Teoria Regulacji Ćwiczenia, Wtorek 17:05-18:45

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Zadanie 1

- $\mathcal{L}\{sin(\omega t)\}$

$$\mathcal{L}\{sin(\omega t)\} = \frac{1}{2j}\left(\frac{1}{s-j\omega} - \frac{1}{s+j\omega}\right) = \frac{\omega}{s^2+\omega^2}$$

- $\mathcal{L}\{cos(\omega t)\}$

$$\mathcal{L}\{cos(\omega t)\} = \frac{1}{2}\left(\frac{1}{s-j\omega} + \frac{1}{s+j\omega}\right) = \frac{s}{s^2+\omega^2}$$

- $\mathcal{L}\{sin(\omega t + \varphi)\}$

$$\mathcal{L}\{sin(\omega t)\cdot cos(\varphi) + sin(\varphi)\cdot cos(\omega t)\} = \frac{cos(\varphi\cdot\omega)+sin(\varphi\cdot s)}{s^2+\omega^2}$$

- $\mathcal{L}\{tsin(\omega t)\}$

$$\mathcal{L}\{\delta(t-T)\} = -\frac{d}{ds}\left(\frac{\omega}{s^2+\omega^2}\right) = \frac{2\omega s}{(s^2+\omega^2)^2}$$

- $\mathcal{L}\{\delta(t-T)\}$

$$\mathcal{L}\{\delta(t-T)\} = e^{-sT}\mathcal{L}\{\delta(t)\} = e^{-sT}\mathcal{L}\{1'(t)\} = e^{-sT}\cdot s\cdot\frac{1}{s} = e^{-sT}$$

Zadanie 2

a) $\frac{1}{(s+1)(s+2)}$

$$\frac{1}{(s+1)(s+2)} = \frac{a}{s+1} + \frac{b}{s+2} = \frac{(a+b)s+b+2a}{(s+1)(s+2)} = \frac{1}{s+1} + \frac{-1}{s+2} = e^{-t} + e^{2t}$$

$$\begin{cases} a = 1 \\ b = -1 \end{cases}$$

c) $\frac{s+3}{(s+1)(s+2)}$

$$\frac{s+3}{(s+1)(s+2)} = \frac{a}{s+1} + \frac{b}{s+2} = \frac{(a+b)s+(2a+b)}{(s+1)(s+2)} = \frac{2}{s+1} + \frac{-1}{s+2} = 2e^{-t} - e^{-2t}$$

$$\begin{cases} a = 2 \\ b = -1 \end{cases}$$

Zadanie 3

a) $\frac{b-a}{(s-a)(s-b)}$

$$\frac{b-a}{(s-a)(s-b)} = \frac{\alpha}{s-a} + \frac{\beta}{s-b} = \frac{(\alpha+\beta)s+(-a\beta-b\alpha)}{(s-a)(s-b)} = \frac{-1}{s-a} + \frac{1}{s-b} = -e^{at} + e^{bt}$$

$$\begin{cases} \alpha = -1 \\ \beta = 1 \end{cases}$$