Lista 2, zadania 3, 4, 5

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Zadanie 3

a) $K(s) = \frac{1}{s(s+1)}$

Odpowiedź skokowa:

$$\lambda(t) = \mathcal{L}^{-1}\left\{\frac{1}{s} \cdot K(s)\right\}$$

$$\lambda(t) = \mathcal{L}^{-1}\left\{\frac{1}{s\left(s^2+s\right)}\right\}$$

$$\lambda(t) = \mathcal{L}^{-1}\left\{\frac{1}{s+1} - \frac{1}{s} + \frac{1}{s^2}\right\}$$

$$\lambda(t) = t - 1 + e^{-t}$$

$$\lambda'(t) = 1 - e^{-t}$$

$$\lambda''(t) = e^{-t}$$

$$\lambda(0) = 0$$

$$\lambda'(0) = 0$$

$$\lambda''(0) = 1$$

Odpowiedź impulsowa:

$$k(t) = \mathcal{L}^{-1}\left\{K(s)\right\}$$

$$k(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^2+s}\right\}$$

$$k(t) = \mathcal{L}^{-1}\left\{-\frac{1}{s+1}+\frac{1}{s}\right\}$$

$$k(t) = 1 - e^{-t}$$

$$k'(t) = e^{-t}$$

$$k''(t) = -e^{-t}$$

$$k(0) = 0$$

$$k'(0) = 1$$

$$k''(0) = -1$$

b) $K(s) = \frac{1}{(s+1)(s^2+1)}$

Odpowiedź skokowa:

$$\lambda(t) = \mathcal{L}^{-1}\left\{\frac{1}{s}\cdot K(s)\right\}$$

$$\lambda(t) = \mathcal{L}^{-1}\left\{\frac{1}{s}\cdot\frac{1}{(s+1)\left(s^2+1\right)}\right\}$$

$$\lambda(t) = \mathcal{L}^{-1}\left\{-\frac{s+1}{2\left(s^2+1\right)} - \frac{1}{2\left(s+1\right)} + \frac{1}{s}\right\}$$

$$\lambda(t) = -\frac{\sin\left(t\right)}{2} - \frac{\cos\left(t\right)}{2} + 1 - \frac{e^{-t}}{2}$$

$$\lambda'(t) = \frac{\sin\left(t\right)}{2} - \frac{\cos\left(t\right)}{2} + \frac{e^{-t}}{2}$$

$$\lambda''(t) = \frac{\sin\left(t\right)}{2} + \frac{\cos\left(t\right)}{2} - \frac{e^{-t}}{2}$$

$$\lambda(0) = 0$$

$$\lambda'(0) = 0$$

$$\lambda''(0) = 1$$

Odpowiedź impulsowa:

$$k(t) = \mathcal{L}^{-1}\left\{K(s)\right\}$$

$$k(t) = \mathcal{L}^{-1}\left\{\frac{1}{\left(s+1\right)\left(s^{2}+1\right)}\right\}$$

$$k(t) = \mathcal{L}^{-1}\left\{-\frac{s-1}{2\left(s^{2}+1\right)} + \frac{1}{2\left(s+1\right)}\right\}$$

$$k(t) = \frac{\sin\left(t\right)}{2} - \frac{\cos\left(t\right)}{2} + \frac{e^{-t}}{2}$$

$$k'(t) = \frac{\sin\left(t\right)}{2} + \frac{\cos\left(t\right)}{2} - \frac{e^{-t}}{2}$$

$$k''(t) = -\frac{\sin\left(t\right)}{2} + \frac{\cos\left(t\right)}{2} + \frac{e^{-t}}{2}$$

$$k(0) = 0$$

$$k'(0) = 0$$

$$k''(0) = 1$$

c) $K(s) = \frac{1}{(s-1)(s+2)}$

Odpowiedź skokowa:

$$\lambda(t) = \mathcal{L}^{-1}\left\{\frac{1}{s} \cdot K(s)\right\}$$

$$\lambda(t) = \mathcal{L}^{-1}\left\{\frac{1}{s\,(s-1)\,(s+2)}\right\}$$

$$\lambda(t) = \mathcal{L}^{-1}\left\{\frac{1}{6\,(s+2)} + \frac{1}{3\,(s-1)} - \frac{1}{2s}\right\}$$

$$\lambda(t) = \frac{e^{t}}{3} - \frac{1}{2} + \frac{e^{-2t}}{6}$$

$$\lambda'(t) = \frac{e^{t}}{3} - \frac{e^{-2t}}{3}$$

$$\lambda''(t) = \frac{e^{t}}{3} + \frac{2e^{-2t}}{3}$$

$$\lambda(0) = 0$$

$$\lambda'(0) = 0$$

$$\lambda''(0) = 1$$

Odpowiedź impulsowa:

$$k(t) = \mathcal{L}^{-1}\left\{K(s)\right\}$$

$$k(t) = \mathcal{L}^{-1}\left\{\frac{1}{(s-1)\,(s+2)}\right\}$$

$$k(t) = \mathcal{L}^{-1}\left\{-\frac{1}{3\,(s+2)} + \frac{1}{3\,(s-1)}\right\}$$

$$k(t) = \frac{e^{t}}{3} - \frac{e^{-2t}}{3}$$

$$k'(t) = \frac{e^{t}}{3} + \frac{2e^{-2t}}{3}$$

$$k''(t) = \frac{e^{t}}{3} - \frac{4e^{-2t}}{3}$$

$$k(0) = 0$$

$$k'(0) = 1$$

$$k''(0) = -1$$