ASSIGNMENT-1

EE24BTECH11032- JOHN BOBBY

1) Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as diameter, then the slope of the line joining A and B can be
(2010)

a) $\frac{-1}{r}$
b) $\frac{1}{r}$
c) $\frac{2}{r}$
d) $\frac{-2}{r}$

2) Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be reciprocal to that of the elipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the elipse, then
(2011)

a) the equation of the hyperbola is $\frac{x^2}{3} - \frac{y^2}{2} = 1$
b) a focus of the hypebola is $(2, 0)$
c) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$
d) the equation of the hyperbola is $x^2 - 3y^2 = 3$

3) Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point $(9, 6)$, then the L is given by
(2010)

a) $y - x + 3 = 0$
b) $y + 3x - 33 = 0$
c) $y + x - 15 = 0$
d) $y - 2x + 12 = 0$

4) Tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$, parallel to the straight line $2x - y = 1$.The points of contact of the tangents to the hyperbola are
(2012)

a) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
b) $\left(\frac{-9}{2\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$
c) $\left(3\sqrt{3}, -2\sqrt{2}\right)$
d) $\left(-3\sqrt{3}, 2\sqrt{2}\right)$

5) Let P and Q be distinct points on the parabola $y^2 = 2x$ such that a circle with PQ as diameter passes through the vertex O of the parabola. If P lies in the first quadrant and the area of the triangle ΔOPQ is $3\sqrt{2}$, then which of the following is (are) the coordinates of P?
(*JEEADV.*2015)

a) $\left(4, 2\sqrt{2}\right)$
b) $\left(9, 3\sqrt{2}\right)$
c) $\left(\frac{1}{4}, \frac{1}{\sqrt{2}}\right)$
d) $\left(1, \sqrt{2}\right)$

6) Let E_1 and E_2 be two elipses whose centres are at the orgin. The major axes of E_1 and E_2 lie along the x-axis and the y-axis,respectively. Let S be the circle $x^2 + (y-1)^2 = 2$.The straight line $x + y = 3$ touches the curves S, E_1 and E_2 at P, Q and R respectively.Suppose that PQ=PR=$\frac{2\sqrt{2}}{3}$. If e_1 and e_2 are the eccentricities of E_1 and E_2,respectively, then the correct expression(s) is (are)
(*JEEADV.*2015)

a) $e_1^2 + e_2^2 = \frac{43}{40}$
b) $e_1 e_2 = \frac{\sqrt{7}}{2\sqrt{10}}$
c) $\left|e_1^2 - e_2^2\right| = \frac{5}{8}$
d) $e_1 e_2 = \frac{\sqrt{3}}{4}$

7) Consider the hyperbola H:$x^2 - y^2 = 1$ and a circle S with centre $\mathbf{N}(x_2, 0)$. Suppose that H and S touch each other at a point $\mathbf{P}(x_1, y_1)$ with $x_1 > 0$ and $y_1 > 0$. The common tangent to H and S at P intersects the x-axis at point M. If (l, m) is the centroid of the triangle PMN, then correct expressions(s) is(are)
(*JEEADV.*2015)

a) $\frac{dl}{dx_1} = 1 - \frac{1}{3x^2}$ for $x_1 > 1$
b) $\frac{dm}{dx_1} = \frac{x_1}{3\sqrt{x_1^2 - 1}}$ for $x_1 > 1$
c) $\frac{dl}{dx_1} = 1 + \frac{1}{3x^2}$ for $x_1 > 1$
d) $\frac{dm}{dy_1} = \frac{1}{3}$ for $y_1 > 0$

8) The circle $C_1: x^2 + y^2 = 3$, with centre at O, intersects the parabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle C_1, at P touches other two circles C_2 and C_3 at R_2 and R_3, respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and the centres Q_2 and Q_3,respectively. If Q_2 and Q_3 lie on the y-axis, then
(*JEEADV.*2016)

a) $Q_2Q_3 = 12$
b) $R_2R_3 = 4\sqrt{6}$
c) area of the triangle OR_2R_3 is $6\sqrt{2}$
d) area of the triangle PQ_2Q_3 is $4\sqrt{2}$

9) Let P be the point on the parabola $y^2 = 4x$ which is at the shortest distance from the center S of the circle $x^2+y^2-4x-16y+64 = 0$. Let Q be the point on the circle dividing the line segment SP internally. Then
(*JEEADV.*2016)

a) SP=$2\sqrt{5}$
b) SQ:QP=$(\sqrt{5}+1) : 2$
c) the x-intercept of the normal to the parabola at P is 6.
d) the slope of the tangent to the circle at Q is $\frac{1}{2}$

10) If $2x - y + 1 = 0$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$ then which of the following cannot be sides of a right angled triangle?
(*JEEADV.*2017)

a) $a, 4, 1$
b) $a, 4, 2$
c) $2a, 8, 1$
d) $2a, 4, 1$

11) If a chord, which is not tangent, of the parabola $y^2 = 16x$ has equation $2x+y = p$, and midpoint (h, k), then which of the following is(are) possible value(s) of p,h and k?
(*JEEADV.*2017)

a) $p = -2, h = 2, k = -4$
b) $p = -1, h = 1, k = 3$
c) $p = 2, h = 3, k = -4$
d) $p = 5, h = 4, k = -3$

12) Consider two straight lines, each of which is tangent to both the circle $x^2 + y^2 = \frac{1}{2}$ and the parabola $y^2 = 4x$. Let these lines intersect at the point Q. Consider the elipse whose center is at orgin $\mathbf{O}(0, 0)$ and whose semi-major axis is OQ. If the length of the minor axis of the elipse is $\sqrt{2}$, then which of the following statement(s) is(are) TRUE?
(*JEEADV.*2018)

a) For the elipse, the eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latus rectum is 1
b) For the elipse, the eccentricity is $\frac{1}{2}$ and the length of the latus rectum is $\frac{1}{2}$
c) The area of the region bounded by the elipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{4\sqrt{2}}(\pi - 2)$
d) The area of the region bounded by the elipse between the line $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{16}(\pi-2)$

I. SECTION -E SUBJECTIVE PROBLEMS

1) Suppose that the normals drawn at three different points on the parabola $y^2 = 4x$ pass through the point(h, k). Show that $h > 2$.
$(1981 - 4Marks)$

2) A is a point on the parabola $y^2 = 4ax$.The normal at A cuts the parabola again at point B. If AB subtends a right angle at the vertex of the parabola.find the slope of AB.
$(1982 - 4Marks)$

3) Three normals are drawn from the point $(c, 0)$ to the curve $y^2 = x$. Show that c must be greater than $\frac{1}{2}$. One normal is always the x-axis. Find c for which the other two normals are perpendicular to each other.
$(1991 - 4Marks)$

4) Through the vertex O of parabola $y^2 = 4x$,chords OP and OQ are drawn at right angles to one another. Show that for all positions of P, PQ cuts the axis of the parabola at a fixed point. Also find the locus of the middle point of PQ.
$(1994 - 4Marks)$

5) Show that the locus of a point that divides a chord of slope 2 of the parabola $y^2 = 4x$ internally in the ratio 1:2 is a parabola. Find

the vertex of this parabola.
(1995 – 5*Marks*)