Winkelfunktionen

 $\sin(x+y) = \sin x \cos y + \cos x \sin y \quad | \quad \sin(z \pm \pi) = -\sin z$ $\cos(z \pm \pi) = -\cos z$ $\sin(x - y) = \sin x \cos y - \cos x \sin y$ $\cos(x+y) = \cos x \cos y - \sin x \sin y$ $\sin(\pi/2 - x) = \cos x$ $\cos(x - y) = \cos x \cos y + \sin x \sin y \mid \cos(\pi/2 - x) = \sin x$ $\sin(nx) = 2\cos x \sin((n-1)x) - \sin((n-2)x) \quad |\sin(-z)| = -\sin z$ $\cos(nx) = 2\cos x \cos((n-1)x) - \cos((n-2)x) \mid \cos(-z) = \cos z$ $\cos^2 z + \sin^2 z = 1$ | $\cosh^2 z - \sinh^2 z = 1$ | $\cosh(iz) = \cos z$ $e^{iz} = \cos z + i \sin z$ | $e^z = \cosh z + \sinh z$ sinh(iz) = i sin z $2\cos z = e^{iz} + e^{-iz}$ $2\cosh z = e^z + e^{-z}$ $\cos(iz) = \cosh z$ $2i \sin z = e^{iz} - e^{-iz}$ | $2 \sinh z = e^{z} - e^{-z}$ $\sin(iz) = i \sinh z$

$$\begin{aligned} \mathbf{e}^{z} &= \sum_{k=0}^{\infty} \frac{z^{k}}{k!} \quad \mathbf{e}^{z} = \lim_{n \to \infty} \left(1 + \frac{z}{n} \right)^{n} \quad \mathbf{ln} \ z = \lim_{h \to 0} \frac{z^{h} - 1}{h} \\ \sin z &= \sum_{k=0}^{\infty} (-1)^{k} \frac{z^{2k+1}}{(2k+1)!} \quad \sinh z = \sum_{k=0}^{\infty} \frac{z^{2k+1}}{(2k+1)!} \\ \cos z &= \sum_{k=0}^{\infty} (-1)^{k} \frac{z^{2k}}{(2k)!} \quad \cosh z = \sum_{k=0}^{\infty} \frac{z^{2k}}{(2k)!} \\ \frac{z}{\mathbf{e}^{z} - 1} &= \sum_{k=0}^{\infty} \overline{B}_{k} \frac{z^{k}}{k!} \quad \mathbf{ln} (1 - x) = (-1) \sum_{k=1}^{\infty} \frac{x^{k}}{k} \quad (-1 \le x < 1) \\ \frac{z}{1 - \mathbf{e}^{-z}} &= \sum_{k=0}^{\infty} B_{k} \frac{z^{k}}{k!} \quad (z + 1)^{a} &= \sum_{k=0}^{\infty} \binom{a}{k} z^{k} \quad (a \in \mathbb{C}, |z| < 1) \\ \frac{1}{1 - z} &= \sum_{k=0}^{\infty} z^{k} \quad \frac{1}{(1 - z)^{n}} &= \sum_{k=0}^{\infty} \binom{n + k - 1}{k} z^{k} \quad (|z| < 1) \\ f[a](z) &:= \mathbf{e}^{(z - a)D} f(a) &= \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!} (z - a)^{k} \end{aligned}$$

Differential rechnung $|x_{n+1} = x_n - f(x_n)/f'(x_n)$

$$f'(x) := \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \left| \begin{array}{l} T(x) = f(x_0) + f'(x_0)(x - x_0) \\ N(x) = f(x_0) - \frac{1}{f'(x_0)}(x - x_0) \end{array} \right|$$

$$(e^x)' = e^x \quad \left| \begin{array}{l} (fg)' = f'g + g'f \\ \ln' x = 1/x \\ (a^x)' = a^x \ln a \\ (g \circ f)' = (g' \circ f)f' \\ (x^n)' = nx^{n-1} \quad \left| \begin{array}{l} (f^{-1})' = 1/(f' \circ f^{-1}) \\ \text{sin' } x = \cos x \\ \cos' x = -\sin x \\ \text{sinh' } x = \cosh x \\ \cosh' x = \sinh x \quad \cot' x = 1 - \cot^2 x = -1/\sin^2 x \\ \text{arcsin' } x = 1/\sqrt{1-x^2} \\ \text{arccos' } x = -1/\sqrt{1-x^2} \\ \text{arccos' } x = -1/\sqrt{1-x^2} \\ \text{arcsinh' } x = 1/\sqrt{x^2+1} \\ \text{arcosh' } x = 1/\sqrt{x^2-1} \\ \text{arcoth' } x = 1/(1-x^2) \\ \text$$

