NCERT 8 EX-13

EE24BTECH11032 - JOHN BOBBY

Question

Question: Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by $x = 0$,$x = 4$,$y = 0$,$y = 4$ into 3 equal parts.

Theoretical Method

The variables used in $y^2 = 4x$ are given below

Variable	**Description**	**values**
$\mathbf{V}$	Quadratic form of the matrix	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
$\mathbf{u}$	Linear coefficient vector	$\begin{pmatrix} -2 \\ 0 \end{pmatrix}$
f	constant term	0

The variables used in $x^2 = 4y$ are given below

Variable	**Description**	**values**
$\mathbf{V}$	Quadratic form of the matrix	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
$\mathbf{u}$	Linear coefficient vector	$\begin{pmatrix} 0 \\ -2 \end{pmatrix}$
f	constant term	0

The point of intersection of the line with the parabolas is

$$x_i = h + k_i m \tag{1}$$

where, k_i is a constant and is calculated as follows:

$$k_i = \frac{1}{m^\top Vm} \left(-m^\top (Vh + u) \pm \sqrt{\left[m^\top (Vh + u)\right]^2 - g(h)(m^\top Vm)} \right). \tag{2}$$

For line $x = 0$ and $y = 0$ we get the intersection points with conic as $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

For line $x = 4$ and $y = 4$ we get the intersection points with conic as $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$

Area between $y^2 = 4x$ and $x^2 = 4y$ $= \int_0^4 2\sqrt{x}dx - \int_0^4 \frac{x^2}{4}dx = \frac{32}{3} - \frac{16}{3} = \frac{16}{3}$
Area between $x^2 = 4y$ and $x = 0$ and $x = 4$$= \int_0^4 2\sqrt{x}dx = \frac{16}{3}$
Area between $y^2 = 4x$ and $y = 0$ and $y = 4$ $= \int_0^4 \frac{y^2}{4}dy = \frac{16}{3}$
We can see that the 3 areas are equal.

Trapezoidal Method

Using trapezoidal rule

$$\int_a^b f(x)\,dx \approx (b-a)\left(\frac{f(a)+f(b)}{2}\right)$$

Applying trapezoid rule for all values of x between 0 and 4 where h is the step size

$$A = \frac{1}{2}h\left(y\left(x_1\right)+y\left(x_0\right)\right)+\frac{1}{2}h\left(y\left(x_2\right)+y\left(x_1\right)\right)+\cdots+\frac{1}{2}h\left(y\left(x_n\right)+y\left(x_{n-1}\right)\right) \tag{3}$$

Let $A(x_n)$ be the area enclosed by the curve $y(x)$ from $x = x_0$ to $x = x_n$, $(x_0, x_1, \ldots x_n)$ be equidistant points with step-size h.

$$A\left(x_n+h\right) = A\left(x_n\right)+\frac{1}{2}h\left(y\left(x_n+h\right)+y\left(x_n\right)\right) \tag{4}$$

We can repeat this till we get required area.
Discretizing the steps, making $A(x_n) = A_n, y(x_n) = y_n$ we get,

$$A_{n+1} = A_n + \frac{1}{2}h\left(y_{n+1} + y_n\right) \tag{5}$$

We can write y_{n+1} in terms of y_n using first principle of derivative.
$y_{n+1} = y_n + hy'_n$

$$A_{n+1} = A_n + \frac{1}{2}h\left(\left(y_n + hy'_n\right) + y_n\right) \tag{6}$$

$$A_{n+1} = A_n + \frac{1}{2}h\left(2y_n + hy'_n\right) \tag{7}$$

$$A_{n+1} = A_n + hy_n + \frac{1}{2}h^2y'_n \tag{8}$$

$$x_{n+1} = x_n + h \tag{9}$$

For $y^2 = 4x$, $y_n = 2\sqrt{x_n}$ and $y'_n = \frac{1}{\sqrt{x_n}}$
For $x^2 = 4y$, $y_n = \frac{x^2}{4}$ and $y'_n = \frac{x}{2}$
The general difference equation will be given by

$$A_{n+1} = A_n + hy_n + \frac{1}{2}h^2y'_n \tag{10}$$

C-Code

```c
#include <math.h>
void trapezoidal_1(double *x, double *y, int n, double h) {
    y[0] = 0;
    for (int i = 0; i < n - 1; i++) {
        y[i+1]=y[i]+h*x[i]*x[i]/4+(h*h*x[i])/4;
    }
}
void trapezoidal_2(double *x, double *y, int n, double h) {
    y[1] = 0;
    for (int i = 1; i < n - 1; i++) {
        y[i+1]=y[i]+h*2*sqrt(x[i])+(h*h)/(2*sqrt(x[i]));
    }
}
void function_1(double *x,double *y,int n){
        for(int i=0;i<n;i++){
                y[i]=x[i]*x[i]/4;
        }
}
void function_2(double *x,double *y,int n){
        for(int i=0;i<n;i++){
                y[i]=2*sqrt(x[i]);
        }
}
```

