Teoria Regulacji Ćwiczenia, Wtorek 17:05-18:45

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Lista 3

Zadanie 1

Niech $M(s) = a_m s^m + a_{m-1}s^{m-1} + ... + a_1 s + a_0$ to nasz wielomian charakterystyczny. Niech $a_m > 0$

a) $\frac{1}{s^4+7s^3+17s^2+17s+16}$ Wszystkie współczynniki są dodatnie co oznacza, że system MOŻE być stabilny. Tworzymy macierz:

$$H = \begin{bmatrix} 7 & 17 & 0 & 0 \\ 1 & 17 & 6 & 0 \\ 0 & 7 & 17 & 0 \\ 0 & 1 & 17 & 6 \end{bmatrix}$$

Kryterium Hurwitza:

$$\begin{cases} \Delta_1 = 7 \\ \Delta_2 = 102 \\ \Delta_3 = 1440 \\ \Delta_4 = a_0 \cdot \Delta_3 = 8640 \end{cases}$$

$$\Delta_1, \Delta_2, \Delta_3, \Delta_4, > 0$$

Układ jest stabilny, brak pierwiastków w części rzeczywistej dodatniej oraz urojonej

Kryterium Michajłowa:

$$M(s) = s^4 + 7s^3 + 17s^2 + 17s + 6$$

$$M(j\omega) = (\omega^4 - 17\omega^2 + 6) + (-7\omega^3 + 17\omega)j$$

$$Re(M(j\omega)) = 0$$

Niech $t = \omega^2$ i $\omega^2 > 0$

$$t^2 - 17t + 6 = 0$$

$$\begin{cases} t_1 = \frac{17+\sqrt{265}}{2} \\ t_2 = \frac{17-\sqrt{265}}{2} \end{cases}$$

$$\begin{cases} \omega_1 = \sqrt{\frac{17+\sqrt{265}}{2}} \\ \omega_2 = -\sqrt{\frac{17+\sqrt{265}}{2}} \\ \omega_3 = \sqrt{\frac{17-\sqrt{265}}{2}} \\ \omega_4 = -\sqrt{\frac{17-\sqrt{265}}{2}} \end{cases}$$

$$Im(M(j\omega)) = 0$$

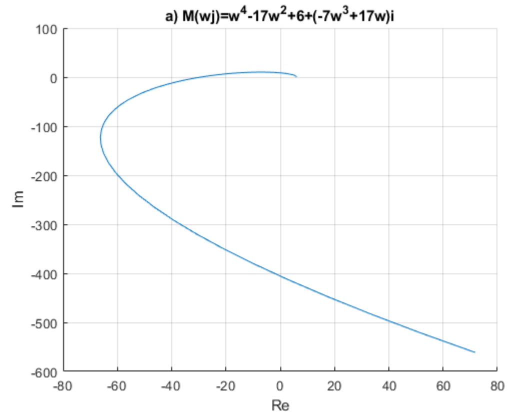
$$-7\omega^3 + 17\omega = 0$$

$$\begin{cases} \omega_5 = 0 \\ \omega_6 = \sqrt{\frac{17}{7}} \\ \omega_7 = -\sqrt{\frac{17}{7}} \end{cases}$$

Bierzemy pod uwagę tylko $\omega >= 0$

$$\begin{cases} \omega_1 = 4.1 \\ \omega_3 = 0.6 \\ \omega_5 = 0 \\ \omega_6 = 1.56 \end{cases}$$

$$\Delta arg M(j\omega)_{0 \leqslant \omega \leqslant \infty} = m\frac{\pi}{2} = 2\pi$$



b) $\frac{s-2}{s^4+6s^3+13s^2+12s+4}$ Może być stabilny.

$$H = \begin{bmatrix} 6 & 12 & 0 & 0 \\ 1 & 13 & 4 & 0 \\ 0 & 6 & 12 & 0 \\ 0 & 1 & 13 & 4 \end{bmatrix}$$

$$\begin{cases} \Delta_1 = 6 \\ \Delta_2 = 76 \\ \Delta_3 = 4 \cdot 144 + 6 \cdot 12 \\ \Delta_4 = a_0 \cdot \Delta_3 = 4(4 \cdot 12^2 + 6 \cdot 12) \end{cases}$$

Kryterium Hurtwitza:

$$\Delta_1, \Delta_2, \Delta_3, \Delta_4, > 0$$

Układ stabilny, brak pierwiastków w części rzeczywistej dodatniej oraz urojonej

Kryterium Michajłowa:

$$M(s) = s^4 + 6s^3 + 13s^2 + 12s + 4$$

$$M(j\omega) = (\omega^4 - 13\omega^2 + 4) + (-6\omega^3 + 12\omega)j$$

$$Re(M(j\omega)) = 0$$

Niech $t = \omega^2$ i $\omega^2 > 0$

$$t^2 - 17t + 6 = 0$$

$$\begin{cases} t_1 = \frac{17+\sqrt{265}}{2} \\ t_2 = \frac{17-\sqrt{265}}{2} \end{cases}$$

