

Implementation of special functions

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The arithmetic geometric mean $\text{agm}(x, y)$ is defined and calculated as the limit of the iteration:

$$\begin{bmatrix} a_0 \\ g_0 \end{bmatrix} := \begin{bmatrix} x \\ y \end{bmatrix}, \quad \begin{bmatrix} a_{n+1} \\ g_{n+1} \end{bmatrix} := \begin{bmatrix} \frac{1}{2}(a_n + g_n) \\ \sqrt{a_n g_n} \end{bmatrix}. \quad (1)$$

This calculation is even valid for complex numbers, but one has to take care of the branch cut of the square root.

The iteration can be stopped if a_n and g_n are sufficiently close to each other. If this condition fails for some reason, to have a more stable algorithm, a maximum number n_{\max} of iterations should be specified. Numerical experiments show that $n_{\max} = 14$ is enough for 64-bit floating point arithmetic with

$$(x, y) \in [10^{-307}, 10^{308}] \times [10^{-307}, 10^{308}]. \quad (2)$$

The complete elliptic integral of the first kind is defined as

$$K(m) := \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - m \sin^2 \theta}}. \quad (3)$$

It is calculated by the arithmetic geometric mean:

$$K(m) = \frac{\pi}{2 \text{agm}(1, \sqrt{1-m})}. \quad (4)$$