Integralrechnung

$$\int_{a}^{b} f(x) dx := \lim_{n \to \infty} \sum_{k=1}^{n} f(a + k \frac{b-a}{n}) \frac{b-a}{n} \quad (f \in C[a, b])$$

$$\int_{a}^{b} f'(x) dx = [f(x)]_{a}^{b} := f(b) - f(a) \quad (f \in C^{1}[a, b])$$

$$\int_{a}^{b} f(g(x)) g'(x) dx = \int_{g(a)}^{g(b)} f(u) du \quad \left(f \in C^{1}[a, b], I \right)$$

$$\int_{a}^{b} f'(x) g(x) dx = [f(x)g(x)]_{a}^{b} - \int_{a}^{b} f(x) g'(x) dx \quad (f, g \in C^{1})$$

$$t = \tan(\frac{x}{2}), \quad \sin x = \frac{2t}{1+t^{2}}, \quad \cos x = \frac{1-t^{2}}{1+t^{2}}, \quad dx = \frac{2dt}{1+t^{2}}$$

$$L\{f(t)\} := \int_{0}^{\infty} f(t) e^{-pt} dt, \quad F\{f(t)\} := \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

$$\frac{d}{dx} \int_{a(x)}^{b(x)} f(x, t) dt = \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x, t) dt + g(x)$$

$$g(x) = f(x, b(x))b'(x) - f(x, a(x))a'(x)$$

Komplexe Zahlen

$$\begin{split} z &= r\mathrm{e}^{\mathrm{i}\varphi} = a + b\mathrm{i} \\ \overline{z} &= r\mathrm{e}^{-\mathrm{i}\varphi} = a - b\mathrm{i} \\ \mathrm{Re}\,z &= a = r\cos\varphi \\ \mathrm{Im}\,z &= b = r\sin\varphi \end{split} \qquad \begin{aligned} |z| &= r = \sqrt{a^2 + b^2} \\ \mathrm{arg}(z) &= \varphi = \mathrm{sgn}(b) \arccos(a/r) \\ z_1 + z_2 &= (a_1 + a_2) + (b_1 + b_2)\mathrm{i} \\ z_2 - z_2 &= (a_1 - a_2) + (b_1 - b_2)\mathrm{i} \\ z_1 z_2 &= r_1 r_2 \mathrm{e}^{\mathrm{i}(\varphi_1 + \varphi_2)} = (a_1 a_2 - b_1 b_2) + (a_1 b_2 + a_2 b_1)\mathrm{i} \\ \frac{z_1}{z_2} &= \frac{r_1}{r_2} \mathrm{e}^{\mathrm{i}(\varphi_1 - \varphi_2)} = \frac{a_1 a_2 + b_1 b_2}{a_2^2 + b_2^2} + \frac{a_2 b_1 - a_1 b_2}{a_2^2 + b_2^2} \mathrm{i} \\ \frac{1}{z} &= \frac{1}{r} \mathrm{e}^{-\mathrm{i}\varphi} = \frac{a}{a^2 + b^2} - \frac{b}{a^2 + b^2} \mathrm{i} \end{aligned}$$

$$x^2 + px + q = 0 \Leftrightarrow 2x = -p \pm \sqrt{p^2 - 4q}$$

 $f(x) = f(2a - x)$ (Achsensymmetrie)
 $f(x) = 2b - f(2a - x)$ (Punktsymmetrie)

$$\begin{array}{l} \textbf{Lineare Algebra} \mid \det(\lambda A) = \lambda^n \det(A), \ \det(A^{-1}) = \frac{1}{\det A} \\ \langle Av, w \rangle = \langle v, A^H w \rangle & (AB)^H = B^H A^H \\ \langle v, w \rangle = |v| |w| \cos \varphi & (AB)^{-1} = B^{-1} A^{-1} \\ |v \times w| = |v| |w| \sin \varphi & \det(AB) = \det(A) \det(B) \\ \text{proj}[w](v) = \frac{\langle v, w \rangle}{\langle w, w \rangle} w, \quad w_k := v_k - \sum_{i=1}^{k-1} \operatorname{proj}[w_i](v_k) \\ \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix} \\ \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}, \quad R(\varphi) = \begin{pmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{pmatrix}$$

