Paraconsistent Epistemic And Contextual Evaluation (PEACE): A Meta-Logical Framework for Contextual Reasoning Beyond Classical Proof

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Abstract

We present **Paraconsistent Epistemic And Contextual Evaluation (PEACE)**, a trivalent, context-sensitive, meta-logical supplement to classical proof. PEACE elevates epistemic humility, contextual reasoning, and heuristic perspectives above strict deduction, while preserving classical rigor when information is unambiguous. The third value B (*both/undecided*) is the default assignment and is *meta-collapsible* to T or F when relevant perspectives converge with sufficient confidence. We formalize the semantics, provide operational truth tables consistent with the collapse dynamics, and state inference principles that integrate seamlessly with classical logic when context is complete. Applications (Goldbach, Collatz, Riemann envelopes) illustrate how PEACE enables disciplined reasoning where classical systems face paradox, under-specification, or computational infeasibility.

1. Introduction

Classical logic enforces bivalence (T, F). In practice, mathematical and scientific reasoning often begins in ambiguity: terms under-specified, contexts implicit, or evidence conflicting. We propose **PEACE**, a logic in which the truth value B ("both true and/or false": unknown, contradictory, or undecided) is the *default*. Under *meta-logic* (perspective-guided collapse), B moves (*linearly and reproducibly*) toward T or F when admissible perspectives align; it may *revert* to B if new contradictions or contextual shifts appear. Thus PEACE supplements classical proof with a principled front-end for honest ambiguity.

Design synthesis. PEACE integrates *paraconsistency* (contain contradictions without explosion), *epistemology* (confidence and calibration), *contextualism* (meaning is context-indexed), and *perspectivism* (multiple admissible heuristic lenses). When context is complete and perspectives agree, PEACE *reduces* to classical logic.

2. Truth Values and Default Stance

Definition 1 (Truth Values)**.** *Let* TV $= \{\mathsf{T}, \mathsf{F}, \mathsf{B}\}$*. Interpretations:*

- T*: true only;* F*: false only;*
- B*: both/undecided — the* default *assignment (unknown, contradictory, or context-dependent).*

Remark 1 (Dual role and dynamics of B)**.** B *behaves dually as* unknown *(insufficient context) and as* contradiction *(support for both* φ *and* $\neg\varphi$ *under different admissible perspectives). It is* sticky *(persists until adequate context) and* dynamic *(collapsible to* T *or* F*;* T/F *may revert to* B *under new information).*

3. Context Completeness (Cc)

Definition 2 (Context Completeness)**.** *For a statement* φ *in context* $c \in C$*, define*

$$\mathrm{Cc}(\varphi, c) = \frac{|K_{avail}(\varphi, c) \cap K_{req}(\varphi)|}{|K_{req}(\varphi)|},$$

where K_{req} *is a minimal set of information required to evaluate* φ *and* K_{avail} *is the information currently present.*

Operationally (for human/LLM pipelines),

$$\mathrm{Cc}(\varphi, c) = 1 - \frac{Q(\varphi, c)}{Q_{\max}},$$

with $Q(\varphi, c)$ the number of clarifying questions still needed and $Q_{\max}$ a practical cap (e.g. 2–3).

Remark 2 (Thresholds)**.** *If* $\mathrm{Cc}(\varphi, c) < 0.3$*, assign* B *stably. If* $\mathrm{Cc}(\varphi, c) \geq 0.8$*, perspectives may collapse* B *to* T *or* F*. Thresholds are tunable but fixed for reproducibility.*

4. Perspectives and Meta-Logic

Definition 3 (Perspective)**.** *A* perspective κ *is an admissible heuristic (epistemic, contextual, logical, or domain-specific) that returns a tentative valuation and a confidence:*

$$\kappa : \varphi \mapsto (v_\kappa(\varphi, c),\ w_\kappa(\varphi, c)) \quad \textit{with } v_\kappa \in \mathrm{TV},\ w_\kappa \in [0, 1].$$

Only finitely many perspectives are applied per evaluation (to avoid combinatorial explosion). Each w_κ is updated over time (reinforcement on successful guidance). Irreducible disagreement among admissible perspectives signals insufficient context and leaves B in place.

