Winkelfunktionen

$\sin(x+y) = \sin x\cos y + \cos x \sin y$ | $\sin(z \pm \pi) = -\sin z$
$\sin(x-y) = \sin x\cos y - \cos x \sin y$ | $\cos(z \pm \pi) = -\cos z$
$\cos(x+y) = \cos x\cos y - \sin x \sin y$ | $\sin(\pi/2 - x) = \cos x$
$\cos(x-y) = \cos x\cos y + \sin x \sin y$ | $\cos(\pi/2 - x) = \sin x$

$\sin(nx) = 2\cos x \sin((n-1)x) - \sin((n-2)x)$ | $\sin(-z) = -\sin z$
$\cos(nx) = 2\cos x \cos((n-1)x) - \cos((n-2)x)$ | $\cos(-z) = \cos z$

$\cos^2 z + \sin^2 z = 1$ | $\cosh^2 z - \sinh^2 z = 1$ | $\cosh(\mathrm{i}z) = \cos z$
$\mathrm{e}^{\mathrm{i}z} = \cos z + \mathrm{i}\sin z$ | $\mathrm{e}^z = \cosh z + \sinh z$ | $\sinh(\mathrm{i}z) = \mathrm{i}\sin z$
$2\cos z = \mathrm{e}^{\mathrm{i}z} + \mathrm{e}^{-\mathrm{i}z}$ | $2\cosh z = \mathrm{e}^z + \mathrm{e}^{-z}$ | $\cos(\mathrm{i}z) = \cosh z$
$2\mathrm{i}\sin z = \mathrm{e}^{\mathrm{i}z} - \mathrm{e}^{-\mathrm{i}z}$ | $2\sinh z = \mathrm{e}^z - \mathrm{e}^{-z}$ | $\sin(\mathrm{i}z) = \mathrm{i}\sinh z$

Reihen

$$\mathrm{e}^z = \sum_{k=0}^{\infty} \frac{z^k}{k!} \quad \Big| \quad \mathrm{e}^z = \lim_{n\to\infty}\left(1+\frac{z}{n}\right)^n \quad \Big| \quad \ln z = \lim_{h\to 0}\frac{z^h - 1}{h}$$

$\sin z = \sum_{k=0}^{\infty}(-1)^k \frac{z^{2k+1}}{(2k+1)!}$ | $\sinh z = \sum_{k=0}^{\infty} \frac{z^{2k+1}}{(2k+1)!}$
$\cos z = \sum_{k=0}^{\infty}(-1)^k \frac{z^{2k}}{(2k)!}$ | $\cosh z = \sum_{k=0}^{\infty} \frac{z^{2k}}{(2k)!}$

$$\frac{z}{\mathrm{e}^z - 1} = \sum_{k=0}^{\infty} \overline{B}_k \frac{x^k}{k!}, \qquad \frac{z}{1-\mathrm{e}^{-z}} = \sum_{k=0}^{\infty} B_k \frac{x^k}{k!}$$

$$f[a](z) := \mathrm{e}^{(z-a)D} f(a) = \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(z-a)^k$$

Differentialrechnung

$$f'(x) := \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$(\mathrm{e}^x)' = \mathrm{e}^x$ | $(fg)' = f'g + g'f$
$\ln' x = 1/x$ | $(f/g)' = (f'g - g'f)/g^2$
$(a^x)' = a^x \ln a$ | $(g \circ f)' = (g' \circ f) f'$
$(x^n)' = nx^{n-1}$ | $(f^{-1})' = 1/(f' \circ f^{-1})$

$\sin' x = \cos x$ | $\tan' x = 1 + \tan^2 x = 1/\cos^2 x$
$\cos' x = -\sin x$ | $\cot' x = -1 - \cot^2 x = -1/\sin^2 x$
$\sinh' x = \cosh x$ | $\tanh' x = 1 - \tanh^2 x = 1/\cosh^2 x$
$\cosh' x = \sinh x$ | $\coth' x = 1 - \coth^2 x = -1/\sinh^2 x$

$\arcsin' x = 1/\sqrt{1-x^2}$ | $\arctan' x = 1/(1+x^2)$
$\arccos' x = -1/\sqrt{1-x^2}$ | $\mathrm{arccot}' x = -1/(1+x^2)$
$\mathrm{arsinh}' x = 1/\sqrt{x^2+1}$ | $\mathrm{artanh}' x = 1/(1-x^2)$
$\mathrm{arcosh}' x = 1/\sqrt{x^2-1}$ | $\mathrm{arcoth}' x = 1/(1-x^2)$

$\nabla(|\mathbf{x}|^2) = 2\mathbf{x}$ | $\nabla(fg) = g\nabla f + f\nabla g$
$\nabla|\mathbf{x}| = \mathbf{x}/|\mathbf{x}|$ | $\nabla\langle f, g\rangle = (Df)^T g + (Dg)^T f$
$\nabla(\frac{1}{g}) = -\frac{\nabla g}{g^2}$ | $\nabla(f/g) = (g\nabla f - f\nabla g)/g^2$
$\nabla \times \nabla f = 0$ | $\langle \nabla, f\mathbf{v}\rangle = \langle \nabla f, \mathbf{v}\rangle + f\langle \nabla, \mathbf{v}\rangle$
$\langle \nabla, \nabla \times \mathbf{v}\rangle = 0$ | $\nabla \times (f\mathbf{v}) = f(\nabla \times \mathbf{v}) - \mathbf{v} \times \nabla f$