Polarkoordinaten Kugelkoordinaten

$x = r \cos \varphi$	$x = r\sin\theta\cos\varphi$
$y = r \sin \varphi$	$y = r\sin\theta\sin\varphi$
$\varphi \in (-\pi, \pi]$	$z = r \cos \theta$
$\det J = r$	$\varphi \in (-\pi, \pi], \ \theta \in [0, \pi]$
Zylinderkoordinaten	$\det J = r^2 \sin \theta$
$x = r_{xy} \cos \varphi$	$\theta = \beta - \pi/2$

$y = r_{xy} \sin \varphi$ z = z $\beta \in [-\pi/2, \pi/2]$ $\cos \theta = \sin \beta$ $\sin \theta = \cos \beta$ $\det J = r_{xy}$

Vektoranalysis

$$\begin{array}{ll} \nabla(|\mathbf{x}|^2) = 2\mathbf{x} & \nabla(fg) = g\nabla f + f\nabla g \\ \nabla|\mathbf{x}| = \mathbf{x}/|\mathbf{x}| & \nabla(f,g) = (Df)^T g + (Dg)^T f \\ \nabla(\frac{1}{g}) = -\frac{\nabla g}{g^2} & \nabla(f/g) = (g\nabla f - f\nabla g)/g^2 \\ \nabla \times \nabla f = 0 & \langle \nabla, \nabla \times \mathbf{v} \rangle = 0 & \nabla \times (f\mathbf{v}) = f(\nabla \times \mathbf{v}) - \mathbf{v} \times \nabla f \\ \nabla \times \nabla \times \mathbf{v} = \nabla(\nabla, \mathbf{v}) - \Delta \mathbf{v} \\ \langle \nabla, v \times w \rangle = \langle \mathbf{w}, \nabla \times \mathbf{v} \rangle - \langle \mathbf{v}, \nabla \times \mathbf{w} \rangle \\ \int_{\gamma} f ds := \int_{a}^{b} f(t) |\gamma'(t)| dt, \ \int_{\gamma} \langle \mathbf{F}, d\mathbf{x} \rangle := \int_{a}^{b} \langle \mathbf{F}(\mathbf{x}(t)), \mathbf{x}'(t) \rangle dt \\ \int_{\varphi(U)} f(\mathbf{x}) d\mathbf{x} = \int_{U} f(\varphi(\mathbf{u})) |\det D\varphi(\mathbf{u})| d\mathbf{u} \end{array}$$

Extremwerte

$$f(x) = \text{extrem} \Rightarrow f'(x) = 0, \ f(p) = \text{extrem} \Rightarrow df_p = 0$$

 $f(x, y) = \text{extrem unter } g(x, y) = 0 \Rightarrow df = \lambda dg$
 $J[\mathbf{x}] := \int_a^b L(t, \mathbf{x}(t), \mathbf{x}'(t)) dt = \text{extrem} \Rightarrow \frac{\partial L}{\partial x_k} = \frac{d}{dt} \frac{\partial L}{\partial x_k'}$

Interpolation

Linear:
$$p(x) = y_1 + \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Quadratisch: $p(x) = y_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1)$
 $a_1 = \frac{y_1 - y_0}{x_1 - x_0}, \quad a_2 = \frac{1}{x_2 - x_1} \left(\frac{y_2 - y_0}{x_2 - x_0} - a_1\right)$

$$y = \overline{y} + \frac{s_{xy}}{s_x}(x - \overline{x}), \ s_x = \sum_{k=1}^{n} (x_k - \overline{x})^2, \ s_{xy} = \sum_{k=1}^{n} (x_k - \overline{x})(y_k - \overline{y})$$

Logik

A	B	$A \wedge B$	$A \vee B$	$A \rightarrow B$	$A \leftrightarrow B$	$A \oplus B$	$A \uparrow B$
0	0	0	0	1	1	0	1
0	1	0	1	1	0	1	1
1	0	0	1	0	0	1	1
1	1	1	1	1	1	0	0

