Computer Science 202a
Homework #1, due in class Thursday, Sept. 15, 2016

Please turn in your homework in TWO SEPARATE PARTS: Part I is problems 1-3 and Part II is problems 4-6. Put your name on both parts. Please list (in Part I) any persons (including course staff) or resources you consulted with in connection with this assignment. Partial credit will be given if the grader can easily understand enough of your answer to award it.

PART I

1. (15 points) Construct truth tables for each of the following propositional expressions, and say for each whether it is a tautology, a contradiction or a contingency.

   (a) $(p \wedge \neg q) \vee (\neg p \wedge q)$

   (b) $(p \leftrightarrow q) \leftrightarrow (\neg p \leftrightarrow \neg q)$

   (c) $(p \oplus q) \rightarrow ((p \oplus q) \oplus r)$

   (d) $(p \vee q) \rightarrow ((p \wedge r) \vee (q \wedge \neg r))$

   (e) $(\neg p \wedge q) \wedge (q \leftrightarrow p)$

2. (15 points) Imagine that you have observed meetings involving five people, identified by the letters A, B, C, D, E, and you have induced the following rules about the meetings:

   (a) If A and B are present then so is C or D (or both).

   (b) If D is present then so are A and E.

   (c) If C and E are present, then B is not present

   Find all meetings of a group of four of these individuals that do not violate any of these rules. Justify your answer.

3. (20 points) Logician Raymond Smullyan describes the island of knights and knaves as populated by two kinds of people: those who always lie (knaves) and those who always tell the truth (knights). Imagine that you are visiting the island and you meet three of its inhabitants, conveniently named A, B and C.

   For each of the following items, determine whether A, B, and C could make the statements attributed to them. If they could make the statements, determine all the assignments of types (knight or knave) to A, B, and C in which the list of statements is possible. Justify your answers.

   (a) A says "B and C are knights." B says "That is correct, C and I are knights." C says "No, that is not correct!"

   (b) A says "There are an odd number of knights among us." B says "Both A and I are knights." C says "No, both A and B are knaves."

   (c) A says "Exactly two of us are knights." B says "Exactly two of us are knaves." C says "Not all of us are of the same type."

   (d) A says "If I am a knave, then so is B." B says "A and C are of the same type." C says "A and B are of opposite types."

PART II

4. (15 points) For the following pairs of formulas, determine whether they are logically equivalent. Justify your answer.

(a) $(p \vee \neg q)$, $\neg(p \to q)$

(b) $(p \to (p \wedge q))$, $p \to q$

(c) $((p \vee q) \wedge (\neg p \vee r))$, $(q \wedge r)$

(d) $(p \leftrightarrow (q \oplus r))$, $(r \to (p \oplus q))$

(e) $((p \to r) \wedge (q \to r))$, $((p \vee q) \to r)$

5. (15 points) Translate each of the following statements about the domain of nonnegative integers into logical formulas, using the following function and predicate symbols. The function symbol $a(x, y)$ stands for $x + y$ and the function symbol $m(x, y)$ stands for $x \cdot y$. The predicate symbol $L(x, y)$ stands for $x \leq y$ and the predicate symbol $E(x, y)$ stands for $x = y$.

(a) For all x and y if $x \leq y$ and $y \leq x$ then $x = y$.

(b) There does not exist a number such that every number is less than or equal to it.

(c) If x is less than or equal to y then there is a number z such that $x + z$ is equal to y.

(d) If x is less than or equal to y then for any z, $x + z$ is less than or equal to $y + z$.

(e) The product of any two numbers is always greater than or equal to at least one of them.

6. (20 points) Choose and define function and predicate symbols, and translate each of the following arguments into statements in predicate logic. Taking all but the last statement of each argument as the hypotheses, and the last statement as the conclusion, assess the validity of the argument. If it is valid, give an informal proof that the conclusion is a logical consequence of the hypotheses. If it is invalid, give an example of a domain that makes all the hypotheses true and the conclusion false.

(a) (The domain is the animals in a particular zoo. Thus, in $\forall x$ and $\exists x$, the variable x refers to an animal in this zoo.)

 i. Every animal that flies has wings.
 ii. Every bird has feathers and wings.
 iii. No bird has six legs.
 iv. There is an animal that has six legs and flies.
 v. There is an animal that has wings but is not a bird.

(b) (The domain is some nonempty set S of numbers. Thus in $\forall x$ and $\exists x$ the variable x refers to an element of this set.)

 i. For all elements x and y, if x is good for y then y is not good for x.
 ii. For all elements x, y and z, if x is good for y and y good for z then x is good for z.
 iii. For every element x there is an element y that is good for x.
 iv. For every pair of elements x and y such that x is good for y, there exists an element z such that x is good for z and z is good for y.
 v. For every element x there is an element y such that x is good for y.