5. Formal Semantics and Truth Tables

Valuation, Collapse, and Propagation

Let Φ be the set of well-formed formulas and C the set of contexts.

Definition 4 (Valuation)**.** *A (context-indexed) valuation is a function*

$$\mathrm{Val} : \Phi \times C \to \mathrm{TV}.$$

By default, new claims receive $\mathrm{Val}(\varphi, c) = \mathsf{B}$ *until sufficient context is available.*

Definition 5 (Context dependence & falsification asymmetry)**.** *For φ in c:*

$$\mathrm{Val}(\varphi, c) = \begin{cases} \mathsf{T} & \textit{if } \varphi \textit{ holds in } c \textit{ under all admissible perspectives,} \\ \mathsf{F} & \textit{if } \text{any } \textit{admissible perspective refutes } \varphi \textit{ in } c, \\ \mathsf{B} & \textit{otherwise.} \end{cases}$$

Definition 6 (Collapse operator)**.** *Let $\mathcal{P}$ be the set of admissible perspectives for φ in c. Define the (deterministic) collapse operator $\mathcal{K}$ by*

$$\mathcal{K}(\mathsf{B}, \mathcal{P}, c) \in \{\mathsf{T}, \mathsf{F}\} \quad \textit{iff} \quad \textit{the consensus of } \mathcal{P} \textit{ exceeds a fixed confidence threshold;}$$

otherwise $\mathcal{K}(\mathsf{B}, \mathcal{P}, c) = \mathsf{B}$*.*

Definition 7 (Dynamic update)**.** *Let t index evaluation rounds. Then*

$$\mathrm{Val}_{t+1}(\varphi, c) = \begin{cases} \mathcal{K}(\mathsf{B}, \mathcal{P}, c) & \textit{if } \mathrm{Val}_t(\varphi, c) = \mathsf{B} \textit{ and collapse occurs,} \\ \mathsf{B} & \textit{if new contradictions/context undermine } \mathsf{T} \textit{ or } \mathsf{F}, \\ \mathrm{Val}_t(\varphi, c) & \textit{otherwise.} \end{cases}$$

Connective semantics (inductive propagation). For compound formulas in context c:

$$\mathrm{Val}(\neg\varphi, c) = \begin{cases} \mathsf{T} & \text{if } \mathrm{Val}(\varphi, c) = \mathsf{F}, \\ \mathsf{F} & \text{if } \mathrm{Val}(\varphi, c) = \mathsf{T}, \\ \mathsf{B} & \text{if } \mathrm{Val}(\varphi, c) = \mathsf{B}, \end{cases}$$

$$\mathrm{Val}(\varphi \wedge \psi, c) = \begin{cases} \mathsf{F} & \text{if } \mathrm{Val}(\varphi, c) = \mathsf{F} \text{ or } \mathrm{Val}(\psi, c) = \mathsf{F}, \\ \mathsf{T} & \text{if } \mathrm{Val}(\varphi, c) = \mathsf{T} \text{ and } \mathrm{Val}(\psi, c) = \mathsf{T}, \\ \mathsf{B} & \text{otherwise,} \end{cases}$$

$$\mathrm{Val}(\varphi \vee \psi, c) = \begin{cases} \mathsf{T} & \text{if } \mathrm{Val}(\varphi, c) = \mathsf{T} \text{ or } \mathrm{Val}(\psi, c) = \mathsf{T}, \\ \mathsf{F} & \text{if } \mathrm{Val}(\varphi, c) = \mathsf{F} \text{ and } \mathrm{Val}(\psi, c) = \mathsf{F}, \\ \mathsf{B} & \text{otherwise,} \end{cases}$$

$$\mathrm{Val}(\varphi \to \psi, c) = \begin{cases} \mathsf{F} & \text{if } \mathrm{Val}(\varphi, c) = \mathsf{T} \text{ and } \mathrm{Val}(\psi, c) = \mathsf{F}, \\ \mathsf{T} & \text{if } \mathrm{Val}(\varphi, c) = \mathsf{F} \text{ or } \mathrm{Val}(\psi, c) = \mathsf{T}, \\ \mathsf{B} & \text{otherwise.} \end{cases}$$

