

# MACHINE LEARNING FOR ANATOMICAL NEUROIMAGING

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## 1 INTRODUCTION

- Why apply pattern recognition to structural MRI?
- Common ways to represent anatomical features
- No Free Lunch and prior knowledge
- Dimensionality reduction

## 2 MORPHOMETRICS

## 3 SIMILARITY BETWEEN BRAINS

## 4 CONCLUSION: WHAT NEXT?

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## TO EXPLAIN OR TO PREDICT?

*“The only relevant test of the validity  
of a hypothesis is comparison of  
prediction with experience.”*

Milton Friedman

# TO EXPLAIN OR TO PREDICT?

*“The predictive point of view is a prototypical point of view to explain the basic activity of statistical analysis.”*

— Akaike

*“The only useful function of a statistician is to make predictions.”*

— Deming

*“The prediction of observables or potential observables is of much greater relevance than the estimate of what are often artificial constructs-parameters”*

— Geisser

Galit Shmueli. “To Explain or to Predict?”. Statistical Science 25(3):289–310 (2010).

## EVIDENCE-BASED SCIENCE

...also just known as “science”.

- Researchers claim to find differences between groups. Do those findings actually discriminate?
- How can we most accurately diagnose a disorder from image data?
- Pharma wants biomarkers. How do we most effectively identify them?
- There are lots of potential imaging biomarkers. Which are most (cost) effective?

Pattern recognition provides a framework to compare data (or preprocessing strategy) to determine the most accurate approach.

# INTER-SUBJECT VARIABILITY

Why focus on anatomy?

- Many medical applications involve understanding differences among individuals/populations.
- In image data, most of the differences we can see are anatomical in nature.
- Understanding growth and development requires us to look at growth and development (anatomy).

# BAYESIAN APPROACHES MAY BE BETTER FOR CLINICAL APPLICATIONS

- Deals with different priors.
  - Consider a method with 90% sensitivity and specificity.
  - Consider using this to screen for a disease afflicting 1% of the population.
  - On average, out of 100 people there would be 10 wrongly assigned to the disease group.
  - A positive diagnosis suggests only about a 10% chance of having the disease.

$$P(\text{Disease}|\text{Pred+}) = \frac{P(\text{Pred+}|\text{Disease})P(\text{Disease})}{P(\text{Pred+}|\text{Disease})P(\text{Disease}) + P(\text{Pred+}|\text{Healthy})P(\text{Healthy})}$$

- Better decision-making by accounting for utility functions.
- Confidence may differ from subject to subject.

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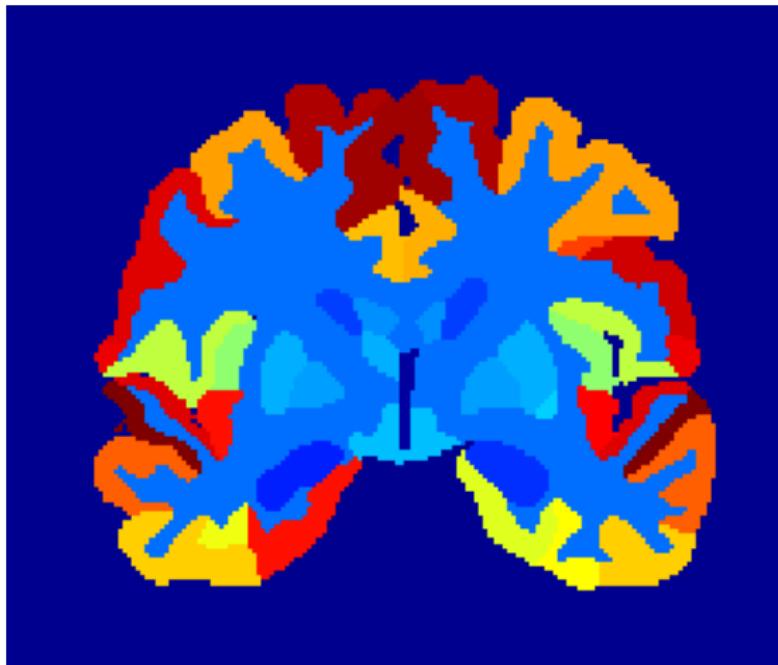
## 4 CONCLUSION: WHAT NEXT?

# FEATURE ENGINEERING

*First-timers are often surprised by how little time in a machine learning project is spent actually doing machine learning. But it makes sense if you consider how time-consuming it is to gather data, integrate it, clean it and pre-process it, and how much trial and error can go into feature design. Also, machine learning is not a one-shot process of building a data set and running a learner, but rather an iterative process of running the learner, analyzing the results, modifying the data and/or the learner, and repeating. Learning is often the quickest part of this, but that's because we've already mastered it pretty well! Feature engineering is more difficult because it's domain-specific, while learners can be largely general-purpose. However, there is no sharp frontier between the two, and this is another reason the most useful learners are those that facilitate incorporating knowledge.*

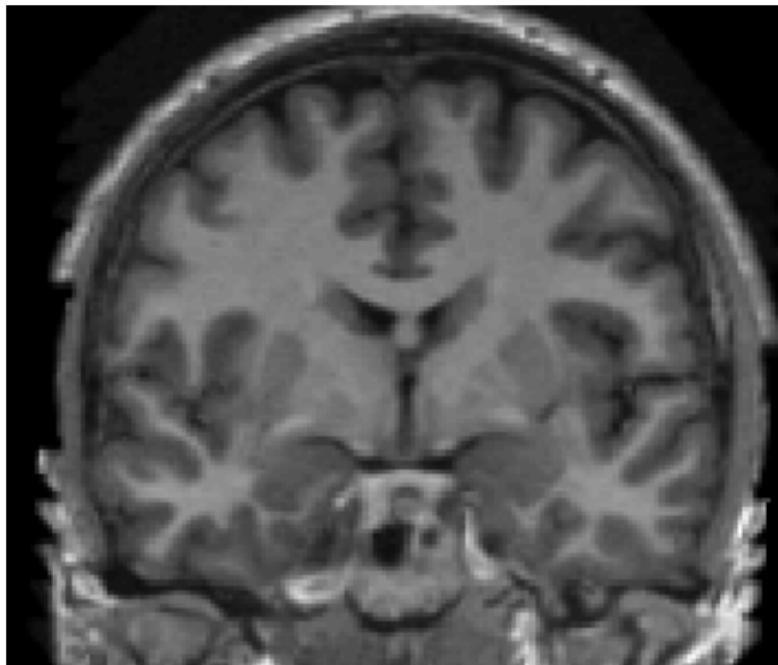
## REGION VOLUMES

Label propagation or other methods can be used to subdivide brain into regions.



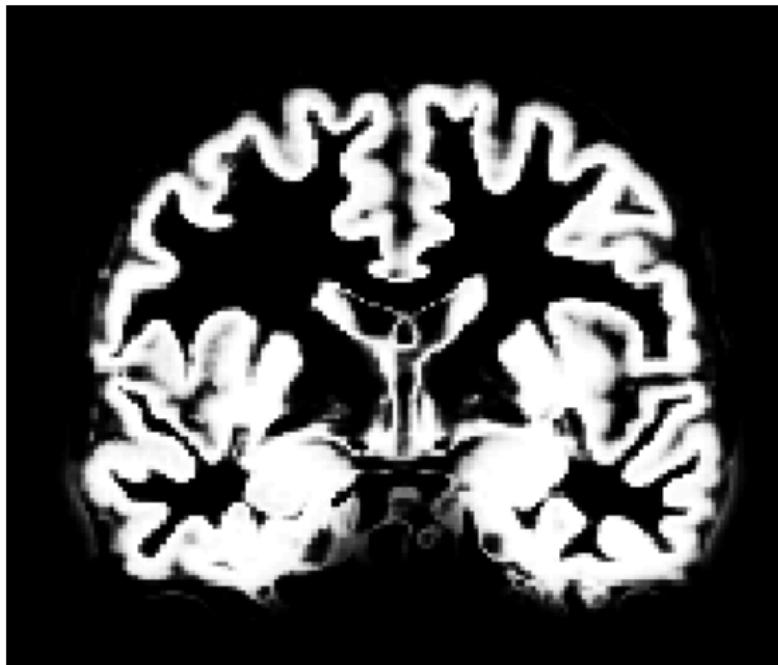
# PIXEL VALUES

Raw pixel data could be another option.  
Data needs to be “spatially normalised” (and possibly skull-stripped).  
Results may not generalise well to data from other scanners.



# TISSUE MAPS

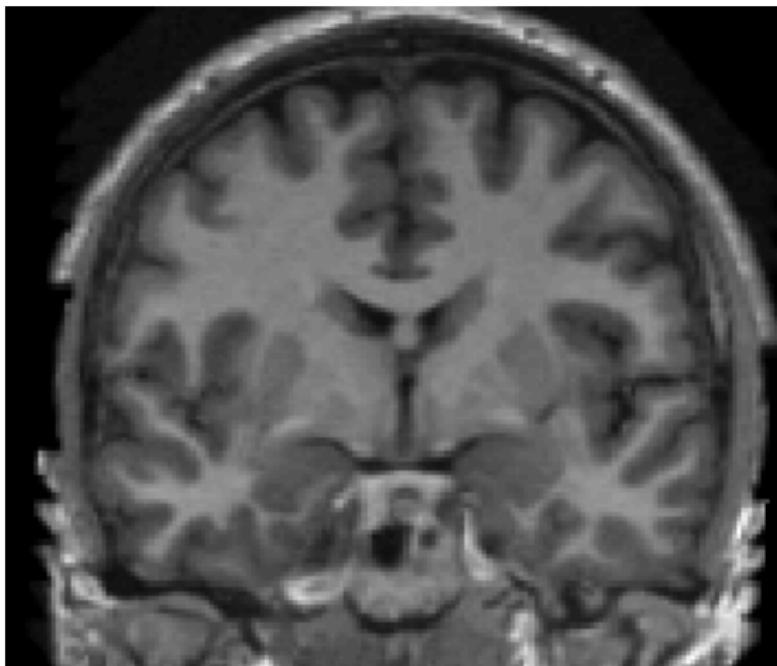
Grey matter maps can work fairly well.  
Data needs to be “spatially normalised”.  
Many neurological problems show up as grey matter atrophy.



# OTHER FEATURES

Other features include:

- Cortical thickness.
- Shape features.
- Principal / Independent component weights.
- Lesion maps.
- etc



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## No FREE DUCKLINGS

**No Free Lunch theorem** says that learning is impossible without prior knowledge ([http://en.wikipedia.org/wiki/No\\_free\\_lunch\\_in\\_search\\_and\\_optimization](http://en.wikipedia.org/wiki/No_free_lunch_in_search_and_optimization)).

**Ugly Duckling theorem** says that things are all equivalently similar to each other without prior knowledge ([http://en.wikipedia.org/wiki/Ugly\\_duckling\\_theorem](http://en.wikipedia.org/wiki/Ugly_duckling_theorem)).

What prior knowledge do we have about the variability among people that can be measured using MRI?  
How do we use this knowledge?

# DIFFERENT WAYS OF MEASURING DISTANCES



## David Mumford

Mathematician

David Bryant Mumford is an American mathematician known for distinguished work in algebraic geometry, and then for research into vision and pattern theory. He won the Fields Medal and was a MacArthur Fellow. [Wikipedia](#)

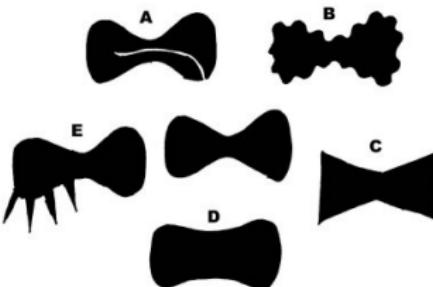
**Born:** June 11, 1937 (age 76), Worth village, West Sussex, Crawley

**Children:** Steve Mumford

**Education:** Phillips Exeter Academy, Harvard University

**Awards:** Fields Medal, Wolf Prize in Mathematics, MacArthur Fellowship, The Shaw Prize in Mathematical Sciences, National Medal of Science for Mathematics and Computer Science

*Empirical Statistics and Stochastic Models for Visual Signals*



**Figure 1.11** Each of the shapes A,B,C,D and E is similar to the central shape, but in different ways. Different metrics on the space of shape bring out these distinctions.

