

Lista 2, zadania 3, 4, 5

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Zadanie 3

a) $K(s) = \frac{1}{s(s+1)}$

Odpowiedź skokowa:

$$\lambda(t) = \mathcal{L}^{-1} \left\{ \frac{1}{s} \cdot K(s) \right\}$$

$$\lambda(t) = \mathcal{L}^{-1} \left\{ \frac{1}{s(s^2 + s)} \right\}$$

$$\lambda(t) = \mathcal{L}^{-1} \left\{ \frac{1}{s+1} - \frac{1}{s} + \frac{1}{s^2} \right\}$$

$$\lambda(t) = t - 1 + e^{-t}$$

$$\lambda'(t) = 1 - e^{-t}$$

$$\lambda''(t) = e^{-t}$$

$$\lambda(0) = 0$$

$$\lambda'(0) = 0$$

$$\lambda''(0) = 1$$

Odpowiedź impulsowa:

$$k(t) = \mathcal{L}^{-1} \{K(s)\}$$

$$k(t) = \mathcal{L}^{-1} \left\{ \frac{1}{s^2 + s} \right\}$$

$$k(t) = \mathcal{L}^{-1} \left\{ -\frac{1}{s+1} + \frac{1}{s} \right\}$$

$$k(t) = 1 - e^{-t}$$

$$k'(t) = e^{-t}$$

$$k''(t) = -e^{-t}$$

$$k(0) = 0$$

$$k'(0) = 1$$

$$k''(0) = -1$$

b) $K(s) = \frac{1}{(s+1)(s^2+1)}$

Odpowiedź skokowa:

$$\lambda(t) = \mathcal{L}^{-1} \left\{ \frac{1}{s} \cdot K(s) \right\}$$

$$\lambda(t) = \mathcal{L}^{-1} \left\{ \frac{1}{s} \cdot \frac{1}{(s+1)(s^2+1)} \right\}$$

$$\lambda(t) = \mathcal{L}^{-1} \left\{ -\frac{s+1}{2(s^2+1)} - \frac{1}{2(s+1)} + \frac{1}{s} \right\}$$

$$\lambda(t) = -\frac{\sin(t)}{2} - \frac{\cos(t)}{2} + 1 - \frac{e^{-t}}{2}$$

$$\lambda'(t) = \frac{\sin(t)}{2} - \frac{\cos(t)}{2} + \frac{e^{-t}}{2}$$

$$\lambda''(t) = \frac{\sin(t)}{2} + \frac{\cos(t)}{2} - \frac{e^{-t}}{2}$$

$$\lambda(0) = 0$$

$$\lambda'(0) = 0$$

$$\lambda''(0) = 0$$

Odpowiedź impulsowa:

$$k(t) = \mathcal{L}^{-1} \{K(s)\}$$

$$k(t) = \mathcal{L}^{-1} \left\{ \frac{1}{(s+1)(s^2+1)} \right\}$$

$$k(t) = \mathcal{L}^{-1} \left\{ -\frac{s-1}{2(s^2+1)} + \frac{1}{2(s+1)} \right\}$$

$$k(t) = \frac{\sin(t)}{2} - \frac{\cos(t)}{2} + \frac{e^{-t}}{2}$$

$$k'(t) = \frac{\sin(t)}{2} + \frac{\cos(t)}{2} - \frac{e^{-t}}{2}$$

$$k''(t) = -\frac{\sin(t)}{2} + \frac{\cos(t)}{2} + \frac{e^{-t}}{2}$$

$$k(0) = 0$$

$$k'(0) = 0$$

$$k''(0) = 1$$

c) $K(s) = \frac{1}{(s-1)(s+2)}$

Odpowiedź skokowa:

$$\lambda(t) = \mathcal{L}^{-1} \left\{ \frac{1}{s} \cdot K(s) \right\}$$

$$\lambda(t) = \mathcal{L}^{-1} \left\{ \frac{1}{s(s-1)(s+2)} \right\}$$

$$\lambda(t) = \mathcal{L}^{-1} \left\{ \frac{1}{6(s+2)} + \frac{1}{3(s-1)} - \frac{1}{2s} \right\}$$

$$\lambda(t) = \frac{e^t}{3} - \frac{1}{2} + \frac{e^{-2t}}{6}$$

$$\lambda'(t) = \frac{e^t}{3} - \frac{e^{-2t}}{3}$$

$$\lambda''(t) = \frac{e^t}{3} + \frac{2e^{-2t}}{3}$$

$$\lambda(0) = 0$$

$$\lambda'(0) = 0$$

$$\lambda''(0) = 1$$

Odpowiedź impulsowa:

$$k(t) = \mathcal{L}^{-1} \{K(s)\}$$

$$k(t) = \mathcal{L}^{-1} \left\{ \frac{1}{(s-1)(s+2)} \right\}$$

$$k(t) = \mathcal{L}^{-1} \left\{ -\frac{1}{3(s+2)} + \frac{1}{3(s-1)} \right\}$$

$$k(t) = \frac{e^t}{3} - \frac{e^{-2t}}{3}$$

$$k'(t) = \frac{e^t}{3} + \frac{2e^{-2t}}{3}$$

$$k''(t) = \frac{e^t}{3} - \frac{4e^{-2t}}{3}$$

$$k(0) = 0$$

$$k'(0) = 1$$

$$k''(0) = -1$$

Zadanie 4

a)

$$K(s) = \frac{s+2}{(s+1)(s+3)} = \frac{L(s)}{M(s)}$$

$$Y(s) = K(s)U(s) = \frac{L(s)}{M(s)}U(s)$$

$$Y(s) [s^2 + 4s + 3] = [s + 2]U(s)$$

$$y'' + 4y' + 3y = u' + 2u$$

b)

$$y'' + 4y' + 3y = 0$$

$$y'' = -4y' - 3y$$

$$\dot{\xi} = A\xi$$

$$\xi = \begin{bmatrix} y \\ y' \end{bmatrix}$$

$$\dot{\xi} = \begin{bmatrix} y' \\ y'' \end{bmatrix}$$

$$\begin{bmatrix} y' \\ y'' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} \begin{bmatrix} y \\ y' \end{bmatrix}$$

Zadanie 5

a) Drugi biegun jest sprzężony, dlatego wynosi on:

$$s_2 = \sigma - j\omega$$

b)