Operational Truth Tables (Meta-Collapsible B)

All rows involving B are *operational*: they persist as B until context collapse converts them to classical values. To keep presentation intact, each table is forced to remain on-page.

Table 1: Negation

p	$\neg p$
T	F
F	T
B	B

Table 2: Conjunction

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F
B	T	B (collapsible)
B	F	F (refuter dominates)
T	B	B (collapsible)
F	B	F (refuter dominates)
B	B	B (stable until collapse)

Table 3: Disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F
B	T	T (witness suffices)
B	F	B (collapsible)
T	B	T (witness suffices)
F	B	B (collapsible)
B	B	B (stable until collapse)

Table 4: Implication

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T
B	T	T (witness suffices)
B	F	B (collapsible; leans F)
T	B	B (collapsible)
F	B	T (classical dominance)
B	B	B (stable until collapse)

Operational note. A falsifier (F) dominates conjunction and (when antecedent is T) implication; a verifier (T) dominates disjunction. Rows with B propagate *ambiguity* until collapse resolves them.

6. Inference Principles

Classical admissibility. If $\mathrm{Cc}(\varphi, c) \geq 0.8$ and admissible perspectives agree, derivations proceed under standard classical rules (modus ponens, etc.).

PEACE meta-rules.

M1. Default to B. Every new claim receives B until context supports collapse.

M2. Collapse. If admissible perspectives exceed a fixed confidence threshold, B $\mapsto$ T or F (deterministically).

M3. Reversion. T or F may revert to B if new contradictions or contextual shifts arise.

M4. Verification asymmetry. One admissible counterexample suffices for F.

M5. Confidence updating. Perspective weights w_κ are increased when their guidance matches outcomes; irrelevant perspectives are ignored.

M6. Conflict & questions. Irreducible disagreement triggers clarifying questions; if unresolved, retain B.

M7. Layered meta-logic. Apply (i) epistemic heuristics, (ii) contextual alignment, (iii) logical consistency, in that order; each layer may refine or revert valuations.

7. Applications (Sketches)

Goldbach. Probes on even N; success if a decomposition is found. Heuristics: Hardy–Littlewood expected representations $\Rightarrow$ small-budget success probabilities. Category errors (over-claiming global verification) avoided by meta-logic.

Collatz. Probes on random large odd n under the accelerated odd→odd map; success if trajectory dips below threshold within a step budget. Heuristics: drift models; $v_2(3n+1)$ correlations; ensemble forecasts.

Riemann envelopes. Probes on $\theta(x) - x$, $\pi(x) - \mathrm{li}(x)$, or Mertens $M(n)$ bounds over moving windows. Heuristics: asymptotic error models and RMT-inspired distributions; violations are decisive F.

8. Foundations and Barriers

- **Trivalence.** Ambiguity is first-class via B; contradictions do not explode.
- **Epistemic humility.** Confidence is earned (calibration/consensus), not asserted.
- **Contextualism.** Meaning is indexed by context; PEACE requires explicit "surface" selection before inference.
- **Formal barriers.** PEACE sidesteps completeness demands (Gödel) by operating meta-logically; it halts search at sufficient context, mitigating combinatorial blowup; liar-type paradoxes stabilize at B.

9. Conclusion

PEACE formalizes disciplined ambiguity. It lets inquiry begin honestly at B, collapse deterministically when warranted, and revert gracefully when context changes. Where classical proof alone is brittle (paradox, infinity, intractability), PEACE provides a rigorous *front end* that preserves sound conclusions and prevents artificial confidence.

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References

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