Winkelfunktionen

$$\sin(x+y) = \sin x\cos y + \cos x \sin y \qquad \sin(z \pm \pi) = -\sin z$$
$$\sin(x-y) = \sin x\cos y - \cos x \sin y \qquad \cos(z \pm \pi) = -\cos z$$
$$\cos(x+y) = \cos x\cos y - \sin x \sin y \qquad \sin(\pi/2 - x) = \cos x$$
$$\cos(x-y) = \cos x\cos y + \sin x \sin y \qquad \cos(\pi/2 - x) = \sin x$$
$$\sin(nx) = 2\cos x \sin((n-1)x) - \sin((n-2)x) \qquad \sin(-z) = -\sin z$$
$$\cos(nx) = 2\cos x \cos((n-1)x) - \cos((n-2)x) \qquad \cos(-z) = \cos z$$
$$\cos^2 z + \sin^2 z = 1 \qquad \cosh^2 z - \sinh^2 z = 1 \qquad \cosh(\mathrm{i}z) = \cos z$$
$$\mathrm{e}^{\mathrm{i}z} = \cos z + \mathrm{i}\sin z \qquad \mathrm{e}^{z} = \cosh z + \sinh z \qquad \sinh(\mathrm{i}z) = \mathrm{i}\sin z$$
$$2\cos z = \mathrm{e}^{\mathrm{i}z} + \mathrm{e}^{-\mathrm{i}z} \qquad 2\cosh z = \mathrm{e}^{z} + \mathrm{e}^{-z} \qquad \cos(\mathrm{i}z) = \cosh z$$
$$2\mathrm{i}\sin z = \mathrm{e}^{\mathrm{i}z} - \mathrm{e}^{-\mathrm{i}z} \qquad 2\sinh z = \mathrm{e}^{z} - \mathrm{e}^{-z} \qquad \sin(\mathrm{i}z) = \mathrm{i}\sinh z$$

Reihen

$$\mathrm{e}^z = \sum_{k=0}^{\infty} \frac{z^k}{k!} \qquad \mathrm{e}^z = \lim_{n\to\infty}\left(1 + \frac{z}{n}\right)^n \qquad \ln z = \lim_{h\to 0}\frac{z^h - 1}{h}$$
$$\sin z = \textstyle\sum_{k=0}^{\infty}(-1)^k \frac{z^{2k+1}}{(2k+1)!} \qquad \sinh z = \sum_{k=0}^{\infty}\frac{z^{2k+1}}{(2k+1)!}$$
$$\cos z = \textstyle\sum_{k=0}^{\infty}(-1)^k \frac{z^{2k}}{(2k)!} \qquad \cosh z = \sum_{k=0}^{\infty}\frac{z^{2k}}{(2k)!}$$
$$\frac{z}{\mathrm{e}^z - 1} = \sum_{k=0}^{\infty}\overline{B}_k\frac{z^k}{k!} \qquad \ln(1-x) = (-1)\sum_{k=1}^{\infty}\frac{x^k}{k} \quad (-1 \le x < 1)$$
$$\frac{z}{1 - \mathrm{e}^{-z}} = \sum_{k=0}^{\infty} B_k\frac{z^k}{k!} \qquad (z+1)^a = \sum_{k=0}^{\infty}\binom{a}{k} z^k \quad (a \in \mathbb{C}, |z|<1)$$
$$\frac{1}{1-z} = \sum_{k=0}^{\infty} z^k \qquad \frac{1}{(1-z)^n} = \sum_{k=0}^{\infty}\binom{n+k-1}{k} z^k \quad (|z|<1)$$
$$f[a](z) := \mathrm{e}^{(z-a)D}f(a) = \sum_{k=0}^{\infty}\frac{f^{(k)}(a)}{k!}(z-a)^k$$