# KERNEL MATRICES

Linear kernel matrices may be computed from the raw features.

$$\mathbf{K} = \mathbf{X}\mathbf{X}^T$$

A simple spatial feature selection may be considered as the following, where  $\mathbf{W}$  is a diagonal matrix of ones and zeros:

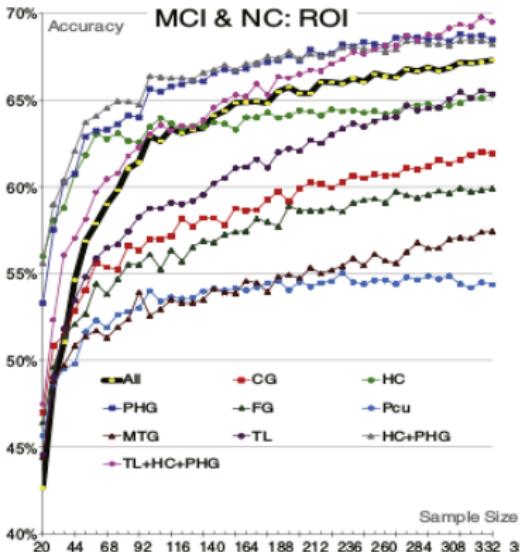
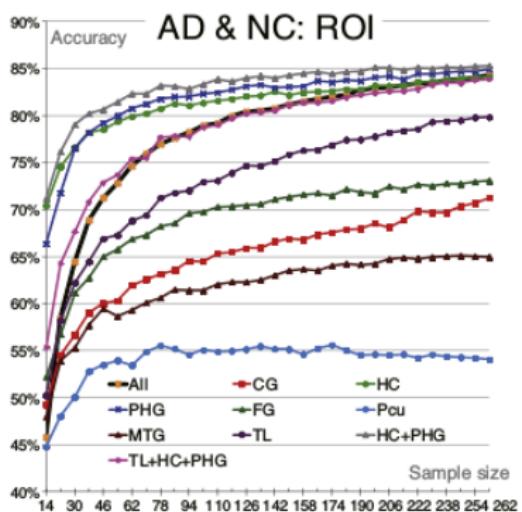
$$\mathbf{K} = \mathbf{X}\mathbf{W}\mathbf{X}^T$$

However,  $\mathbf{W}$  may be more complicated, for example encoding spatial smoothing, high-pass filtering or any number of other things.

# PRIOR KNOWLEDGE ABOUT BRAIN REGIONS INVOLVED

- The best way would be to augment the training data with data from previous studies.
- Lack of data-sharing means this is generally not possible, so we need to extract information from publications.
- The neuroimaging literature is mostly blobs.
- These give pointers about how best to weight the data ( $\mathbf{W} = \text{diag}(\mathbf{w})$ ,  $w_i \in \mathbb{R}^+$ ).

# WEIGHTING SUSPECTED REGIONS MORE HEAVILY



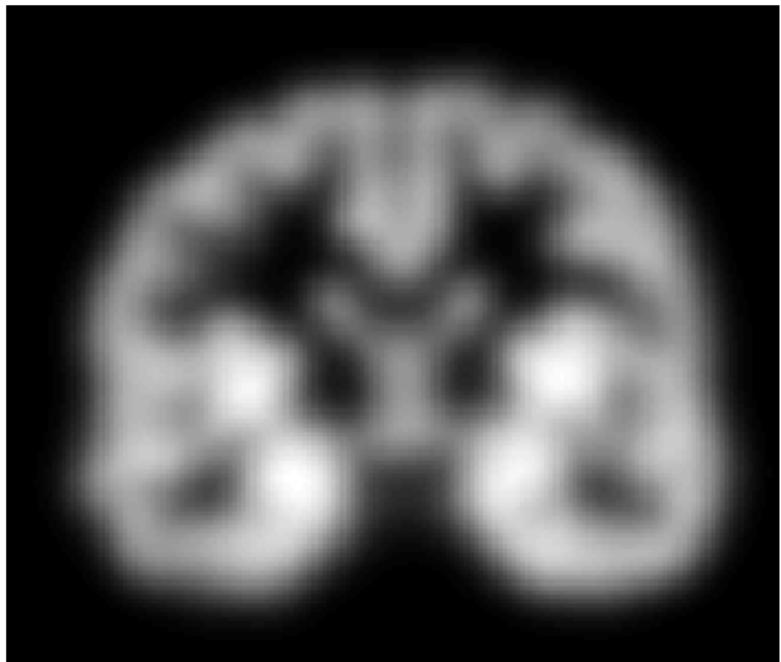
Chu et al. "Does feature selection improve classification accuracy? Impact of sample size and feature selection on classification using anatomical magnetic resonance images". *NeuroImage* 60:59–70 (2012).

# SMOOTHING CAN HELP

If we know that  
higher frequency  
signal is more likely to  
be noise.

$$\mathbf{K} = \mathbf{XW}\mathbf{X}^T$$

$\mathbf{W}$  no longer  
diagonal.



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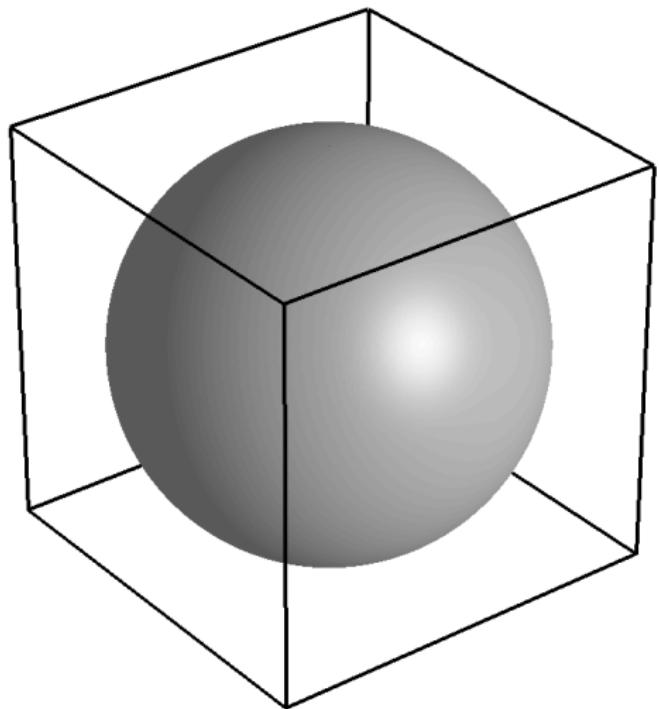
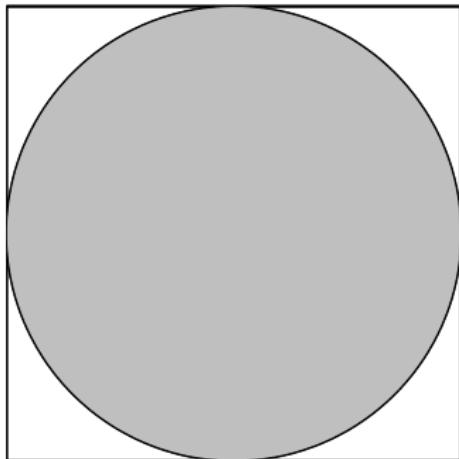
## 3 SIMILARITY BETWEEN BRAINS

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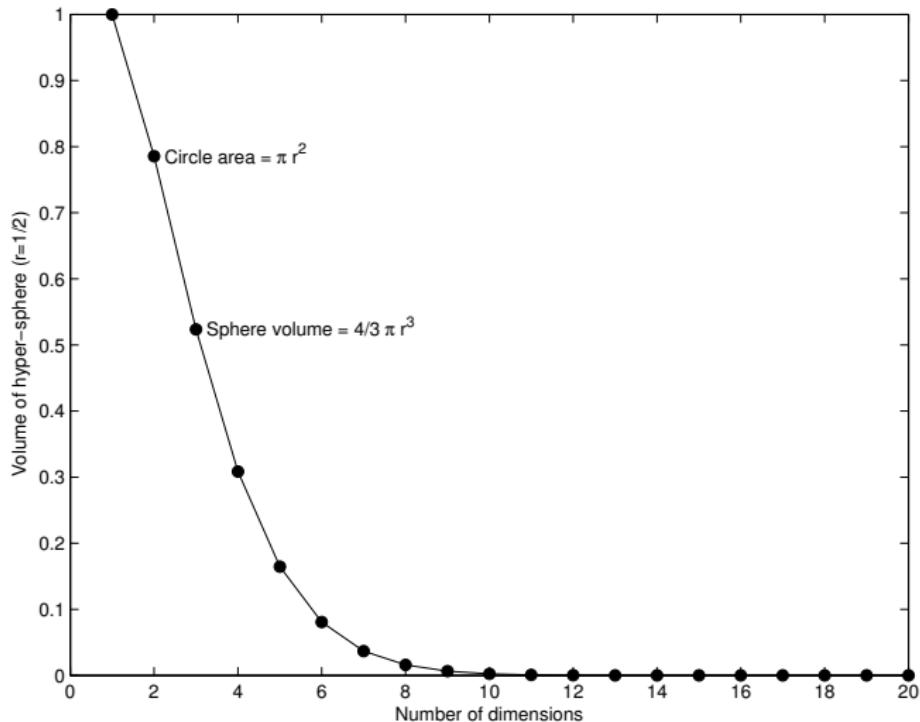
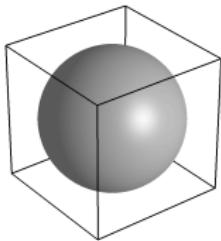
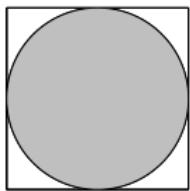
# CURSE OF DIMENSIONALITY

Large  $p$ , small  $n$ .

# UNINTUITIVE BEHAVIOUR IN HIGH-DIMENSIONS



# UNINTUITIVE BEHAVIOUR IN HIGH-DIMENSIONS



# DIMENSIONALITY $\neq$ NUMBER OF VOXELS

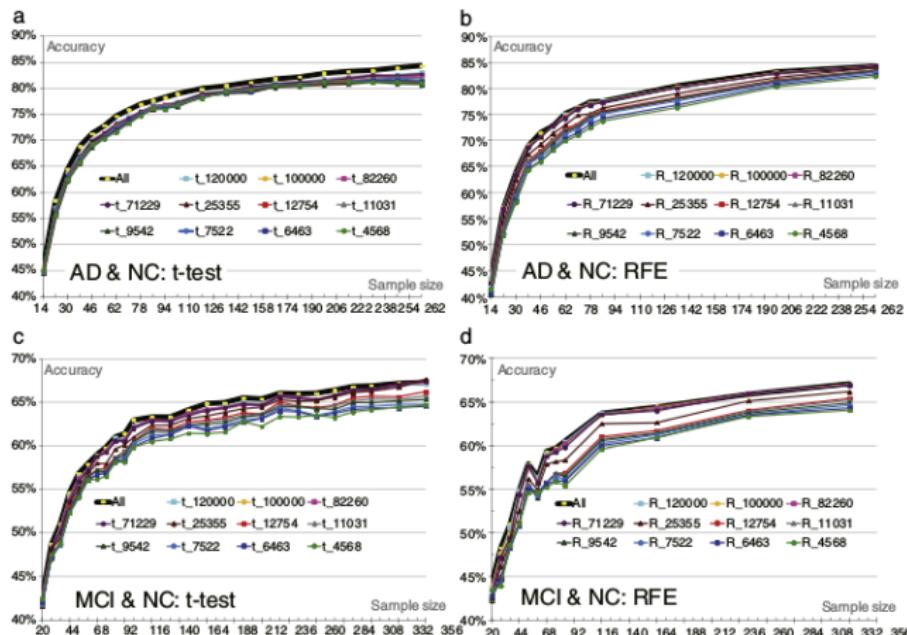
- Lots of effort on data-driven feature selection methods.
  - Involves estimating weighting matrix such that  $\mathbf{W} = \text{diag}(\mathbf{w})$ ,  $w_i \in \{0, 1\}$ .
  - Lots of parameters needed to achieve this.
- Many papers claim excellent results.
- Little evidence to suggest that most voxel-based feature selection methods help.
  - Little or no increase in predictive accuracy.
  - Commonly perceived as being more “interpretable”.

