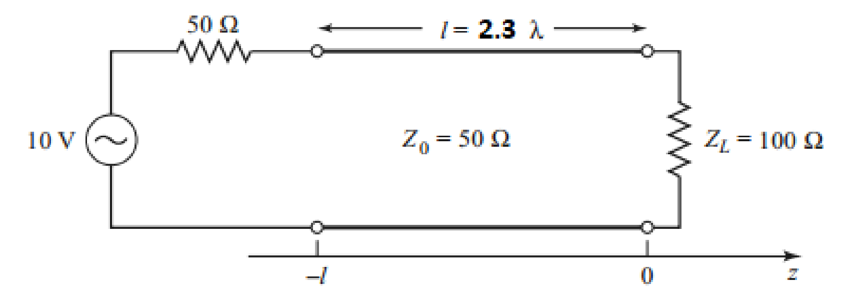
29.-



$$\Gamma(\ell) = \Gamma e^{-2j\beta\ell} e^{-2\alpha\ell} = \Gamma e^{-2\gamma\ell}$$

$$Z_{\text{in}} = Z_0 \frac{Z_L + Z_0 \tanh \gamma \ell}{Z_0 + Z_L \tanh \gamma \ell}$$

$$P_{\text{in}} = \frac{|V_o^+|^2}{2Z_0}\left(1 - |\Gamma(\ell)|^2\right)e^{2\alpha\ell}$$

$$P_L = \frac{|V_o^+|^2}{2Z_0}\left(1 - |\Gamma|^2\right)$$

$$P_{\text{loss}} = P_{\text{in}} - P_L = \frac{|V_o^+|^2}{2Z_0}\left[\left(e^{2\alpha\ell} - 1\right) + |\Gamma|^2\left(1 - e^{-2\alpha\ell}\right)\right]$$

pozar_0_exercise_02_29.m

```
Z0=50        % ohm
Zgen=50
Vgen=10      % Volt
ZL=100

syms lambda
alpha_dB=.5/lambda  % [dB/lambda]
Np2dB=10*log10(exp(1)^2)
alpha_Np=alpha_dB/Np2dB
```

alpha_dB =1/(2*lambda)
Np2dB = 8.685889638065037
alpha_Np =281474976710656/(4889721167171369*lambda)

```
beta=2*pi/lambda % [m^-1]
L=2.3*lambda       % length of transmission line [m]

gamma=alpha_Np+1j*beta
gamma_L=gamma*L
gamma_L_rad=double(gamma_L)  % =   0.1323 +14.4513i
```

gamma_L_rad = 0.132398642847158 +14.451326206513048i

sometimes for the complex propagation constant gamma, units Np/m and degree are mixed as the solutions manual does. For this exercise the solutions manual shows gamma=.1325 +1j*108º. 108 degree is the remainder of imag(gamma_degree)/360

```
14.4513*180/pi-floor(14.4513*180/pi/360)*360
```

= 1.079984984774065e+02

reflection coefficients on both sides of the transmission line

```
refl_Load=(ZL-Z0)/(ZL+Z0)
refl_gen=refl_Load*exp(-2*gamma_L_rad)
```

refl_Load = 0.333333333333333
refl_gen = -0.206936161907528 + 0.150347922208021i

TL input impedance (from generator)

```
Zin=Z0*(ZL+Z0*tanh(gamma_L_rad))/(Z0+ZL*tanh(gamma_L_rad))
```

Zin = 31.588363022059227 +10.163454575205781i

TL input power

```
Pin=(abs(Vgen*Zin/(Zgen+Zin)))^2/(2*Z0*(1-(abs(refl_gen))^2)*exp(2*real(gamma_L_rad))
```

Pin = 0.198382847405214

$$P_{\text{in}} = \frac{1}{2}\text{Re}\left\{V(-\ell)I^*(-\ell)\right\} = \frac{|V_o^+|^2}{2Z_0}\left(e^{2\alpha\ell} - |\Gamma|^2 e^{-2\alpha\ell}\right)$$

$$= \frac{|V_o^+|^2}{2Z_0}\left(1 - |\Gamma(\ell)|^2\right)e^{2\alpha\ell}$$

PLoad power reaching load

```
PLoad=(abs(Vgen*Zin/(Zgen+Zin)))^2/(2*Z0)*(1-(abs(refl_Load))^2)
```