Differentialrechnung | $x_{n+1} = x_n - f(x_n)/f'(x_n)$

$$f'(x) := \lim_{h\to 0}\frac{f(x+h) - f(x)}{h} \qquad \begin{aligned} T(x) &= f(x_0) + f'(x_0)(x - x_0) \\ N(x) &= f(x_0) - \tfrac{1}{f'(x_0)}(x - x_0)\end{aligned}$$
$$(\mathrm{e}^x)' = \mathrm{e}^x \qquad (fg)' = f'g + g'f$$
$$\ln' x = 1/x \qquad (f/g)' = (f'g - g'f)/g^2$$
$$(a^x)' = a^x \ln a \qquad (g\circ f)' = (g'\circ f)f'$$
$$(x^n)' = nx^{n-1} \qquad (f^{-1})' = 1/(f'\circ f^{-1})$$
$$\sin' x = \cos x \qquad \tan' x = 1 + \tan^2 x = 1/\cos^2 x$$
$$\cos' x = -\sin x \qquad \cot' x = -1 - \cot^2 x = -1/\sin^2 x$$
$$\sinh' x = \cosh x \qquad \tanh' x = 1 - \tanh^2 x = 1/\cosh^2 x$$
$$\cosh' x = \sinh x \qquad \coth' x = 1 - \coth^2 x = -1/\sinh^2 x$$
$$\arcsin' x = 1/\sqrt{1-x^2} \qquad \arctan' x = 1/(1+x^2)$$
$$\arccos' x = -1/\sqrt{1-x^2} \qquad \operatorname{arccot}' x = -1/(1+x^2)$$
$$\operatorname{arsinh}' x = 1/\sqrt{x^2+1} \qquad \operatorname{artanh}' x = 1/(1-x^2)$$
$$\operatorname{arcosh}' x = 1/\sqrt{x^2-1} \qquad \operatorname{arcoth}' x = 1/(1-x^2)$$

Integralrechnung

$$\int_a^b f(x)\,\mathrm{d}x := \lim_{n\to\infty}\textstyle\sum_{k=1}^{n} f\!\left(a + k\frac{b-a}{n}\right)\frac{b-a}{n} \qquad (f \in C[a,b])$$
$$\int_a^b f'(x)\,\mathrm{d}x = [f(x)]_a^b := f(b) - f(a) \qquad (f \in C^1[a,b])$$
$$\int_a^b f(g(x))\,g'(x)\,\mathrm{d}x = \int_{g(a)}^{g(b)} f(u)\,\mathrm{d}u \qquad \left(\begin{smallmatrix} f\in C(I,\mathbb{R}), \\ g\in C^1([a,b],I)\end{smallmatrix}\right)$$
$$\int_a^b f'(x)g(x)\,\mathrm{d}x = [f(x)g(x)]_a^b - \int_a^b f(x)g'(x)\,\mathrm{d}x \quad (f,g \in C^1)$$
$$t = \tan(\tfrac{x}{2}),\ \sin x = \tfrac{2t}{1+t^2},\ \cos x = \tfrac{1-t^2}{1+t^2},\ \mathrm{d}x = \tfrac{2\mathrm{d}t}{1+t^2}$$
$$L\{f(t)\} := \int_0^{\infty} f(t)\mathrm{e}^{-pt}\mathrm{d}t,\ F\{f(t)\} := \tfrac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)\mathrm{e}^{-\mathrm{i}\omega t}\mathrm{d}t$$
$$\frac{\mathrm{d}}{\mathrm{d}x}\int_{a(x)}^{b(x)} f(x,t)\,\mathrm{d}t = g(x) + \int_{a(x)}^{b(x)}\frac{\partial}{\partial x}f(x,t)\,\mathrm{d}t$$
wobei $g(x) = f(x, b(x))b'(x) - f(x, a(x))a'(x)$

Komplexe Zahlen

$$z = r\mathrm{e}^{\mathrm{i}\varphi} = a + b\mathrm{i} \qquad |z| = r = \sqrt{a^2 + b^2}$$
$$\overline{z} = r\mathrm{e}^{-\mathrm{i}\varphi} = a - b\mathrm{i} \qquad \arg(z) = \varphi = \operatorname{sgn}(b)\arccos(a/r)$$
$$\operatorname{Re} z = a = r\cos\varphi \qquad z_1 + z_2 = (a_1 + a_2) + (b_1 + b_2)\mathrm{i}$$
$$\operatorname{Im} z = b = r\sin\varphi \qquad z_2 - z_2 = (a_1 - a_2) + (b_1 - b_2)\mathrm{i}$$
$$z_1 z_2 = r_1 r_2 \mathrm{e}^{\mathrm{i}(\varphi_1 + \varphi_2)} = (a_1a_2 - b_1b_2) + (a_1b_2 + a_2b_1)\mathrm{i}$$
$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\mathrm{e}^{\mathrm{i}(\varphi_1 - \varphi_2)} = \frac{a_1a_2 + b_1b_2}{a_2^2 + b_2^2} + \frac{a_2b_1 - a_1b_2}{a_2^2 + b_2^2}\mathrm{i}$$
$$\frac{1}{z} = \frac{1}{r}\mathrm{e}^{-\mathrm{i}\varphi} = \frac{a}{a^2+b^2} - \frac{b}{a^2+b^2}\mathrm{i}$$

