

# ASSIGNMENT-1

EE24BTECH11032- JOHN BOBBY

- 1) Let  $A$  and  $B$  be two distinct points on the parabola  $y^2 = 4x$ . If the axis of the parabola touches a circle of radius  $r$  having  $AB$  as diameter, then the slope of the line joining  $A$  and  $B$  can be (2010)
  - a)  $\frac{-1}{r}$
  - b)  $\frac{1}{r}$
  - c)  $\frac{5}{r}$
  - d)  $\frac{2}{r}$
- 2) Let the eccentricity of the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  be reciprocal to that of the ellipse  $x^2 + 4y^2 = 4$ . If the hyperbola passes through a focus of the ellipse, then (2011)
  - a) the equation of the hyperbola is  $\frac{x^2}{3} - \frac{y^2}{2} = 1$
  - b) a focus of the hyperbola is  $(2, 0)$
  - c) the eccentricity of the hyperbola is  $\sqrt{\frac{5}{3}}$
  - d) the equation of the hyperbola is  $x^2 - 3y^2 = 3$
- 3) Let  $L$  be a normal to the parabola  $y^2 = 4x$ . If  $L$  passes through the point  $(9, 6)$ , then the  $L$  is given by (2010)
  - a)  $y - x + 3 = 0$
  - b)  $y + 3x - 33 = 0$
  - c)  $y + x - 15 = 0$
  - d)  $y - 2x + 12 = 0$
- 4) Tangents are drawn to the hyperbola  $\frac{x^2}{9} - \frac{y^2}{4} = 1$ , parallel to the straight line  $2x - y = 1$ . The points of contact of the tangents to the hyperbola are (2012)
  - a)  $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
  - b)  $\left(\frac{-9}{2\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$
  - c)  $(3\sqrt{3}, -2\sqrt{2})$
  - d)  $(-3\sqrt{3}, 2\sqrt{2})$
- 5) Let  $P$  and  $Q$  be distinct points on the parabola  $y^2 = 2x$  such that a circle with  $PQ$  as diameter passes through the vertex  $O$  of the parabola. If  $P$  lies in the first quadrant and the area of the triangle  $\Delta OPQ$  is  $3\sqrt{2}$ , then which of the following is (are) the coordinates of  $P$ ? (JEE ADV.2015)
  - a)  $(4, 2\sqrt{2})$
  - b)  $(9, 3\sqrt{2})$
  - c)  $\left(\frac{1}{4}, \frac{1}{\sqrt{2}}\right)$
  - d)  $(1, \sqrt{2})$
- 6) Let  $E_1$  and  $E_2$  be two ellipses whose centres are at the origin. The major axes of  $E_1$  and  $E_2$  lie along the  $x$ -axis and the  $y$ -axis, respectively. Let  $S$  be the circle  $x^2 + (y - 1)^2 = 2$ . The straight line  $x + y = 3$  touches the curves  $S$ ,  $E_1$  and  $E_2$  at  $P$ ,  $Q$  and  $R$  respectively. Suppose that  $PQ = PR = \frac{2\sqrt{2}}{3}$ . If  $e_1$  and  $e_2$  are the eccentricities of  $E_1$  and  $E_2$ , respectively, then the correct expression(s) is (are) (JEE ADV.2015)
  - a)  $e_1^2 + e_2^2 = \frac{43}{40}$
  - b)  $e_1 e_2 = \frac{\sqrt{7}}{2\sqrt{10}}$