Disjunktion	Konjunktion	Bezeichnung
$A \lor A \equiv A$ $A \lor 0 \equiv A$ $A \lor 1 \equiv 1$ $A \lor \overline{A} \equiv 1$	$A \land A \equiv A$ $A \land 1 \equiv A$ $A \land 0 \equiv 0$ $A \land \overline{A} \equiv 0$	Idempotenzgesetze Neutralitätsgesetze Extremalgesetze Komplementärgesetze
$A \lor B \equiv B \lor A$ $(A \lor B) \lor C \equiv A \lor (B \lor C)$ $\overline{A \lor B} \equiv \overline{A} \land \overline{B}$ $A \lor (A \land B) \equiv A$	$A \wedge B \equiv B \wedge A$ $(A \wedge B) \wedge C \equiv A \wedge (B \wedge C)$ $\overline{A \wedge B} \equiv \overline{A} \vee \overline{B}$ $A \wedge (A \vee B) \equiv A$	Kommutativgesetze Assoziativgesetze De Morgansche Regeln Absorptionsgesetze

$$\begin{array}{c|c} (A \to B) \equiv \overline{A} \vee B & | & (A \leftrightarrow B) \equiv (A \to B) \wedge (B \to A) \\ (A \to B) \equiv (\overline{B} \to \overline{A}) & | & (A \leftrightarrow B) \equiv (\overline{A} \vee B) \wedge (\overline{B} \vee A) \\ A \vee \forall_x P_x \equiv \forall_x (A \vee P_x) & | & \forall_x (P_x \wedge Q_x) \equiv \forall_x P_x \wedge \forall_x Q_x \\ A \wedge \exists_x P_x \equiv \exists_x (A \wedge P_x) & | & \exists_x (P_x \vee Q_x) \equiv \exists_x P_x \vee \exists_x Q_x \\ (I \models M) :\Leftrightarrow \forall \varphi \in M \colon I(\varphi) \\ \end{array}$$

$$(\models \varphi) : \Leftrightarrow \forall I \colon I(\varphi) \mid (M \models \varphi) : \Leftrightarrow \forall I \colon ((I \models M) \Rightarrow I(\varphi))$$

$$\operatorname{erf}(\varphi) : \Leftrightarrow \exists I \colon I(\varphi) \mid \operatorname{erf}(M) : \Leftrightarrow \exists I \colon (I \models M)$$

$$\operatorname{erf}(\{\varphi_1,\ldots,\varphi_n\}) \Leftrightarrow \operatorname{erf}(\varphi_1\wedge\ldots\wedge\varphi_n)$$

$$\operatorname{erf}(\varphi_1 \vee \ldots \vee \varphi_n) \Leftrightarrow \operatorname{erf}(\varphi_1) \vee \ldots \vee \operatorname{erf}(\varphi_n)$$

$$(M \vdash \varphi) \Rightarrow (M \models \varphi)$$
 (Korrektheit)

$$(M \models \varphi) \Rightarrow (M \vdash \varphi)$$
 (Vollständigkeit)

$$(M \cup \{\varphi\} \vdash \psi) \Leftrightarrow (M \vdash \varphi \rightarrow \psi)$$

$$(M \cup \{\varphi\} \models \psi) \Leftrightarrow (M \models \varphi \rightarrow \psi)$$

Mengenlehre

$$\begin{array}{l} A \cap B := \{x \mid x \in A \wedge x \in B\} \\ A \cup B := \{x \mid x \in A \vee x \in B\} \\ A \setminus B := \{x \mid x \in A \wedge x \notin B\} \\ A \setminus B := \{x \mid x \in A \wedge x \notin B\} \\ A \cap B := \{x \mid x \in A \wedge x \notin B\} \\ A \cap B := \{x \mid x \in A \wedge x \notin B\} \\ A \cap B := \{x \mid x \in A \wedge x \notin B\} \\ A \cap B := \{x \mid x \in A \cap x \notin B\} \\ A \cap B := \{x \mid x \in A \cap x \notin B\} \\ A \cap B := \{x \mid x \in A \cap x \notin B\} \\ A \cap B := \{x \mid x \in A \cap x \notin B\} \\ A \cap B := \{x \mid x \in A \cap x \notin B\} \\ A \cap B := \{x \mid x \in A \cap x \notin B\} \\ A \cap B := \{x \mid x \in A \cap x \notin B\} \\ A \cap B := \{x \mid x \in A \cap x \notin B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in A \cap x \in B\} \\ A \cap B := \{x \mid x \in A \cap x \in A \cap x \in A\} \\ A \cap B := \{x \mid x \in A \cap x \in A \cap x \in A \cap x \in A\} \\ A \cap B := \{x \mid x \in A \cap x \in A \cap x \in A\} \\ A \cap B := \{x \mid x \in A \cap x \in A \cap x \in A \cap x \in A\} \\ A \cap B := \{x \mid x \in A \cap x \in A \cap x \in A \cap x \in A\} \\ A \cap B := \{x \mid x \in A \cap x \in A \cap x \in A\} \\ A \cap B$$