Algebra

$$x^2 + px + q = 0 \iff 2x = -p \pm \sqrt{p^2 - 4q}$$
$$f(x) = f(2a - x) \qquad \text{(Achsensymmetrie)}$$
$$f(x) = 2b - f(2a - x) \qquad \text{(Punktsymmetrie)}$$

Lineare Algebra | $\det(\lambda A) = \lambda^n \det(A),\ \det(A^{-1}) = \frac{1}{\det A}$

$$\langle Av, w\rangle = \langle v, A^H w\rangle \qquad (AB)^H = B^H A^H$$
$$\langle v, w\rangle = |v||w|\cos\varphi \qquad (AB)^{-1} = B^{-1}A^{-1}$$
$$|v \times w| = |v||w|\sin\varphi \qquad \det(AB) = \det(A)\det(B)$$
$$\operatorname{proj}[w](v) = \tfrac{\langle v,w\rangle}{\langle w,w\rangle}w, \quad w_k := v_k - \textstyle\sum_{i=1}^{k-1}\operatorname{proj}[w_i](v_k)$$
$$\begin{pmatrix} a_1 \\ a_2 \\ a_3\end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3\end{pmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1\end{pmatrix}$$
$$\begin{pmatrix} a & b \\ c & d\end{pmatrix}^{-1} = \tfrac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a\end{pmatrix}, \quad R(\varphi) = \begin{pmatrix}\cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi\end{pmatrix}$$

Polarkoordinaten

$x = r\cos\varphi$
$y = r\sin\varphi$
$\varphi \in (-\pi, \pi]$
$\det J = r$

Zylinderkoordinaten

$x = r_{xy}\cos\varphi$
$y = r_{xy}\sin\varphi$
$z = z$
$\det J = r_{xy}$

Kugelkoordinaten

$x = r\sin\theta\,\cos\varphi$
$y = r\sin\theta\,\sin\varphi$
$z = r\cos\theta$
$\varphi \in (-\pi, \pi],\ \theta \in [0, \pi]$
$\det J = r^2\sin\theta$

$\theta = \beta - \pi/2$
$\beta \in [-\pi/2, \pi/2]$
$\cos\theta = \sin\beta$
$\sin\theta = \cos\beta$

Vektoranalysis

$$\nabla(|\mathbf{x}|^2) = 2\mathbf{x} \qquad \nabla(fg) = g\nabla f + f\nabla g$$
$$\nabla|\mathbf{x}| = \mathbf{x}/|\mathbf{x}| \qquad \nabla\langle f, g\rangle = (Df)^T g + (Dg)^T f$$
$$\nabla(\tfrac{1}{g}) = -\tfrac{\nabla g}{g^2} \qquad \nabla(f/g) = (g\nabla f - f\nabla g)/g^2$$
$$\nabla\times\nabla f = 0 \qquad \langle\nabla, f\mathbf{v}\rangle = \langle\nabla f, \mathbf{v}\rangle + f\langle\nabla, \mathbf{v}\rangle$$
$$\langle\nabla, \nabla\times\mathbf{v}\rangle = 0 \qquad \nabla\times(f\mathbf{v}) = f(\nabla\times\mathbf{v}) - \mathbf{v}\times\nabla f$$
$$\nabla\times\nabla\times\mathbf{v} = \nabla\langle\nabla, \mathbf{v}\rangle - \Delta\mathbf{v}$$
$$\langle\nabla, v\times w\rangle = \langle\mathbf{w}, \nabla\times\mathbf{v}\rangle - \langle\mathbf{v}, \nabla\times\mathbf{w}\rangle$$
$$\int_\gamma f\mathrm{d}s := \int_a^b f(t)\,|\gamma'(t)|\,\mathrm{d}t, \quad \int_\gamma\langle\mathbf{F}, \mathrm{d}\mathbf{x}\rangle := \int_a^b\langle\mathbf{F}(\mathbf{x}(t)), \mathbf{x}'(t)\rangle\,\mathrm{d}t$$
$$\int_{\varphi(U)} f(\mathbf{x})\,\mathrm{d}\mathbf{x} = \int_U f(\varphi(\mathbf{u}))\,|\det D\varphi(\mathbf{u})|\,\mathrm{d}\mathbf{u}$$

