Teoria Regulacji Ćwiczenia, Wtorek 17:05-18:45

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Zadanie n6

Założenia: $y(0^-) = 0$ oraz $y'(0^-) = 1$

- $y'' + 3y' + 2y = 0$

$$\mathcal{L}\left\{y'' + 3y' + 2y\right\} = \mathcal{L}\left\{0\right\}$$

$$s^2Y(s) - sy(0) - y'(0) + 3sY(s) - y(0) + 2Y(s) = 0$$

$$Y(s)\left(s^2 + 3s + 2\right) - 1 = 0$$

$$Y(s) = \frac{1}{(s+1)(s+2)}$$

$$Y(s) = \frac{A}{s+1} + \frac{B}{s+2}$$

$$\{\, A = 1B = -1$$

$$Y(s) = \frac{1}{s+1} + \frac{-1}{s+2}$$

$$\mathcal{L}^{-1}\left\{Y(s)\right\} = y(t) = e^{-t} - e^{-2t}$$

- $y'' + 3y' + 2y = 4$

$$\mathcal{L}\left\{y'' + 3y' + 2y\right\} = \mathcal{L}\left\{4\right\}$$

$$s^2Y(s) - sy(0) - y'(0) + 3sY(s) - y(0) + 2Y(s) = \frac{4}{s}$$

$$Y(s)\left(s^2 + 3s + 2\right) - 1 = \frac{4}{s}$$

$$Y(s) = \frac{s+4}{s(s+1)(s+2)}$$

$$Y(s) = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$$

$$\{\, A = 2B = -3C = 1$$

$$Y(s) = \frac{2}{s} + \frac{-3}{s+1} + \frac{1}{s+2}$$

$$\mathcal{L}^{-1}\left\{Y(s)\right\} = y(t) = -3e^{-t} + e^{-2t} + 2$$

- $y'' + 3y' + 2y = t$

$$\mathcal{L}\left\{y'' + 3y' + 2y\right\} = \mathcal{L}\left\{t\right\}$$

$$s^2Y(s) - sy(0) - y'(0) + 3sY(s) - y(0) + 2Y(s) = \frac{1}{s^2}$$

$$Y(s)\left(s^2 + 3s + 2\right) - 1 = \frac{1}{s^2}$$

$$Y(s) = \frac{s^2+1}{s^2(s+1)(s+2)}$$

$$Y(s) = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+1} + \frac{D}{s+2}$$

$$\{\, A = \frac{1}{2}B = \frac{-3}{4}C = 2D = \frac{-5}{4}$$

$$Y(s) = \frac{\frac{1}{2}}{s^2} + \frac{\frac{-3}{4}}{s} + \frac{2}{s+1} + \frac{\frac{-5}{4}}{s+2}$$

$$\mathcal{L}^{-1}\left\{Y(s)\right\} = y(t) = 2e^{-t} - \frac{5}{4}e^{-2t} + \frac{1}{2}t - \frac{3}{4}$$

- $y'' + 3y' + 2y = sin(\omega t)$

$$\mathcal{L}\left\{y'' + 3y' + 2y\right\} = \mathcal{L}\left\{sin(\omega t)\right\}$$

$$Y(s)\left(s^2 + 3s + 2\right) - 1 = \frac{\omega}{s^2 + \omega^2}$$

$$Y(s) = \frac{s^2 + \omega^2 + \omega}{(s^2 + \omega^2)(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{Cs + D}{s^2 + \omega^2}$$

$$\{ A = \frac{\omega^2 + \omega + 1}{\omega^2 + 1} B = \frac{-\omega^2 - \omega = 4}{\omega^2 + 4} C = \frac{-3\omega}{\omega^4 + 5\omega^2 + 4} D = \frac{-\omega^3 + 2\omega}{\omega^4 + 5\omega^2 + 4}$$

$$Y(s) = \frac{\frac{\omega^2+\omega+1}{\omega^2+1}}{s+1} + \frac{\frac{-\omega^2-\omega=4}{\omega^2+4}}{s+2} + \frac{\frac{-3\omega}{\omega^4+5\omega^2+4}s + \frac{-\omega^3+2\omega}{\omega^4+5\omega^2+4}}{s^2 + \omega^2}$$

$$\mathcal{L}^{-1}\left\{Y(s)\right\} = y(t)$$

$$y(t) = \frac{\omega^2 + \omega + 1}{\omega^2 + 1} e^{-t} + \frac{-\omega^2 - \omega - 4}{\omega^2 + 4} e^{-2t} + \frac{1}{\omega^4 + 5\omega^2 + 4} \cdot \left[-3\omega cos(\omega t) + (-\omega^2 + 2) sin(\omega t)\right]$$

- $y'' + 2y' + 2y = 0$

$$\mathcal{L}\left\{y'' + 2y' + 2y\right\} = \mathcal{L}\left\{0\right\}$$

$$Y(s)\left(s^2 + 2s + 2\right) - 1 = 0$$

$$Y(s) = \frac{1}{s^2 + 2s + 2} = \frac{1}{(s+1)^2 + 1}$$

$$\mathcal{L}^{-1}\left\{Y(s)\right\} = y(t) = e^t sin(t)$$

- $y'' + 2y' + 2y = 1$

$$\mathcal{L}\left\{y'' + 2y' + 2y\right\} = \mathcal{L}\left\{1\right\}$$

$$Y(s)\left(s^2 + 2s + 2\right) - 1 = \frac{1}{s}$$

$$Y(s) = \frac{s+1}{s\left(s^2+2s+2\right)} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+2}$$

$$\{\, A = \frac{1}{2} B = \frac{-1}{2} C = 0$$

$$Y(s) = \frac{1}{2s} + \frac{\frac{1}{2}s - 1 + 1}{s^2+2s+2}$$

$$\mathcal{L}^{-1}\left\{Y(s)\right\} = y(t) = \frac{1}{2}\left(1 - e^{-t}\left[cos(t) - sin(t)\right]\right)$$