NCERT 8 Example 13

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Question: Find the area between $f(x) = \cos x$ and lines $x = 0$ and $x = 2\pi$
Solution:

I. Trapezoidal Method

Using trapezoidal rule

$$\int_a^b f(x)\,dx \approx (b-a)\left(\frac{f(a)+f(b)}{2}\right)$$

Applying trapezoid rule for all values of x between 0 and 2π
where h is the step size

$$A = \frac{1}{2}h\left(y(x_1)+y(x_0)\right) + \frac{1}{2}h\left(y(x_2)+y(x_1)\right) + \cdots + \frac{1}{2}h\left(y(x_n)+y(x_{n-1})\right)$$

Let $A(x_n)$ be the area enclosed by the curve $y(x)$ from $x = x_0$ to $x = x_n$, $(x_0, x_1, \dots x_n)$ be equidistant points with step-size h.

$$A(x_n+h) = A(x_n) + \frac{1}{2}h\left(y(x_n+h)+y(x_n)\right) \tag{1}$$

We can repeat this till we get required area.
Discretizing the steps, making $A(x_n) = A_n, y(x_n) = y_n$ we get,

$$A_{n+1} = A_n + \frac{1}{2}h\left(y_{n+1}+y_n\right) \tag{2}$$

We can write y_{n+1} in terms of y_n using first principle of derivative. $y_{n+1} = y_n + hy'_n$

$$A_{n+1} = A_n + \frac{1}{2}h\left((y_n+hy'_n)+y_n\right) \tag{3}$$

$$A_{n+1} = A_n + \frac{1}{2}h\left(2y_n+hy'_n\right) \tag{4}$$

$$A_{n+1} = A_n + hy_n + \frac{1}{2}h^2y'_n \tag{5}$$

$$x_{n+1} = x_n + h \tag{6}$$

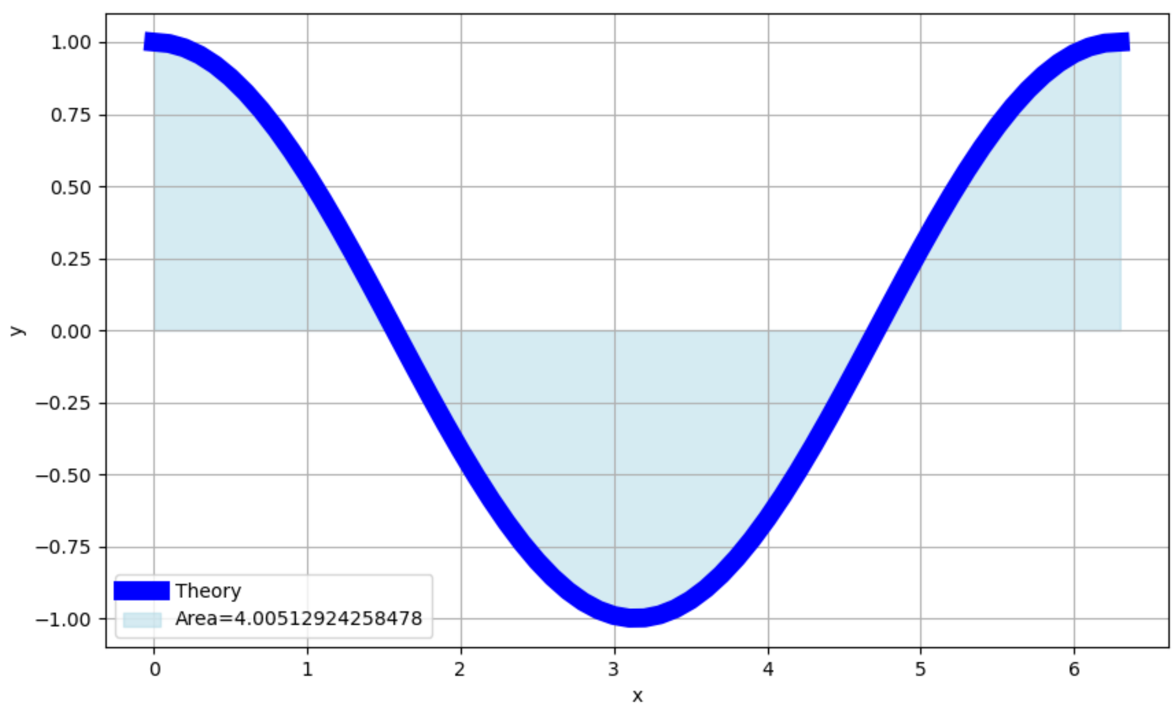
In the given question, $y_n = \cos x_n$ and $y'_n = -\sin x_n$
The general difference equation will be given by

$$A_{n+1} = A_n + hy_n + \frac{1}{2}h^2y'_n \tag{7}$$

$$A_{n+1} = A_n + h\left(\cos x_n\right) + \frac{1}{2}h^2\left(-\sin x_n\right) \tag{8}$$

$$x_{n+1} = x_n + h \tag{9}$$

Iterating till we reach $x_n = \frac{\pi}{2}$ will return $\frac{1}{4}$th of the required area.
Area obtained computationally: 4.0051 sq. units
Area obtained theoreticall: 4 sq.unis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