PLoad = 0.144789944222718

lost power

```
Ploss=Pin-PLoad
```

Ploss = 0.053592903182496

Comment 1: perhaps of use too:
maximum available power from generator
Pmax_gen=.5*(abs(Vgen))^2/(4*real(Zgen))

Pmax = 0.25

Alternatively

$$P(z) = P_o e^{-2\alpha z} \qquad \alpha = \frac{P_\ell(z=0)}{2P_o}$$

alpha_dB and **alpha_Np** are per metre.

alpha=double(alpha_Np*L)

alpha = 0.132398642847158

Pout=Pin*exp(-2*alpha)

Pout = 0.152231357248704

The solutions manual assumes $|V_o^+| = \frac{10}{2} e^{-\alpha \ell} = 4.38\,v.$
that would be all matched; generator to TL and TL to load.

absV0plus_all_match=abs(Vgen/2*exp(-real(gamma_L_rad)))

absV0plus_all_match = 4.379958588165586

Instead, from

$$V(z) = V_o^+\left(e^{-\gamma z} + \Gamma e^{\gamma z}\right)$$

$$Z_{\text{in}} = \frac{V(-\ell)}{I(-\ell)} = Z_0 \frac{Z_L + Z_0 \tanh \gamma \ell}{Z_0 + Z_L \tanh \gamma \ell}.$$

$$V_o^+ = V_g \frac{Z_{\text{in}}}{Z_{\text{in}} + Z_g} \frac{1}{\left(e^{\gamma \ell} + \Gamma_\ell e^{-\gamma \ell}\right)}$$

$$V_o^+ = V_g \frac{Z_0}{Z_0 + Z_g} \frac{e^{-\gamma \ell}}{\left(1 - \Gamma_\ell \Gamma_g e^{-2\gamma \ell}\right)}$$

It's reasonable to consider that V0plus =Vgen*Zin/(Zgen+Zin)*(exp(g_L)+refl_gen*exp(-gamma_L_rad))
should be used, 2.89a in [POZAR] pg81

Vin=Vgen*Zin/(Zin+Zgen)*1/(exp(gamma_L_rad)+refl_gen*exp(-gamma_L_rad))
absVin=abs(Vin)

Vin =
0.155584072758048 - 3.279133570422918i
absVin = 3.282822471040817

would be the correct start voltage for the procedure followed in the solutions manual.
But because 2.92/93/94 are readily available, the answers have already been provided.
This Pin is the same as the above used

Pin=(abs(Vgen*Zin/(Zgen+Zin)))^2/(2*Z0*(1-(abs(refl_gen))^2)*exp(2*real(gamma_L_rad))

And it's not the other apparently correct
Vgen*Zin/(Zin+Zgen)
abs(Vgen*Zin/(Zin+Zgen))

= 3.965319190462363 + 0.751739611040107i
= 4.035947066681602

Loss_dB=double(alpha_dB*L)
10*log10(Pin)-10*log10(PLoad)

= 1.367657186248119 dB

10*log10(exp(2*alpha))

= 1.150000000000000

0.21 dB missing, the perturbation method is not exact

Generator and TL are matched, but TL and load are not. And it's not the other apparently correct
Vgen*Zin/(Zin+Zgen)

abs(Vgen*Zin/(Zin+Zgen))

3.965319190462363 + 0.751739611040107i

Comment: Zin with lossless TL would be
Zin_lossless=Z0*(ZL+1j*Z0*tan(imag(gamma_L_rad)))/(Z0+1j*ZL*tan(imag(gamma_L_rad)))

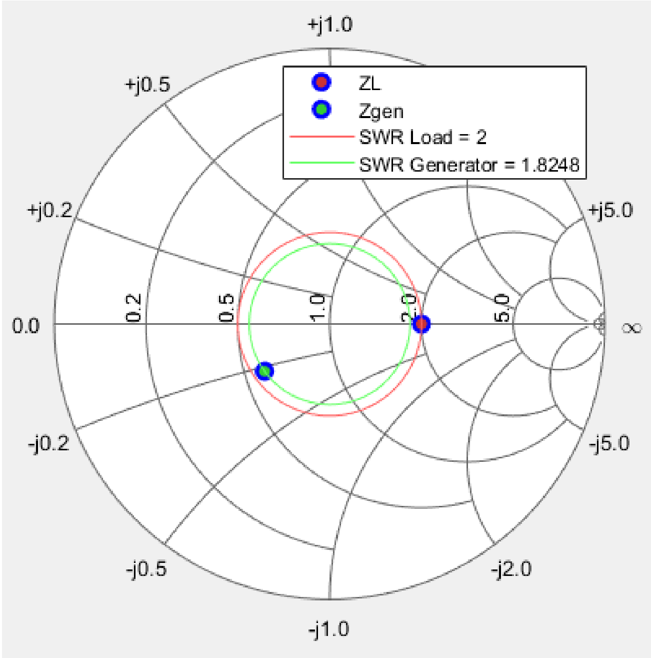
same as
Zin_lossless=Z0*(ZL+1j*Z0*tan(2*pi*2.3))/(Z0+1j*ZL*tan(2*pi*2.3))