Extremwerte

$$f(x) = \text{extrem} \Rightarrow f'(x) = 0,\ f(p) = \text{extrem} \Rightarrow \mathrm{d}f_p = 0$$
$$f(x,y) = \text{extrem unter } g(x,y) = 0 \Rightarrow \mathrm{d}f = \lambda\mathrm{d}g$$
$$J[\mathbf{x}] := \int_a^b L(t, \mathbf{x}(t), \mathbf{x}'(t))\,\mathrm{d}t = \text{extrem} \Rightarrow \frac{\partial L}{\partial x_k} = \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial x_k'}$$

Interpolation

Linear: $p(x) = y_1 + \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

Quadratisch: $p(x) = y_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1)$

$$a_1 = \frac{y_1 - y_0}{x_1 - x_0}, \quad a_2 = \frac{1}{x_2 - x_1}\left(\frac{y_2 - y_0}{x_2 - x_0} - a_1\right)$$

Regression

$$y = \overline{y} + \frac{s_{xy}}{s_{xx}}(x - \overline{x}), \quad s_{xx} = \sum_{k=1}^{n}(x_k - \overline{x})^2, \quad s_{xy} = \sum_{k=1}^{n}(x_k - \overline{x})(y_k - \overline{y})$$

Logik

A	B	$A \wedge B$	$A \vee B$	$A \rightarrow B$	$A \leftrightarrow B$	$A \oplus B$	$A \uparrow B$
0	0	0	0	1	1	0	1
0	1	0	1	1	0	1	1
1	0	0	1	0	0	1	1
1	1	1	1	1	1	0	0

Disjunktion	**Konjunktion**	**Bezeichnung**
$A \vee A \equiv A$	$A \wedge A \equiv A$	Idempotenzgesetze
$A \vee 0 \equiv A$	$A \wedge 1 \equiv A$	Neutralitätsgesetze
$A \vee 1 \equiv 1$	$A \wedge 0 \equiv 0$	Extremalgesetze
$A \vee \overline{A} \equiv 1$	$A \wedge \overline{A} \equiv 0$	Komplementärgesetze
$A \vee B \equiv B \vee A$	$A \wedge B \equiv B \wedge A$	Kommutativgesetze
$(A \vee B) \vee C \equiv A \vee (B \vee C)$	$(A \wedge B) \wedge C \equiv A \wedge (B \wedge C)$	Assoziativgesetze
$\overline{A \vee B} \equiv \overline{A} \wedge \overline{B}$	$\overline{A \wedge B} \equiv \overline{A} \vee \overline{B}$	De morgansche Regeln
$A \vee (A \wedge B) \equiv A$	$A \wedge (A \vee B) \equiv A$	Absorptionsgesetze

$(A \rightarrow B) \equiv \overline{A} \vee B$ | $(A \leftrightarrow B) \equiv (A \rightarrow B) \wedge (B \rightarrow A)$

$(A \rightarrow B) \equiv (\overline{B} \rightarrow \overline{A})$ | $(A \leftrightarrow B) \equiv (\overline{A} \vee B) \wedge (\overline{B} \vee A)$

$A \vee \forall_x P_x \equiv \forall_x (A \vee P_x)$ | $\forall_x (P_x \wedge Q_x) \equiv \forall_x P_x \wedge \forall_x Q_x$

$A \wedge \exists_x P_x \equiv \exists_x (A \wedge P_x)$ | $\exists_x (P_x \vee Q_x) \equiv \exists_x P_x \vee \exists_x Q_x$

$(I \models M) :\Leftrightarrow \forall \varphi{\in}M\colon I(\varphi)$

$(\models \varphi) :\Leftrightarrow \forall I\colon I(\varphi)$ | $(M \models \varphi) :\Leftrightarrow \forall I\colon ((I \models M) \Rightarrow I(\varphi))$

$\mathrm{erf}(\varphi) :\Leftrightarrow \exists I\colon I(\varphi)$ | $\mathrm{erf}(M) :\Leftrightarrow \exists I\colon (I \models M)$

