Paraconsistent Epistemic And Contextual Evaluation (PEACE)

Abstract

We present *Paraconsistent Epistemic And Contextual Evaluation* (PEACE), a meta-logical framework that nests on top of classical logic. It allows safe reasoning in the presence of paradoxes, contradictions, and uncertainty by integrating dialetheic paraconsistency, perspectivism, and contextualism. Classical reasoning is preserved as a special case inside stable contexts, while paradoxical or context-laden claims are identified as *category errors* when evaluated purely classically. PEACE provides an epistemically humble methodology for universal claim evaluation.

1 Base Logic: Meta-Dialetheic Core

1.1 Truth Values

The base truth value set is:

$$V = \{T, F, B\}$$

where:

- T: true only
- F: false only
- B: both true-content and false-content (*meta-dialetheic default*)

The designated values are:

$$\mathsf{D} = \{T, B\}$$

meaning that any value with true-content is considered "true enough" for consequence.

1.2 Content Semantics

Define content functions:

$$t(T) = 1, \quad t(F) = 0, \quad t(B) = 1$$
$$f(T) = 0, \quad f(F) = 1, \quad f(B) = 1$$

Connectives are defined compositionally:

Negation

$$t(\neg A) := f(A), \quad f(\neg A) := t(A)$$

Thus:

$$\neg T = F, \quad \neg F = T, \quad \neg B = B$$

Conjunction

$$t(A \wedge B) := t(A) \wedge t(B)$$
$$f(A \wedge B) := f(A) \vee f(B)$$

Disjunction

$$t(A \vee B) := t(A) \vee t(B)$$
$$f(A \vee B) := f(A) \wedge f(B)$$

Implication Defined as:

$$A \rightarrow B := \neg A \vee B$$

1.3 Semantic Consequence

We define:

$$\Gamma \vDash \varphi \quad \text{iff} \quad \forall v \ \big(\forall \gamma \in \Gamma,\ v(\gamma) \in \mathsf{D} \Rightarrow v(\varphi) \in \mathsf{D}\big)$$

This is paraconsistent: from A and $\neg A$ we cannot infer arbitrary B.

2 Perspectives

A *perspective* is a pair:

$$P = (I_P, \kappa_P)$$

where:

- I_P: an interpretation or normative constraint on vocabulary in φ
- κ_P: a verdict function $\kappa_P : \text{Formulas} \to \{T, F, B\}$

2.1 Admissibility and Load-Bearing

Admissibility (A): P is admissible for φ in context c iff:

1. *Relevance*: I_P constrains some symbol or speech-act in φ
2. *Coherence*: I_P preserves the content calculus

Load-Bearing (L): P is load-bearing for φ iff it changes $t(\cdot)$ or $f(\cdot)$ of at least one semantically decisive part of φ.

2.2 Conservativity Rule

If P is admissible but not load-bearing, then:

$$\kappa_P(\varphi) := \text{value}(\varphi)$$

This prevents "decorative" perspectives from altering truth values.

2.3 Neutral Policy

Prior to perspective application, all atoms default to B unless restricted by strong, context-independent evidence.

3 Contexts

A context c induces a relevance preorder $\preceq_c$ on admissible perspectives.

3.1 Influential Set

$$\mathcal{P}_c^*(\varphi) := \{P \mid P \text{ admissible and load-bearing for } \varphi\}$$

3.2 Contextual Verdict

Single-valued:

$$\text{Verdict}_c(\varphi) := \begin{cases} \kappa_{P^*}(\varphi) & \text{if } \mathcal{P}_c^*(\varphi) \neq \varnothing \text{ and } P^* \text{ is } \preceq_c\text{-maximal} \\ \text{value}(\varphi) & \text{otherwise} \end{cases}$$

Set-valued: Return the set of $\kappa_P(\varphi)$ for all $\preceq_c$-maximal P; optionally aggregate $\{T, F\}$ to B.

4 Entailment Variants

- **Designated entailment:** $\models$ as defined above.
- **Cautious entailment:** $\Gamma \models_T \varphi$ iff all models designate φ as T.
- **Contextual entailment:** $\Gamma \models_c \varphi$ iff φ is designated in all $\preceq_c$-maximal influential perspectives.

