NCERT 9.4.3

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Question: Find the solution for the differential equation $\frac{dy}{dx} + y = 1$ using trapezoidal rule.
Solution:
Let

$$f(x, y) = 1 - y \tag{1}$$

$$y(0) = 0 \tag{2}$$

From Forward Euler method:

$$\frac{y_{n+1} - y_n}{h} = f(x_n, y_n) \tag{3}$$

From Backward Euler method:

$$\frac{y_{n+1} - y_n}{h} = f(x_{n+1}, y_{n+1}) \tag{4}$$

On adding both equation (2) and (3), We get the Trapezoidal Method

$$\frac{y_{n+1} - y_n}{h} = \frac{1}{2}\left[f(x_n, y_n) + f(x_{n+1}, y_{n+1})\right] \tag{5}$$

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y_{n+1})\right] \tag{6}$$

$$y_{n+1} = y_n + \frac{h}{2}\left[1 - y_n + 1 - y_{n+1}\right] = y_n + \frac{h}{2}\left[2 - y_n - y_{n+1}\right] \tag{7}$$

$$\tag{8}$$

On rearranging, we get the difference equation

$$y_{n+1} = \frac{2 - h}{2 + h}y_n + \frac{2h}{2 + h} \tag{9}$$

$$x_{n+1} = x_n + h \tag{10}$$