$\mathrm{erf}(\{\varphi_1, \ldots, \varphi_n\}) \Leftrightarrow \mathrm{erf}(\varphi_1 \wedge \ldots \wedge \varphi_n)$

$\mathrm{erf}(\varphi_1 \vee \ldots \vee \varphi_n) \Leftrightarrow \mathrm{erf}(\varphi_1) \vee \ldots \vee \mathrm{erf}(\varphi_n)$

$(M \vdash \varphi) \Rightarrow (M \models \varphi)$ (Korrektheit)

$(M \models \varphi) \Rightarrow (M \vdash \varphi)$ (Vollständigkeit)

$(M \cup \{\varphi\} \vdash \psi) \Leftrightarrow (M \vdash \varphi \rightarrow \psi)$

$(M \cup \{\varphi\} \models \psi) \Leftrightarrow (M \models \varphi \rightarrow \psi)$

$(M \models \varphi_1) \wedge (M \models \varphi_2) \wedge (\{\varphi_1, \varphi_2\} \models \psi) \Rightarrow (M \models \psi)$

Natürliches Schließen

$$\frac{\Gamma \vdash A \wedge B}{\Gamma \vdash A} \qquad \frac{\Gamma \vdash A}{\Gamma \vdash A \vee B} \qquad \frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \wedge B} \qquad \frac{}{\Gamma, A \vdash A} \qquad \frac{}{\Gamma \vdash \top}$$

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \rightarrow B} \qquad \frac{\Gamma \vdash A \vee B \quad \Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma \vdash C} \qquad \frac{\Gamma, A \vdash \bot}{\Gamma \vdash \neg A}$$

$$\frac{\Gamma \vdash A \rightarrow B \quad \Gamma \vdash A}{\Gamma \vdash B} \qquad \frac{\Gamma \vdash \neg A \quad \Gamma \vdash A}{\Gamma \vdash \bot} \qquad \frac{\Gamma \vdash \bot}{\Gamma \vdash A} \qquad \left(\frac{\Gamma \vdash \neg\neg A}{\Gamma \vdash A}\right)$$

$$\frac{\Gamma \vdash A}{\Gamma \vdash \forall x\colon A} (x \notin \mathrm{FV}(\Gamma)) \qquad \frac{\Gamma \vdash \forall x\colon A}{\Gamma \vdash A[x := t]} \qquad \frac{\Gamma \vdash A}{\Gamma, \Delta \vdash A}$$

$$\frac{\Gamma \vdash A[x := t]}{\Gamma \vdash \exists x\colon A} \qquad \frac{\Gamma \vdash \exists x\colon A \quad \Gamma, A \vdash B}{\Gamma \vdash B} (x \notin \mathrm{FV}(\Gamma) \cup \mathrm{FV}(B))$$

Mengenlehre

$A \cap B := \{x \mid x \in A \wedge x \in B\}$ | $A \subseteq B :\Leftrightarrow \forall_x (x \in A \Rightarrow x \in B)$

$A \cup B := \{x \mid x \in A \vee x \in B\}$ | $A = B :\Leftrightarrow \forall_x (x \in A \Leftrightarrow x \in B)$

$A \setminus B := \{x \mid x \in A \wedge x \notin B\}$ | $A = B :\Leftrightarrow A \subseteq B \wedge B \subseteq A$

$\bigcap_{i \in I} A_i := \{x \mid \forall i{\in}I\colon x{\in}A_i\}$ | $f(M) := \{y \mid \exists x{\in}M\colon y{=}f(x)\}$

$\bigcup_{i \in I} A_i := \{x \mid \exists i{\in}I\colon x{\in}A_i\}$ | $f^{-1}(N) := \{x \mid f(x) \in N\}$

$A \times B := \{t \mid \exists x{\in}A\colon \exists y{\in}B\colon t = (x, y)\}$

$A \subseteq B \Leftrightarrow A \cup B = B \Leftrightarrow A \cap B = A \Leftrightarrow A \setminus B = \emptyset$

$f(M \cup N) = f(M) \cup f(N)$ | $f^{-1}(M \cup N) = f^{-1}(M) \cup f^{-1}(N)$

$f(M \cap N) \subseteq f(M) \cap f(N)$ | $f^{-1}(M \cap N) = f^{-1}(M) \cap f^{-1}(N)$

$M \subseteq N \Rightarrow f(M) \subseteq f(N)$ | $M \subseteq N \Rightarrow f^{-1}(M) \subseteq f^{-1}(N)$

