NCERT 6.6.9

EE24BTECH11032- John Bobby

Question:A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m^3. If building of tank costs 70 per sq. meter for the base and 45 per square meter for sides. What is the cost of least expensive tank?

Solution: Let the length be x and the breadth be y

I. Theoretical Approach

$$\text{Volume} = (2)(x)(y) = 8 \tag{1}$$
$$xy = 4 \tag{2}$$
$$\text{Total Cost} = 70(xy) + 45 \times 2(2x + 2y) = 280 + 180(x + y) \tag{3}$$

From equation (2)

$$\text{Total Cost} = 280 + 180x + \frac{720}{x} \tag{4}$$

Differentiating wrt to x on both sides,

$$\frac{dy}{dx} = 180 - \frac{720}{x^2} \tag{5}$$

To be a critical point $\frac{dy}{dx}$ must be zero,

$$180 - \frac{720}{x^2} = 0 \tag{6}$$
$$x^2 = \frac{720}{180} \tag{7}$$
$$x = \pm 2 \tag{8}$$
$$\tag{9}$$

$x = 2$ as length cant be negative
Checking $\frac{d^2y}{dx^2}$ to be positive for minimum

$$\frac{d^2y}{dx^2} = \frac{1440}{x^3} \tag{10}$$
$$\frac{d^2y}{dx^2} > 0 \text{ for } x = 2 \tag{11}$$

Thus $x = 2$ is the minimum

II. Gradient Descent

$$x_{n+1} = x_n - \alpha f'(x_n) \tag{12}$$

$$x_{n+1} = x_n - \alpha\left(180 - \frac{720}{{x_n}^2}\right) \tag{13}$$

Where α is the learning rate,
This iteration will stop until we reach a stable state ($f'(x) \approx 0$)
We get,
$x_{min} = 2$
We can also solve using geometric programming by using the cvpxy module in python. On running the code we get the minimum value of x to be 2.
For using quadratic programming we can assume a new variable y as $\frac{1}{x}$, but the constraint will be non-linear but it can be solved as cvxpy supports this structure.