a)
b)Tomasz Gniazdowski 248988

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b) Koło bez poślizgu poprzecznego i wzdłużnego

$q = (x, y, \theta, \phi)^T$

$g_1 = (c\theta, s\theta, 0, 1)^T$

$g_2 = (0, 0, 1, 0)^T$

- n - wymiar przestrzeni $= 4$
  m - l. generatorów $= 2$
  l - l. ograniczeń nichol $= n - m = 2$

- $D_0(q) = \begin{bmatrix} c\theta & 0 \\ s\theta & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\det D_0^{(2\times2)} \neq 0 \Rightarrow$ $\dim D_0 = 2$

- $D_1(q) = \operatorname{span}\{g_1, g_2, g_{12}\}$

$$g_{12} = [g_1, g_2] = \frac{\partial g_2}{\partial q}\cdot g_1 - \frac{\partial g_1}{\partial q}\cdot g_2 =$$

$$- \begin{array}{cccc} x & y & \theta & \phi \\ \end{array}$$

$$= -\begin{bmatrix} 0 & 0 & -s\theta & 0 \\ 0 & 0 & c\theta & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} s\theta \\ -c\theta \\ 0 \\ 0 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} c\theta & 0 & s\theta \\ s\theta & 0 & -c\theta \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$ $\det D_1^{(3\times3)} \neq 0 \Rightarrow$ $\dim D_1 = 3$

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(2b)

$$D_2 = \text{span}\{g_1, g_2, g_{12}, g_{112}, g_{212}\}$$

$$g_{112} = [g_1, g_{12}] = \frac{dg_{12}}{dq} g_1 - \frac{dg_1}{dq} g_{12} =$$

$$\begin{bmatrix} 0 & 0 & c\theta & 0 \\ 0 & 0 & s\theta & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} c\theta \\ s\theta \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & -s\theta & 0 \\ 0 & 0 & c\theta & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} s\theta \\ -c\theta \\ 0 \\ 0 \end{bmatrix} =$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$g_{212} = [g_2, g_{12}] = \frac{dg_{12}}{dq} g_2 - \frac{dg_2}{dq} g_{12} =$$

$$\begin{bmatrix} 0 & 0 & c\theta & 0 \\ 0 & 0 & s\theta & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} s\theta \\ -c\theta \\ 0 \\ 0 \end{bmatrix} =$$

$$\begin{bmatrix} c\theta \\ s\theta \\ 0 \\ 0 \end{bmatrix}$$

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3b

$$D_2 = \begin{array}{c} \left[\begin{array}{c|c|c|c} c\theta & 0 & s\theta & c\theta \\ \hline s\theta & 0 & -c\theta & s\theta \\ \hline 0 & 1 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 \end{array}\right] \\ \begin{array}{cccc} g_1 & g_2 & g_{12} & g_{212} \end{array} \end{array}$$

$\det D_2 \neq 0 \Rightarrow$
$\dim D_2 = 4$

Wektor wzrostu dystrybucji

$$[2,\ 3,\ 4]$$

Stopień nieholonomiczności $= 2$, bo $Dim_2 = 4 = n$

$$A(q) \cdot \dot{q} = 0$$

Jeżeli $r_p(q) = 0$, to układ jest nieholonomiczny.
Stopień określony powyżej (nieholonomiczności).

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(1c) c) samochód kinematyczny

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$$q = \begin{bmatrix} x \\ y \\ \theta \\ \phi \end{bmatrix}$$

$$g_1 = \begin{bmatrix} Lc\theta c\phi \\ Ls\theta c\phi \\ s\phi \\ 0 \end{bmatrix} \qquad g_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

- $n = 4$
  $m = 2$
  $l = 4 - 2 = 2$

- $$D_0 = \begin{bmatrix} Lc\theta c\phi & 0 \\ Ls\theta c\phi & 0 \\ s\phi & 0 \\ 0 & 1 \end{bmatrix}$$

$\det D_0^{(2\times 2)} \neq 0 \Rightarrow$
$\underline{\dim D_0 = 2}$

$$D_1 = \operatorname{span}\{g_1, g_2, g_{12}\}$$

$$g_{12} = [g_1, g_2] =$$

$$\frac{dg_2}{q} \cdot g_1 - \frac{dg_1}{q} \cdot g_2 = - \begin{bmatrix} 0 & 0 & -Lc\phi s\theta & -Lc\theta s\phi \\ 0 & 0 & Lc\theta c\phi & -Ls\theta s\phi \\ 0 & 0 & 0 & c\phi \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} =$$