26.928588541323347 +11.871170407231441i

Zin_lossless =
26.928588541323329 +11.871170407231373i

Now with Smith Chart:

```
Z0=50; ZL=100;

sm1=smithchart;ax=gca;hold all;
refl_ZL=(ZL-Z0)/(ZL+Z0);

plot(ax,real(refl_ZL),imag(refl_ZL),...
                          'o','Color',[1 0 0],...
                          'LineWidth',2,...
                          'MarkerEdgeColor','b',...
                          'MarkerFaceColor',[.8 .2 .2],...
                          'MarkerSize',7)                 % ZL


 plot(ax,refl_mod(end).*cos(a(end)),refl_mod(end).*sin(a(end)),...
                          'o','Color',[1 0 0],...
                          'LineWidth',2,...
                          'MarkerEdgeColor','b',...
                          'MarkerFaceColor',[.2 .8 .2],...
                          'MarkerSize',7)     % Zin at generator

[x_swr,y_swr]=Smith_plotGammaCircle(ax,ZL,Z0,[1 .4 .4])

refl_in=...
refl_mod(end).*cos(a(end))+1j*refl_mod(end).*sin(a(end))
Zin_gen=Z0*(1+refl_in)/(1-refl_in)

[x_swr,y_swr]=Smith_plotGammaCircle(ax,Zin_gen,Z0,[.4 1 .4])

SWR_Load=(1+abs(refl_ZL))/(1-abs(refl_ZL))
SWR_gen=(1+abs(refl_in))/(1-abs(refl_in))

str_swr_load=['SWR Load = ' num2str(SWR_Load)]
str_swr_gen=['SWR Generator = ' num2str(SWR_gen)]

legend(ax,'ZL','Zgen',str_swr_load,str_swr_gen)
```




```
legend(ax,'off')

Smith_plotRefLine2PhaseCircle(ax,ZL,Z0,[.6 1 .6])
Smith_plotRefLine2PhaseCircle(ax,Zin_gen,Z0,[.6 1 .6])

syms lambda
alpha_dB=.5/lambda % [dB/lambda]
Np2dB=10*log10(exp(1)^2);alpha_Np=alpha_dB/Np2dB
beta=2*pi/lambda;L=2.3*lambda

g=alpha_Np+1j*beta   % g=gamma=alpha+1j*beta
g_L=g*L;g_L=double(simplify(g_L))
```

angle to run along TL: beta*L=pi rad means 360º around Smith Chart
so to match the imag(g_L) angle with Smith chart angle, double it.

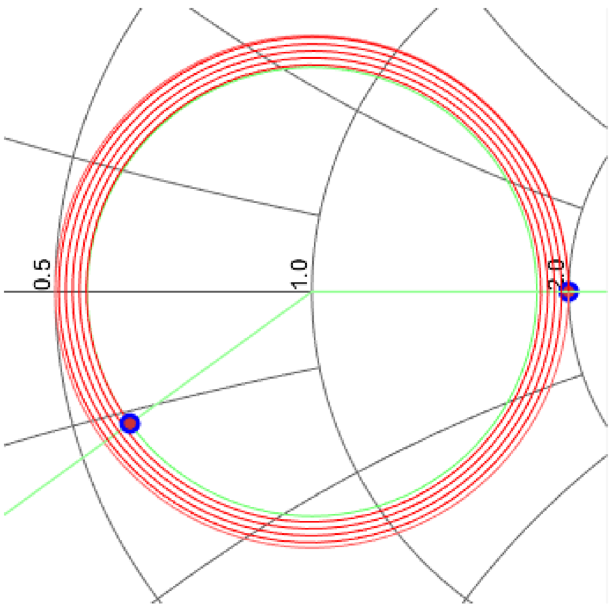
```
N1=100
da=2*pi/N1  % angle differential for 2*pi around Smith Chart = lambda/2

amount_full_turns=floor(14.4513*180/pi/360)
```

