$$\sin(x+y)=\sin x\cos y+\cos x\sin y$$
$$\sin(x-y)=\sin x\cos y-\cos x\sin y$$
$$\cos(x+y)=\cos x\cos y-\sin x\sin y$$
$$\cos(x-y)=\cos x\cos y+\sin x\sin y$$

$$\sin(z\pm\pi)=-\sin z$$
$$\cos(z\pm\pi)=-\cos z$$
$$\cos z=\cos(-z)$$
$$\sin(-z)=-\sin z$$

$$\cos^2 z+\sin^2 z=1$$
$$\mathrm{e}^{\mathrm{i}z}=\cos z+\mathrm{i}\sin z$$
$$2\cos z=\mathrm{e}^{\mathrm{i}z}+\mathrm{e}^{-\mathrm{i}z}$$
$$2\mathrm{i}\sin z=\mathrm{e}^{\mathrm{i}z}-\mathrm{e}^{-\mathrm{i}z}$$

$$\cosh^2 z-\sinh^2 z=1$$
$$\mathrm{e}^z=\cosh z+\sinh z$$
$$2\cosh z=\mathrm{e}^z+\mathrm{e}^{-z}$$
$$2\sinh z=\mathrm{e}^z-\mathrm{e}^{-z}$$

$$\cosh(\mathrm{i}z)=\cos z$$
$$\sinh(\mathrm{i}z)=\mathrm{i}\sin z$$
$$\cos(\mathrm{i}z)=\cosh z$$
$$\sin(\mathrm{i}z)=\mathrm{i}\sinh z$$

$$\mathrm{e}^z=\sum_{k=0}^{\infty}\frac{z^k}{k!}\quad\Big|\quad \mathrm{e}^z=\lim_{n\to\infty}\left(1+\frac{z}{n}\right)^n\quad\Big|\quad \ln z=\lim_{h\to 0}\frac{z^h-1}{h}$$

$$\sin z=\textstyle\sum_{k=0}^{\infty}(-1)^k\frac{z^{2k+1}}{(2k+1)!}$$
$$\cos z=\textstyle\sum_{k=0}^{\infty}(-1)^k\frac{z^{2k}}{(2k)!}$$
$$\sinh z=\textstyle\sum_{k=0}^{\infty}\frac{z^{2k+1}}{(2k+1)!}$$
$$\cosh z=\textstyle\sum_{k=0}^{\infty}\frac{z^{2k}}{(2k)!}$$

$$\sum_{k=m}^{n-1}q^k=\frac{q^m-q^n}{q-1},\qquad \sum_{k=m}^{n-1}k^p q^k=\left(q\frac{\mathrm{d}}{\mathrm{d}q}\right)^p\frac{q^m-q^n}{q-1}$$

$$\textstyle\sum_{k=1}^{n}k=(n/2)(n+1)$$
$$\textstyle\sum_{k=1}^{n}k^2=(n/6)(n+1)(2n+1)$$
$$\textstyle\sum_{k=1}^{n}k^3=(n/2)^2(n+1)^2$$

$$\textstyle\sum_{k=m}^{n-1}(\Delta a)_k=a_n-a_m$$
$$(\Delta a)_k:=a_{k+1}-a_k$$

$$(a+b)^n=\sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k,\qquad \binom{n}{k}:=\frac{1}{k!}n^{\underline{k}}=\frac{n!}{k!(n-k)!}$$

$$\frac{z}{\mathrm{e}^z-1}=\sum_{k=0}^{\infty}\overline{B}_k\frac{x^k}{k!},\qquad \frac{z}{1-\mathrm{e}^{-z}}=\sum_{k=0}^{\infty}B_k\frac{x^k}{k!}$$

$$f[a](z):=\mathrm{e}^{(z-a)D}f(a)=\sum_{k=0}^{\infty}\frac{f^{(k)}(a)}{k!}(z-a)^k$$

$f(x)=f(2a-x)$ (Achsensymmetrie)

