# Lista 2, zadania 3, 4, 5

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## Zadanie 3

a) $K(s) = \frac{1}{s(s+1)}$

Odpowiedź skokowa:

$$\lambda(t) = \mathcal{L}^{-1}\left\{\frac{1}{s} \cdot K(s)\right\}$$

$$\lambda(t) = \mathcal{L}^{-1}\left\{\frac{1}{s\left(s^2+s\right)}\right\}$$

$$\lambda(t) = \mathcal{L}^{-1}\left\{\frac{1}{s+1} - \frac{1}{s} + \frac{1}{s^2}\right\}$$

$$\lambda(t) = t - 1 + e^{-t}$$

$$\lambda'(t) = 1 - e^{-t}$$

$$\lambda''(t) = e^{-t}$$

$$\lambda(0) = 0$$

$$\lambda'(0) = 0$$

$$\lambda''(0) = 1$$

Odpowiedź impulsowa:

$$k(t) = \mathcal{L}^{-1}\left\{K(s)\right\}$$

$$k(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^2+s}\right\}$$

$$k(t) = \mathcal{L}^{-1}\left\{-\frac{1}{s+1}+\frac{1}{s}\right\}$$

$$k(t) = 1 - e^{-t}$$

$$k'(t) = e^{-t}$$

$$k''(t) = -e^{-t}$$

$$k(0) = 0$$

$$k'(0) = 1$$

$$k''(0) = -1$$

b) $K(s) = \frac{1}{(s+1)(s^2+1)}$

Odpowiedź skokowa:

$$\lambda(t) = \mathcal{L}^{-1}\left\{\frac{1}{s}\cdot K(s)\right\}$$

$$\lambda(t) = \mathcal{L}^{-1}\left\{\frac{1}{s}\cdot\frac{1}{(s+1)\left(s^2+1\right)}\right\}$$

$$\lambda(t) = \mathcal{L}^{-1}\left\{-\frac{s+1}{2\left(s^2+1\right)} - \frac{1}{2\left(s+1\right)} + \frac{1}{s}\right\}$$

$$\lambda(t) = -\frac{\sin\left(t\right)}{2} - \frac{\cos\left(t\right)}{2} + 1 - \frac{e^{-t}}{2}$$

$$\lambda'(t) = \frac{\sin\left(t\right)}{2} - \frac{\cos\left(t\right)}{2} + \frac{e^{-t}}{2}$$

$$\lambda''(t) = \frac{\sin\left(t\right)}{2} + \frac{\cos\left(t\right)}{2} - \frac{e^{-t}}{2}$$

$$\lambda(0) = 0$$

$$\lambda'(0) = 0$$

$$\lambda''(0) = 0$$

Odpowiedź impulsowa:

$$k(t) = \mathcal{L}^{-1}\left\{K(s)\right\}$$

$$k(t) = \mathcal{L}^{-1}\left\{\frac{1}{(s+1)\left(s^2+1\right)}\right\}$$

$$k(t) = \mathcal{L}^{-1}\left\{-\frac{s-1}{2\left(s^2+1\right)} + \frac{1}{2\left(s+1\right)}\right\}$$

$$k(t) = \frac{\sin\left(t\right)}{2} - \frac{\cos\left(t\right)}{2} + \frac{e^{-t}}{2}$$

$$k'(t) = \frac{\sin\left(t\right)}{2} + \frac{\cos\left(t\right)}{2} - \frac{e^{-t}}{2}$$

$$k''(t) = -\frac{\sin\left(t\right)}{2} + \frac{\cos\left(t\right)}{2} + \frac{e^{-t}}{2}$$

$$k(0) = 0$$

$$k'(0) = 0$$

$$k''(0) = 1$$

c) $K(s) = \frac{1}{(s-1)(s+2)}$

Odpowiedź skokowa:

$$\lambda(t) = \mathcal{L}^{-1}\left\{\frac{1}{s} \cdot K(s)\right\}$$

$$\lambda(t) = \mathcal{L}^{-1}\left\{\frac{1}{s\,(s-1)\,(s+2)}\right\}$$

$$\lambda(t) = \mathcal{L}^{-1}\left\{\frac{1}{6\,(s+2)} + \frac{1}{3\,(s-1)} - \frac{1}{2s}\right\}$$

$$\lambda(t) = \frac{e^t}{3} - \frac{1}{2} + \frac{e^{-2t}}{6}$$

$$\lambda'(t) = \frac{e^t}{3} - \frac{e^{-2t}}{3}$$

$$\lambda''(t) = \frac{e^t}{3} + \frac{2e^{-2t}}{3}$$

$$\lambda(0) = 0$$

$$\lambda'(0) = 0$$

$$\lambda''(0) = 1$$

Odpowiedź impulsowa:

$$k(t) = \mathcal{L}^{-1}\left\{K(s)\right\}$$

$$k(t) = \mathcal{L}^{-1}\left\{\frac{1}{\left(s-1\right)\left(s+2\right)}\right\}$$

$$k(t) = \mathcal{L}^{-1}\left\{-\frac{1}{3\left(s+2\right)} + \frac{1}{3\left(s-1\right)}\right\}$$

$$k(t) = \frac{e^{t}}{3} - \frac{e^{-2t}}{3}$$

$$k'(t) = \frac{e^{t}}{3} + \frac{2e^{-2t}}{3}$$

$$k''(t) = \frac{e^{t}}{3} - \frac{4e^{-2t}}{3}$$

$$k(0) = 0$$

$$k'(0) = 1$$

$$k''(0) = -1$$

## Zadanie 4

a)

$$K(s) = \frac{1}{(s+1)(s+3)} = \frac{L(s)}{M(s)}$$

$$Y(s) = K(s)U(s) = \frac{L(s)}{M(s)}U(s)$$

$$Y(s)\left[s^2+4s+3\right] = [s+2]U(s)$$

$$y'' + 4y' + 3y = u' + 2u$$

b)

$$y'' + 4y' + 3y = 0$$

$$y'' = -4y' - 3y$$

$$\dot{\xi} = A\xi$$

$$\xi = \begin{bmatrix} y \\ y' \end{bmatrix}$$

$$\dot{\xi} = \begin{bmatrix} y' \\ y'' \end{bmatrix}$$

$$\begin{bmatrix} y' \\ y'' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} \begin{bmatrix} y \\ y' \end{bmatrix}$$

## Zadanie 5

### a)

Drugi biegun jest sprzezony, dlatego wynosi on:

$$s_2 = \sigma - j\omega$$

### b)

$$K(s) = \frac{1}{M(s)} = \frac{1}{(s-s_1)(s-s_2)} = \frac{1}{s^2 - 2s\delta + \delta + \omega^2}$$

$\alpha$)

$$\omega = 0, \delta > 0$$

$$\frac{1}{s^2 + \delta^2 - 2s\delta} = \frac{1}{s-\delta} = e^{\delta t} \cdot t \cdot 1$$

$\beta$)

$$\omega = 0, \delta < 0$$

$$\frac{1}{(s+\delta)^2} = \frac{1}{(s+\delta)^2} = e^{-\delta t} \cdot t \cdot 1(t)$$

$\gamma$)

$$\omega \neq 0, \delta > 0$$

$$\frac{1}{s^2 - 2s\delta + \delta + \omega^2} = \frac{1}{\omega} \cdot \frac{\omega}{(s-\delta)^2 + \omega^2}$$

$$y(t) = \frac{1}{\omega} e^{\delta t} \cdot sin(\omega t) \cdot 1(t)$$

$\delta$)

$$\omega \neq 0, \delta < 0$$

$$\frac{1}{(s+\delta)^2 + \omega^2} = \frac{1}{\omega} \frac{\omega}{(s+\delta)^2 + \omega^2}$$

$$y(t) = \frac{1}{\omega} \cdot e^{-\delta t} sin(\omega t) \cdot 1(t)$$