$(g \circ f)(M) = g(f(M))$ | $(g \circ f)^{-1}(M) = f^{-1}(g^{-1}(M))$

Kombinatorik

$$\sum_{k=m}^{n-1} q^k = \frac{q^n - q^m}{q - 1}, \qquad \sum_{k=m}^{n-1} k^p q^k = \left(q \frac{\mathrm{d}}{\mathrm{d}q}\right)^p \frac{q^n - q^m}{q - 1}$$

$\sum_{k=1}^n k = (n/2)(n+1)$ | $\sum_{k=m}^{n-1} (\Delta a)_k = a_n - a_m$

$\sum_{k=1}^n k^2 = (n/6)(n+1)(2n+1)$ | $(\Delta a)_k := a_{k+1} - a_k$

$\sum_{k=1}^n k^3 = (n/2)^2(n+1)^2$ | $n! = n \cdot (n-1)!$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k, \qquad \binom{n}{k} := \frac{1}{k!} n^{\underline{k}} = \frac{n!}{k!(n-k)!}$$

$n! = \Gamma(n+1)$, $\Gamma(z+1) = z\Gamma(z)$, $\Gamma(z)\Gamma(1-z) = \frac{\pi}{\sin(\pi z)}$

$\binom{n+1}{k+1} = \binom{n}{k} + \binom{n+1}{k}$, $\binom{n}{k} = \binom{n}{n-k}$, $\binom{n}{0} = \binom{n}{n} = 1$

$n! \approx \sqrt{2\pi n}\, n^n \exp\left(\frac{1}{12n} - n\right) \approx \sqrt{2\pi n}\, (n/e)^n$

Twelvefold way. Fächer: $n := |N|$, Karten: $k := |K|$

	$f\colon K \rightarrow N$	$f \in \mathrm{Inj}(K, N)$	$f \in \mathrm{Sur}(K, N)$
f	n^k	$n^{\underline{k}}$	$n! \genfrac\{\}{0pt}{}{k}{n}$
$f \circ S_k$	$\binom{n+k-1}{k}$	$\binom{n}{k}$	$\binom{k-1}{k-n}$
$S_n \circ f$	$\sum_{i=0}^n \genfrac\{\}{0pt}{}{k}{i}$	$[k \leq n]$	$\genfrac\{\}{0pt}{}{k}{n}$
$S_n \circ f \circ S_k$	$p_n(n+k)$	$[k \leq n]$	$p_n(k)$

S_k: Karten nicht unterscheidbar | Inj: max. 1 Karte pro Fach

S_n: Fächer nicht unterscheidbar | Sur: mind. 1 Karte pro Fach

$\genfrac\{\}{0pt}{}{n}{k} = k \genfrac\{\}{0pt}{}{n-1}{k} + \genfrac\{\}{0pt}{}{n-1}{k-1}$ | $\genfrac\{\}{0pt}{}{n}{0} = \genfrac[]{0pt}{}{n}{0} = [n{=}0]$

$\genfrac[]{0pt}{}{n}{k} = (n-1) \genfrac[]{0pt}{}{n-1}{k} + \genfrac[]{0pt}{}{n-1}{k-1}$ | $\genfrac\{\}{0pt}{}{n}{1} = [n{>}0]$

$x^n = \sum_{k=0}^n \genfrac\{\}{0pt}{}{n}{k} x^{\underline{k}}$ | $\genfrac\{\}{0pt}{}{n}{2} = (2^{n-1} - 1)[n{>}0]$

Wahrscheinlichkeitsrechnung

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B \mid A)$

A unabhängig zu $B :\Leftrightarrow P(A \cap B) = P(A)P(B)$

$P(A) = \sum_{k=1}^n P(A \mid B_k)P(B_k)$ für Zerlegung (B_k) von Ω

$P(A) = \int_{-\infty}^{\infty} P(A \mid X = x)\, \mathrm{d}F_X(x)$

$P(g(X) = y) = \sum_{x \in g^{-1}(\{y\})} P(X = x)$

$P(g(X, Y) = z) = \sum_{(x,y) \in g^{-1}(\{z\})} P(X = x, Y = y)$

$E(X) = \sum_{\omega \in \Omega} X(\omega)P(\{\omega\}) = \sum_x xP(X = x)$

$E(g(X)) = \sum_{\omega \in \Omega} g(X(\omega))P(\{\omega\}) = \sum_x g(x)P(X = x)$