- c)  $|e_1^2 - e_2^2| = \frac{5}{8}$   
d)  $e_1 e_2 = \frac{\sqrt{3}}{4}$
- 7) Consider the hyperbola  $H: x^2 - y^2 = 1$  and a circle  $S$  with centre  $(Nx_2, 0)$ . Suppose that  $H$  and  $S$  touch each other at a point  $P(x_1, y_1)$  with  $x_1 > 0$  and  $y_1 > 0$ . The common tangent to  $H$  and  $S$  at  $P$  intersects the  $x$ -axis at point  $M$ . If  $(l, m)$  is the centroid of the triangle  $PMN$ , then correct expression(s) is(are) (JEE ADV.2015)
- a)  $\frac{dl}{dx_1} = 1 - \frac{1}{3x_1^2}$  for  $x_1 > 1$   
b)  $\frac{dm}{dx_1} = \frac{x_1}{3\sqrt{x_1^2 - 1}}$  for  $x_1 > 1$   
c)  $\frac{dl}{dx_1} = 1 + \frac{1}{3x_1^2}$  for  $x_1 > 1$   
d)  $\frac{dm}{dy_1} = \frac{1}{3}$  for  $y_1 > 0$
- 8) The circle  $C_1: x^2 + y^2 = 3$ , with centre at  $O$ , intersects the parabola  $x^2 = 2y$  at the point  $P$  in the first quadrant. Let the tangent to the circle  $C_1$ , at  $P$  touches other two circles  $C_2$  and  $C_3$  at  $R_2$  and  $R_3$ , respectively. Suppose  $C_2$  and  $C_3$  have equal radii  $2\sqrt{3}$  and the centres  $Q_2$  and  $Q_3$ , respectively. If  $Q_2$  and  $Q_3$  lie on the  $y$ -axis, then (JEE ADV.2016)
- a)  $Q_2 Q_3 = 12$   
b)  $R_2 R_3 = 4\sqrt{6}$   
c) area of the triangle  $OR_2 R_3$  is  $6\sqrt{2}$   
d) area of the triangle  $PQ_2 Q_3$  is  $4\sqrt{2}$
- 9) Let  $P$  be the point on the parabola  $y^2 = 4x$  which is at the shortest distance from the center  $S$  of the circle  $x^2 + y^2 - 4x - 16y + 64 = 0$ . Let  $Q$  be the point on the circle dividing the line segment  $SP$  internally. Then (JEE ADV.2016)
- a)  $SP = 2\sqrt{5}$   
b)  $SQ : QP = (\sqrt{5} + 1) : 2$   
c) the  $x$ -intercept of the normal to the parabola at  $P$  is 6.  
d) the slope of the tangent to the circle at  $Q$  is  $\frac{1}{2}$
- 10) If  $2x - y + 1 = 0$  is a tangent to the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$  then which of the following cannot be sides of a right angled triangle? (JEE ADV.2017)
- a)  $a, 4, 1$   
b)  $a, 4, 2$   
c)  $2a, 8, 1$   
d)  $2a, 4, 1$
- 11) If a chord, which is not tangent, of the parabola  $y^2 = 16x$  has equation  $2x + y = p$ , and midpoint  $(h, k)$ , then which of the following is(are) possible value(s) of  $p, h$  and  $k$ ? (JEE ADV.2017)
- a)  $p = -2, h = 2, k = -4$   
b)  $p = -1, h = 1, k = 3$   
c)  $p = 2, h = 3, k = -4$   
d)  $p = 5, h = 4, k = -3$
- 12) Consider two straight lines, each of which is tangent to both the circle  $x^2 + y^2 = \frac{1}{2}$  and the parabola  $y^2 = 4x$ . Let these lines intersect at the point  $Q$ . Consider the ellipse whose center is at origin  $O(0, 0)$  and whose semi-major axis is  $OQ$ . If the length of the minor axis of the ellipse is  $\sqrt{2}$ , then which of the following statement(s) is(are) TRUE? (JEE ADV.2018)
- a) For the ellipse, the eccentricity is  $\frac{1}{\sqrt{2}}$  and the length of the latus rectum is 1  
b) For the ellipse, the eccentricity is  $\frac{1}{2}$  and the length of the latus rectum is  $\frac{1}{2}$   
c) The area of the region bounded by the ellipse between the lines  $x = \frac{1}{\sqrt{2}}$  and  $x = 1$  is  $\frac{1}{4\sqrt{2}}(\pi - 2)$   
d) The area of the region bounded by the ellipse between the line  $x = \frac{1}{\sqrt{2}}$  and  $x = 1$  is  $\frac{1}{16}(\pi - 2)$

## I. SECTION -E SUBJECTIVE PROBLEMS

- 1) Suppose that the normals drawn at three different points on the parabola  $y^2 = 4x$  pass through the point  $(h, k)$ . Show that  $h > 2$ . (1981-4 Marks)
- 2)  $A$  is a point on the parabola  $y^2 = 4ax$ . The normal at  $A$  cuts the parabola again at point  $B$ . If  $AB$  subtends a right angle at the vertex of the parabola. find the slope of  $AB$ . (1982-4 Marks)
- 3) Three normals are drawn from the point  $(c, 0)$  to the curve  $y^2 = x$ . Show that  $c$  must be greater than  $\frac{1}{2}$ . One normal is always the x-axis. Find  $c$  for which the other two normals are perpendicular to each other. (1991-4 Marks)
- 4) Through the vertex  $O$  of parabola  $y^2 = 4x$ , chords  $OP$  and  $OQ$  are drawn at right angles to one another. Show that for all positions of  $P$ ,  $PQ$  cuts the axis of the parabola at a fixed point. Also find the locus of the middle point of  $PQ$ . (1994-4 Marks)
- 5) Show that the locus of a point that divides a chord of slope 2 of the parabola  $y^2 = 4x$  internally in the ratio 1 : 2 is a parabola. Find the vertex of this parabola. (1995-5 Marks)