# “DATA-DRIVEN FEATURE SELECTION”

*“In our evaluation, two methods included a feature selection step: Voxel-STAND and Voxel-COMPARE. Overall, these methods did not perform substantially better than simpler ones... ... A more robust way to decrease the dimensionality of the features way would be to use more prior knowledge of the disease.”*

Cuingnet et al. “Automatic classification of patients with Alzheimer’s disease from structural MRI: A comparison of ten methods using the ADNI database”. *NeuroImage* 56(2):766–781 (2011).

# “DATA-DRIVEN FEATURE SELECTION”



Chu et al. “Does feature selection improve classification accuracy? Impact of sample size and feature selection on classification using anatomical magnetic resonance images”. *NeuroImage* 60:59–70 (2012).

# “DATA-DRIVEN FEATURE SELECTION”

...did not help the winning entry.

PITTSBURGH

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BRAIN ACTIVITY INTERPRETATION  
COMPETITION 2007

**2007 PBAIC Results**

First, second, and third place winners were announced at the Competition Workshop at the Organization for Human Brain Mapping being held June 10-14, 2007 in Chicago, Illinois, USA. Prizes will be awarded (1st: \$10,000, Tied-2nd: \$3,500; 3rd: \$2,000; and Neuroscience Award: \$5,000). Awards were presented during the morning workshop on Thursday, June 14, 2007. For more information, visit the [OHBM website](#).

[Audio](#) | [Video](#) | [Poster](#) | [Slides](#) | [Photos](#)

**Top 3 Winners**

**1st Place:** \$10,000 Prize  
[Carlton Chu, Yizhao Ni, Geoffrey Tan, John Ashburner](#)  
University College London  
Title: *Kernel Methods for fMRI Pattern Prediction – applications of Relevance Vector Regression and Kernel Ridge Regression*

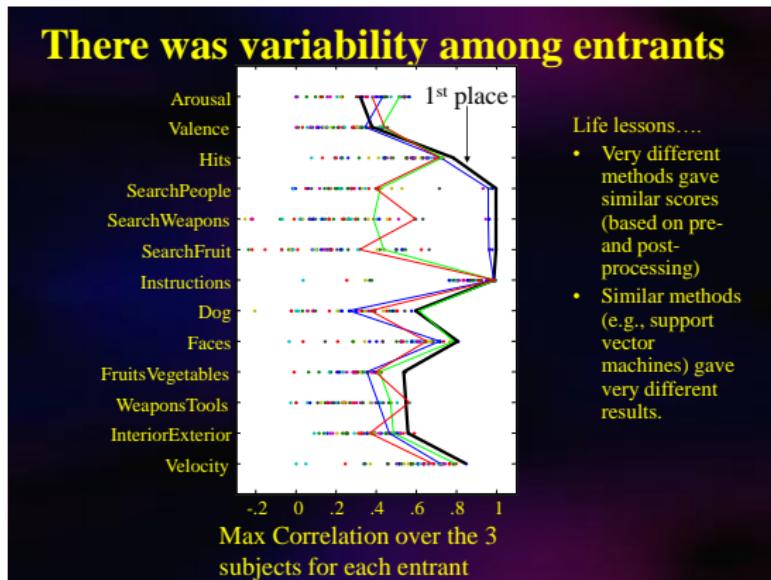
**2nd Place (tied):** \$3,500 Prize  
[Dennis Chingrey and The Princeton EBC Team](#)  
Princeton University  
Title: *One Size Does Not Fit All: Regressor and Subject Specific Techniques for Predicting Behavior in a Structured Environment*

**2nd Place (tied):** \$3,500 Prize  
[Giancarlo Valente, Federico De Martino, Fabrizio Esposito and the Maastricht Team](#)  
University of Maastricht  
Title: *Predictions of PBAIC 2007 Ratings with Linear Relevance Vector Machine regression*



<http://www.lrdc.pitt.edu/ebc/2007/2007.html>

# “DATA-DRIVEN FEATURE SELECTION”



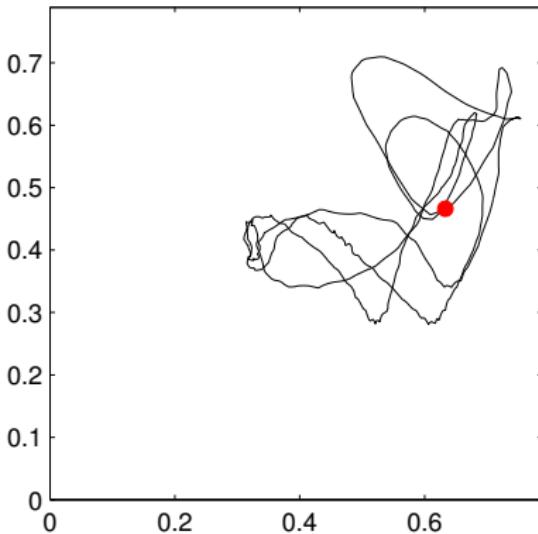
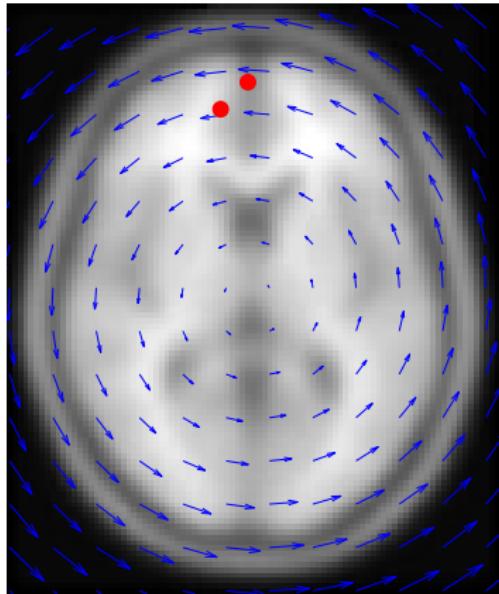
<http://www.lrdc.pitt.edu/ebc/2007/2007.html>

# MODELLING THE NONLINEARITIES

- Can capture many nonlinearities by appropriately modelling the data.
- Allows nonlinear effects to be modelled by a linear classifier.
- Gives more interpretable characterisations of differences.
- May lead to more accurate predictions.

# TRANSFORMED IMAGES FALL ON MANIFOLDS

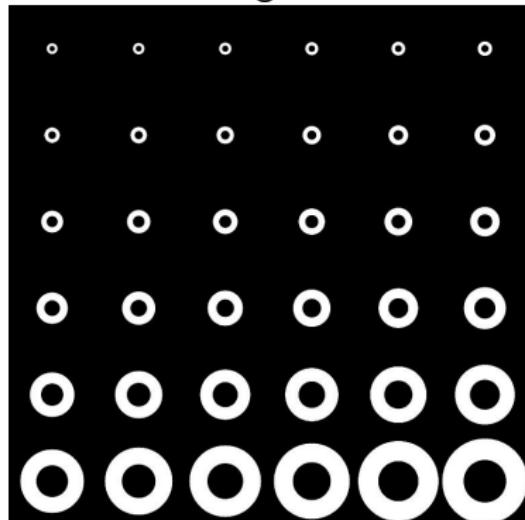
Rotating an image leads to points on a 1D manifold.



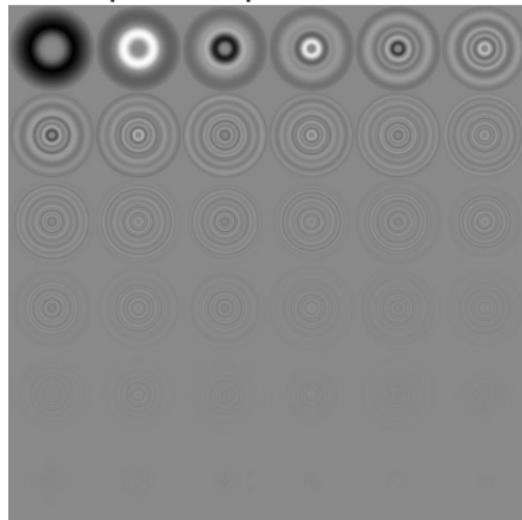
Rigid-body motion leads to a 6-dimensional manifold (not shown).

# ONE MODE OF GEOMETRIC VARIABILITY

Simulated images



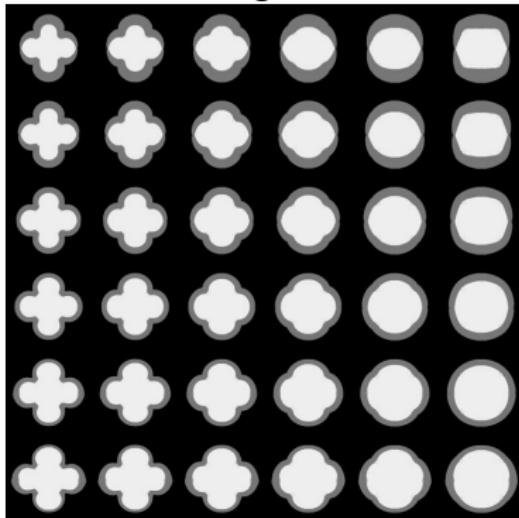
Principal components



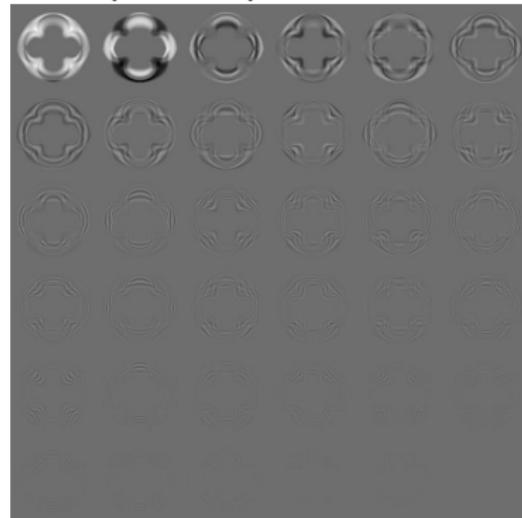
A suitable model would reduce this variability to a single dimension.

# TWO MODES OF GEOMETRIC VARIABILITY

Simulated images



Principal components



A suitable model would reduce this variability to two dimensions.

## 1 INTRODUCTION

## 2 MORPHOMETRICS

- Traditional morphometrics
- Geometric morphometrics
- Allometric relations

## 3 SIMILARITY BETWEEN BRAINS

## 4 CONCLUSION: WHAT NEXT?

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## “TRADITIONAL” MORPHOMETRICS

Traditional morphometrics analyzes lengths, widths, masses, angles, ratios and areas. In general, traditional morphometric data are measurements of size.

A drawback of using many measurements of size is that most will be highly correlated; as a result, there are few independent variables despite the many measurements.

Early analysis methods were often univariate, performing statistical analysis on each feature in isolation.

Wikipedia contributors, "Morphometrics," Wikipedia, The Free Encyclopedia,  
<http://en.wikipedia.org/w/index.php?title=Morphometrics&oldid=641987541> (accessed March 26, 2015).

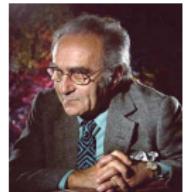
# CRITICISM OF UNIVARIATE APPROACHES

*“This unhappy result can be traced to the piecemeal tests which have hitherto been used. A bone or a tooth is a unit ; it is not a discrete assembly of independent measurements.”*

— Jacob Bronowski & W.M. Long (Nature, 1951)

*“The right statistical method must treat the set of variates as a single coherent matrix ; and this is, in fact, the technique of multivariate analysis.”*

— Jacob Bronowski & W.M. Long (Nature, 1951)



# BIOLOGICAL VARIABILITY IS MULTIVARIATE



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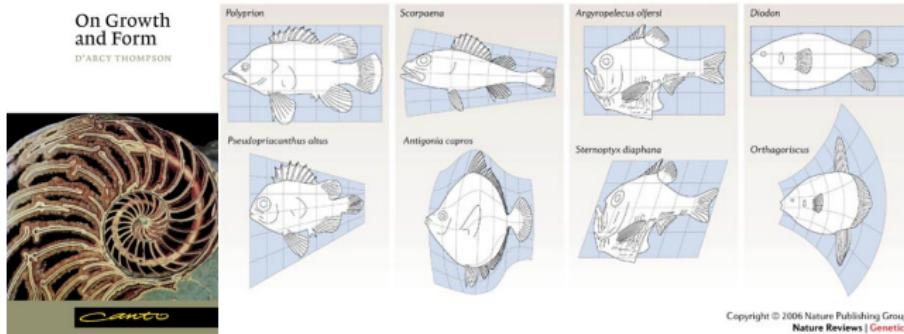
# GEOMETRIC MORPHOMETRICS

*"We are now in the midst of a revolution in morphometric methodology. The new approaches are more effective in capturing information about the shape of an organism and result in more powerful statistical procedures for testing for differences in shape. They are also more effective in enabling a researcher to visualize differences in shape and in suggesting simple traditional measurements that could be used in future studies."*

FJ Rohlf & LF Marcus. "A Revolution in Morphometrics". Trends in Ecology & Evolution 8.4: 129-132 (1993).

