01)

```
void imprime(int *v, int n) {
    if(n != 0) → Θ(1)
        cout << v[n-1] << " "; Θ(1)
    imprime(v, n-1); T(n-1)
}
```

$T(n) = T(n-1) + \Theta(1); \quad T(0) = c$

$T(n) = T(n-1) + c$

$T(1) = T(0) + c = c + c = 2c$

$T(2) = T(1) + c = 2c + c = 3c$

$T(3) = T(2) + c = 3c + c = 4c$

$T(n) = T(n-1) + c = (n-1)c + c = nc - c + c = n \cdot c$

$\therefore T(n) \in \Theta(n)$

4 10

```
void f1(int *array, int begin, int end) {
    if (begin >= end) → Θ(1)
        return;

    int mid = begin + (end - begin) / 2; → Θ(1)
    f1(array, begin, mid); → T(n/2)
    f1(array, mid + 1, end); → T(n/2)
    f2(array, begin, mid, end);
}                             ↳ Θ(n)
// Obs.: f2 tem complexidade linear
```

$T(n) = 2\,T\left(\frac{n}{2}\right) + \Theta(1) + \Theta(n)$

$= 2\,T\left(\frac{n}{2}\right) + \Theta(n) \quad \rightarrow \quad a\,T\left(\frac{n}{b}\right) + \Theta(n)$

- Usando o teorema mestre,

$a = 2$
$b = 2$

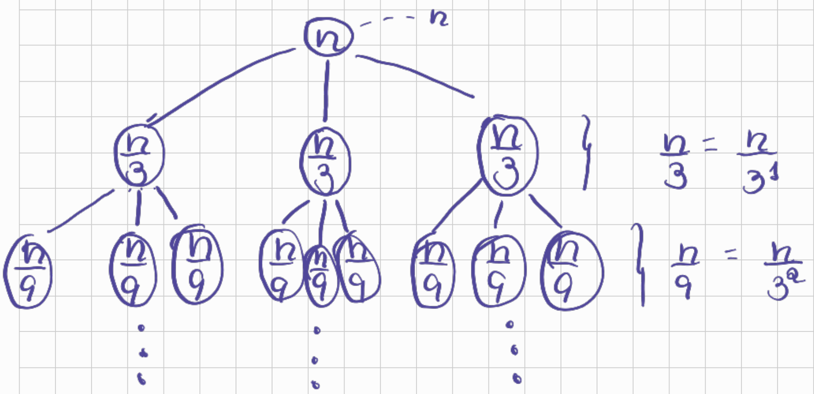
$f(n) = \Theta(n^k) \therefore \Theta(n) \rightarrow k = 1$

$\therefore 2 = 2^1$ (2^o Caso)

Assim, $T(n) \in n \lg n$

02) $T(1) = 1$

a) $T(n) = 3T\left(\frac{n}{3}\right) + n$



custo do nó: $\frac{n}{3^{nível}} \cdot \underbrace{3^{nível}}_{nós} \rightarrow$ Custo total: $\sum_{i=0}^{\log_3 n} \left(\cancel{3^i} \cdot \frac{n}{\cancel{3^i}} \right) = n \log_3 n$

Logo, $T(n) \in \Theta(n \log_3 n)$

b) $T(n) = 3T\left(\frac{n}{3}\right) + 1$

$$\begin{cases} T(n) = 3T\left(\frac{n}{3}\right) + 1, & n \neq 1 \\ T(1) = 1, & n = 1 \end{cases}$$

$T(n) = 3T\left(\frac{n}{3}\right) + 1 \dashrightarrow T\left(\frac{n}{3}\right) = 3T\left(\frac{n}{9}\right) + 1$

$= 9T\left(\frac{n}{9}\right) + 1 + 3 \dashrightarrow T\left(\frac{n}{9}\right) = 3T\left(\frac{n}{27}\right) + 1$

$= 27T\left(\frac{n}{27}\right) + 1 + 3 + 9 \dashrightarrow T\left(\frac{n}{27}\right) = 3T\left(\frac{n}{81}\right) + 1$

$= 81T\left(\frac{n}{81}\right) + 1 + 3 + 9 + 27$

...

$= 3^k T\left(\frac{n}{3^k}\right) + \sum_{i=0}^{k} 3^i$

p/ $\frac{n}{3^k} = 1 \rightarrow 3^k = n \rightarrow \log_3 n = k$

$\therefore T(n) = n \underbrace{T\left(\frac{n}{n}\right)}_{T(1)} + \sum_{i=0}^{\log_3 n} 3^i = n + \underbrace{\sum_{i=0}^{\log_3 n} 3^i}_{PG} \rightarrow S(n) = \frac{a_1 (q^n - 1)}{q - 1} = \frac{1(3^{\log_3 n} - 1)}{3 - 1} = \frac{n-1}{2}$

$\therefore T(n) = n + \frac{n-1}{2} = \frac{3n-1}{2}$ Logo, $T(n) \in \Theta(n)$

3) N I O V Z A P K R Q S

0	1	2	3	4	5	6	7	8	9	10	11	12
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

0	1	2	3	4	5	6	7	8	9	10	11	12
-1	N	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

$h(N) = 14 \bmod 13 = 1$

0	1	2	3	4	5	6	7	8	9	10	11	12
-1	N	-1	-1	-1	-1	-1	-1	-1	I	-1	-1	-1

$h(I) = 9 \bmod 13 = 9$

0	1	2	3	4	5	6	7	8	9	10	11	12
-1	N	O	-1	-1	-1	-1	-1	-1	I	-1	-1	-1

$h(O) = 15 \bmod 13 = 2$

0	1	2	3	4	5	6	7	8	9	10	11	12
-1	N	O	-1	-1	-1	-1	-1	-1	I	V	-1	-1

$h(V) = 22 \bmod 13 = 9$

0	1	2	3	4	5	6	7	8	9	10	11	12
Z	N	O	-1	-1	-1	-1	-1	-1	I	V	-1	-1

$h(Z) = 26 \bmod 13 = 0$

0	1	2	3	4	5	6	7	8	9	10	11	12
Z	N	O	A	-1	-1	-1	-1	-1	I	V	-1	-1

$h(A) = 1 \bmod 13 = 1$

0	1	2	3	4	5	6	7	8	9	10	11	12
Z	N	O	A	P	-1	-1	-1	-1	I	V	-1	-1

$h(P) = 16 \bmod 13 = 3$

0	1	2	3	4	5	6	7	8	9	10	11	12
Z	N	O	A	P	-1	-1	-1	-1	I	V	K	-1

$h(K) = 11 \bmod 13 = 11$

0	1	2	3	4	5	6	7	8	9	10	11	12
Z	N	O	A	P	R	-1	-1	-1	I	V	K	-1

$h(R) = 18 \bmod 13 = 5$

0	1	2	3	4	5	6	7	8	9	10	11	12
Z	N	O	A	P	R	Q	-1	-1	I	V	K	-1

$h(Q) = 17 \bmod 13 = 4$

0	1	2	3	4	5	6	7	8	9	10	11	12
Z	N	O	A	P	R	Q	S	-1	I	V	K	-1

$h(S) = 19 \bmod 13 = 6$

0	1	2	3	4	5	6	7	8	9	10	11	12
Z	N	O	A	P	R	Q	S	-1	I	V	K	-1

N: 0
I: 0
O: 0
V: 1
Z: 0
A: 2
P: 1
K: 0
R: 0
Q: 2
S: 1