$f(x)=2b-f(2a-x)$ (Punktsymmetrie)

$$(\mathrm{e}^x)'=\mathrm{e}^x$$
$$\ln' x=1/x$$
$$(a^x)'=a^x\ln a$$
$$(x^n)'=nx^{n-1}$$

$$(fg)'=f'g+g'f$$
$$(f/g)'=(f'g-g'f)/g^2$$
$$(g\circ f)'=(g'\circ f)f'$$
$$(f^{-1})'=1/(f'\circ f^{-1})$$

$$\sin' x=\cos x$$
$$\cos' x=-\sin x$$
$$\sinh' x=\cosh x$$
$$\cosh' x=\sinh x$$

$$\tan' x=1+\tan^2 x=1/\cos^2 x$$
$$\cot' x=-1-\cot^2 x=-1/\sin^2 x$$
$$\tanh' x=1-\tanh^2 x=1/\cosh^2 x$$
$$\coth' x=1-\coth^2 x=-1/\sinh^2 x$$

$$\arcsin' x=1/\sqrt{1-x^2}$$
$$\arccos' x=-1/\sqrt{1-x^2}$$
$$\operatorname{arsinh}' x=1/\sqrt{x^2+1}$$
$$\operatorname{arcosh}' x=1/\sqrt{x^2-1}$$

$$\arctan' x=1/(1+x^2)$$
$$\operatorname{arccot}' x=-1/(1+x^2)$$
$$\operatorname{artanh}' x=1/(1-x^2)$$
$$\operatorname{arcoth}' x=1/(1-x^2)$$

$$\nabla(|\mathbf{x}|^2)=2\mathbf{x}$$
$$\nabla|\mathbf{x}|=\mathbf{x}/|\mathbf{x}|$$
$$\nabla(\tfrac{1}{g})=-\tfrac{\nabla g}{g^2}$$

$$\nabla(fg)=g\nabla f+f\nabla g$$
$$\nabla\langle f,g\rangle=(Df)^T g+(Dg)^T f$$
$$\nabla(f/g)=(g\nabla f-f\nabla g)/g^2$$

$$f'(x):=\lim_{h\to 0}\tfrac{f(x+h)-f(x)}{h}$$

$$\int_a^b f(g(x))\,g'(x)\,\mathrm{d}x=\int_{g(a)}^{g(b)}f(u)\,\mathrm{d}u\quad\left(\substack{f\in C(I,\mathbb{R}),\\ g\in C^1([a,b],I)}\right)$$

$$\int_a^b f'(x)g(x)\,\mathrm{d}x=[f(x)g(x)]_a^b-\int_a^b f(x)g'(x)\,\mathrm{d}x\quad(f,g\in C^1)$$

$$t=\tan(\tfrac{x}{2}),\ \sin x=\tfrac{2t}{1+t^2},\ \cos x=\tfrac{1-t^2}{1+t^2},\ \mathrm{d}x=\tfrac{2\mathrm{d}t}{1+t^2}$$

$$L\{f(t)\}:=\int_0^{\infty}f(t)\mathrm{e}^{-pt}\mathrm{d}t,\ F\{f(t)\}:=\tfrac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(t)\mathrm{e}^{-\mathrm{i}\omega t}\mathrm{d}t$$

$$\int_\gamma f\,\mathrm{d}s:=\int_a^b f(t)\,|\gamma'(t)|\,\mathrm{d}t,\ \int_\gamma\langle\mathbf{F},\mathrm{d}\mathbf{x}\rangle:=\int_a^b\langle\mathbf{F}(\mathbf{x}(t)),\mathbf{x}'(t)\rangle\,\mathrm{d}t$$

$$z=r\mathrm{e}^{\mathrm{i}\varphi}=a+b\mathrm{i}$$
$$\overline{z}=r\mathrm{e}^{-\mathrm{i}\varphi}=a-b\mathrm{i}$$
$$\operatorname{Re}z=a=r\cos\varphi$$
$$\operatorname{Im}z=b=r\sin\varphi$$

$$|z|=r=\sqrt{a^2+b^2}$$
$$\arg(z)=\varphi=\operatorname{sgn}(b)\arccos(a/r)$$
$$z_1+z_2=(a_1+a_2)+(b_1+b_2)\mathrm{i}$$
$$z_2-z_2=(a_1-a_2)+(b_1-b_2)\mathrm{i}$$

