

ASSIGNMENT-1

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- 1) Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as diameter, then the slope of the line joining A and B can be (2010)
 - a) $\frac{-1}{r}$
 - b) $\frac{1}{r}$
 - c) $\frac{2}{r}$
 - d) $\frac{5}{r}$
- 2) Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be reciprocal to that of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the ellipse, then (2011)
 - a) the equation of the hyperbola is $\frac{x^2}{3} - \frac{y^2}{2} = 1$
 - b) a focus of the hyperbola is $(2, 0)$
 - c) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$
 - d) the equation of the hyperbola is $x^2 - 3y^2 = 3$
- 3) Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point $(9, 6)$, then the L is given by (2010)
 - a) $y - x + 3 = 0$
 - b) $y + 3x - 33 = 0$
 - c) $y + x - 15 = 0$
 - d) $y - 2x + 12 = 0$
- 4) Tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$, parallel to the straight line $2x - y = 1$. The points of contact of the tangents to the hyperbola are (2012)
 - a) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
 - b) $\left(\frac{-9}{2\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$
 - c) $(3\sqrt{3}, -2\sqrt{2})$
 - d) $(-3\sqrt{3}, 2\sqrt{2})$
- 5) Let P and Q be distinct points on the parabola $y^2 = 2x$ such that a circle with PQ as diameter passes through the vertex O of the parabola. If P lies in the first quadrant and the area of the triangle ΔOPQ is $3\sqrt{2}$, then which of the following is (are) the coordinates of P ? (JEE ADV.2015)
 - a) $(4, 2\sqrt{2})$
 - b) $(9, 3\sqrt{2})$
 - c) $\left(\frac{1}{4}, \frac{1}{\sqrt{2}}\right)$
 - d) $(1, \sqrt{2})$
- 6) Let E_1 and E_2 be two ellipses whose centres are at the origin. The major axes of E_1 and E_2 lie along the x -axis and the y -axis, respectively. Let S be the circle $x^2 + (y - 1)^2 = 2$. The straight line $x + y = 3$ touches the curves S , E_1 and E_2 at P , Q and R respectively. Suppose that $PQ = PR = \frac{2\sqrt{2}}{3}$. If e_1 and e_2 are the eccentricities of E_1 and E_2 , respectively, then the correct expression(s) is (are) (JEE ADV.2015)
 - a) $e_1^2 + e_2^2 = \frac{43}{40}$
 - b) $e_1 e_2 = \frac{\sqrt{7}}{2\sqrt{10}}$
 - c) $|e_1^2 - e_2^2| = \frac{5}{8}$
 - d) $e_1 e_2 = \frac{\sqrt{3}}{4}$
- 7) Consider the hyperbola $H: x^2 - y^2 = 1$ and a circle S with centre $(Nx_2, 0)$. Suppose that H and S touch each other at a point $P(x_1, y_1)$ with $x_1 > 0$ and $y_1 > 0$. The common tangent to H and S at P intersects the x -axis at point M . If (l, m) is the centroid of the triangle PMN , then correct expressions(s) is(are) (JEE ADV.2015)
 - a) $\frac{dl}{dx_1} = 1 - \frac{1}{3x^2}$ for $x_1 > 1$
 - b) $\frac{dm}{dx_1} = \frac{x_1}{3\sqrt{x_1^2 - 1}}$ for $x_1 > 1$
 - c) $\frac{dl}{dx_1} = 1 + \frac{1}{3x^2}$ for $x_1 > 1$
 - d) $\frac{dm}{dy_1} = \frac{1}{3}$ for $y_1 > 0$
- 8) The circle $C_1: x^2 + y^2 = 3$, with centre at O , intersects the parabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle C_1 , at P touches other two circles C_2 and C_3 at R_2 and R_3 , respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and the centres Q_2 and Q_3 , respectively. If Q_2 and Q_3 lie on the y -axis, then (JEE ADV.2016)
 - a) $Q_2 Q_3 = 12$
 - b) $R_2 R_3 = 4\sqrt{6}$

- c) area of the triangle OR_2R_3 is $6\sqrt{2}$
 d) area of the triangle PQ_2Q_3 is $4\sqrt{2}$
- 9) Let P be the point on the parabola $y^2 = 4x$ which is at the shortest distance from the center S of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$. Let Q be the point on the circle dividing the line segment SP internally. Then (JEE ADV.2016)
- $SP = 2\sqrt{5}$
 - $SQ : QP = (\sqrt{5} + 1) : 2$
 - the x-intercept of the normal to the parabola at P is 6.
 - the slope of the tangent to the circle at Q is $\frac{1}{2}$
- 10) If $2x - y + 1 = 0$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$ then which of the following cannot be sides of a right angled triangle? (JEE ADV.2017)
- $a, 4, 1$
 - $a, 4, 2$
 - $2a, 8, 1$
 - $2a, 4, 1$
- 11) If a chord, which is not tangent, of the parabola $y^2 = 16x$ has equation $2x + y = p$, and midpoint (h, k) , then which of the following is(are) possible value(s) of p, h and k ? (JEE ADV.2017)
- $p = -2, h = 2, k = -4$
 - $p = -1, h = 1, k = 3$
 - $p = 2, h = 3, k = -4$
 - $p = 5, h = 4, k = -3$
- 12) Consider two straight lines, each of which is tangent to both the circle $x^2 + y^2 = \frac{1}{2}$ and the parabola $y^2 = 4x$. Let these lines intersect at the point Q . Consider the ellipse whose center is at origin $O(0, 0)$ and whose semi-major axis is OQ . If the length of the minor axis of the ellipse is $\sqrt{2}$, then which of the following statement(s) is(are) TRUE? (JEE ADV.2018)
- For the ellipse, the eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latus rectum is 1
 - For the ellipse, the eccentricity is $\frac{1}{2}$ and the length of the latus rectum is $\frac{1}{2}$
 - The area of the region bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{4\sqrt{2}}(\pi - 2)$
 - The area of the region bounded by the ellipse between the line $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{16}(\pi - 2)$

I. SECTION -E SUBJECTIVE PROBLEMS

- Suppose that the normals drawn at three different points on the parabola $y^2 = 4x$ pass through the point (h, k) . Show that $h > 2$. (1981-4 Marks)
- A is a point on the parabola $y^2 = 4ax$. The normal at A cuts the parabola again at point B . If AB subtends a right angle at the vertex of the parabola. find the slope of AB . (1982-4 Marks)
- Three normals are drawn from the point $(c, 0)$ to the curve $y^2 = x$. Show that c must be greater than $\frac{1}{2}$. One normal is always the x-axis. Find c for which the other two normals are perpendicular to each other. (1991-4 Marks)
- Through the vertex O of parabola $y^2 = 4x$, chords OP and OQ are drawn at right angles to one another. Show that for all positions of P , PQ cuts the axis of the parabola at a fixed point. Also find the locus of the middle point of PQ . (1994-4 Marks)
- Show that the locus of a point that divides a chord of slope 2 of the parabola $y^2 = 4x$ internally in the ratio 1 : 2 is a parabola. Find the vertex of this parabola. (1995-5 Marks)