$E(g(X, Y) = \sum_{(x,y)} g(x, y)P(X = x, Y = y)$

$E(X) = \int_{-\infty}^{\infty} xf(x)\, \mathrm{d}x = \int_0^{\infty} (1 - F(x))\, \mathrm{d}x - \int_{-\infty}^{0} F(x)\, \mathrm{d}x$

$E(g(X)) = \int_{-\infty}^{\infty} g(x)f(x)\, \mathrm{d}x$ | $P(A) = E(1_A)$

$E(X \mid A) = E(1_A X)/P(A)$ | $P(A \mid B) = E(1_A \mid B)$

$P(X \leq x) = F(x) = \int_{-\infty}^{x} f(t)\, \mathrm{d}t$

$P(a \leq X \leq b) = F(b) - F(a) = \int_a^b f(t)\, \mathrm{d}t$

Normalverteilung

$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \mathrm{e}^{-t^2/2}\, \mathrm{d}t = \frac{1}{2} + \frac{1}{2}\, \mathrm{erf}(\frac{x}{\sqrt{2}})$

$F(x) = \Phi(\frac{x-\mu}{\sigma})$

Mechanik

$\mathbf{v} = \mathbf{x}'(t)$	$\omega = \varphi'(t)$
$\mathbf{a} = \mathbf{v}'(t)$	$\alpha = \omega'(t)$
$\mathbf{F} = \mathbf{p}'(t)$	$\mathbf{M} = \mathbf{L}'(t)$
$\mathbf{p} = m\mathbf{v}$	$L = J\omega$
$\mathbf{F} = m\mathbf{a}$	$M = J\alpha$
$P = \langle \mathbf{F}, \mathbf{v} \rangle$	$P = \langle \mathbf{M}, \boldsymbol{\omega} \rangle$
$E_{\text{kin}} = \frac{1}{2} m \lvert\mathbf{v}\rvert^2$	$E_{\text{rot}} = \frac{1}{2} J\omega^2$

$s = \varphi r$	$\mathbf{M} = \mathbf{r} \times \mathbf{F}$	$E_{\text{pot}} = mgh$
$v = \omega r$	$\mathbf{L} = \mathbf{r} \times \mathbf{p}$	$E_{\text{kin}} + E_{\text{pot}} = \text{const.}$
$a = \alpha r$	$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$	$F = Ds \quad \text{(Feder)}$

Gleichstrom

$U = RI$	$Q = It$	$GR = 1$
$I = GU$	$W = Pt$	
$P = UI$	$W = QU$	

Wechselstrom

$\underline{U} = \underline{Z}\underline{I}$	$\underline{Z} = R + jX$	$Z^2 = R^2 + X^2$
$\underline{I} = \underline{Y}\underline{U}$	$\underline{Y} = G + jQ$	$R = Z\cos\varphi$
$\underline{S} = \underline{U}\underline{I}$	$\underline{S} = P + jB$	$X = Z\sin\varphi$

$\underline{Z} = R$	Widerstand	$\omega = 2\pi f$
$\underline{Z} = jX_C$	Kondensator	$X_C = -1/(\omega C)$
$\underline{Z} = jX_L$	Spule	$X_L = \omega L$

$u_s = \sqrt{2}\, U_{\text{eff}}$	$u = u_s \sin(\omega t + \varphi_0)$
$i_s = \sqrt{2}\, I_{\text{eff}}$	$i = i_s \sin(\omega t + \varphi_0)$

Allgemeine Gleichungen

$u = Ri$	$p = ui$
$i = Cu'(t)$	
$u = Li'(t)$	

Elektrostatisches Feld

$$F = \frac{1}{4\pi\varepsilon} \frac{Q_1 Q_2}{r^2} \quad \Big| \quad \mathbf{F}_1 = \frac{1}{4\pi\varepsilon} Q_1 Q_2 \frac{\mathbf{x}_1 - \mathbf{x}_2}{\lvert\mathbf{x}_1 - \mathbf{x}_2\rvert^3}$$

$\mathbf{F} = q\mathbf{E}$	$Q = CU$	$U = \varphi(B) - \varphi(A)$
$\mathbf{D} = \varepsilon\mathbf{E}$	$\varepsilon = \varepsilon_0 \varepsilon_r$	$W = QU$