Python-Code

```
import numpy as np
import matplotlib.pyplot as plt
import ctypes
# Load the shared library
lib = ctypes.CDLL('./lib.so')
# Set argument and return types for the C functions
lib.trapezoidal_1.argtypes = [ctypes.POINTER(ctypes.c_double),
↪  ctypes.POINTER(ctypes.c_double), ctypes.c_int, ctypes.c_double]
lib.trapezoidal_2.argtypes = [ctypes.POINTER(ctypes.c_double),
↪  ctypes.POINTER(ctypes.c_double), ctypes.c_int, ctypes.c_double]
lib.function_1.argtypes=[ctypes.POINTER(ctypes.c_double),
↪  ctypes.POINTER(ctypes.c_double), ctypes.c_int]
lib.function_2.argtypes=[ctypes.POINTER(ctypes.c_double),
↪  ctypes.POINTER(ctypes.c_double), ctypes.c_int]
# Parameters
x_start = 0
x_end = 4
h = 0.1
n_steps = 41
```

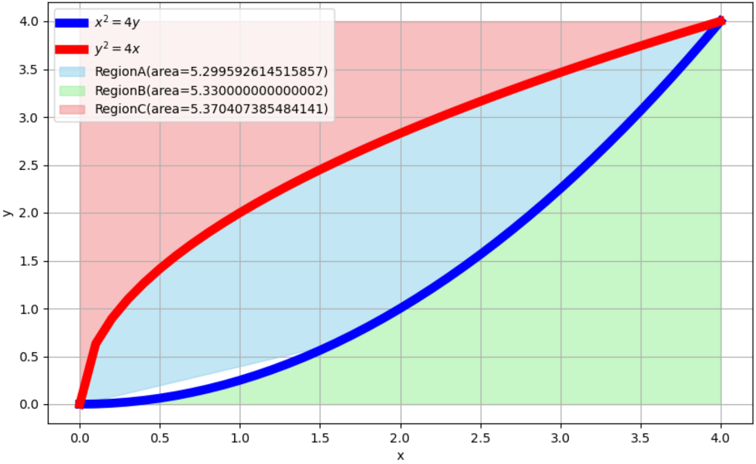
Python-Code

```python
#Setting up the arrays
area_x = np.linspace(x_start, x_end, n_steps)
area_y_1 = np.zeros(n_steps)
area_y_2 = np.zeros(n_steps)
y_1=np.zeros(n_steps)
y_2=np.zeros(n_steps)
#Conversion to ctypes array
area_x_ctypes = area_x.ctypes.data_as(ctypes.POINTER(ctypes.c_double))
area_y_1_ctypes = area_y_1.ctypes.data_as(ctypes.POINTER(ctypes.c_double))
area_y_2_ctypes = area_y_2.ctypes.data_as(ctypes.POINTER(ctypes.c_double))
y_1_ctypes = y_1.ctypes.data_as(ctypes.POINTER(ctypes.c_double))
y_2_ctypes = y_2.ctypes.data_as(ctypes.POINTER(ctypes.c_double))
# Call the C functions
lib.function_1(area_x_ctypes, y_1_ctypes, n_steps)
lib.trapezoidal_1(area_x_ctypes,area_y_1_ctypes, n_steps,h)
lib.function_2(area_x_ctypes, y_2_ctypes, n_steps)
lib.trapezoidal_2(area_x_ctypes,area_y_2_ctypes, n_steps,h)
#Area variables
Area1=area_y_1[40]
Area2=area_y_2[40]
```

Python-Code

```
# Plotting
plt.figure(figsize=(10, 6))
plt.plot(area_x, y_1, label="Function1", linestyle='-', color='b',linewidth=7)
plt.plot(area_x, y_2, label="Function2", linestyle='-', color='r',linewidth=7)
plt.fill_betweenx(y_2, area_x, np.sqrt(4 * y_2), where=(y_2 >= 0),
↪  color='skyblue', alpha=0.5)
plt.fill_between(area_x, y_1, 0, where=(area_x >= 0), color='lightgreen',
↪  alpha=0.5)
plt.fill_betweenx(y_2, 0, area_x, where=(y_2 >= 0), color='lightcoral',
↪  alpha=0.5)
plt.xlabel("x")
plt.ylabel("y")
#plt.legend()
plt.legend(['$x^2=4y$','$y^2=4x$',f'RegionA(area={Area2-Area1})',f'RegionB(area={A
plt.grid()
#plt.show()
plt.savefig('plot.png')
```

Plot



$x^2 = 4y$
$y^2 = 4x$
RegionA(area=5.299592614515857)
RegionB(area=5.330000000000002)
RegionC(area=5.370407385484141)
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
y
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
x