9.9.2.30

EE24BTECH11032 - John Bobby

Question: Calculate the area under the curve $y = 2\sqrt{x}$ included with the lines $x = 1$ and $x = 0$.

Variable	Value
$\mathbf{m_1}$	$\begin{pmatrix}0\\1\end{pmatrix}$
$\mathbf{m_2}$	$\begin{pmatrix}0\\1\end{pmatrix}$
$\mathbf{h_1}$	$\begin{pmatrix}0\\0\end{pmatrix}$
$\mathbf{h_2}$	$\begin{pmatrix}1\\0\end{pmatrix}$
$\mathbf{V}$	$\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$
$\mathbf{u}$	$\begin{pmatrix}-2\\0\end{pmatrix}$
f	0

TABLE 0: Input Parameters

Solution: For a line $\mathbf{x} = \mathbf{h} + k\mathbf{m}$,the intersection of the line with a conic with parameters $\mathbf{V}, \mathbf{u}, \mathbf{h}, \mathbf{m}$ and h is given by $\mathbf{x} = \mathbf{h} + k_i\mathbf{m}$
$L_1 : x = 0$
$L_2 : x = 1$

$$k_i = \frac{1}{\mathbf{m}^\top\mathbf{V}\mathbf{m}}\left(-\mathbf{m}^\top(\mathbf{V}\mathbf{h}+\mathbf{u}) + \sqrt{\left[\mathbf{m}^\top(\mathbf{V}\mathbf{h}+\mathbf{u})\right]^2 - g(\mathbf{h})(\mathbf{m}^\top\mathbf{V}\mathbf{m})}\right) \tag{0.1}$$

on solving for L_1 and L_2

$$k_1 = 0, k_2 = 2 \tag{0.2}$$

$$\mathbf{A} = \mathbf{h_1} + k_1\mathbf{m_1} = \begin{pmatrix}0\\0\end{pmatrix} + 0\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix} \tag{0.3}$$

$$\mathbf{B} = \mathbf{h_2} + k_2\mathbf{m_2} = \begin{pmatrix}1\\0\end{pmatrix} + 2\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\2\end{pmatrix} \tag{0.4}$$

Thus the area under the curve included with the lines $x = 1$ and $x = 0$ is given by

$$\int_0^1 2\sqrt{x}\,dx = \left(\frac{4}{3}x^{\frac{3}{2}}\right)_0^1 = \frac{4}{3} \tag{0.5}$$

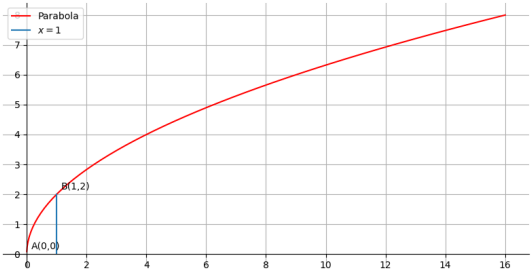

Fig. 0.1: Plot of Parabola