NCERT 9.4.3

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Question: Find the solution for the differential equation $\frac{dy}{dx} + y = 1$

I. Mathematical Approach

$$\frac{dy}{dx} = 1 - y$$

On rearranging the terms,

$$\frac{dy}{1-y} = dx$$
$$\int \frac{dy}{1-y} = \int dx$$
$$-\log|y-1| + c_1 = x + c_2$$

On simplification

$$-\log|y-1| = x + c$$
$$|y-1| = \pm e^{-x}$$
$$y = 1 \pm e^{-x}$$

For the numerical approach we are assuming that the function passes through origin
Thus is the function y is,

$$y = 1 - e^{-x}$$

II. Numerical Approach

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{y(x+h) - y(x)}{h}$$

If h is sufficiently small,

$$y(x+h) - y(x) = h\frac{dy}{dx}$$
$$y(x+h) = y(x) + h\frac{dy}{dx}$$

Thus the value of $y(x+h)$ can be predicted if we know the value of the derivative at that point
For this question

1) The interval $[0, 100]$ is divided into 1000 equal parts each of width 0.1units
2) On starting from a known point (x, y) the value for $y(x + 0.1)$ is calculated. This procedure is repeated until the value of x reaches 100.
3) Knowing all the y values for the equally spaced x values in interval, the solution plot can be plotted.

The plot is given below