$$\begin{split} \sum_{k=m}^{n-1} q^k &= \frac{q^n - q^m}{q-1}, \quad \sum_{k=m}^{n-1} k^p q^k = \left(q \frac{\mathrm{d}}{\mathrm{d}q}\right)^p \frac{q^n - q^m}{q-1} \\ \sum_{k=1}^n k &= (n/2)(n+1) \\ \sum_{k=1}^n k^2 &= (n/6)(n+1)(2n+1) \\ \sum_{k=1}^n k^3 &= (n/2)^2(n+1)^2 \\ (a+b)^n &= \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k, \quad \binom{n}{k} := \frac{1}{k!} n^k = \frac{n!}{k!(n-k)!} \\ n! &= \Gamma(n+1), \quad \Gamma(z+1) = z\Gamma(z), \quad \Gamma(z)\Gamma(1-z) = \frac{\pi}{\sin(\pi z)} \\ \binom{n+1}{k+1} &= \binom{n}{k} + \binom{n+1}{k}, \quad \binom{n}{k} &= \binom{n}{n-k}, \quad \binom{n}{0} &= \binom{n}{n} &= 1 \\ n! &\approx \sqrt{2\pi n} \, n^n \exp\left(\frac{1}{12n} - n\right) \approx \sqrt{2\pi n} \, (n/e)^n \end{split}$$

Twelvefold way. Fächer: n := |N|, Karten: k := |K|

	$f: K \rightarrow N$	$f \in \mathrm{Inj}(K,N)$	$f \in \operatorname{Sur}(K, N)$
\overline{f}	n^k	n <u>k</u>	$n!\binom{k}{n}$
$f \circ S_k$	$\binom{n+k-1}{k}$	$\binom{n}{k}$	$\binom{k-1}{k-n}$
$S_n \circ f$	$\sum_{i=0}^{n} \begin{Bmatrix} k \\ i \end{Bmatrix}$	$[k \leq n]$	${k \brace n}$
$S_n \circ f \circ S_k$	$p_n(n+k)$	$[k \leq n]$	$p_n(k)$

 S_k : Karten nicht unterscheidbar | Inj: max. 1 Karte pro Fach S_n : Fächer nicht unterscheidbar | Sur: mind. 1 Karte pro Fach

Wahrscheinlichkeitsrechnung

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B \mid A)$$

$$A \text{ unabhängig zu } B :\Leftrightarrow P(A \cap B) = P(A)P(B)$$

$$P(A) = \sum_{k=1}^{n} P(A \mid B_k)P(B_k) \text{ für Zerlegung } (B_k) \text{ von } \Omega$$

$$P(A) = \int_{-\infty}^{\infty} P(A \mid X = x) \, dF_X(x)$$

$$E(X) = \sum_{\omega \in \Omega} X(\omega)P(\{\omega\}) = \sum_{x \in \Omega'} xP(X = x)$$

$$E(X) = \int_{-\infty}^{\infty} xf(x) \, dx = \int_{0}^{\infty} (1 - F(x)) \, dx - \int_{-\infty}^{0} F(x) \, dx$$

$$E(g(X)) = \int_{-\infty}^{\infty} g(x)f(x) \, dx \quad | P(A) = E(1_A)$$

$$E(X \mid A) = E(1_A X)/P(A) \quad | P(A \mid B) = E(1_A \mid B)$$

Normalverteilung
$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2} dt = \frac{1}{2} + \frac{1}{2} \operatorname{erf}(\frac{x}{\sqrt{2}})$$

