q)\right]$$

Dla i-tego przegubu obrotowego:

$$J_{mi}(q)=\begin{pmatrix}R_{0,3k}^{i-1}\times(T_0^n-T_0^{i-1})\\ R_{0,3k}^{i-1}\end{pmatrix}$$

Dla i-tego przegubu przesuwnego:

$$J_{mi}(q)=\begin{pmatrix}R_{0,3k}^{i-1}\\ 0\end{pmatrix}$$

$$J_{m1}(q)=\begin{bmatrix}R_{0,3k}^0\times(T_0^3-T_0^0)\\ R_{0,3k}^0\end{bmatrix},\quad J_{m2}(q)=\begin{bmatrix}R_{0,3k}^1\\ 0\end{bmatrix},\quad J_{m3}(q)=\begin{bmatrix}R_{0,3k}^2\times(T_0^3-T_0^2)\\ R_{0,3k}^2\end{bmatrix}$$

Na podstawie kinematyk cząstkowych:

$$R_{0,3k}^0=\begin{pmatrix}0\\0\\1\end{pmatrix},\quad T_0^0=\begin{pmatrix}0\\0\\0\end{pmatrix},\quad T_0^3=\begin{pmatrix}-s_1(a_3c_3+a_2)+q_2c_1\\ c_1(a_3c_3+a_2)+q_2s_1\\ d_1+a_3s_3\end{pmatrix},\quad R_{0,3k}^1=\begin{pmatrix}c_1\\s_1\\0\end{pmatrix}$$

$$R_{0,3k}^2=\begin{pmatrix}c_1\\s_1\\0\end{pmatrix},\quad T_0^2=\begin{pmatrix}-a_2s_1+q_2c_1\\ a_2c_1+q_2s_1\\ d_1\end{pmatrix},\quad R_{0,3k}^0=\begin{bmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},\quad R_{0,3k}^2=\begin{bmatrix}0 & 0 & s_1\\ 0 & 0 & -c_1\\ -s_1 & c_1 & 0\end{bmatrix}$$

$$T_0^3-T_0^0=\begin{pmatrix}-s_1(a_3c_3+a_2)+q_2c_1\\ c_1(a_3c_3+a_2)+q_2s_1\\ d_1+a_3s_3\end{pmatrix}-\begin{pmatrix}0\\0\\0\end{pmatrix}=\begin{pmatrix}-s_1(a_3c_3+a_2)+q_2c_1\\ c_1(a_3c_3+a_2)+q_2s_1\\ d_1+a_3s_3\end{pmatrix}$$

$$T_0^3 - T_0^2 = \begin{pmatrix} -s_1(a_3c_3+a_2)+q_2c_1 \\ c_1(a_3c_3+a_2)+q_2s_1 \\ d_1+a_3s_3 \end{pmatrix} - \begin{pmatrix} -a_2s_1+q_2c_1 \\ a_2c_1+q_2s_1 \\ d_1 \end{pmatrix} = \begin{pmatrix} -s_1a_3c_3 - s_1a_2 + q_2c_1 + a_2s_1 - q_2c_1 \\ c_1a_3c_3 + a_2c_1 + q_2s_1 - a_2c_1 - q_2s_1 \\ d_1 + a_3s_3 - d_1 \end{pmatrix}$$

$$T_0^3 - T_0^2 = \begin{pmatrix} -s_1a_3c_3 \\ c_1a_3c_3 \\ a_3s_3 \end{pmatrix}$$

$$R^0_{0,3k} \times (T_0^3 - T_0^0) = [R^0_{0,3k}](T_0^3 - T_0^0) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} -s_1(a_3c_3+a_2)+q_2c_1 \\ c_1(a_3c_3+a_2)+q_2s_1 \\ d_1+a_3s_3 \end{pmatrix}$$

$$R^0_{0,3k} \times (T_0^3 - T_0^0) = \begin{pmatrix} -c_1(a_3c_3+a_2)-q_2s_1 \\ -s_1(a_3c_3+a_2)+q_2c_1 \\ 0 \end{pmatrix}$$

$$R^2_{0,3k} \times (T_0^3 - T_0^2) = [R^2_{0,3k}](T_0^3 - T_0^2) = \begin{bmatrix} 0 & 0 & s_1 \\ 0 & 0 & -c_1 \\ -s_1 & c_1 & 0 \end{bmatrix} \begin{pmatrix} -s_1a_3c_3 \\ c_1a_3c_3 \\ a_3s_3 \end{pmatrix} =$$

$$R^2_{0,3k} \times (T_0^3 - T_0^2) = \begin{pmatrix} s_1a_3s_3 \\ -c_1a_3s_3 \\ s_1^2a_3c_3 + c_1^2a_3c_3 \end{pmatrix} = \begin{pmatrix} s_1a_3s_3 \\ -c_1a_3s_3 \\ (s_1^2+c_1^2)a_3c_3 \end{pmatrix} = \begin{pmatrix} s_1a_3s_3 \\ -c_1a_3s_3 \\ a_3c_3 \end{pmatrix}$$

Wyliczenie kolumn:

$$J_{m1}(q) = \begin{pmatrix} R^0_{0,3k} \times (T_0^3 - T_0^0) \\ R^0_{0,3k} \end{pmatrix} = \begin{pmatrix} -c_1(a_3c_3+a_2)-q_2s_1 \\ -s_1(a_3c_3+a_2)+q_2c_1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$J_{m2}(q) = \begin{pmatrix} R^1_{0,3k} \\ 0 \end{pmatrix} = \begin{pmatrix} c_1 \\ s_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad J_{m3}(q) = \begin{pmatrix} R^2_{0,3k} \times (\Gamma^3_0 - \Gamma^2_0) \\ R^2_{0,3k} \end{pmatrix} = \begin{pmatrix} s_1 a_3 s_3 \\ -c_1 a_3 s_3 \\ a_3 c_3 \\ c_1 \\ s_1 \\ 0 \end{pmatrix}$$

$$J_m(q) = [J_{m1}(q)\ J_{m2}(q)\ J_{m3}(q)] = \begin{bmatrix} -c_1(a_3c_3+a_2)-q_2s_1 & c_1 & s_1a_3s_3 \\ -s_1(a_3c_3+a_2)+q_2c_1 & s_1 & -c_1a_3s_3 \\ 0 & 0 & a_3c_3 \\ 0 & 0 & c_1 \\ 0 & 0 & s_1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$J_m(q)\dot{q} = \begin{bmatrix} -c_1(a_3c_3+a_2)-q_2s_1 & c_1 & s_1a_3s_3 \\ -s_1(a_3c_3+a_2)+q_2c_1 & s_1 & -c_1a_3s_3 \\ 0 & 0 & a_3c_3 \\ 0 & 0 & c_1 \\ 0 & 0 & s_1 \\ 1 & 0 & 0 \end{bmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{pmatrix} = \begin{pmatrix} -(c_1(a_3c_3+a_2)+q_2s_1)\dot{q}_1 + c_1\dot{q}_2 + s_1a_3s_3\dot{q}_3 \\ (-s_1(a_3c_3+a_2)+q_2c_1)\dot{q}_1 + s_1\dot{q}_2 - c_1a_3s_3\dot{q}_3 \\ a_3c_3\dot{q}_3 \\ c_1\dot{q}_3 \\ s_1\dot{q}_3 \\ \dot{q}_1 \end{pmatrix}$$

Odpowiedź w przedostatniej linii J_m(q)=... . Mnożenie jakobianu przez q’ jest niepotrzebne.

4.
   a.
   b.