

NCERT 9.4.3

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Question: Find the solution for the differential equation $\frac{dy}{dx} + y = 1$

I. MATHEMATICAL APPROACH

$$\frac{dy}{dx} = 1 - y$$

On rearranging the terms,

$$\begin{aligned}\frac{dy}{1-y} &= dx \\ \int \frac{dy}{1-y} &= \int dx \\ -\log|y-1| + c_1 &= x + c_2\end{aligned}$$

On simplification

$$\begin{aligned}-\log|y-1| &= x + c \\ |y-1| &= \pm e^{-x} \\ y &= 1 \pm e^{-x}\end{aligned}$$

For the numerical approach we are assuming that the function passes through origin
Thus is the function y is,

$$y = 1 - e^{-x}$$

II. NUMERICAL APPROACH

$$\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{y(x+h) - y(x)}{h}$$

If h is sufficiently small,

$$\begin{aligned}y(x+h) - y(x) &= h \frac{dy}{dx} \\ y(x+h) &= y(x) + h \frac{dy}{dx}\end{aligned}$$

Thus the value of $y(x+h)$ can be predicted if we know the value of the derivative at that point
For this question

- 1) The interval $[0, 5]$ is divided into 51 equal parts each of width 0.1units
- 2) On starting from a known point (x, y) the value for $y(x+0.1)$ is calculated. This procedure is repeated until the value of x reaches 5.
- 3) Knowing all the y values for the equally spaced x values in interval, the solution plot can be plotted.

The plot is given below