# D'ARCY THOMPSON'S APPROACH

*“...diverse and dissimilar fishes can be referred as a whole to identical functions of very different co-ordinate systems...”*



We can compute relative shapes using image registration.

# GEOMETRIC MORPHOMETRICS

- ① Data are recorded to capture the geometry in the form of 2D or 3D **coordinates of landmark points**.
- ② Geometric relationships among landmarks are not inherent in the raw coordinates themselves. **The relationship among points is captured by fitting an appropriate function to them** in 2 or 3D.
- ③ **The analyses are designed to indicate directions of maximum variation** and hence may suggest which conventional variables one should emphasize in verbal descriptions of the results.
- ④ Displays of the results of the analyses are emphasized, using **differences or changes that can be shown on pictorial representations** of the organisms studied.



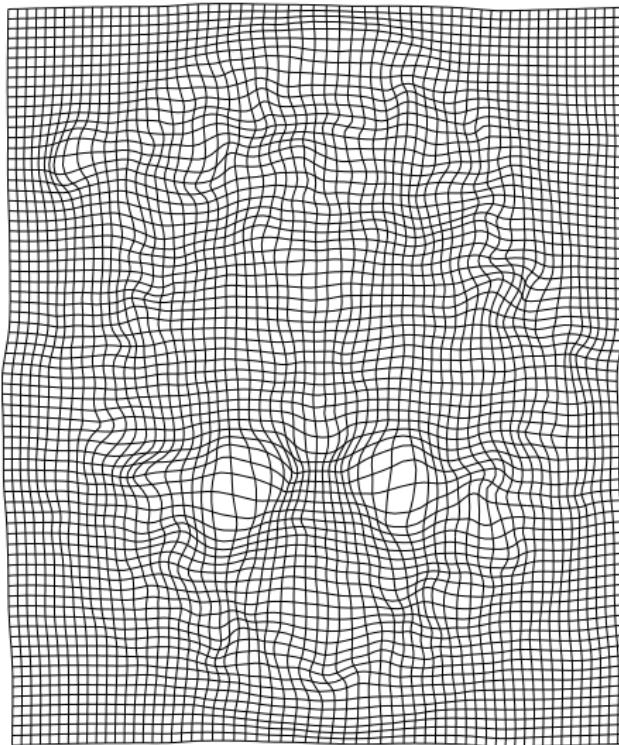
# GEOMETRIC MORPHOMETRICS

Classic multivariate statistical methods used:

- Procrustes analysis
- Principal Component Analysis (PCA)
- Canonical Correlation Analysis (CCA)
- Multivariate ANalysis of COVAriance (MANCOVA)
- Discriminant functions

These approaches could be augmented by pattern recognition and other machine learning techniques.

# ENCODING GEOMETRY



- Manual definition of landmarks is laborious, subjective and not very reproducible.
- Relatively few landmarks in the brain.
- Image registration is easier.
- Can apply the tools of geometric morphometrics (statistical shape analysis) to automatically estimated deformations.

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# LOGARITHMIC RELATIONSHIPS

## PROBLEMS OF RELATIVE GROWTH

### CHAPTER I

#### CONSTANT DIFFERENTIAL GROWTH-RATIOS

Huxley. "Problems of relative growth."

Methuen & co, London (1932).

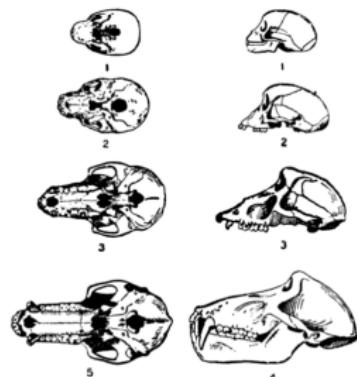


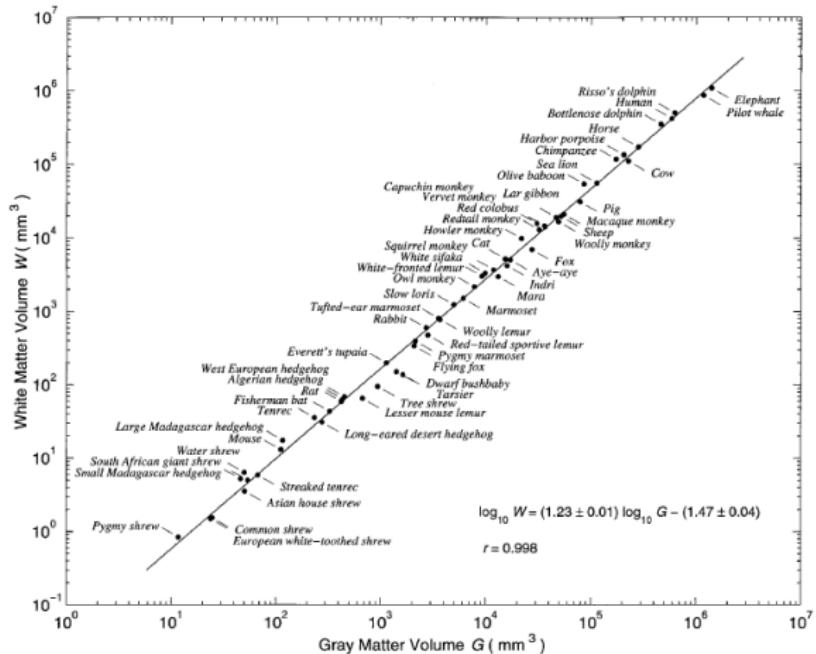
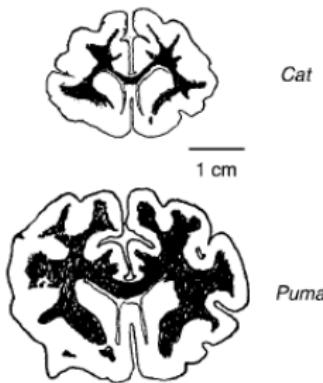
FIG. 10.—Baboon skulls of various sizes, to show the increase in relative size of facial region with absolute size of skull.

Julian Huxley demonstrated logarithmic relationships between magnitude variables (eg height, weight, length, area, volume).

$$\log y = a \log x + \log k$$

*"the logarithmic method of plotting brings into true relief an important point which is entirely obscured by the usual method of plotting on the absolute scale – namely the fact that growth is concerned essentially with the*

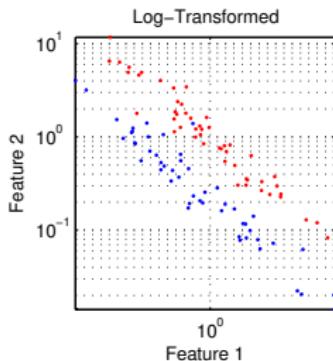
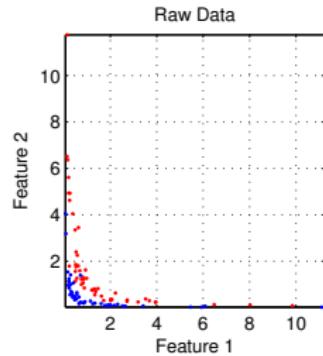
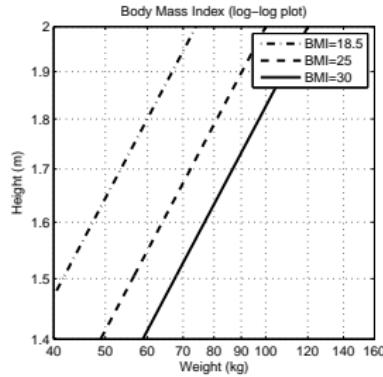
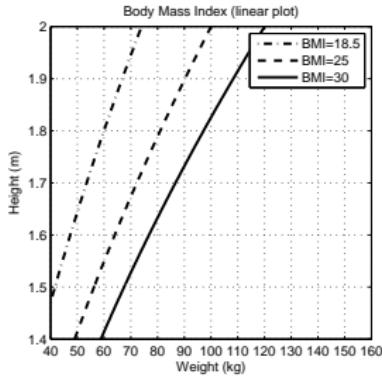
# LOGARITHMIC RELATIONSHIPS



Zhang and Sejnowski. "A universal scaling law between gray matter and white matter of cerebral cortex." Proceedings of the National Academy of Sciences 97(10):5621–5626 (2000).

# LOGARITHMIC RELATIONSHIPS

- Linear methods are more interpretable.
- Nonlinear methods usually increase dimensionality.
- Better to preprocess to obtain features that behave more linearly.



# EXPONENTIALS

There are many types of exponentials and their inverses.

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- Non-linear distances (manifolds)
- Image registration for measuring distances
- Tangent space representations
- Scalar momenta
- Empirical examples

## 4 CONCLUSION: WHAT NEXT?

## ① INTRODUCTION

## ② MORPHOMETRICS

## ③ SIMILARITY BETWEEN BRAINS

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- Image registration for measuring distances
- Tangent space representations
- Scalar momenta
- Empirical examples

## ④ CONCLUSION: WHAT NEXT?

# NON-EUCLIDEAN GEOMETRY

- Distances are not always measured along a straight line.
- Sometimes we want distances measured on a manifold.
- Shortest path on a manifold is along a *geodesic*.



Linear trajectory



Nonlinear trajectory



# METRIC DISTANCES

Distances should satisfy the properties of a *metric*:

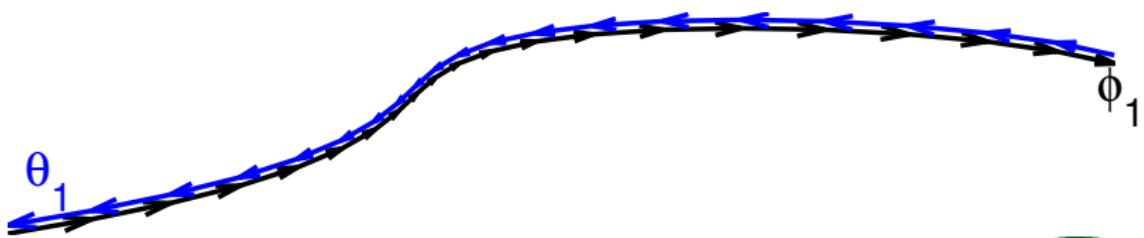
- ①  $d(\mathbf{x}, \mathbf{y}) \geq 0$  (non-negativity)
- ②  $d(\mathbf{x}, \mathbf{y}) = 0$  if and only if  $\mathbf{x} = \mathbf{y}$  (identity of indiscernibles)
- ③  $d(\mathbf{x}, \mathbf{y}) = d(\mathbf{y}, \mathbf{x})$  (symmetry)
- ④  $d(\mathbf{x}, \mathbf{z}) \leq d(\mathbf{x}, \mathbf{y}) + d(\mathbf{y}, \mathbf{z})$  (triangle inequality).

Satisfying (3) requires inverse-consistent image registration.

Satisfying (4) requires a specific class of image registration models.



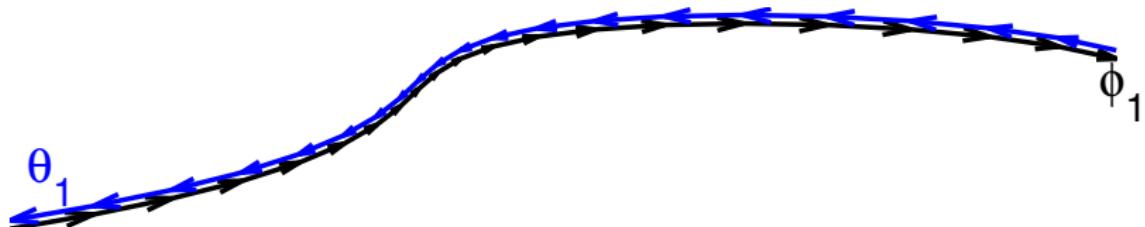
# COMPUTING A METRIC DISTANCE



Decompose a curved path into a series of short line segments, and add the lengths of the segments together.