$$F(x) = \Phi(\frac{x-\mu}{\sigma})$$

Mechanik

$\mathbf{v} = \mathbf{x}'(t)$	$\omega = \varphi'(t)$
$\mathbf{a} = \mathbf{v}'(t)$	$\alpha = \omega'(t)$
$\mathbf{F} = \mathbf{p}'(t)$	$\mathbf{M} = \mathbf{L}'(t)$
$\mathbf{p} = m\mathbf{v}$	$L = J\omega$
$\mathbf{F} = m\mathbf{a}$	$M = J\alpha$
$P = \langle \mathbf{F}, \mathbf{v} \rangle$	$P = \langle \mathbf{M}, \boldsymbol{\omega} \rangle$
$E_{\rm kin} = \frac{1}{2}m \mathbf{v} ^2$	$E_{\rm rot} = \frac{1}{2}J\omega^2$
$s = \varphi r \mid \mathbf{M} = 1$	$\mathbf{r} \times \mathbf{F} \mid E_{\text{pot}} = mgh$
$v = \omega r \mid \mathbf{L} = \mathbf{r}$	\times p $\mid E_{\rm kin} + E_{\rm pot} = {\rm const.}$
	$\mathbf{p} \times \mathbf{r} \mid F = D\mathbf{s}$ (Feder)

Gleichstrom

$$U = RI \mid Q = It \mid GR = 1$$

 $I = GU \mid W = Pt \mid$
 $P = UI \mid W = QU \mid$

Wechselstrom

Wethselstom
$$\underline{U} = \underline{ZI} \mid \underline{Z} = R + jX \mid Z^2 = R^2 + X^2$$

$$\underline{I} = \underline{YU} \mid \underline{Y} = G + jQ \mid R = Z\cos\varphi$$

$$\underline{S} = \underline{UI} \mid \underline{S} = P + jB \mid X = Z\sin\varphi$$

$$\underline{Z} = R \mid \text{Widerstand} \quad \omega = 2\pi f$$

$$\underline{Z} = jX_C \mid \text{Kondensator} \quad X_C = -1/(\omega C)$$

$$\underline{Z} = jX_L \mid \text{Spule} \quad X_L = \omega L$$

$$u_s = \sqrt{2} U_{\text{eff}} \mid u = u_s \sin(\omega t + \varphi_0)$$

$$i_s = \sqrt{2} I_{\text{eff}} \mid i = i_s \sin(\omega t + \varphi_0)$$

Allgemeine Gleichungen

$$u = Ri$$

 $i = Cu'(t)$
 $u = Li'(t)$ $p = ui$

Elektrostatisches Feld

$$\begin{split} F &= \frac{1}{4\pi\varepsilon} \frac{Q_1 Q_2}{r^2} \; \bigg| \; \mathbf{F}_1 = \frac{1}{4\pi\varepsilon} Q_1 Q_2 \frac{\mathbf{x}_1 - \mathbf{x}_2}{|\mathbf{x}_1 - \mathbf{x}_2|^3} \\ \mathbf{F} &= q \mathbf{E} \; \bigg| \; Q = C U \; \bigg| \; U = \varphi(B) - \varphi(A) \\ \mathbf{D} &= \varepsilon \mathbf{E} \; \bigg| \; \varepsilon = \varepsilon_0 \varepsilon_r \; \bigg| \; W = Q U \\ \mathbf{E} &= -\nabla \varphi \\ \varepsilon_0 E^2 &= 2 w_e \end{split}$$

Plattenkondensator

$$U = Ed \mid C = \varepsilon A/d$$

Homogenes Feld in der Spule

$$Hl = NI \mid Bl = \mu NI \mid \Theta = NI$$

Magnetostatisches Feld

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B} \quad \Phi = BA$$

$$F = q\upsilon B \quad \mathbf{B} = \mu \mathbf{H}$$

$$F = BIl \quad \mu = \mu_0 \mu_r$$

$$H = I/(2\pi r) \quad \text{(Feld um einen geraden Leiter)}$$

$$B^2 = 2\mu_0 w_m$$

Elektrodynamik

$$\begin{aligned} \mathbf{E} &= -\nabla \varphi \\ \varepsilon \Delta \varphi &= -\rho(x) \\ \varepsilon_0 E^2 &= 2w_e \\ B^2 &= 2\mu_0 w_m \end{aligned}$$

