

9.9.2.30

EE24BTECH11032 - John Bobby

Question: Calculate the area under the curve $y = 2\sqrt{x}$ included with the lines $x = 1$ and $x = 0$.

Variable	Value	Description
\mathbf{m}_1	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	direction vector of L_1
\mathbf{m}_2	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	direction vector of L_2
\mathbf{h}_1	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	vector passing through L_1
\mathbf{h}_2	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	vector passing through L_2
\mathbf{V}	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$	Conic parameter
\mathbf{u}	$\begin{pmatrix} -2 \\ 0 \end{pmatrix}$	Conic parameter
f	0	Conic parameter

TABLE 0: Input Parameters

Solution: For a line $\mathbf{x} = \mathbf{h} + k\mathbf{m}$, the intersection of the line with a conic with parameters $\mathbf{V}, \mathbf{u}, \mathbf{h}, \mathbf{m}$ and h is given by $\mathbf{x} = \mathbf{h} + k_i\mathbf{m}$

$$L_1 : x = 0$$

$$L_2 : x = 1$$

$$k_i = \frac{1}{\mathbf{m}^\top \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^\top (\mathbf{V} \mathbf{h} + \mathbf{u}) + \sqrt{[\mathbf{m}^\top (\mathbf{V} \mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h}) (\mathbf{m}^\top \mathbf{V} \mathbf{m})} \right) \quad (0.1)$$

on solving for L_1 and L_2

$$k_1 = 0, k_2 = 2 \quad (0.2)$$

$$\mathbf{A} = \mathbf{h}_1 + k_1 \mathbf{m}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 0 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (0.3)$$

$$\mathbf{B} = \mathbf{h}_2 + k_2 \mathbf{m}_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (0.4)$$

Thus the area under the curve included with the lines $x = 1$ and $x = 0$ is given by

$$\int_0^1 2\sqrt{x} dx = \left(\frac{4}{3} x^{\frac{3}{2}} \right)_0^1 = \frac{4}{3} \quad (0.5)$$

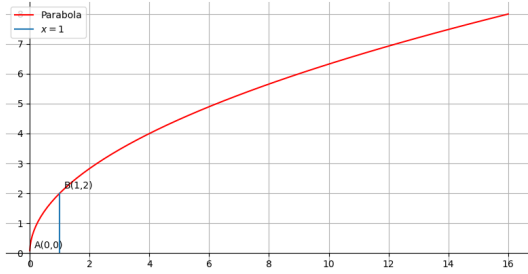


Fig. 0.1: Plot of Parabola