# COMPUTING LARGE DEFORMATIONS



We can consider a large deformation as the composition of a series of small deformations:

$$\varphi_1 = (\text{Id} + \mathbf{v}_{t_{N-1}}) \circ (\text{Id} + \mathbf{v}_{t_{N-2}}) \circ \dots \circ (\text{Id} + \mathbf{v}_{t_1}) \circ (\text{Id} + \mathbf{v}_0)$$

The inverse of the deformation can be computed from:

$$\vartheta_1 = (\text{Id} - \mathbf{v}_0) \circ (\text{Id} - \mathbf{v}_{t_1}) \circ \dots \circ (\text{Id} - \mathbf{v}_{t_{N-2}}) \circ (\text{Id} - \mathbf{v}_{t_{N-1}})$$

# METRIC DISTANCES FROM LARGE DEFORMATIONS

By modelling trajectories as piecewise linear, distances can be computed by adding the distances from the small deformations:

$$d = \frac{1}{N} \sum_{n=0}^{N-1} \|\mathbf{L}\mathbf{v}_{t_n}\|$$

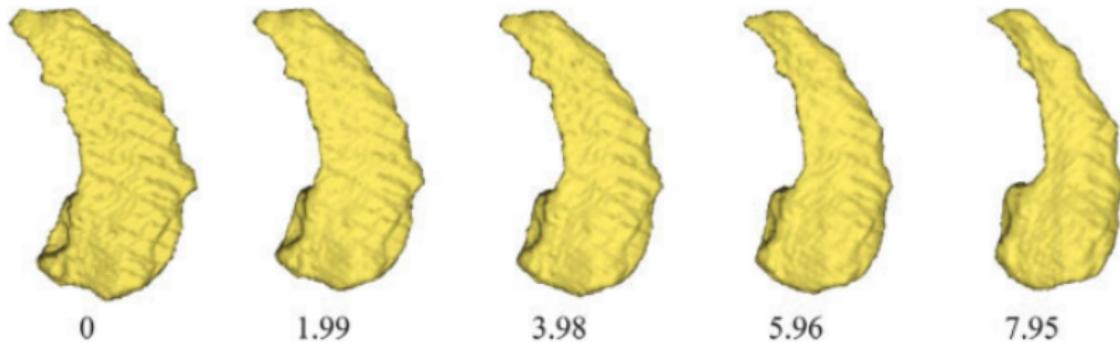
If  $N$  approaches infinity (and we use small deformations of  $Id + \frac{1}{N}\mathbf{v}_t$ ), the evolution of a deformation may be conceptualised as integrating the following equation:

$$\frac{d\varphi}{dt} = \mathbf{v}_t(\varphi)$$

Geodesic distances (from zero) are then measured by:

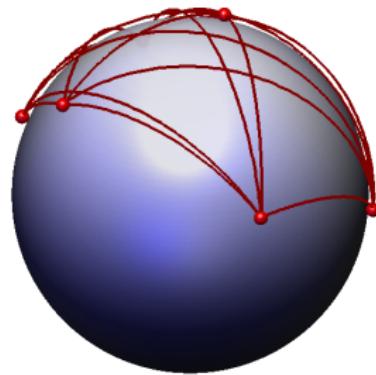
$$d = \int_{t=0}^1 \|\mathbf{L}\mathbf{v}_t\| dt$$

# METRIC DISTANCES FROM LARGE DEFORMATIONS



Miller et al. "Collaborative computational anatomy: an MRI morphometry study of the human brain via diffeomorphic metric mapping." Human Brain Mapping 30(7):2132–2141 (2009).

# METRIC DISTANCES FROM LARGE DEFORMATIONS



Miller et al. "Collaborative computational anatomy: an MRI morphometry study of the human brain via diffeomorphic metric mapping." Human Brain Mapping 30(7):2132–2141 (2009).

## ① INTRODUCTION

## ② MORPHOMETRICS

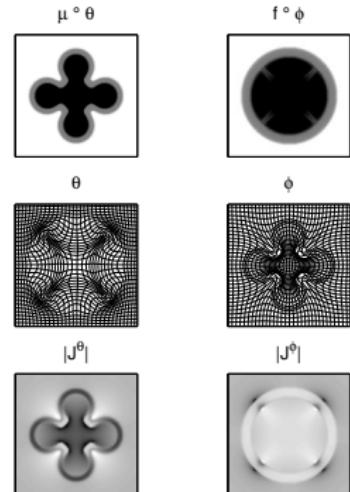
## ③ SIMILARITY BETWEEN BRAINS

- Non-linear distances (manifolds)
- **Image registration for measuring distances**
- Tangent space representations
- Scalar momenta
- Empirical examples

## ④ CONCLUSION: WHAT NEXT?

# IMAGE REGISTRATION

- Image registration measures distances between images.
- Often involves minimising the sum of two terms:
  - Distance between the image intensities.
  - Distance of the deformation from the identity.
- The sum of these terms gives the distance.



# LDMM

*Large Deformation Diffeomorphic Metric Mapping* is an image registration algorithm that minimises the following:

$$\mathcal{E} = \frac{1}{2} \int_{t=0}^1 \|\mathbf{L}\mathbf{v}_t\|^2 dt + \frac{1}{2\sigma^2} \|f - \mu(\varphi_1^{-1})\|^2$$

where  $\varphi_0 = \text{Id}$ ,  $\frac{d\varphi}{dt} = \mathbf{v}_t(\varphi_t)$

The first term minimises the squared distance measure of the deformations, whereas the second term simply minimises the difference between the warped template and the individual scan.  
The objective is to estimate a series of velocity fields ( $\mathbf{v}_t$ ).

Beg, MF, Miller, MI, Trouvé, A & Younes, L. *Computing large deformation metric mappings via geodesic flows of diffeomorphisms*. International Journal of Computer Vision 61(2):139–157 (2005).

## ① INTRODUCTION

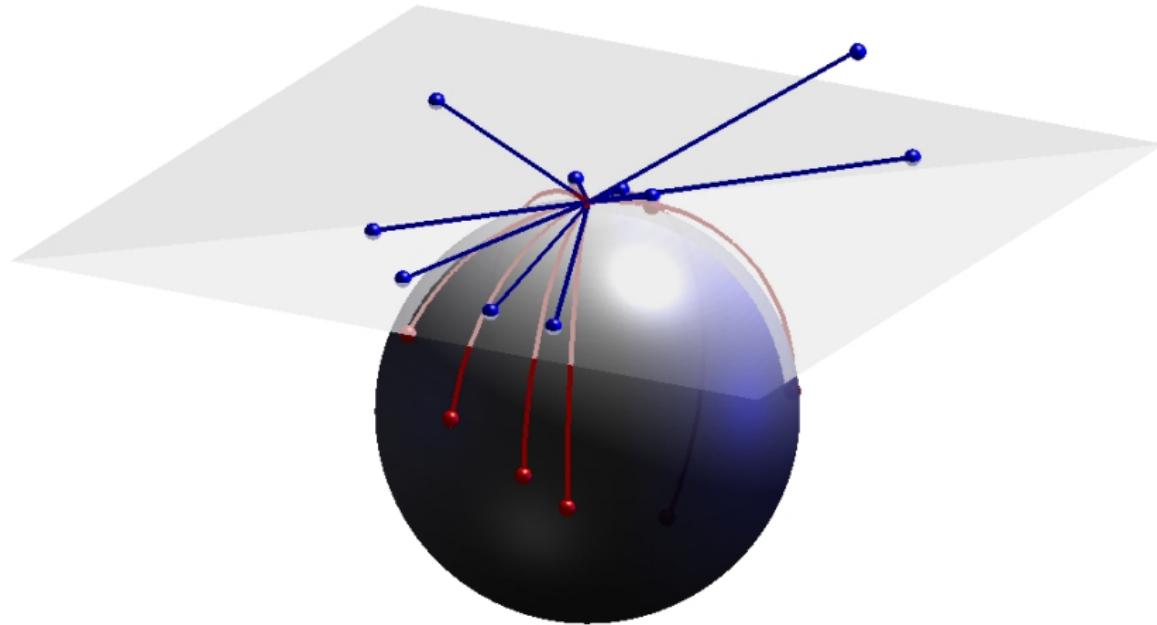
## ② MORPHOMETRICS

## ③ SIMILARITY BETWEEN BRAINS

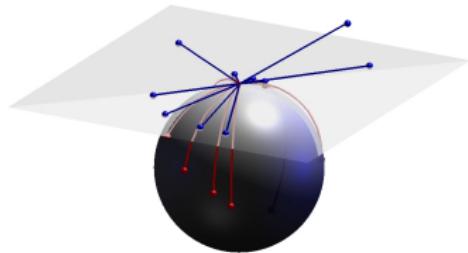
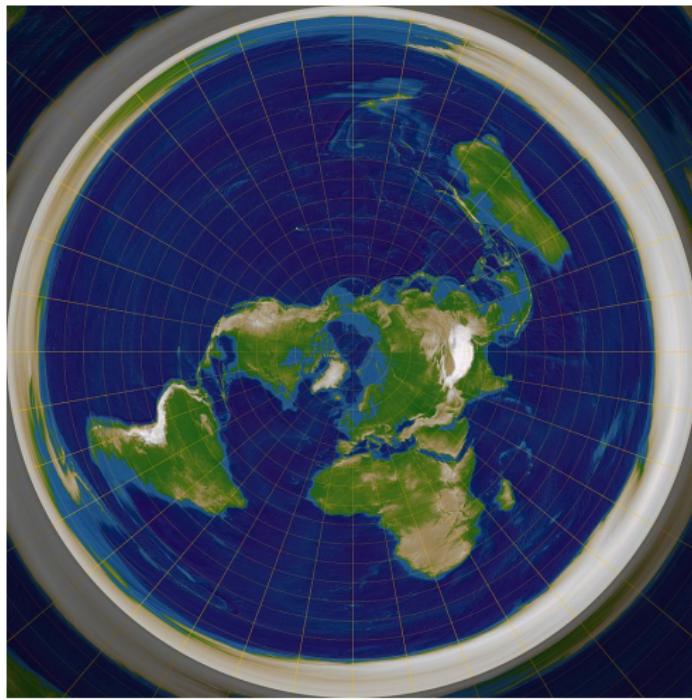
- Non-linear distances (manifolds)
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## ④ CONCLUSION: WHAT NEXT?

# LINEAR APPROXIMATIONS TO NONLINEAR PROBLEMS



# EXPONENTIAL MAP



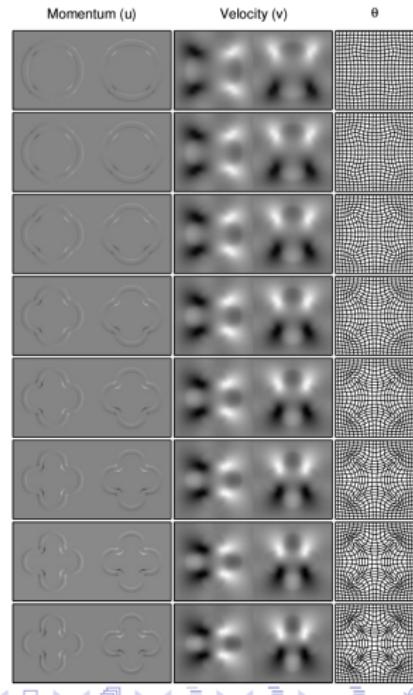
"Azimuthal Equidistant N90" by RokerHRO - Own work. Licensed under CC BY-SA 3.0 via Wikimedia Commons - [http://commons.wikimedia.org/wiki/File:Azimuthal\\_Equidistant\\_N90.jpg](http://commons.wikimedia.org/wiki/File:Azimuthal_Equidistant_N90.jpg)

# LDMM VIA “GEODESIC SHOOTING”

In practice, we just need to estimate an initial velocity ( $\mathbf{v}_0$ ), from which we compute the initial momentum by  $\mathbf{u}_0 = \mathbf{L}^\dagger \mathbf{L} \mathbf{v}_0$ .