Maxwell-Gleichungen

$$\begin{split} \langle \nabla, \mathbf{D} \rangle &= \rho_f(x) \\ \langle \nabla, \mathbf{B} \rangle &= 0 \\ \nabla \times \mathbf{E} &= -\partial_t \mathbf{B} \\ \nabla \times \mathbf{H} &= \mathbf{J}_f + \partial_t \mathbf{D} \end{split} \quad \begin{aligned} \langle \nabla, \varepsilon_0 \mathbf{E} \rangle &= \rho(x) \\ \langle \nabla, \mathbf{B} \rangle &= 0 \\ \nabla \times \mathbf{E} &= -\partial_t \mathbf{B} \\ \nabla \times \mathbf{B} &= \mu_0 (\mathbf{J} + \varepsilon_0 \partial_t \mathbf{E}) \end{aligned}$$

Spezielle Relativitätstheorie

$$\begin{split} \gamma &= \frac{1}{\sqrt{1 - \beta^2}}, \quad \beta = v/c \\ \gamma &= \cosh \varphi, \quad \beta \gamma = \sinh \varphi, \quad \beta = \tanh \varphi \\ ct' &= \gamma(ct - \beta x), \quad x' = \gamma(x - vt), \quad (y, z)' = (y, z) \\ t &= \gamma \tau \qquad \left| \begin{array}{c} E_{\rm kin} = E - E_0 \\ E_{\rm kin} = \gamma mc^2 - mc^2 \\ E &= \gamma mc^2 \end{array} \right| \begin{bmatrix} E_{\rm kin} = \gamma mc^2 - mc^2 \\ E^2 &= (pc)^2 + (mc^2)^2 \end{bmatrix} \\ \Lambda_v &= \begin{bmatrix} \gamma & -\beta \gamma & 0 & 0 \\ -\beta \gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ g &= {\rm diag}(1, -1, -1, -1) \\ (\partial_\mu) &= (\partial_{ct}, \partial_x, \partial_y, \partial_z) \\ (\partial^\mu) &= (\partial_{ct}, -\partial_x, -\partial_y, -\partial_z) \\ \textbf{Optik} \end{split}$$

$$\frac{1}{f} = \frac{1}{g} + \frac{1}{b}, \quad A = \frac{B}{G} = \frac{b}{g}$$

$$n_1 \sin(\varphi_1) = n_2 \sin(\varphi_2)$$

$$c_0 = nc$$

Thermodynamik

$$R = N_A k_B \mid m = nM \mid V = nV_m$$

$$R = R_s M \mid m = Nm_T \mid N = nN_A$$

$$pV = nRT$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$Q = mc\Delta T$$

Konstanten

$$\begin{split} \varepsilon_0 &= 8.8542 \times 10^{-12} \, \text{C/(V m)} \\ \mu_0 &= 4\pi \times 10^{-7} \, \text{H/m} \\ c_0 &= 2.9979 \times 10^8 \, \text{m/s} \\ e &= 1.6022 \times 10^{-19} \, \text{C} \\ G &= 6.674 \times 10^{-11} \, \text{m}^3/(\text{kg s}^2) \\ N_A &= 6.0221 \times 10^{23} \, \text{mol}^{-1} \\ k_B &= 1.3806 \times 10^{-23} \, \text{J/K} \\ R &= 8.3145 \, \text{J/(mol K)} \\ 0 \, \text{K} &= -273.15 \, ^{\circ} \text{C} \\ u &= 1.6605 \times 10^{-27} \, \text{kg} \\ h &= 6.6261 \times 10^{-34} \, \text{Js} \\ \bar{h} &= 1.0546 \times 10^{-34} \, \text{Js} \\ \bar{\sigma} &= 5.6704 \times 10^{-8} \, \text{W/(m}^2 \text{K}^4) \\ m_e &= 9.1094 \times 10^{-31} \, \text{kg} \\ m_p &= 1.6726 \times 10^{-27} \, \text{kg} \\ m_n &= 1.6749 \times 10^{-27} \, \text{kg} \end{split}$$

 $m_{\alpha} = 6.6447 \times 10^{-27} \text{ kg}$