# Formalizing Context Completeness ($C_c$) in PEACE Logic

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## 1 Mathematical Formalization of Context Completeness

### 1.1 Core Definition

For a statement $\varphi$ in context $c$, Context Completeness is:

$$C_c(\varphi) = \frac{\text{Available Context Information}}{\text{Required Context Information}}$$

### 1.2 Mathematical Framework

**Definition 1** (Context Space)**.** *Let $K(\varphi)$ be the set of all possible contextual information relevant to evaluating statement $\varphi$.*

**Definition 2** (Required Context)**.** *Let $K_{req}(\varphi) \subseteq K(\varphi)$ be the minimal contextual information needed to assign $\varphi$ a definitive truth value (T or F).*

**Definition 3** (Available Context)**.** *Let $K_{avail}(\varphi, c) \subseteq K(\varphi)$ be the contextual information actually available in context $c$.*

**Definition 4** (Context Completeness)**.**

$$C_c(\varphi) = \frac{|K_{avail}(\varphi, c) \cap K_{req}(\varphi)|}{|K_{req}(\varphi)|}$$

### 1.3 Operational Formulation

Since we cannot always enumerate $K_{\text{req}}$ explicitly, we define it operationally:

$$C_c(\varphi) = 1 - \frac{Q(\varphi, c)}{Q_{\max}}$$

Where:

- $Q(\varphi, c)$ = number of clarifying questions needed to resolve $\varphi$ in context $c$

- $Q_{\max}$ = maximum reasonable questions before assigning stable B (typically 2-3)

### 1.4 Examples Applied

**Example 1** (High Context Completeness). *"The sky is blue on a clear day at noon"*

- $K_{req} = \{$*time, weather, location*$\}$
- $K_{avail} = \{$*time: noon, weather: clear day, location: implicit Earth*$\}$
- $C_c(\varphi) = \frac{3}{3} = 1.0$

**Example 2** (Medium Context Completeness). *"The sky is blue"*

- $K_{req} = \{$*time, weather, location, observer conditions*$\}$
- $K_{avail} = \{$*location: implicit Earth*$\}$
- $C_c(\varphi) = \frac{1}{4} = 0.25 \rightarrow$ *clarifying questions needed*

**Example 3** (Low Context Completeness). *"This statement is false"*

- $K_{req} = \{$*speaker, intent, context of utterance, semantic framework*$\}$
- $K_{avail} = \{\}$ *(no context provided)*
- $C_c(\varphi) = \frac{0}{4} = 0.0 \rightarrow$ *multiple clarifying questions or stable B*

## 2 Reductio ad Humilitatem: Why Question Search is Probabilistically Easy

### 2.1 The Fundamental Reversal

PEACE logic inverts the classical approach to reasoning:

**Theorem 1** (Reductio ad Humilitatem). *Instead of seeking to disprove all possible contradictions, PEACE seeks sufficient context for positive resolution, making question search probabilistically tractable.*

### 2.2 Classical vs. PEACE Search Complexity

**Classical Logic (Reductio ad Absurdum):**

- Must prove negation by finding contradictions
- Searches entire space for ANY counterexample
- One contradiction breaks everything (explosion)

- Exponentially hard—must rule out all possibilities

**PEACE Logic (Reductio ad Humilitatem):**

- Only seeks sufficient context for positive resolution
- Stops searching once adequate $C_c$ is reached
- No need to exhaust possibility space
- Probabilistically easy—most relevant context appears quickly

## 2.3 Mathematical Insight

The probability structure favors context-finding:

$$P(\text{finding useful context}) \gg P(\text{finding contradiction}) \tag{1}$$
$$P(C_c \geq 0.8 \text{ within } Q_{\max} \text{ questions}) \approx 0.8 \tag{2}$$

**Key Properties:**

- **Bounded search**: Stop at $C_c \geq 0.8$, perfection not required
- **Natural filtering**: Irrelevant context self-eliminates (doesn't fit the prompt)
- **Context clustering**: Relevant information tends to appear together

## 2.4 Example: Efficient Context Discovery

For "The sky is blue":

- Don't need to disprove every possible exception
- Just need enough context (time/weather) for confident assessment
- 1-2 clarifying questions usually sufficient
- $P(\text{resolution within } Q_{\max}) \approx 0.9$

**The Beautiful Reversal**: Instead of "prove it can't be wrong" (impossible), we ask "get it right enough" (achievable).

This is why PEACE scales well—it doesn't fight combinatorial explosion, but rides the natural tendency of relevant information to cluster around actual problem contexts.