We set the deformation at time 0 to an identity transform ( $\varphi_0 = Id$ ), and then evolve the following dynamical system for unit time:

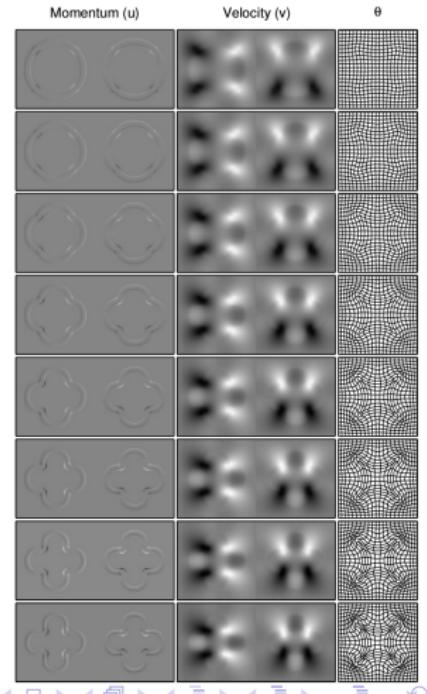
$$\begin{aligned}\mathbf{u}_t &= \det |\mathbf{D}\varphi_t^{-1}| (\mathbf{D}\varphi_t^{-1})^T (\mathbf{u}_0 \circ \varphi_t^{-1}) \\ \mathbf{v}_t &= (\mathbf{L}^\dagger \mathbf{L})^{-1} \mathbf{u}_t \\ \frac{d\varphi}{dt} &= \mathbf{v}_t(\varphi_t)\end{aligned}$$



# LDMM VIA “GEODESIC SHOOTING”

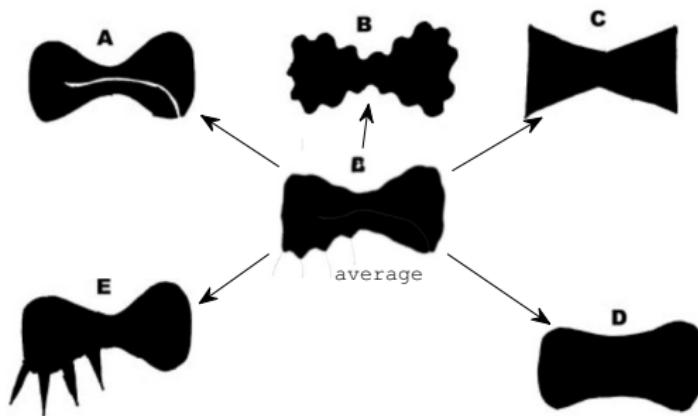
The final deformation ( $\varphi_1$ ) is a type of exponential of the initial velocity ( $v_0$ ).

Exponential map (Riemannian geometry). (2015, January 13). In Wikipedia, The Free Encyclopedia. Retrieved 18:04, March 31, 2015, from [http://en.wikipedia.org/w/index.php?title=Exponential\\_map\\_\(Riemannian\\_geometry\)&oldid=642372186](http://en.wikipedia.org/w/index.php?title=Exponential_map_(Riemannian_geometry)&oldid=642372186)



# “GROUPOWISE REGISTRATION”

Minimising distortions by centering around the mean.



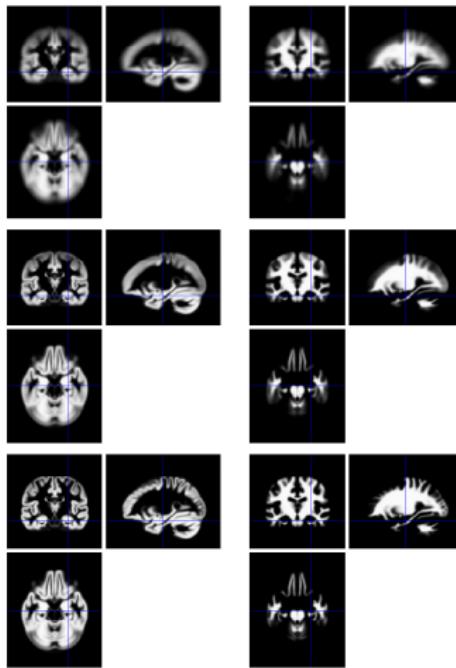
# “GROUPOWISE REGISTRATION”



# “GROUPWISE REGISTRATION”

Ignoring the many technical details, the procedure involves alternating between:

- Create the mean of aligned images.
- Align all images to be slightly closer to the mean.



# KERNEL MATRIX

Construction of kernel matrix accounts for the regularisation used by the image registration:

$$\begin{aligned} k(\mathbf{v}, \mathbf{w}) &= \langle \mathbf{L}^\dagger \mathbf{L} \mathbf{v}, \mathbf{w} \rangle \\ &= \langle \mathbf{L} \mathbf{v}, \mathbf{L} \mathbf{w} \rangle \end{aligned}$$

Wang, Lei, et al. "Large deformation diffeomorphism and momentum based hippocampal shape discrimination in dementia of the Alzheimer type." Medical Imaging, IEEE Transactions on 26(4):462–470 (2007).

## 1 INTRODUCTION

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## 4 CONCLUSION: WHAT NEXT?

# GEODESIC SHOOTING

At the solution, gradients of the objective function should vanish:

$$\mathbf{L}^\dagger \mathbf{L} \mathbf{v}_0 + \frac{1}{\sigma^2} \det |\mathbf{D}\varphi_1| (f \circ \varphi_1 - \mu) (\nabla \mu) = 0$$

Re-expressing this, we see that the initial velocity and momentum is given by:

$$\mathbf{L}^\dagger \mathbf{L} \mathbf{v}_0 = \mathbf{u}_0 = \frac{1}{\sigma^2} (\nabla \mu) \det |\mathbf{D}\varphi_1| (\mu - f \circ \varphi_1)$$

# “SCALAR MOMENTUM”

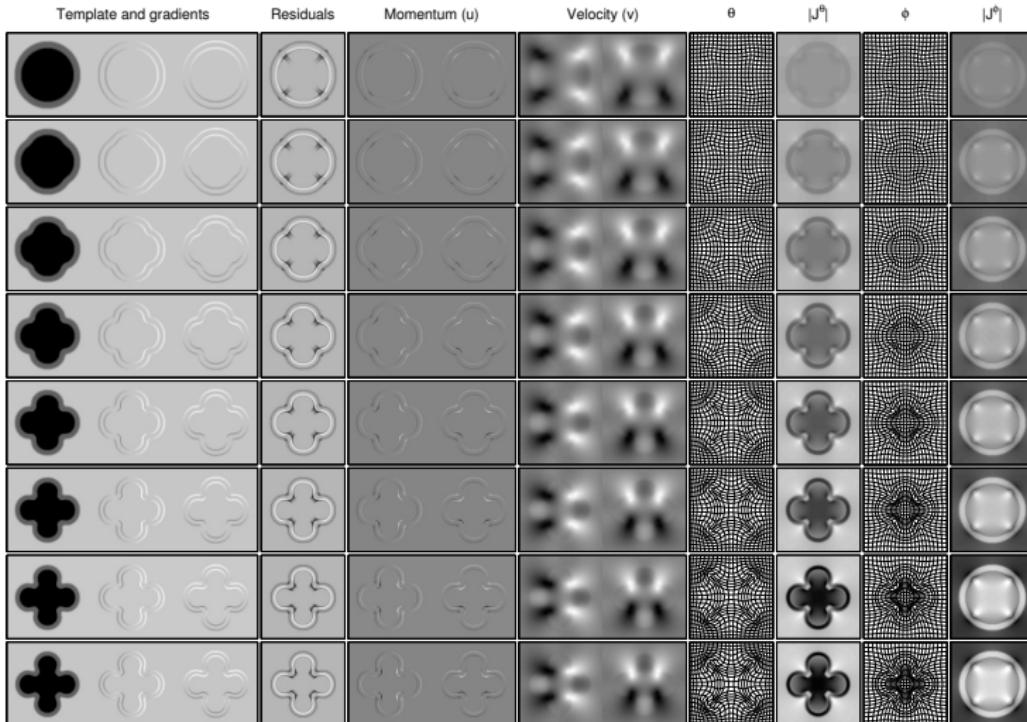
$$\mathbf{u}_0 = \frac{1}{\sigma^2} (\nabla \mu) \det |\mathbf{D}\varphi_1| (\mu - f \circ \varphi_1)$$

If a population of subjects are all aligned with the same template image,  $\frac{1}{\sigma^2} (\nabla \mu)$  will be the same for all subjects. Deviations from the template are encoded by the “*scalar momentum*”,  $\det |\mathbf{D}\varphi_1| (\mu - f \circ \varphi_1)$ . This is a scalar field, and in principle is all that is needed (along with the template) to reconstruct the original images.

Miller et al. “Collaborative computational anatomy: an MRI morphometry study of the human brain via diffeomorphic metric mapping.” *Human Brain Mapping* 30(7):2132–2141 (2009).

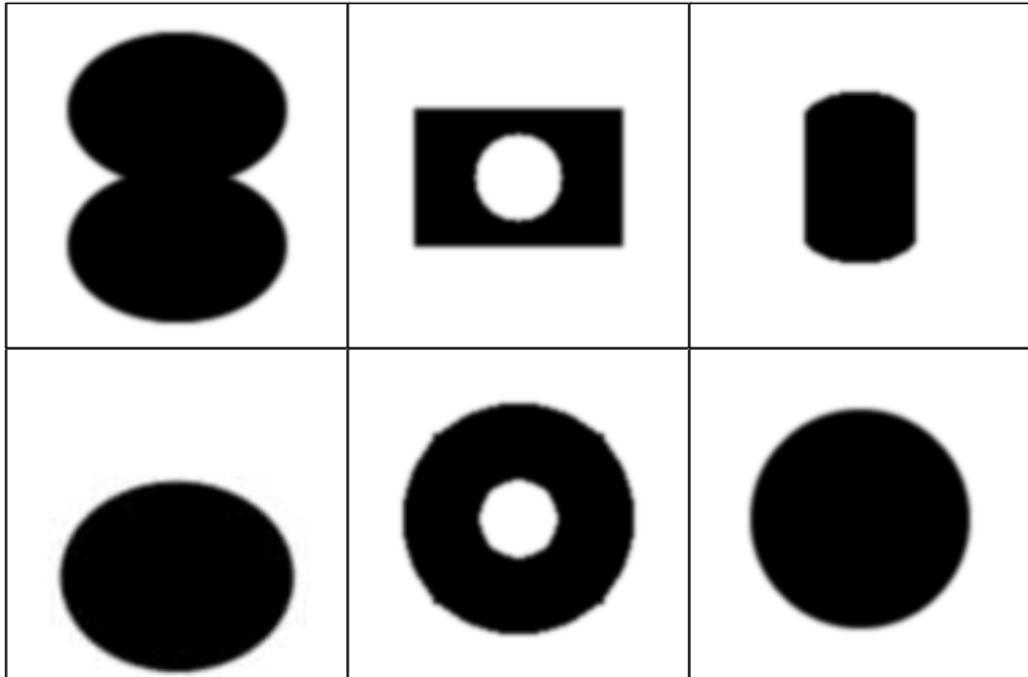
Singh, Fletcher, Preston, Ha, King, Marron, Wiener & Joshi (2010). *Multivariate Statistical Analysis of Deformation Momenta Relating Anatomical Shape to Neuropsychological Measures*. T. Jiang et al. (Eds.): MICCAI 2010, Part III, LNCS 6363, pp. 529–537, 2010.