5 Category Error Detection

Define $CatErr(\varphi, c)$ to hold if any of:

1. Self-reference enabling diagonalization
2. Indexicals, vague predicates, figurative operators
3. Normative/pragmatic load (felicity conditions)
4. Strong conflicting evidence: $t(\varphi) = f(\varphi) = 1$

If $CatErr(\varphi, c)$, pure classical evaluation is a *category error*; switch to PEACE meta-evaluation.

6 Evidence Model

Each atom p has evidence scores $(e^+(p), e^-(p)) \in [0,1]^2$. Threshold rules map these to T, F, or B.

7 Theorems

- **Non-explosion**: There exist v with $v(A) = B$, $v(B) = F$ such that $A, \neg A \not\models B$.
- **Neutral fixed point for Liar**: Any $S \leftrightarrow \neg True(S)$ yields $S = B$ under P_{meta}.
- **Conservativity**: If P is admissible but not load-bearing, $\kappa_P(\varphi) = \text{value}(\varphi)$.
- **Classical preservation**: On B-free fragments, $\models$ coincides with classical entailment.

8 Operational Recipe

Given claim φ and context c:

1. Check for $CatErr(\varphi, c)$. If yes, use PEACE.
2. Initialize atoms via evidence; else default B.
3. Compute neutral value value(φ).
4. Identify admissible load-bearing perspectives.
5. Rank via $\preceq_c$; return verdict(s).
6. Reason using chosen entailment variant.

9 Examples and Applications

In this section, we illustrate *Paraconsistent Epistemic And Contextual Evaluation* (PEACE) by applying it to well-known paradoxes and context-laden claims. Each example includes a formal PEACE analysis and a plain-language explanation so that readers without a formal logic background can still understand the reasoning.

We introduce the notion of *Context Completeness* $C_c(\varphi) \in [0, 1]$, measuring how fully a claim φ specifies the background needed for evaluation. The guiding aphorism is:

> **Aphorism:** The less complete the context, the harder it is to evaluate a claim correctly. When C_c is low, more perspectives are admissible and load-bearing, and the verdict set grows larger. A claim with very low C_c will tend to stabilize at the neutral value B in PEACE.

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9.1 Example 1: The Liar Paradox

Claim: "This sentence is false." Let $L \equiv \neg True(L)$.

Neutral evaluation: In the base PEACE logic with transparent truth, the fixed point is $L = B$.

Context completeness: High ($C_c \approx 0.9$) — the paradox is fully specified; only its resolution method is in question.

Perspectives:

- P_{meta}: transparent semantics $\to B$.
- P_{prag}: assertional felicity norms $\to F$ (self-falsifying assertions are defective).
- P_{fig}: "is false" means "has any false-content" $\to T$.
- Decorative humor: B.

Plain-language: Depending on how we *take* the statement, it can be both true and false at once, just false because it is a bad assertion, or true because it correctly describes its own falsity.

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9.2 Example 2: Russell's Paradox

Claim: "The set of all sets that do not contain themselves."

Neutral evaluation: Under naive comprehension, the self-membership test yields B.

Context completeness: Low ($C_c \approx 0.3$) — the claim omits the foundational framework (e.g., ZFC vs. naive set theory).

Perspectives:

- P_{meta}: naive set semantics $\to B$.
- P_{formal}: ZFC set theory $\to F$ (no such set exists).
- P_{fig}: metaphor for self-exclusion $\to T$.
- Decorative humor: B.

Plain-language: If we don't say what kind of sets we're talking about, we can't settle the issue. In one mathematics context, the set can't exist; in another, it both exists and doesn't. As a metaphor, it might make perfect sense.

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9.3 Example 3: Sorites (Heap) Paradox

Claim: "A collection of n grains of sand is a heap."

Neutral evaluation: With a vague predicate "heap," intermediate n values yield B.

Context completeness: Very low ($C_c \approx 0.2$) — the claim does not define "heap" or threshold n.

Perspectives:

- P_{meta}: vague predicate semantics $\to B$ for mid-range n.
- P_{precise}: defines explicit threshold (e.g., $n \geq 1000$) $\to T$ or F depending on n.
- $P_{\text{figurative}}$: heap as metaphor (e.g., "a heap of trouble") $\to$ *variable*.

