9.9.2.30

EE24BTECH11032 - John Bobby

Question: Calculate the area under the curve $y = 2\sqrt{x}$ included with the lines $x = 1$ and $x = 0$.

Solution: For a line $\mathbf{x} = \mathbf{h} + k\mathbf{m}$,the intersection of the line with a conic with parameters $\mathbf{V}, \mathbf{u}, \mathbf{h}, \mathbf{m}$ and h is given by $\mathbf{x} = \mathbf{h} + k_i\mathbf{m}$
$L_1 : x = 0$
$L_2 : x = 1$

$$k_i = \frac{1}{\mathbf{m}^\top \mathbf{V}\mathbf{m}} \left(-\mathbf{m}^\top \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) + \sqrt{\left[\mathbf{m}^\top \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) \right]^2 - g\left(\mathbf{h} \right) \left(\mathbf{m}^\top \mathbf{V}\mathbf{m} \right)} \right) \tag{0.1}$$

on solving for L_1 and L_2

$$k_1 = 0, k_2 = 2 \tag{0.2}$$

$$\mathbf{A} = \mathbf{h_1} + k_1\mathbf{m_1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 0 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.3}$$

$$\mathbf{B} = \mathbf{h_2} + k_2\mathbf{m_2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{0.4}$$

Thus the area under the curve included with the lines $x = 1$ and $x = 0$ is given by

$$\int_0^1 2\sqrt{x}\, dx = \left(\frac{4}{3} x^{\frac{3}{2}} \right)_0^1 = \frac{4}{3} \tag{0.5}$$

Variable	Description	Value
$\mathbf{m_1}$	direction vector of L_1	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
$\mathbf{m_2}$	direction vector of L_2	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
$\mathbf{h_1}$	vector passing through L_1	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
$\mathbf{h_2}$	vector passing through L_2	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
$\mathbf{V}$	Conic parameter	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
$\mathbf{u}$	Conic parameter	$\begin{pmatrix} -2 \\ 0 \end{pmatrix}$
f	Conic parameter	0

TABLE 0: Input Parameters

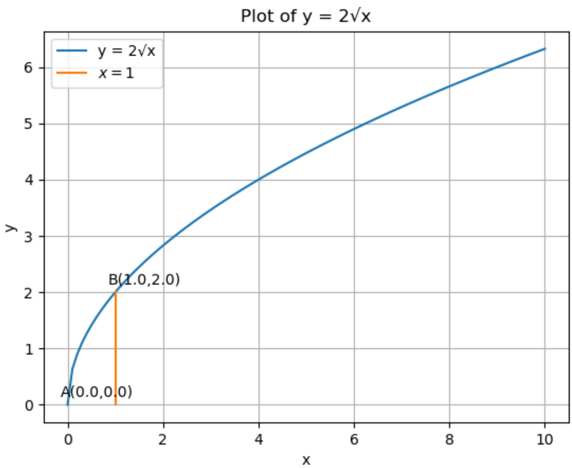

Fig. 0.1: Plot of Parabola