$$z_1z_2=r_1r_2\mathrm{e}^{\mathrm{i}(\varphi_1+\varphi_2)}=(a_1a_2-b_1b_2)+(a_1b_2+a_2b_1)\mathrm{i}$$
$$\frac{z_1}{z_2}=\frac{r_1}{r_2}\mathrm{e}^{\mathrm{i}(\varphi_1-\varphi_2)}=\frac{a_1a_2+b_1b_2}{a_2^2+b_2^2}+\frac{a_2b_1-a_1b_2}{a_2^2+b_2^2}\mathrm{i}$$
$$\frac{1}{z}=\frac{1}{r}\mathrm{e}^{-\mathrm{i}\varphi}=\frac{a}{a^2+b^2}-\frac{b}{a^2+b^2}\mathrm{i}$$
$$x^2+px+q=0:\ \ x=-\tfrac{p}{2}\pm\tfrac{1}{2}\sqrt{p^2-4q}$$
$$\begin{bmatrix}a&b\\c&d\end{bmatrix}^{-1}=\frac{1}{ad-bc}\begin{bmatrix}d&-b\\-c&a\end{bmatrix},\quad R(\varphi)=\begin{bmatrix}\cos\varphi&-\sin\varphi\\\sin\varphi&\cos\varphi\end{bmatrix}$$

Polarkoordinaten

$$x=r\cos\varphi$$
$$y=r\sin\varphi$$
$$\varphi\in(-\pi,\pi]$$
$$\det J=r$$

Zylinderkoordinaten

$$x=r_{xy}\cos\varphi$$
$$y=r_{xy}\sin\varphi$$
$$z=z$$
$$\det J=r_{xy}$$

Kugelkoordinaten

$$x=r\sin\theta\,\cos\varphi$$
$$y=r\sin\theta\,\sin\varphi$$
$$z=r\cos\theta$$
$$\varphi\in(-\pi,\pi],\ \theta\in[0,\pi]$$
$$\det J=r^2\sin\theta$$
$$\theta=\beta-\pi/2$$
$$\beta\in[-\pi/2,\pi/2]$$
$$\cos\theta=\sin\beta$$
$$\sin\theta=\cos\beta$$

$$\langle Av,w\rangle=\langle v,A^Hw\rangle$$
$$\langle v,w\rangle=|v||w|\cos\varphi$$
$$|v\times w|=|v||w|\sin\varphi$$

$$(AB)^H=B^HA^H$$
$$(AB)^{-1}=B^{-1}A^{-1}$$
$$\det(AB)=\det(A)\det(B)$$

$$\operatorname{proj}[w](v)=\tfrac{\langle v,w\rangle}{\langle w,w\rangle}w,\quad w_k:=v_k-\textstyle\sum_{i=1}^{k-1}\operatorname{proj}[w_i](v_k)$$
$$y=\overline{y}+\frac{s_{xy}}{s_x}(x-\overline{x}),\ s_x=\sum_{k=1}^{n}(x_k-\overline{x})^2,\ s_{xy}=\sum_{k=1}^{n}(x_k-\overline{x})(y_k-\overline{y})$$
$$n!=n\cdot(n-1)!,\quad\Gamma(z+1)=z\Gamma(z)$$
$$\tbinom{n+1}{k+1}=\tbinom{n}{k}+\tbinom{n+1}{k},\quad\tbinom{n}{k}=\tbinom{n}{n-k},\quad\tbinom{n}{0}=1$$

A	B	$A \wedge B$	$A \vee B$	$A \rightarrow B$	$A \leftrightarrow B$	$A \oplus B$	$A \uparrow B$
0	0	0	0	1	1	0	1
0	1	0	1	1	0	1	1
1	0	0	1	0	0	1	1
1	1	1	1	1	1	0	0

Disjunktion	**Konjunktion**	**Bezeichnung**
$A \vee A \equiv A$	$A \wedge A \equiv A$	Idempotenzgesetze
$A \vee 0 \equiv A$	$A \wedge 1 \equiv A$	Neutralitätsgesetze
$A \vee 1 \equiv 1$	$A \wedge 0 \equiv 0$	Extremalgesetze
$A \vee \overline{A} \equiv 1$	$A \wedge \overline{A} \equiv 0$	Komplementärgesetze
$A \vee B \equiv B \vee A$	$A \wedge B \equiv B \wedge A$	Kommutativgesetze
$(A \vee B) \vee C \equiv A \vee (B \vee C)$	$(A \wedge B) \wedge C \equiv A \wedge (B \wedge C)$	Assoziativgesetze
$\overline{A \vee B} \equiv \overline{A} \wedge \overline{B}$	$\overline{A \wedge B} \equiv \overline{A} \vee \overline{B}$	De Morgansche Regeln
$A \vee (A \wedge B) \equiv A$	$A \wedge (A \vee B) \equiv A$	Absorptionsgesetze