Plain-language: Without saying exactly how many grains make a heap, we can't decide for sure. If you set a rule, you can decide — but different rules give different answers.

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9.4 Example 4: Barber Paradox

Claim: "The barber shaves all and only those men who do not shave themselves."

Neutral evaluation: Self-reference in membership to the shaved group yields B.

Context completeness: Moderate ($C_c \approx 0.5$) — missing whether barber can shave himself or is outside the group.

Perspectives:

- P_{meta}: formal semantics $\to B$.
- P_{legal}: resolves by rule (barber is exempt) $\to F$ or T depending on rule.
- P_{story}: fictional resolution where barber alternates days $\to$ *both*.

Plain-language: As stated, the barber both shaves and does not shave himself. You need an extra rule to resolve it — without one, it's both true and false.

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9.5 Example 5: Schrödinger's Cat

Claim: "The cat in the box is alive."

Neutral evaluation: In quantum interpretation before observation, both alive and dead $\to B$.

Context completeness: High ($C_c \approx 0.8$) if physics model specified; low if not.

Perspectives:

- P_{quantum}: superposition $\to B$.
- $P_{\text{classical}}$: after observation, definite T or F.
- $P_{\text{figurative}}$: alive = "still relevant" $\to$ *context*.

Plain-language: Before looking, the cat is in a both/and state; after looking, it's one or the other. In everyday talk, we don't think that way — we assume it's one or the other all along.

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9.6 Example 6: Ship of Theseus

Claim: "The ship with all parts replaced over time is the same ship."

Neutral evaluation: Identity over time without clear criteria yields B.

Context completeness: Moderate ($C_c \approx 0.5$) — missing explicit identity conditions.

Perspectives:

- $P_{\text{classical\_identity}}$: strict part-identity $\rightarrow F$.
- $P_{\text{functional}}$: function continuity $\rightarrow T$.
- P_{cultural}: cultural narrative $\rightarrow T$ or B.

Plain-language: Whether it's "the same" depends on what you care about — the material, the function, or the story.

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9.7 General Observations

In all cases, the PEACE framework:

1. Identifies missing or ambiguous context via low C_c.
2. Generates a set of admissible perspectives.
3. Applies load-bearing tests to limit verdict changes.
4. Preserves classical reasoning inside any single perspective with T/F values only.
5. Flags classical-only evaluation as a category error when $CatErr(\varphi, c)$ holds.

9.8 Example 7: P vs NP as a Category Error

Claim: "For all decision problems in NP, there exists a polynomial-time algorithm that solves them" (P = NP).

Neutral evaluation in PEACE: In the classical formalism of computational complexity theory, NP is defined as the set of decision problems for which a given *certificate* can be *verified* in polynomial time by a deterministic

Turing machine. This formalization *explicitly strips away all context* from the problem other than a bare string membership condition. It is a deliberate idealization: NP questions are encoded as total functions on formal strings; all semantic, pragmatic, or domain-specific structure is removed.

Context completeness: Extremely low for real-world NP problems once formalized in this way: $C_c \approx 0.1$. The act of reduction to the NP definition destroys much of the original context in which humans (or other algorithms) actually solve such problems.

PEACE diagnosis:

1. In real settings, many NP problems are *contextually grounded*: they have semantic constraints, domain-specific heuristics, and background knowledge that guide efficient solutions.

2. The NP definition forces all such problems into a *contextless classical shell*, treating them as arbitrary string sets.

3. From the PEACE standpoint, this is a *category error*: the classical NP definition evaluates a claim ("there exists a polytime solver") in a context stripped of exactly the information that makes such a solver possible.

4. This is directly analogous to Russell's Paradox: the definition creates a setting where the question cannot be answered in its original intended sense without importing back the lost context.

Perspectives:

- $P_{\text{meta\_formal}}$: Within the stripped-down formalism, P vs NP is a perfectly well-posed classical statement — but only about decontextualized, syntactic problems. Verdict: *Unknown*.

- P_{applied}: Reintroduces real-world context for NP problems; notes that the stripped definition no longer matches the actual problem humans care about. Verdict: B (both possible and impossible, depending on whether context is reinstated).