$\mathbf{E} = -\nabla\varphi$

$\varepsilon_0 E^2 = 2w_e$

Plattenkondensator

$U = Ed \;\big|\; C = \varepsilon A/d$

Homogenes Feld in der Spule

$Hl = NI \;\big|\; Bl = \mu NI \;\big|\; \Theta = NI$

Magnetostatisches Feld

$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$	$\Phi = BA$
$F = qvB$	$\mathbf{B} = \mu\mathbf{H}$
$F = BIl$	$\mu = \mu_0 \mu_r$

$H = I/(2\pi r) \quad$ (Feld um einen geraden Leiter)

$B^2 = 2\mu_0 w_m$

Elektrodynamik

$\mathbf{E} = -\nabla\varphi$

$\varepsilon\Delta\varphi = -\rho(x)$

$\varepsilon_0 E^2 = 2w_e$

$B^2 = 2\mu_0 w_m$

Maxwell-Gleichungen

$\langle \nabla, \mathbf{D} \rangle = \rho_f(x)$	$\langle \nabla, \varepsilon_0 \mathbf{E} \rangle = \rho(x)$
$\langle \nabla, \mathbf{B} \rangle = 0$	$\langle \nabla, \mathbf{B} \rangle = 0$
$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}$	$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}$
$\nabla \times \mathbf{H} = \mathbf{J}_f + \partial_t \mathbf{D}$	$\nabla \times \mathbf{B} = \mu_0(\mathbf{J} + \varepsilon_0 \partial_t \mathbf{E})$

Spezielle Relativitätstheorie

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}, \quad \beta = v/c$$

$\gamma = \cosh\varphi, \quad \beta\gamma = \sinh\varphi, \quad \beta = \tanh\varphi$

$ct' = \gamma(ct - \beta x), \quad x' = \gamma(x - vt), \quad (y, z)' = (y, z)$

$t = \gamma\tau$	$E_{\text{kin}} = E - E_0$
$p = \gamma mv$	$E_{\text{kin}} = \gamma mc^2 - mc^2$
$E = \gamma mc^2$	$E^2 = (pc)^2 + (mc^2)^2$

$$\Lambda_v = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$g = \operatorname{diag}(1, -1, -1, -1)$

$(\partial_\mu) = (\partial_{ct}, \partial_x, \partial_y, \partial_z)$

$(\partial^\mu) = (\partial_{ct}, -\partial_x, -\partial_y, -\partial_z)$

Optik

$$\frac{1}{f} = \frac{1}{g} + \frac{1}{b}, \quad A = \frac{B}{G} = \frac{b}{g}$$

$n_1 \sin(\varphi_1) = n_2 \sin(\varphi_2)$

$c_0 = nc$

Thermodynamik

$R = N_A k_B$	$m = nM$	$V = nV_m$
$R = R_s M$	$m = Nm_T$	$N = nN_A$

$pV = nRT$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$Q = mc\Delta T$

Konstanten

$\varepsilon_0 = 8.8542 \times 10^{-12}\,\mathrm{C/(V\,m)}$

$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H/m}$

$c_0 = 2.9979 \times 10^{8}\,\mathrm{m/s}$

$e = 1.6022 \times 10^{-19}\,\mathrm{C}$

$G = 6.674 \times 10^{-11}\,\mathrm{m^3/(kg\,s^2)}$

$N_A = 6.0221 \times 10^{23}\,\mathrm{mol^{-1}}$

$k_B = 1.3806 \times 10^{-23}\,\mathrm{J/K}$

$R = 8.3145\,\mathrm{J/(mol\,K)}$

$0\,\mathrm{K} = -273.15\,^\circ\mathrm{C}$

$u = 1.6605 \times 10^{-27}\,\mathrm{kg}$

$h = 6.6261 \times 10^{-34}\,\mathrm{Js}$

$\hbar = 1.0546 \times 10^{-34}\,\mathrm{Js}$

$\sigma = 5.6704 \times 10^{-8}\,\mathrm{W/(m^2K^4)}$

$m_e = 9.1094 \times 10^{-31}\,\mathrm{kg}$

$m_p = 1.6726 \times 10^{-27}\,\mathrm{kg}$

$m_n = 1.6749 \times 10^{-27}\,\mathrm{kg}$

$m_\alpha = 6.6447 \times 10^{-27}\,\mathrm{kg}$