# EVOLUTION



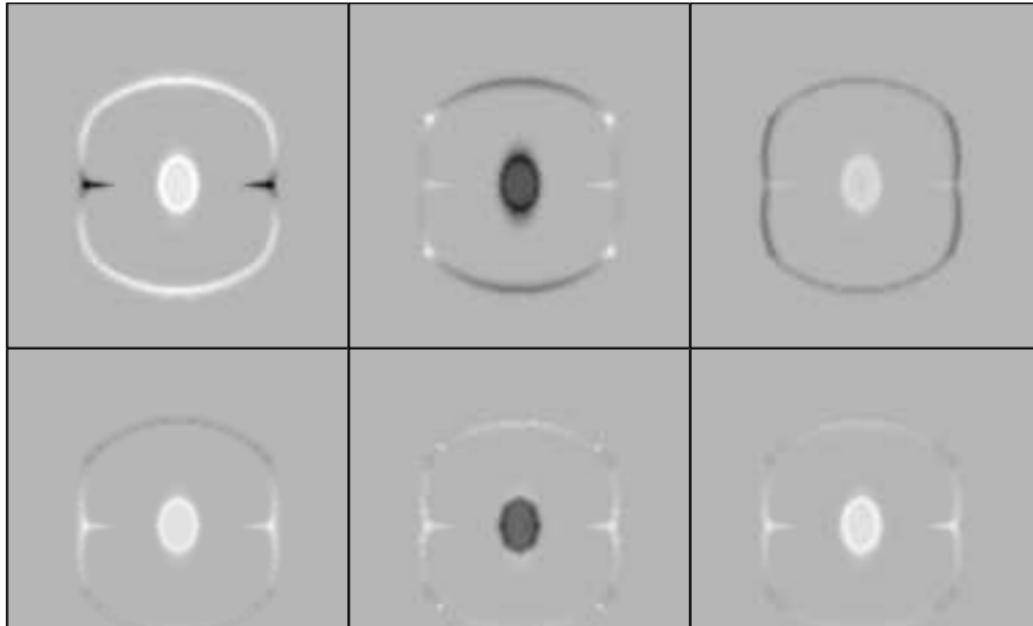
# EXAMPLE IMAGES

Some example images.



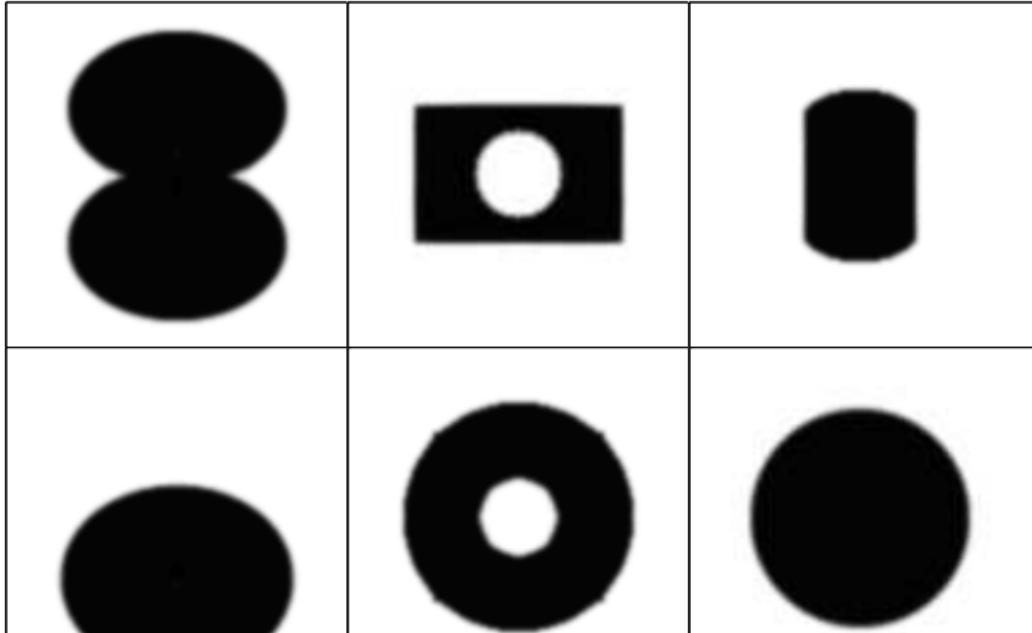
# SCALAR MOMENTUM

Scalar momentum after aligning the examples to a common template.



# RECONSTRUCTED IMAGES

Images reconstructed using just the template and scalar momentum.



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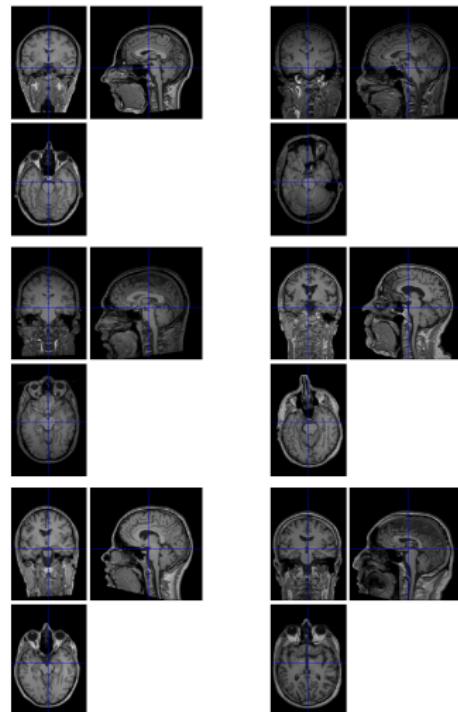
## ④ CONCLUSION: WHAT NEXT?

# REAL DATA

Used 550 T1w brain MRI from  
**IXI** (**I**nformation **X**traction from  
**I**mages) dataset.

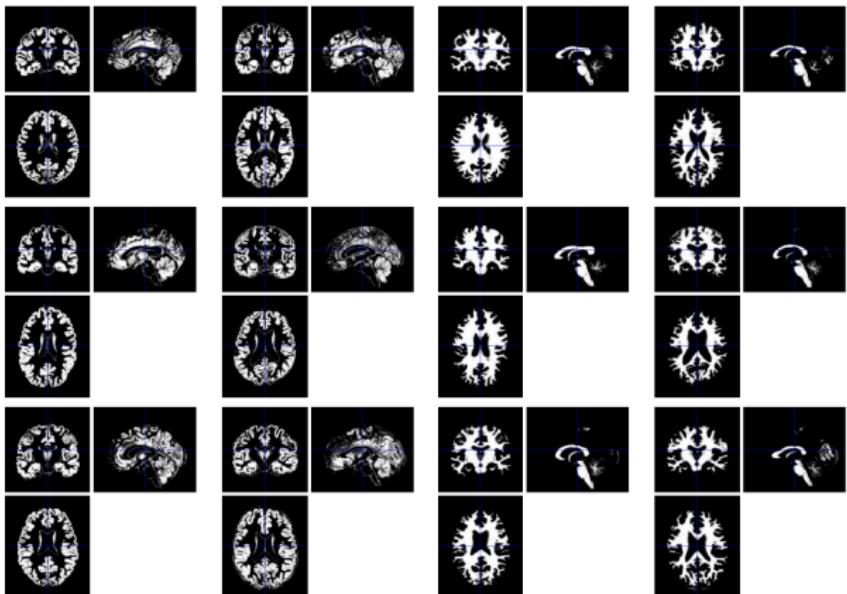
[http://www.  
brain-development.org/](http://www.brain-development.org/)  
Data from three different  
hospitals in London:

- Hammersmith Hospital using a Philips 3T system
- Guy's Hospital using a Philips 1.5T system
- Institute of Psychiatry using a GE 1.5T system



# GREY AND WHITE MATTER

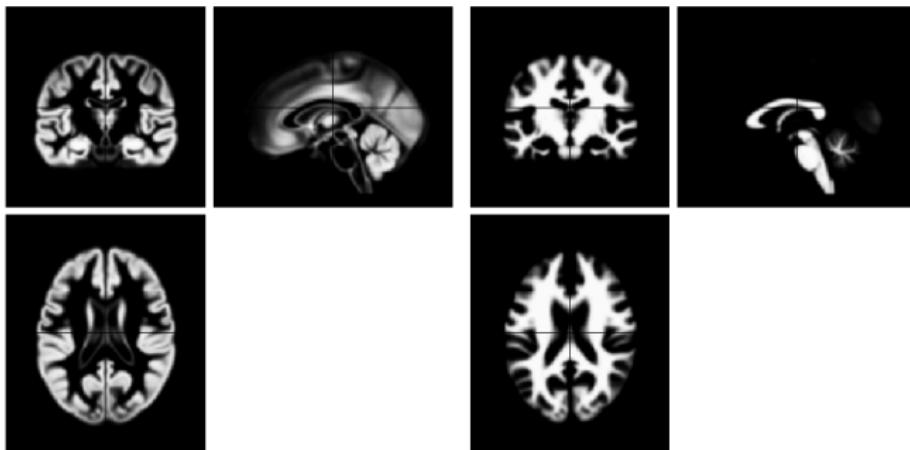
Segmented into  
GM and WM.  
Approximately  
aligned via  
rigid-body.



Ashburner, J & Friston, KJ. *Unified segmentation*. *NeuroImage* 26(3):839–851 (2005).

# DIFFEOMORPHIC ALIGNMENT

All GM and WM were diffeomorphically aligned to their common average-shaped template.



Ashburner, J & Friston, KJ. *Diffeomorphic registration using geodesic shooting and Gauss-Newton optimisation*. *NeuroImage* 55(3):954–967 (2011).

Ashburner, J & Friston, KJ. *Computing average shaped tissue probability templates*. *NeuroImage* 45(2):333–341 (2009).

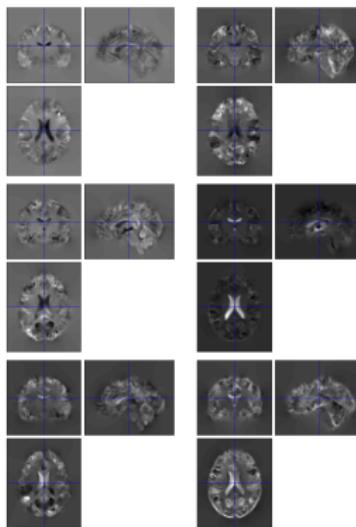
# VOLUMETRIC FEATURES

A number of features were used for pattern recognition.

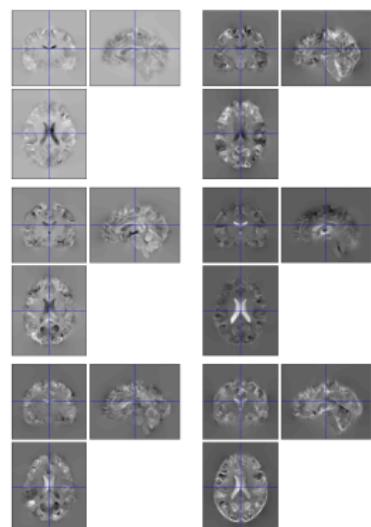
Firstly, two features relating to relative volumes.

Initial velocity divergence is similar to logarithms of Jacobian determinants.