- $P_{\text{philosophical}}$: Treats the question "Is P = NP?" for *real-world* problems as ill-formed under the formal NP definition. Verdict: F (false) in the sense of category error.

Plain-language: The way we define NP in computer science is a kind of trick: we throw away almost everything that makes a problem human-solvable in the real world, and keep only the bare string verification property. Then we ask whether those stripped problems can always be solved quickly. For many practical NP problems, the answer in the real world is "yes" *only because* of the context we threw away. So in PEACE terms, we've changed the question without realizing it — and the new question doesn't actually mean what we think it does. That's why "NP" in the pure formal sense and "NP" in the applied sense behave differently — and why P vs NP, as people *imagine* it, is a paradoxical category error.

Mathematical link: Let $\mathcal{P}_{\text{real}}$ be a real-world NP problem, with contextual information K that enables efficient solving:

$$\mathcal{P}_{\text{real}} = (I, Q, K)$$

where I is the instance space, Q is the decision predicate, and K is the context. The NP reduction map ρ strips K:

$$\rho(\mathcal{P}_{\text{real}}) = (I, Q)$$

In PEACE terms, $C_c(\rho(\mathcal{P}_{\text{real}})) \ll C_c(\mathcal{P}_{\text{real}})$. Verdict stability drops, admissible perspectives proliferate, and contradictions appear between the applied and formal readings of "efficiently solvable".

Theorem 9.1 (PEACE on Formal $P = NP$)**.** Let $\mathcal{P}_{\text{real}}$ denote a real-world decision problem with context K. Let $\rho(\mathcal{P}_{\text{real}})$ be its formal NP encoding with K removed. Then:

$$C_c(\rho(\mathcal{P}_{\text{real}})) \ll C_c(\mathcal{P}_{\text{real}})$$

and $\rho(\mathcal{P}_{\text{real}})$ is, in general, a different computational problem.

Therefore, the formal equation $P = NP$ in complexity theory does not model the real-world claim "Every efficiently verifiable problem can be efficiently solved." The formal $P = NP$ is thus a *false abstraction* of the intended question. In PEACE, its truth in the formal model is irrelevant to, and generally false about, the real domain.

Structural Analogy: Russell's Paradox Meets the Halting Problem

Russell's Paradox arises when one attempts to treat the "set of all sets that do not contain themselves" as if it were an ordinary set in naive comprehension. The definition destroys the necessary context (the type system or axiomatic framework) that would make the claim coherent, producing a self-referential object that cannot exist in the intended sense.

The formal P vs NP question makes an *identical category error in structure*. It takes the class of real-world verification problems — each embedded in a rich context K — and, by the NP definition, strips K away, leaving a contextless encoding $\rho(\mathcal{P}_{\text{real}})$. It then asks whether *these stripped objects* are all solvable in polynomial time, while continuing to speak as though it were asking about the original, context-rich problems.

However, because the P vs NP question is explicitly about *time-bounded* solvability, it also inherits the structural flavor of the *Halting Problem*. The Halting Problem demonstrates that there are questions about algorithmic behavior that cannot be decided in general, because they encode self-reference in the temporal behavior of computation. P vs NP does the same, but with the additional restriction of a polynomial-time bound, creating a *time-constrained self-referential problem*.

In both Russell's Paradox and the Halting Problem:

- The pathology arises from attempting to evaluate an object after removing or ignoring the structural context that ensures coherence.
- Self-reference is smuggled into the definition of the object itself.
- The resulting question cannot be answered in the original intended sense without reinstating the missing context.

PEACE Synthesis: Formal P vs NP is structurally a *hybrid* of Russell's Paradox and the Halting Problem:

- Like Russell's Paradox, it destroys decisive context (K), producing pathological abstractions.
- Like the Halting Problem, it encodes a self-referential decision about algorithmic behavior within a time-bound constraint.

Conclusion: In PEACE terms, the popular P vs NP is not a single coherent question at all. It is a *false abstraction* that combines the context-deletion pathology of Russell's Paradox with the time-bounded self-reference of the Halting Problem. As such, it is trivially false about the intended real-world computational phenomena, and any formal resolution of P vs NP applies only to its patently hypothetical contextless mathematical shell.