

Teoria Regulacji Ćwiczenia, Wtorek 17:05-18:45

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Zadanie n6

Założenia: $y(0^-) = 0$ oraz $y'(0^-) = 1$

$$\bullet \quad y'' + 3y' + 2y = 0$$

$$\mathcal{L}\{y'' + 3y' + 2y\} = \mathcal{L}\{0\}$$

$$s^2Y(s) - sy(0) - y'(0) + 3sY(s) - y(0) + 2Y(s) = 0$$

$$Y(s)(s^2 + 3s + 2) - 1 = 0$$

$$Y(s) = \frac{1}{(s+1)(s+2)}$$

$$Y(s) = \frac{A}{s+1} + \frac{B}{s+2}$$

$$\begin{cases} A = 1 \\ B = -1 \end{cases}$$

$$Y(s) = \frac{1}{s+1} + \frac{-1}{s+2}$$

$$\mathcal{L}^{-1}\{Y(s)\} = y(t) = e^{-t} - e^{-2t}$$

- $y'' + 3y' + 2y = 4$

$$\mathcal{L}\{y'' + 3y' + 2y\} = \mathcal{L}\{4\}$$

$$s^2Y(s) - sy(0) - y'(0) + 3sY(s) - y(0) + 2Y(s) = \frac{4}{s}$$

$$Y(s)(s^2 + 3s + 2) - 1 = \frac{4}{s}$$

$$Y(s) = \frac{s + 4}{s(s + 1)(s + 2)}$$

$$Y(s) = \frac{A}{s} + \frac{B}{s + 1} + \frac{C}{s + 2}$$

$$\begin{cases} A = 2 \\ B = -3 \\ C = 1 \end{cases}$$

$$Y(s) = \frac{2}{s} + \frac{-3}{s + 1} + \frac{1}{s + 2}$$

$$\mathcal{L}^{-1}\{Y(s)\} = y(t) = -3e^{-t} + e^{-2t} + 2$$

- $y'' + 3y' + 2y = t$

$$\mathcal{L}\{y'' + 3y' + 2y\} = \mathcal{L}\{t\}$$

$$s^2Y(s) - sy(0) - y'(0) + 3sY(s) - y(0) + 2Y(s) = \frac{1}{s^2}$$

$$Y(s)(s^2 + 3s + 2) - 1 = \frac{1}{s^2}$$

$$Y(s) = \frac{s^2 + 1}{s^2(s + 1)(s + 2)}$$

$$Y(s) = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s + 1} + \frac{D}{s + 2}$$

$$\begin{cases} A = \frac{1}{2} \\ B = \frac{-3}{4} \\ C = 2 \\ D = \frac{-5}{4} \end{cases}$$

$$Y(s) = \frac{\frac{1}{2}}{s^2} + \frac{\frac{-3}{4}}{s} + \frac{2}{s+1} + \frac{\frac{-5}{4}}{s+2}$$

$$\mathcal{L}^{-1}\{Y(s)\} = y(t) = 2e^{-t} - \frac{5}{4}e^{-2t} + \frac{1}{2}t - \frac{3}{4}$$

$$\bullet \quad y'' + 3y' + 2y = \sin(\omega t)$$

$$\mathcal{L}\{y'' + 3y' + 2y\} = \mathcal{L}\{\sin(\omega t)\}$$

$$Y(s)(s^2 + 3s + 2) - 1 = \frac{\omega}{s^2 + \omega^2}$$

$$Y(s) = \frac{s^2 + \omega^2 + \omega}{(s^2 + \omega^2)(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{Cs + D}{s^2 + \omega^2}$$

$$\begin{cases} A = \frac{\omega^2 + \omega + 1}{\omega^2 + 1} \\ B = \frac{-\omega^2 - \omega - 4}{\omega^2 + 4} \\ C = \frac{-3\omega}{\omega^4 + 5\omega^2 + 4} \\ D = \frac{-\omega^3 + 2\omega}{\omega^4 + 5\omega^2 + 4} \end{cases}$$

$$Y(s) = \frac{\frac{\omega^2 + \omega + 1}{\omega^2 + 1}}{s+1} + \frac{\frac{-\omega^2 - \omega - 4}{\omega^2 + 4}}{s+2} + \frac{\frac{-3\omega}{\omega^4 + 5\omega^2 + 4}s + \frac{-\omega^3 + 2\omega}{\omega^4 + 5\omega^2 + 4}}{s^2 + \omega^2}$$

$$\mathcal{L}^{-1}\{Y(s)\} = y(t)$$

$$y(t) = \frac{\omega^2 + \omega + 1}{\omega^2 + 1}e^{-t} + \frac{-\omega^2 - \omega - 4}{\omega^2 + 4}e^{-2t} + \frac{1}{\omega^4 + 5\omega^2 + 4} \cdot [-3\omega \cos(\omega t) + (-\omega^2 + 2)\sin(\omega t)]$$

$$\bullet \quad y'' + 2y' + 2y = 0$$

$$\mathcal{L}\{y'' + 2y' + 2y\} = \mathcal{L}\{0\}$$

$$Y(s)(s^2 + 2s + 2) - 1 = 0$$

$$Y(s) = \frac{1}{s^2 + 2s + 2} = \frac{1}{(s+1)^2 + 1}$$

$$\mathcal{L}^{-1}\{Y(s)\} = y(t) = e^t \sin(t)$$

- $y'' + 2y' + 2y = 1$

$$\mathcal{L}\{y'' + 2y' + 2y\} = \mathcal{L}\{1\}$$

$$Y(s)(s^2 + 2s + 2) - 1 = \frac{1}{s}$$

$$Y(s) = \frac{s+1}{s(s^2 + 2s + 2)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 2s + 2}$$

$$\begin{cases} A = \frac{1}{2} \\ B = \frac{-1}{2} \\ C = 0 \end{cases}$$

$$Y(s) = \frac{1}{2s} + \frac{\frac{1}{2}s - 1 + 1}{s^2 + 2s + 2}$$

$$\mathcal{L}^{-1}\{Y(s)\} = y(t) = \frac{1}{2}(1 - e^{-t}[\cos(t) - \sin(t)])$$