Jacobian Determinants

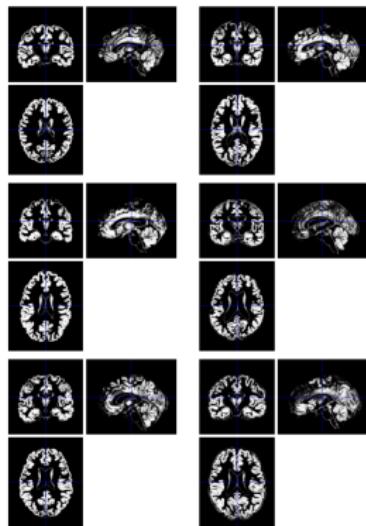


Initial Velocity Divergence

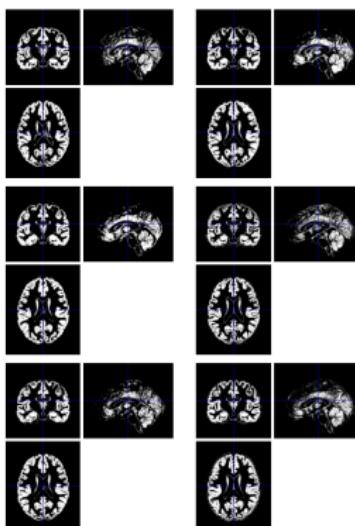


# GREY MATTER FEATURES

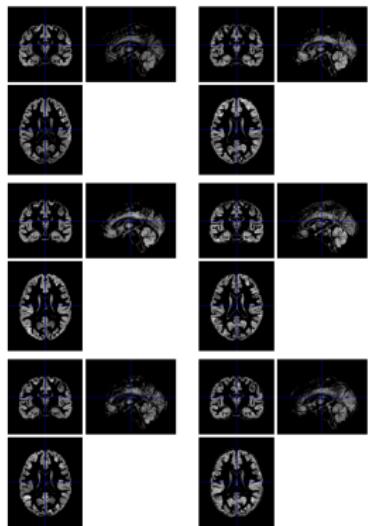
Rigidly Registered  
GM



Nonlinearly  
Registered GM

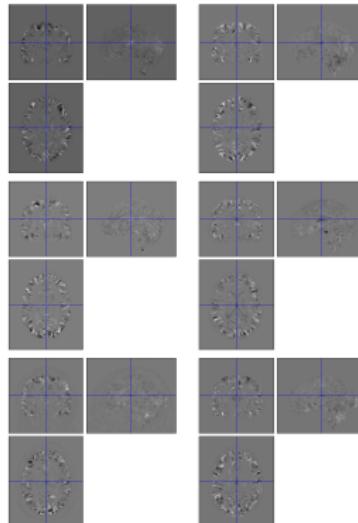


Registered and  
Jacobian Scaled GM



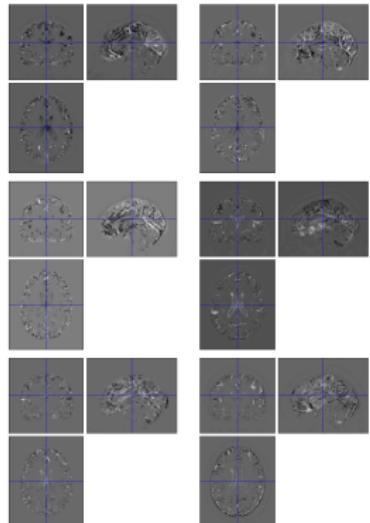
# “SCALAR MOMENTUM” FEATURES

First Momentum Component



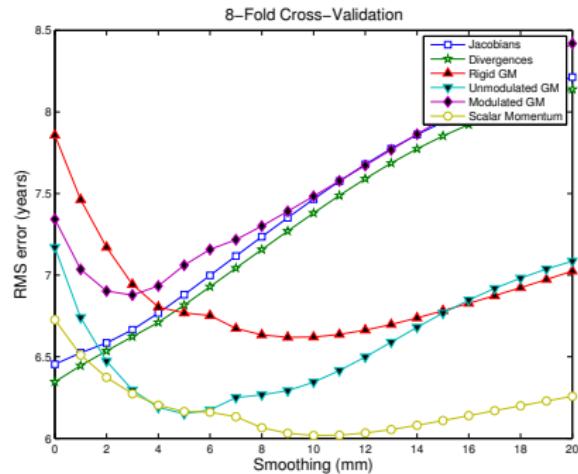
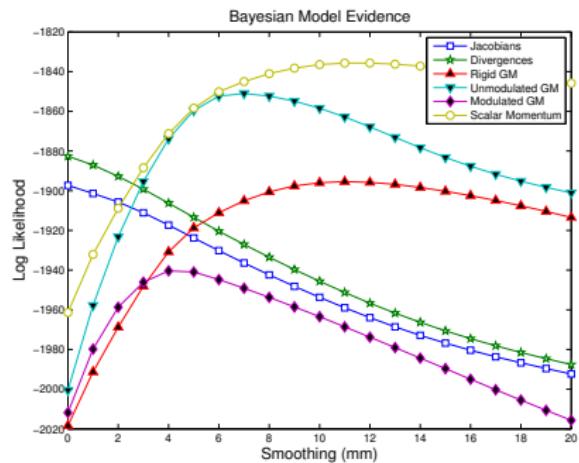
“Scalar momentum” actually has two components because GM was matched with GM and WM was matched with WM.

Second Momentum Component



# AGE REGRESSION

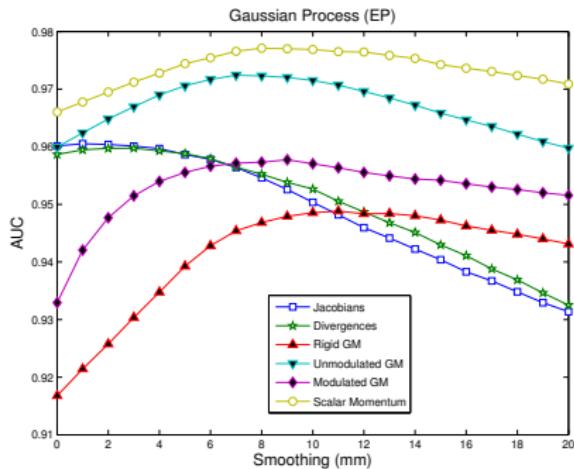
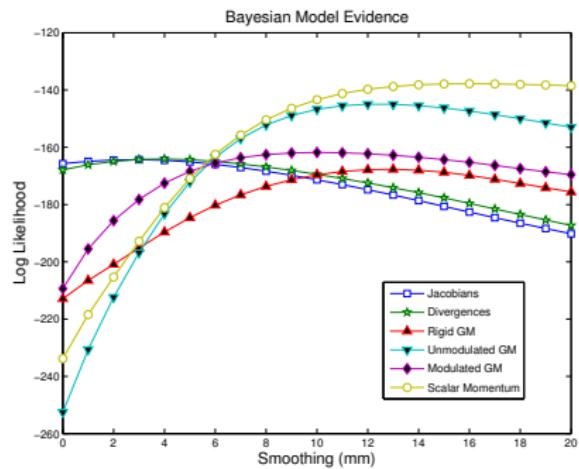
Linear Gaussian Process Regression to predict subject ages.



Rasmussen, CE & Williams, CKI. *Gaussian processes for machine learning*. Springer (2006).

# SEX CLASSIFICATION

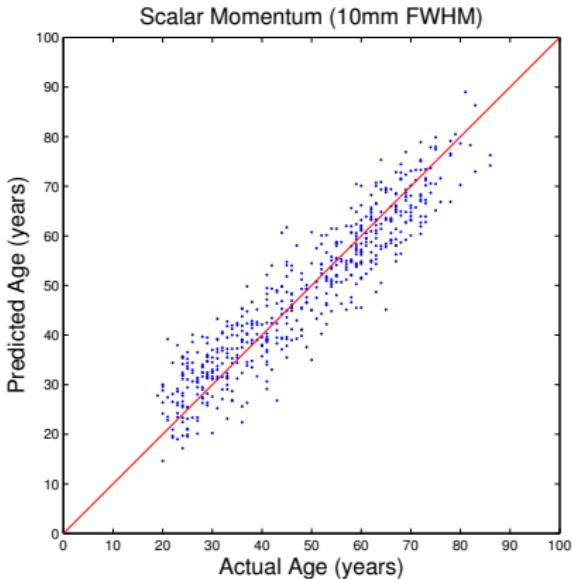
Linear Gaussian Process Classification (EP) to predict sexes.



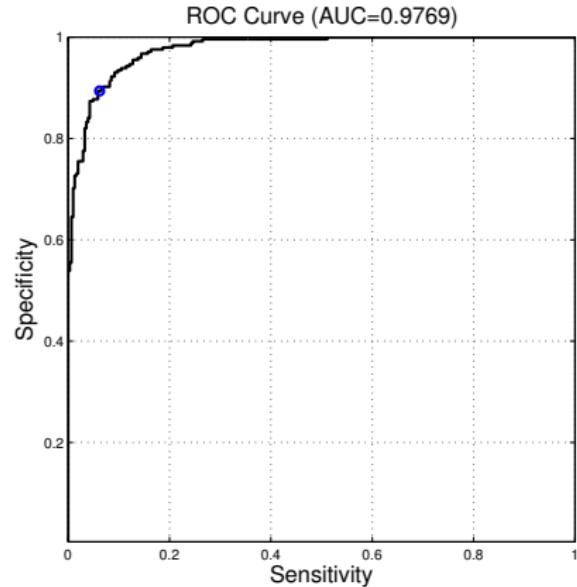
Rasmussen, CE & Williams, CKI. *Gaussian processes for machine learning*. Springer (2006).

# PREDICTIVE ACCURACIES

Age



Sex



# CONCLUSIONS

- Scalar momentum (with about 10mm smoothing) appears to be a useful feature set.
- Jacobian-scaled warped GM is surprisingly poor.
- Amount of spatial smoothing makes a big difference.
- Further dependencies on the details of the registration still need exploring.

## 1 INTRODUCTION

## 2 MORPHOMETRICS

## 3 SIMILARITY BETWEEN BRAINS

## 4 CONCLUSION: WHAT NEXT?

- Dealing with uncertain labels
- Visualising differences

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- Visualising differences

*...all clinically diagnosed AD patients will not have AD pathology, and up to 30% of cognitively normal subjects will have a pathologic diagnosis of AD at autopsy.*

Vemuri et al. "Antemortem MRI based STructural Abnormality iNDex (STAND)-scores correlate with postmortem Braak neurofibrillary tangle stage". *NeuroImage* 42(2):559–567 (2008).

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- Visualising differences

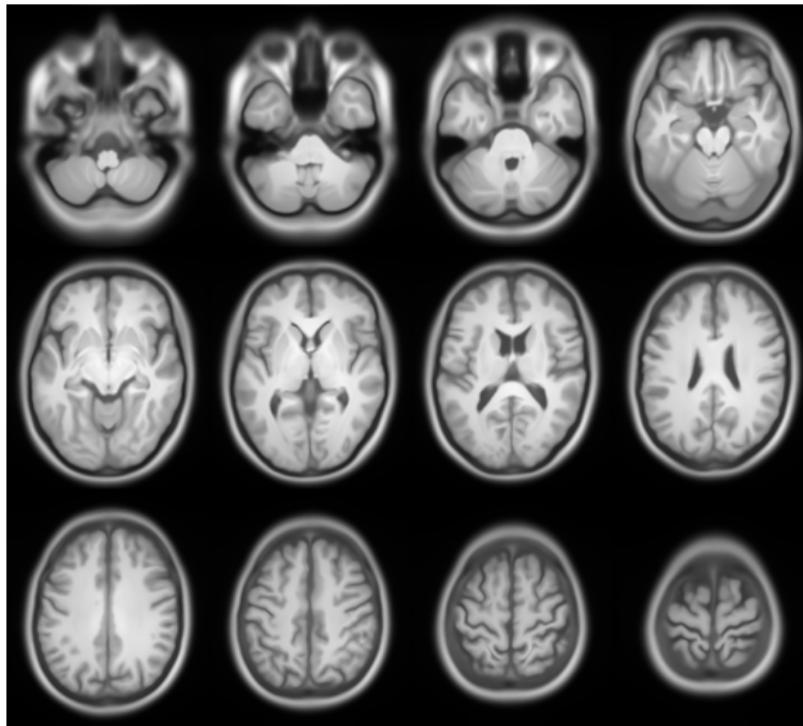
# VISUALISING DIFFERENCES

Neuroimagers like their blobs and summary tables.

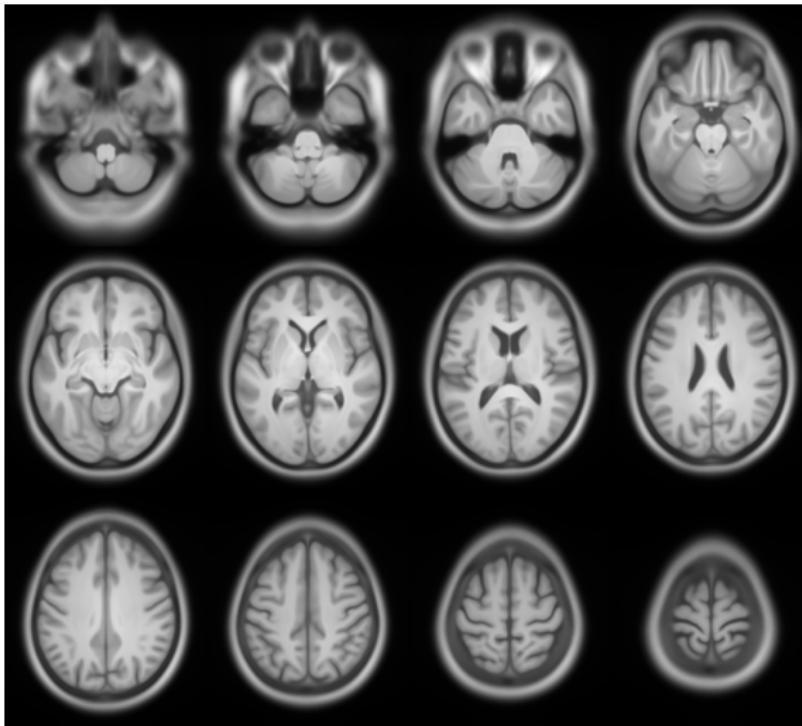
How do we best understand whole-brain multivariate differences?

Here's my attempt.....

# EXAGGERATED MALE BRAIN



# AVERAGE BRAIN



# EXAGGERATED FEMALE BRAIN

