ASSIGNMENT-1

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1) Let **A** and **B** be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as diameter, then the slope of the line joining **A** and **B** can be (2010)
   a) $\frac{-1}{r}$
   b) $\frac{1}{r}$
   c) $\frac{2}{r}$
   d) $\frac{2}{r}$
2) Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be reciprocal to that of the elipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the elipse, then (2011)
   a) the equation of the hyperbola is $\frac{x^2}{3} - \frac{y^2}{2} = 1$
   b) a focus of the hypebola is $(2, 0)$
   c) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$
   d) the equation of the hyperbola is $x^2 - 3y^2 = 3$
3) Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point $(9, 6)$, then the L is given by (2010)
   a) $y - x + 3 = 0$
   b) $y + 3x - 33 = 0$
   c) $y + x - 15 = 0$
   d) $y - 2x + 12 = 0$
4) Tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$, parallel to the straight line $2x - y = 1$. The points of contact of the tangents to the hyperbola are (2012)
   a) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
   b) $\left(\frac{-9}{2\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$
   c) $\left(3\sqrt{3}, -2\sqrt{2}\right)$
   d) $\left(-3\sqrt{3}, 2\sqrt{2}\right)$
5) Let **P** and **Q** be distinct points on the parabola $y^2 = 2x$ such that a circle with PQ as diameter passes through the vertex **O** of the parabola. If **P** lies in the first quadrant and the area of the triangle ΔOPQ is $3\sqrt{2}$, then which of the following is (are) the coordinates of **P**? (JEE ADV.2015)
   a) $\left(4, 2\sqrt{2}\right)$
   b) $\left(9, 3\sqrt{2}\right)$
   c) $\left(\frac{1}{4}, \frac{1}{\sqrt{2}}\right)$
   d) $\left(1, \sqrt{2}\right)$
6) Let E_1 and E_2 be two elipses whose centres are at the orgin. The major axes of E_1 and E_2 lie along the x-axis and the y-axis,respectively. Let S be the circle $x^2 + (y-1)^2 = 2$. The straight line $x + y = 3$ touches the curves S, E_1 and E_2 at **P**, **Q** and **R** respectively. Suppose that $PQ = PR = \frac{2\sqrt{2}}{3}$. If e_1 and e_2 are the eccentricities of E_1 and E_2, respectively, then the correct expression(s) is (are) (JEE ADV.2015)
   a) $e_1^2 + e_2^2 = \frac{43}{40}$
   b) $e_1 e_2 = \frac{\sqrt{7}}{2\sqrt{10}}$

c) $\left|e_1^2 - e_2^2\right| = \frac{5}{8}$
d) $e_1 e_2 = \frac{\sqrt{3}}{4}$

7) Consider the hyperbola H:$x^2 - y^2 = 1$ and a circle S with centre $\mathbf{N}(x_2, 0)$. Suppose that H and S touch each other at a point $\mathbf{P}(x_1, y_1)$ with $x_1 > 0$ and $y_1 > 0$. The common tangent to H and S at **P** intersects the x-axis at point **M**. If (l, m) is the centroid of the triangle PMN, then correct expressions(s) is(are) (JEE ADV.2015)

a) $\frac{dl}{dx_1} = 1 - \frac{1}{3x^2}$ for $x_1 > 1$
b) $\frac{dm}{dx_1} = \frac{x_1}{3\sqrt{x_1^2 - 1}}$ for $x_1 > 1$
c) $\frac{dl}{dx_1} = 1 + \frac{1}{3x^2}$ for $x_1 > 1$
d) $\frac{dm}{dy_1} = \frac{1}{3}$ for $y_1 > 0$

8) The circle C_1:$x^2 + y^2 = 3$, with centre at **O**, intersects the parabola $x^2 = 2y$ at the point **P** in the first quadrant. Let the tangent to the circle C_1, at **P** touches other two circles C_2 and C_3 at $\mathbf{R_2}$ and $\mathbf{R_3}$, respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and the centres $\mathbf{Q_2}$ and $\mathbf{Q_3}$,respectively. If $\mathbf{Q_2}$ and $\mathbf{Q_3}$ lie on the y-axis, then (JEE ADV.2016)

a) $Q_2 Q_3 = 12$
b) $R_2 R_3 = 4\sqrt{6}$
c) area of the triangle OR_2R_3 is $6\sqrt{2}$
d) area of the triangle PQ_2Q_3 is $4\sqrt{2}$

9) Let **P** be the point on the parabola $y^2 = 4x$ which is at the shortest distance from the center S of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$. Let **Q** be the point on the circle dividing the line segment SP internally. Then (JEE ADV.2016)

a) $SP = 2\sqrt{5}$
b) $SQ : QP = (\sqrt{5} + 1) : 2$
c) the x-intercept of the normal to the parabola at **P** is 6
d) the slope of the tangent to the circle at **Q** is $\frac{1}{2}$

10) If $2x - y + 1 = 0$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$ then which of the following cannot be sides of a right angled triangle? (JEE ADV.2017)

a) $a, 4, 1$
b) $a, 4, 2$
c) $2a, 8, 1$
d) $2a, 4, 1$

11) If a chord, which is not tangent, of the parabola $y^2 = 16x$ has equation $2x + y = p$, and midpoint (h, k), then which of the following is(are) possible value(s) of p, h and k? (JEE ADV.2017)

a) $p = -2, h = 2, k = -4$
b) $p = -1, h = 1, k = 3$
c) $p = 2, h = 3, k = -4$
d) $p = 5, h = 4, k = -3$

12) Consider two straight lines, each of which is tangent to both the circle $x^2 + y^2 = \frac{1}{2}$ and the parabola $y^2 = 4x$. Let these lines intersect at the point **Q**. Consider the elipse whose center is at orgin $\mathbf{O}(0, 0)$ and whose semi-major axis is OQ. If the length of the minor axis of the elipse is $\sqrt{2}$, then which of the following statement(s) is(are) TRUE? (JEE ADV.2018)

a) For the elipse, the eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latus rectum is 1
b) For the elipse, the eccentricity is $\frac{1}{2}$ and the length of the latus rectum is $\frac{1}{2}$
c) The area of the region bounded by the elipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{4\sqrt{2}}(\pi - 2)$
d) The area of the region bounded by the elipse between the line $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{16}(\pi - 2)$

I. SECTION -E SUBJECTIVE PROBLEMS

1) Suppose that the normals drawn at three different points on the parabola $y^2 = 4x$ pass through the point(h, k). Show that $h > 2$. (1981-4 Marks)
2) **A** is a point on the parabola $y^2 = 4ax$. The normal at **A** cuts the parabola again at point **B**. If AB subtends a right angle at the vertex of the parabola. Find the slope of AB. (1982-4 Marks)
3) Three normals are drawn from the point $(c, 0)$ to the curve $y^2 = x$. Show that c must be greater than $\frac{1}{2}$. One normal is always the x-axis. Find c for which the other two normals are perpendicular to each other. (1991-4 Marks)
4) Through the vertex **O** of parabola $y^2 = 4x$, chords OP and OQ are drawn at right angles to one another. Show that for all positions of **P**, PQ cuts the axis of the parabola at a fixed point. Also find the locus of the middle point of PQ. (1994-4 Marks)
5) Show that the locus of a point that divides a chord of slope 2 of the parabola $y^2 = 4x$ internally in the ratio $1 : 2$ is a parabola. Find the vertex of this parabola. (1995-5 Marks)