GATE 2007 MA

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1) Let $f(z) = 2z^2 - 1$. Then the maximum value of $|f(z)|$ on the unit disc D=$\{z \in \mathbf{C} : |z| < 1\}$ equals
   a) 1
   b) 2
   c) 3
   d) 4
2) Let $f(z) = \frac{1}{z^2-3z+2}$ Then the coefficient of $\frac{1}{z^3}$ in the Laurent series expansion of $f(x)$ for $|z| > 1$ is
   a) 0
   b) 1
   c) 3
   d) 5
3) Let f:$\mathbf{C} \to \mathbf{C}$ be an arbitrary analytic function satisfying f(0) = 0 and $f(1) = 2$. Then
   a) there exists a sequence $\{z_n\}$ such that $|z_n| > n$ and $|f(z_n)| > n$
   b) there exists a sequence $\{z_n\}$ such that $|z_n| > n$ and $|f(z_n)| < n$
   c) there exists a sequence $\{z_n\}$ such that $|f(z_n)| > n$
   d) there exists a sequence $\{z_n\}$ such that $z_n \to 0$ and $f(z_n) \to 2$
4) Define f:$\mathbf{C} \to \mathbf{C}$ by
   $f(z) = \begin{cases} 0, & if \quad Re(z) = 0 \quad or \quad Im(z) = 0, \\ z, & otherwise \end{cases}$
   Then the set of points where f is analytic is
   a) $\{z : Re(z) \neq 0 \quad and \quad Im(z) \neq 0\}$
   b) $\{z : Re(z) \neq 0\}$
   c) $\{z : Re(z) \neq 0 \quad or \quad Im(z) \neq 0\}$
   d) $\{z : Im(z) \neq 0\}$
5) Let U(n) be the set of all positive integers less than n and relatively prime to n. Then U(n) is the group under multiplication modulo n. For $n = 248$, the number of elements in U(n) is
   a) 60
   b) 120
   c) 180
   d) 240
6) Let R$[x]$ be the polynomial ring in x with real coefficients and let $I = \langle x^2 + 1 \rangle$ be the ideal generated by the polynomial $x^2 + 1$ in R$[x]$. Then
   a) I is a maximum ideal
   b) I is a prime ideal but NOT a maximum ideal
   c) I is NOT a prime ideal
   d) R$[x]/I$ has zero divisors
7) Consider $\mathbf{Z}_5$ and $\mathbf{Z}_{20}$ as rings modulo 5 and 20, respectively. Then the number of homomorphisms $\varphi : \mathbf{Z_5} \to \mathbf{Z_{20}}$ is
   a) 1
   b) 2
   c) 4
   d) 5

8) Let $\mathbf{Q}$ be the field of rational numbers and consider $\mathbf{Z_2}$ as a field modulo 2. Let $f(x) = x^3-9x^2+9x+3$. Then $f(x)$ is
   a) irreducible over $\mathbf{Q}$ but reducible over $\mathbf{Z_2}$
   b) irreducible over both $\mathbf{Q}$ and $\mathbf{Z_2}$
   c) reducible over $\mathbf{Q}$ but irreducible over $\mathbf{Z_2}$
   d) reducible over both $\mathbf{Q}$ and $\mathbf{Z_2}$
9) Consider $\mathbf{Z_5}$ as a field modulo 5 and let $f(x) = x^5 + 4x^4 + 4x^3 + 4x^2 + x + 1$. Then the zeroes of $f(x)$ over $\mathbf{Z_5}$ are 1 and 3 with respective multiplicity
   a) 1 and 4
   b) 2 and 3
   c) 2 and 2
   d) 1 and 2
10) Consider the Hilbert space $l^2 = \left\{\mathbf{x} = \{x_n\} : x_n \in \mathbf{R}, \sum_{n=1}^{\infty} {x_n}^2 < \infty\right\}$. Let $E = \left\{\{x_n\} : |x_n| \leq \frac{1}{n} \quad \text{for all n}\right\}$ be a subset of l^2. Then
   a) $E^o = \left\{\mathbf{x} : |x_n| < \frac{1}{n} \quad \text{for all n}\right\}$
   b) $E^o = E$
   c) $E^o = \left\{\mathbf{x} : |x_n| < \frac{1}{n} \quad \text{for all n but finitely many n}\right\}$
   d) $E^o = \phi$
11) Let X and Y be normed linear spaces and let $T : X \to Y$ be a linear map. Then T is contiuous if
   a) Y is finite dimensional
   b) X is finite dimensional
   c) T is one to one
   d) T is onto
12) Let X be a normed linear space and let $E_1, E_2 \subseteq X$. Define $E_1 + E_2 = \{x + y : x \in E_1, y \in E_2\}$. Then $E_1 + E_2$ is
   a) open if E_1 or E_2 is open
   b) NOT open unless both E_1 and E_2 are open
   c) closed if E_1 or E_2 is closed
   d) closed if both E_1 and E_2 are closed
13) For each $a \in \mathbf{R}$, consider the linear programming problem Max. $z = x_1 + 2x_2 + 3x_3 + 4x_4$ subject to
   $ax_1 + 2x_3 \leq 1$
   $x_1 + ax_2 + 3x_4 \leq 2$
   $x_1, x_2, x_3, x_4 \geq 0.$
   Let $S = \{a \in \mathbf{R} : \text{ the given LP problem has a basic feasible solution}\}$.
   a) $S = \phi$
   b) $S = \mathbf{R}$
   c) $S = (0, \infty)$
   d) $S = (-\infty, \mathbf{0})$
14) Consider the linear programming problem Max. $z = x_1 + 5x_2 + 3x_3$ subject to
   $2x_1 - 3x_2 + 5x_3 \leq 3$
   $3x_1 + 2x_3 \leq 5$
   $x_1, x_2, x_3 \geq 0.$
   Then the dual of this LP problem
   a) has a feasible solution but does NOT have a basic feasible solution
   b) has a basic feasible solution
   c) has infinite number of feasible solutions
   d) has no feasible solution

15) Consider a transportation problem with two warehouses and two markets. The warehouse capacities are $a_1 = 2$ and $a_2 = 4$ and the market demands $b_1 = 3$ and $b_2 = 3$. Let x_n be the quantity shipped from warehouse i to market j and c_{ij} be the corresponding unit cost. Suppose that $c_{11} = 1, c_{21} = 1, c_{22} = 2$. $(x_{11}, x_{12}, x_{21}, x_{22}) = (2, 0, 1, 3)$ is optimal for every
   a) $c_{12} = [1, 2]$
   b) $c_{12} = [0, 3]$
   c) $c_{12} = [1, 3]$
   d) $c_{12} = [2, 4]$

16) The smallest degree of the polynomial that interpolates the data

x	-2	-1	0	1	2	3
$f(x)$	-58	-21	-12	-13	-6	27

   is
   a) 3
   b) 4
   c) 5
   d) 6

17) Suppose that x_0 is sufficiently close to 3. Which of the following iterations $x_{n+1} = g(x_n)$ will converge to the fixed point $x = 3$?
   a) $x_{n+1} = -16 + 6x_n + \frac{3}{x_n}$
   b) $x_{n+1} = \sqrt{3 + 2x_n}$
   c) $x_{n+1} = \frac{3}{x_n - 2}$
   d) $x_{n+1} = \frac{x_n^2 - 3}{2}$