Teoria Regulacji Ćwiczenia, Wtorek 17:05-18:45

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Zadanie 1 z Listy 2

$$Ty'(t) + y(t) = ku(t)$$

a) $u(t) = 0$

$\mathcal{L}\{Ty'(t) + y(t)\} = \mathcal{L}\{0\}$
$Y(s)[Ts+1] - Ty(0) = 0$
$Y(s) = y(0)\frac{1}{s+\frac{1}{T}}$
$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{y(0)\frac{1}{s+\frac{1}{T}}\right\}$
$y(t) = y(0)e^{-\frac{1}{T}t}$

b) $u(t) = \delta(t)$

$\mathcal{L}\{Ty'(t) + y(t)\} = \mathcal{L}\{k\}$
$Y(s)[Ts+1] - Ty(0) = k$
$Y(s) = \left(\frac{k+Ty(0)}{T}\right)\frac{1}{s+\frac{1}{T}}$
$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\left(\frac{k+Ty(0)}{T}\right)\frac{1}{s+\frac{1}{T}}\right\}$
$y(t) = \left(\frac{k+Ty(0)}{T}\right)e^{-\frac{1}{T}t}$

c) $u(t) = 1(t)$

$\mathcal{L}\{Ty'(t) + y(t)\} = \mathcal{L}\{k \cdot 1(t)\}$
$Y(s)[Ts+1] - Ty(0) = \frac{k}{s}$
$Y(s) = \frac{k+sTy(0)}{s(Ts+1)} = \frac{A}{s} + \frac{B}{Ts+1}$

$$\begin{cases} A = k \\ B = T(y(0) - k) \end{cases}$$

$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{k\frac{1}{s} + (y(0) - k)\frac{1}{s+\frac{1}{T}}\right\}$
$y(t) = k + (y(0) - k)e^{-\frac{1}{T}t}$

d) $u(t) = sin(\omega t)$

$$\mathcal{L}\left\{Ty'(t) + y(t)\right\} = \mathcal{L}\left\{k \cdot sin(\omega t)\right\}$$
$$Y(s)\left[Ts+1\right] - Ty(0) = k\tfrac{\omega}{s^2+\omega^2}$$
$$Y(s) = \tfrac{k\omega + Ty(0)\left(s^2+\omega^2\right)}{s^2+\omega^2} = \tfrac{As+B}{s^2+\omega^2} + \tfrac{C}{Ts+1}$$

$$\begin{cases} A = \frac{-Tk\omega}{T^2\omega^2+1} \\ B = \frac{k\omega}{T^2\omega^2+1} \\ C = \frac{T^2k\omega}{T^2\omega^2+1} \end{cases}$$

$$\mathcal{L}^{-1}\left\{Y(s)\right\} = \mathcal{L}^{-1}\left\{\tfrac{k}{T^2\omega^2+1}\left(\tfrac{-T\omega s}{s^2+\omega^2} + \tfrac{\omega}{s^2+\omega^2} + \tfrac{T^2\omega}{Ts+1}\right) + \tfrac{Ty(0)}{Ts+1}\right\}$$
$$y(t) = \tfrac{k}{T^2\omega^2+1}\left(-T\omega cos(\omega t) + sin(\omega t) + T\omega e^{-\frac{1}{T}t}\right) + y(0)\,e^{-\frac{1}{T}t}$$

e) $u(t) = t$

$$\mathcal{L}\left\{Ty'(t) + y(t)\right\} = \mathcal{L}\left\{k \cdot t\right\}$$
$$Y(s)\left[Ts+1\right] - Ty(0) = \tfrac{k}{s}$$
$$Y(s) = \tfrac{k+s^2Ty(0)}{s^2(Ts+1)} = \tfrac{A}{s} + \tfrac{B}{s^2} + \tfrac{C}{Ts+1}$$

$$\begin{cases} A = -Tk \\ B = k \\ C = Ty(0) + T^2k \end{cases}$$

$$\mathcal{L}^{-1}\left\{Y(s)\right\} = \mathcal{L}^{-1}\left\{\tfrac{-Tk}{s} + \tfrac{k}{s^2} + \tfrac{y(0)+Tk}{s+\frac{1}{T}}\right\}$$
$$y(t) = -Tk + kt + \left(y(0) + Tk\right)e^{-\frac{1}{T}t}$$

f) $u(t) = 2\delta(t) + 1(t)$

$$\mathcal{L}\left\{Ty'(t) + y(t)\right\} = \mathcal{L}\left\{k\left(2\delta(t) + 1(t)\right)\right\}$$
$$Y(s)\left[Ts+1\right] - Ty(0) = k\tfrac{2s+1}{s}$$
$$Y(s) = k\tfrac{sTy(0)+2s+1}{s(Ts+1)} = \tfrac{A}{s} + \tfrac{B}{Ts+1}$$

$$\begin{cases} A = k \\ B = 2k + Ty(0) - Tk \end{cases}$$

$$\mathcal{L}^{-1}\left\{Y(s)\right\} = \mathcal{L}^{-1}\left\{\tfrac{k}{s} + \tfrac{2k+Ty(0)-Tk}{Ts+1}\right\}$$
$$y(t) = k + \tfrac{1}{T}\left(2k + Ty(0) - Tk\right)e^{-\frac{1}{T}t}$$