25-01-2023 Shift-2

EE24BTECH11032- John Bobby

1) Let the function $f(x) = 2x^3 + (2p-7)x^2 + 3(2p-9)x - 6$ have a maxima for some value of $x < 0$ and a minima for some value of $x > 0$. Then, the set of all values of p is
   a) $\left(\frac{9}{2}, \infty\right)$
   b) $\left(0, \frac{9}{2}\right)$
   c) $\left(-\infty, \frac{9}{2}\right)$
   d) $\left(-\frac{9}{2}, \frac{9}{2}\right)$
2) Let z be a complex number such that $\left|\frac{z-2i}{z+i}\right| = 2, z \neq -i$. Then z lies on the circle of radius 2 and centre
   a) $(2, 0)$
   b) $(0, 0)$
   c) $(0, 2)$
   d) $(0, -2)$
3) If the function
$$f(x) = \begin{cases} (1 + |\cos x|)\frac{\lambda}{|\cos x|} & , 0 < x < \frac{\pi}{2} \\ \mu & , x = \frac{\pi}{2} \\ e^{\frac{\cot 6x}{\cot 4x}} & , \frac{\pi}{2} < x < \pi \end{cases}$$
is continuous at $x = \frac{\pi}{2}$, then $9\lambda + 6log_e\mu + \mu^6 - e^{6\lambda}$ is equal to
   a) 11
   b) 8
   c) $2e^4 + 8$
   d) 10
4) Let $f(x) = 2x^n + \lambda, \lambda \in \mathbf{R}, n \in \mathbf{N}$, and $f(4) = 133, f(5) = 255$. Then the sum of all the positive integer divisors of $(f(3) - f(2))$ is
   a) 61
   b) 60
   c) 58
   d) 59
5) If the four points, whose position vectors are $3\hat{i} - 4\hat{j} + 2\hat{k}, \hat{i} + 2\hat{j} - \hat{k}$ and $5\hat{i} - 2\alpha\hat{j} + 4\hat{k}$ are coplanar, then α is equal to
   a) $\frac{73}{17}$
   b) $\frac{-107}{17}$
   c) $\frac{-73}{17}$
   d) $\frac{107}{17}$
6) Let $A = \begin{pmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -i \\ 0 & 1 \end{pmatrix}$, where $i = \sqrt{-1}$.If $M = A^\top BA$, then the inverse of the matrix $AM^{2023}A^\top$ is
   a) $\begin{pmatrix} 1 & -2023i \\ 0 & 1 \end{pmatrix}$
   b) $\begin{pmatrix} 1 & 0 \\ -2023i & 1 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 0 \\ 2023i & 1 \end{pmatrix}$
d) $\begin{pmatrix} 1 & 2023i \\ 0 & 1 \end{pmatrix}$

7) Let $\Delta, \nabla \in \{\wedge, \vee\}$ be such that $(p \to q)\,\Delta\,(p \nabla q)$ is a tautology. Then
   a) $\Delta = \wedge, \nabla = \vee$
   b) $\Delta = \vee, \nabla = \wedge$
   c) $\Delta = \vee, \nabla = \vee$
   d) $\Delta = \wedge, \nabla = \wedge$
8) The number of numbers, strictly between 5000 and 10000 can be formed using the digits $1, 3, 5, 7, 9$ without repetition, is
   a) 6
   b) 12
   c) 120
   d) 72
9) The number of functions f:$\{1, 2, 3, 4\} \to \{a \in \mathbf{Z} : |a| \leq 8\}$ satisfying $f(n) + \frac{1}{n} f(n+1) = 1, \forall n \in \{1, 2, 3\}$ is
   a) 3
   b) 4
   c) 1
   d) 2
10) The equations of two sides of a variable trianlge are $x = 0$ and $y = 3$, and its third side is a tangent to the parabola $y^2 = 6x$. The locus of its circumcentre is :
   a) $4y^2 - 18y - 3x - 18 = 0$
   b) $4y^2 + 18y + 3x + 18 = 0$
   c) $4y^2 - 18y + 3x + 18 = 0$
   d) $4y^2 - 18y - 3x + 18 = 0$
11) Let f:$\mathbf{R} \to \mathbf{R}$ be a function defined by $f(x) = log_{\sqrt{m}} \left\{ \sqrt{2}(\sin x - \cos x) + m - 2 \right\}$, for some m, such that the range of f is $[0, 2]$. Then the value of m is
   a) 5
   b) 3
   c) 2
   d) 4
12) Let A,B,C be 3×3 matrices such that A is symmetric and B and C are skew-symmetric. Consider the statements
   $(S1)\, A^{13}B^{26} - B^{26}A^{13}$ is symmetric
   $(S2)\, A^{26}C^{13} - C^{13}A^{26}$ is symmetric
   Then,
   a) Only S2 is true
   b) Only S1 is true
   c) Both S1 and S2 are false
   d) Both S1 and S2 are true
13) Let $y = y(t)$ be a solution of the differential equation $\frac{dy}{dt} + \alpha y = \gamma e^{-\beta t}$
   Where, $\alpha > 0, \beta > 0$ and $\gamma > 0$. Then $\lim_{t \to \infty} y(t)$
   a) is 0
   b) does not exist
   c) is 1
   d) is -1

14) $\sum_{k=0}^{6} {}^{51-k}C_3$ is equal to

a) ${}^{51}C_4 - {}^{45}C_4$

b) ${}^{51}C_3 - {}^{45}C_3$

c) ${}^{52}C_4 - {}^{45}C_4$

d) ${}^{52}C_3 - {}^{45}C_3$

15) The shortest distance between the lines $x + 1 = 2y = -12z$ and $x = y + 2 = 6z - 6$ is

a) 2

b) 3

c) $\frac{5}{2}$

d) $\frac{3}{2}$