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(2c)

$$g_{12} = -\begin{bmatrix} -Lc\theta s\phi \\ -Ls\theta s\phi \\ c\phi \\ 0 \end{bmatrix} = \begin{bmatrix} Lc\theta s\phi \\ Ls\theta s\phi \\ -c\phi \\ 0 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} Lc\theta c\phi & 0 & Lc\theta s\phi \\ Ls\theta c\phi & 0 & Ls\theta s\phi \\ s\phi & 0 & -c\phi \\ 0 & 1 & 0 \end{bmatrix}$$

$\det D_1^{(3\times3)} \neq 0 \Rightarrow$

$\underline{\dim D_1 = 3}$

$$D_2 = \text{span}\{g_1, g_2, g_{12}, g_{112}, g_{212}\}$$

$$g_{212} = [g_2, g_{12}] =$$

$$\frac{dg_{12}}{dq} g_2 - \underbrace{\frac{g_2}{dq}}_{=0} g_{12} = \begin{array}{c} \begin{array}{cccc} x & y & \theta & \phi \end{array} \\ \begin{bmatrix} 0 & 0 & +Ls\phi s\theta & Lc\theta c\phi \\ 0 & 0 & Lc\theta s\phi & Ls\theta c\phi \\ 0 & 0 & 0 & s\phi \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} =$$

$$\begin{bmatrix} Lc\theta c\phi \\ Ls\theta c\phi \\ s\phi \\ 0 \end{bmatrix}$$

$\cancel{g_{112} = [g_1, g_{12}] =}$

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3.C

$$g_{112} = [g_1, \; g_{12}] =$$

$$\frac{\partial g_{12}}{\partial q} g_1 - \frac{\partial g_1}{\partial q} g_{12}$$

$$\begin{array}{cccc} x & y & \theta & \phi \end{array}$$

$$\begin{bmatrix} 0 & 0 & -Ls\phi s\theta & Lc\theta c\phi \\ 0 & 0 & Ls\phi c\theta & Ls\theta c\phi \\ 0 & 0 & \cancel{\theta} & s\phi \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Lc\theta c\phi \\ Ls\theta c\phi \\ s\phi \\ 0 \end{bmatrix} -$$

$$\begin{array}{cccc} x & y & \theta & \phi \end{array}$$

$$\begin{bmatrix} 0 & 0 & -Lc\phi s\theta & -Lc\theta s\phi \\ 0 & 0 & Lc\phi c\theta & -Ls\theta s\phi \\ 0 & 0 & 0 & c\phi \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Lc\theta s\phi \\ Ls\theta s\phi \\ -c\phi \\ 0 \end{bmatrix} = \begin{bmatrix} -Ls\phi s\theta s\phi - (Lc^2\phi \, s\theta) \\ Ls\phi c\theta s\phi + Lc\phi c\theta c\phi \\ 0 - 0 \\ 0 - 0 \end{bmatrix} =$$

$$\begin{bmatrix} -Ls\theta \\ Lc\theta \\ 0 \\ 0 \end{bmatrix}$$

4c

$$D_2 = \begin{bmatrix} Lc\theta c\phi & 0 & Lc\theta s\phi & -Ls\theta \\ Ls\theta c\phi & 0 & Ls\theta s\phi & Lc\theta \\ s\phi & 0 & -c\phi & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$\det D_2 \neq 0 \Rightarrow \dim D_2 = 4$

- wektor wzrostu : $[2, 3, 4]$
- stopień nieholonomiczności $= 2$, $\dim D_2 = n = 4$
- Sterowalny, nieholonomiczny

CS Zeskanowane w CamScanner

d) Jan Bronicki

d) $q = (q_1, q_2, q_3)^T$

$$g_1 = \begin{pmatrix} 1 \\ 0 \\ q_2 \end{pmatrix} \qquad g_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

1) $n = 3$ $\quad l = 1$

$m = 2$

2) $$D_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ q_2 & 0 \end{bmatrix} \qquad \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \neq 0 \qquad \dim D_0 = 2$$

$$D_1 = D_0 + [D_0, D_0] = \operatorname{span}_{C^\infty} [g_1, g_2, g_{12}]$$

$$g_{12} = [g_1, g_2] = \underbrace{\frac{\partial g_2}{\partial q}}_{=0} g_1 - \frac{\partial g_1}{\partial q} g_2 = -\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = -\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

$$D_1 = \det \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ q_2 & 0 & -1 \end{bmatrix} = -1 \neq 0 \qquad \dim D_1 = 3$$

3), 4) $r_0(q) = 2 < r_1(q) = 3 = n$

STOPIEŃ NIEHOLOMICZNOŚCI = 1

Układ jest w pełni holonomiczny bazując na poprzednich punktach

e)

f)