$$\begin{cases} \omega_1 = \sqrt{\frac{17+\sqrt{265}}{2}} \\ \omega_2 = -\sqrt{\frac{17+\sqrt{265}}{2}} \\ \omega_3 = \sqrt{\frac{17-\sqrt{265}}{2}} \\ \omega_4 = -\sqrt{\frac{17-\sqrt{265}}{2}} \end{cases}$$

$$Im(M(j\omega)) = 0$$

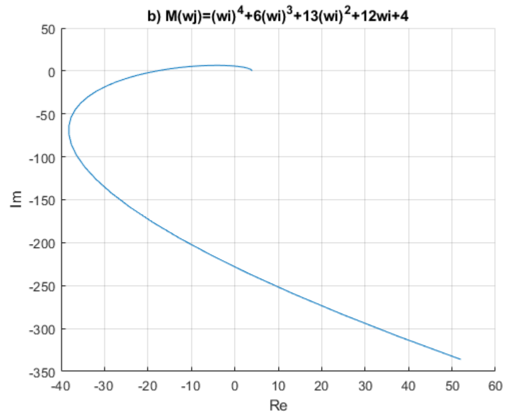
$$-7\omega^3 + 17\omega = 0$$

$$\begin{cases} \omega_5 = 0 \\ \omega_6 = \sqrt{\frac{17}{7}} \\ \omega_7 = -\sqrt{\frac{17}{7}} \end{cases}$$

Bierzemy pod uwagę tylko $\omega >= 0$

$$\begin{cases} \omega_1 = 4.1 \\ \omega_3 = 0.6 \\ \omega_5 = 0 \\ \omega_6 = 1.56 \end{cases}$$

$$\Delta arg M(j\omega)_{0 \leqslant \omega \leqslant \infty} = m\frac{\pi}{2} = 2\pi$$



c) $\frac{s+3}{s^3+4s^2+s-6}$

$$a_0 = -6 < 0 \text{ niestabilny}$$

$$H = \begin{bmatrix} 4 & -6 & 0 \\ 1 & 1 & 0 \\ 0 & 4 & -6 \end{bmatrix}$$

$$\begin{cases} \Delta_0 = 4 \\ \Delta_1 = 10 \\ \Delta_2 = a_0 \cdot \Delta_1 = -60 \end{cases}$$

$$V\left(4,\ \frac{10}{4}, -\frac{60}{10}\right) = 1$$

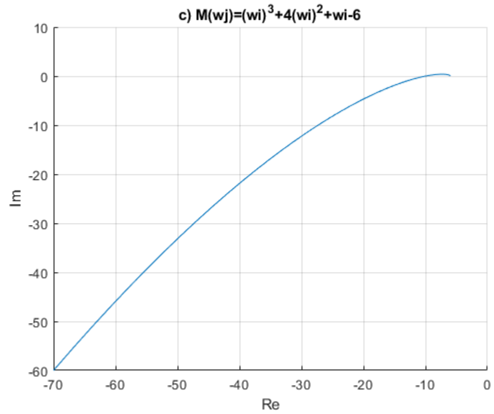
$$\Delta_0, \Delta_1, \Delta_2 \neq 0$$

$$V \neq 0$$

Niestabilny

Michajłowa

$$M(\omega j) = (\omega j)^3 + 4(\omega j)^2 + \omega j - 6$$



d) $\frac{s+4}{s^3+6s^2+11s+6}$

Może być stabilny ponieważ współczynniki są dodatnie.

$$H = \begin{bmatrix} 6 & 6 & 0 \\ 1 & 11 & 0 \\ 0 & 6 & 6 \end{bmatrix}$$

$$\begin{cases} \Delta_0 = 6 \\ \Delta_1 = 60 \\ \Delta_2 = a_0 \cdot \Delta_1 = 360 \end{cases}$$

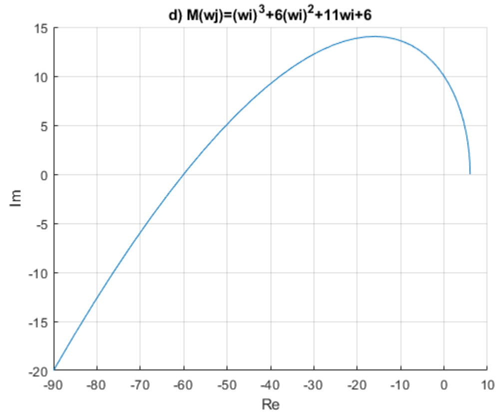
$$V\left(6,\ \frac{60}{6}, \frac{360}{60}\right) = 0$$

$$\Delta_0, \Delta_1, \Delta_2 > 0$$

Stabilne

Michajłowa

$$M(\omega j) = (\omega j)^2 + 6(\omega j)^2 + 11(\omega j) + 6$$



Zadanie 2

$$s^2 + a_1 s + a_0 = 0$$

$$a_1, a_0 > 0$$

$$H_{2x2} = \begin{bmatrix} a_1 & 0 \\ 1 & a_0 \end{bmatrix}$$

$$\begin{cases} \Delta_1 = a_1 > 0 \\ \Delta_2 = a_1 a_0 > 0 \end{cases}$$

Z Hurwitza system jest stabilny zakładając, że $a_1 > 0$ oraz $a_0 > 0$.