$(A \rightarrow B) \equiv \overline{A} \vee B$ | $(A \leftrightarrow B) \equiv (A \rightarrow B) \wedge (B \rightarrow A)$

$(A \rightarrow B) \equiv (\overline{B} \rightarrow \overline{A})$ | $(A \leftrightarrow B) \equiv (\overline{A} \vee B) \wedge (\overline{B} \vee A)$

$A \vee \forall_x P_x \equiv \forall_x (A \vee P_x)$ | $\forall_x (P_x \wedge Q_x) \equiv \forall_x P_x \wedge \forall_x Q_x$

$A \wedge \exists_x P_x \equiv \exists_x (A \wedge P_x)$ | $\exists_x (P_x \vee Q_x) \equiv \exists_x P_x \vee \exists_x Q_x$

$(I \models M) :\Leftrightarrow \forall \varphi \in M \colon I(\varphi)$

$(\models \varphi) :\Leftrightarrow \forall I \colon I(\varphi)$ | $(M \models \varphi) :\Leftrightarrow \forall I \colon ((I \models M) \Rightarrow I(\varphi))$

$\mathrm{erf}(\varphi) :\Leftrightarrow \exists I \colon I(\varphi)$ | $\mathrm{erf}(M) :\Leftrightarrow \exists I \colon (I \models M)$

$\mathrm{erf}(\{\varphi_1, \ldots, \varphi_n\}) \Leftrightarrow \mathrm{erf}(\varphi_1 \wedge \ldots \wedge \varphi_n)$

$\mathrm{erf}(\varphi_1 \vee \ldots \vee \varphi_n) \Leftrightarrow \mathrm{erf}(\varphi_1) \vee \ldots \vee \mathrm{erf}(\varphi_n)$

$(M \vdash \varphi) \Rightarrow (M \models \varphi)$ (Korrektheit)

$(M \models \varphi) \Rightarrow (M \vdash \varphi)$ (Vollständigkeit)

$(M \cup \{\varphi\} \vdash \psi) \Leftrightarrow (M \vdash \varphi \rightarrow \psi)$

$(M \cup \{\varphi\} \models \psi) \Leftrightarrow (M \models \varphi \rightarrow \psi)$

$A \cap B := \{x \mid x \in A \wedge x \in B\}$ | $A \subseteq B :\Leftrightarrow \forall_x (x \in A \Rightarrow x \in B)$

$A \cup B := \{x \mid x \in A \vee x \in B\}$ | $A = B :\Leftrightarrow \forall_x (x \in A \Leftrightarrow x \in B)$

$A \setminus B := \{x \mid x \in A \wedge x \notin B\}$ | $A = B :\Leftrightarrow A \subseteq B \wedge B \subseteq A$

$\bigcap_{i \in I} A_i := \{x \mid \forall i \in I \colon x \in A_i\}$ | $f(M) := \{y \mid \exists x \in M \colon y = f(x)\}$

$\bigcup_{i \in I} A_i := \{x \mid \exists i \in I \colon x \in A_i\}$ | $f^{-1}(N) := \{x \mid f(x) \in N\}$

$A \times B := \{t \mid \exists x \in A \colon \exists y \in B \colon t = (x, y)\}$

$A \subseteq B \Leftrightarrow A \cup B = B \Leftrightarrow A \cap B = A$

$f(M \cup N) = f(M) \cup f(N)$ | $f^{-1}(M \cup N) = f^{-1}(M) \cup f^{-1}(N)$

$f(M \cap N) \subseteq f(M) \cap f(N)$ | $f^{-1}(M \cap N) = f^{-1}(M) \cap f^{-1}(N)$

$M \subseteq N \Rightarrow f(M) \subseteq f(N)$ | $M \subseteq N \Rightarrow f^{-1}(M) \subseteq f^{-1}(N)$

$(g \circ f)(M) = g(f(M))$ | $(g \circ f)^{-1}(M) = f^{-1}(g^{-1}(M))$