angle rads left when no full turns left

```
14.4513-floor(14.4513*180/pi/360)

amount_da_rem=floor((14.4513-floor(14.4513*180/pi/360))/da)
```

total amount angle steps needed to achieve N1 resolution

```
N2=N1*amount_full_turns+amount_da_rem
a0=angle(refl_ZL)
a=double(linspace(a0,2*imag(g_L),N2));

refl_mod=linspace(refl_ZL,refl_ZL*exp(-real(g_L)),N2)
plot(ax,refl_mod.*cos(a),refl_mod.*sin(a),...
                          '-','LineWidth',.5,'Color',[1  0 0])
plot(ax,refl_mod(end).*cos(a(end)),refl_mod(end).*sin(a(end)),...
                          'o','Color',[0 1 0],...
                          'LineWidth',2,...
                          'MarkerEdgeColor','b',...
                    'MarkerFaceColor',[.8 .2 .2],...
                          'MarkerSize',7) % ZL
 axis([-0.4008  0.3832  -0.4103  0.3737])
```



with reference plane on generator-TL

$$V(z) = V_o^+\left(e^{-\gamma z} + \Gamma e^{\gamma z}\right)$$

g*L = 0.1323 +14.451i =
0.1323 + 1.884i

tanh(g*L) = 1.184 - 2.597i

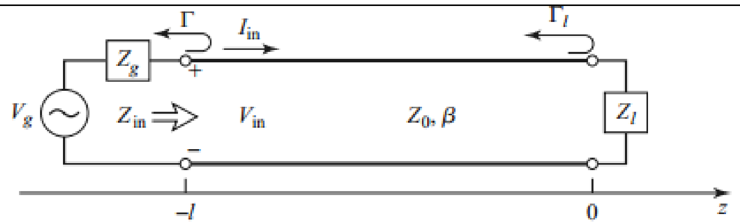
$$\gamma\ell = (\alpha + j\beta)\ell = \boxed{0.1325 + j\,108°}$$

$$Z_{in} = Z_0\frac{Z_L + Z_0\tanh\gamma\ell}{Z_0 + Z_L\tanh\gamma\ell} = 50\,\frac{100 + 50(.845 + j2.19)}{50 + 100(.845 + j2.19)} = 32.5 - j\,12.4\ \Omega$$

Comment: error in solutions manual, the value of tanh(gamma*L) does not correspond to the tanh of the calculated gamma*L
It's actually 'amplified' by 1/exp(alpha*L) but it's not mentioned anywhere.

```
tanh(g_L)
tanh(real(g_L)+1j*(imag(g_L)-4*pi))
```

the following expressions are for lossless transmission line



$$V(-\ell) = V_g\frac{Z_{in}}{Z_{in} + Z_g} = V_o^+\left(e^{j\beta\ell} + \Gamma_\ell e^{-j\beta\ell}\right)$$

$$V_o^+ = V_g\frac{Z_{in}}{Z_{in} + Z_g}\,\frac{1}{\left(e^{j\beta\ell} + \Gamma_\ell e^{-j\beta\ell}\right)}$$

$$V_o^+ = V_g\frac{Z_0}{Z_0 + Z_g}\,\frac{e^{-j\beta\ell}}{\left(1 - \Gamma_\ell\Gamma_g e^{-2j\beta\ell}\right)}$$

cannot be applied to lossy transmission lines because different input voltages are obtained:

```
Vgen*Zin/(Zin+Zgen)*1/(exp(1j*imag(gamma_L_rad))+refl_Load*exp(-1j*imag(gamma_L_rad)))
abs(Vgen*Zin/(Zin+Zgen)*1/(exp(1j*imag(gamma_L_rad))+refl_Load*exp(-1j*imag(gamma_L_rad))))

Vgen*Z0/(Z0+Zgen)*exp(-1j*imag(gamma_L_rad))/(1-refl_Load*refl_gen*exp(-1j*2*imag(gamma_L_rad)))
abs(Vgen*Z0/(Z0+Zgen)*exp(-1j*imag(gamma_L_rad))/(1-refl_Load*refl_gen*exp(-1j*2*imag(gamma_L_rad))))
```