Integralrechnung

$$\int_a^b f(g(x))\, g'(x)\,\mathrm{d}x = \int_{g(a)}^{g(b)} f(u)\,\mathrm{d}u \quad \left(\begin{smallmatrix} f\in C(I,\mathbb{R}), \\ g\in C^1([a,b],I)\end{smallmatrix}\right)$$

$$\int_a^b f'(x)g(x)\,\mathrm{d}x = [f(x)g(x)]_a^b - \int_a^b f(x)g'(x)\,\mathrm{d}x \quad (f, g \in C^1)$$

$$t = \tan(\tfrac{x}{2}),\ \sin x = \tfrac{2t}{1+t^2},\ \cos x = \tfrac{1-t^2}{1+t^2},\ \mathrm{d}x = \tfrac{2\mathrm{d}t}{1+t^2}$$

$$L\{f(t)\} := \int_0^{\infty} f(t)\mathrm{e}^{-pt}\mathrm{d}t,\ F\{f(t)\} := \tfrac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t)\mathrm{e}^{-\mathrm{i}\omega t}\mathrm{d}t$$

$$\int_\gamma f\mathrm{d}s := \int_a^b f(t)\,|\gamma'(t)|\,\mathrm{d}t,\ \int_\gamma \langle \mathbf{F}, \mathrm{d}\mathbf{x}\rangle := \int_a^b \langle \mathbf{F}(\mathbf{x}(t)), \mathbf{x}'(t)\rangle\,\mathrm{d}t$$

Extremwerte

$$f(x) = \text{extrem} \Rightarrow f'(x) = 0,\ f(p) = \text{extrem} \Rightarrow \mathrm{d}f_p = 0$$

$$f(x,y) = \text{extrem unter } g(x,y) = 0 \Rightarrow \mathrm{d}f = \lambda \mathrm{d}g$$

$$J[\mathbf{x}] := \int_a^b L(t, \mathbf{x}(t), \mathbf{x}'(t))\,\mathrm{d}t = \text{extrem} \Rightarrow \frac{\partial L}{\partial x_k} = \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial x_k'}$$

Komplexe Zahlen

$z = r\mathrm{e}^{\mathrm{i}\varphi} = a + b\mathrm{i}$ | $|z| = r = \sqrt{a^2+b^2}$
$\overline{z} = r\mathrm{e}^{-\mathrm{i}\varphi} = a - b\mathrm{i}$ | $\arg(z) = \varphi = \mathrm{sgn}(b)\arccos(a/r)$
$\mathrm{Re}\, z = a = r\cos\varphi$ | $z_1 + z_2 = (a_1 + a_2) + (b_1 + b_2)\mathrm{i}$
$\mathrm{Im}\, z = b = r\sin\varphi$ | $z_2 - z_2 = (a_1 - a_2) + (b_1 - b_2)\mathrm{i}$

$$z_1 z_2 = r_1 r_2 \mathrm{e}^{\mathrm{i}(\varphi_1+\varphi_2)} = (a_1a_2 - b_1b_2) + (a_1b_2 + a_2b_1)\mathrm{i}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\mathrm{e}^{\mathrm{i}(\varphi_1-\varphi_2)} = \frac{a_1a_2 + b_1b_2}{a_2^2 + b_2^2} + \frac{a_2b_1 - a_1b_2}{a_2^2+b_2^2}\mathrm{i}$$

$$\frac{1}{z} = \frac{1}{r}\mathrm{e}^{-\mathrm{i}\varphi} = \frac{a}{a^2+b^2} - \frac{b}{a^2+b^2}\mathrm{i}$$

Algebra

$$x^2 + px + q = 0: \ \ x = -\tfrac{p}{2} \pm \tfrac{1}{2}\sqrt{p^2 - 4q}$$

$f(x) = f(2a - x)$ (Achsensymmetrie)
$f(x) = 2b - f(2a - x)$ (Punktsymmetrie)

Polarkoordinaten

$x = r\cos\varphi$
$y = r\sin\varphi$
$\varphi \in (-\pi, \pi]$
$\det J = r$

Zylinderkoordinaten

$x = r_{xy}\cos\varphi$
$y = r_{xy}\sin\varphi$
$z = z$
$\det J = r_{xy}$

Kugelkoordinaten

$x = r\sin\theta\,\cos\varphi$
$y = r\sin\theta\,\sin\varphi$
$z = r\cos\theta$
$\varphi \in (-\pi, \pi],\ \theta \in [0, \pi]$
$\det J = r^2\sin\theta$

$\theta = \beta - \pi/2$
$\beta \in [-\pi/2, \pi/2]$
$\cos\theta = \sin\beta$
$\sin\theta = \cos\beta$

Lineare Algebra

$\langle Av, w\rangle = \langle v, A^H w\rangle$ | $(AB)^H = B^H A^H$
$\langle v, w\rangle = |v||w|\cos\varphi$ | $(AB)^{-1} = B^{-1}A^{-1}$
$|v \times w| = |v||w|\sin\varphi$ | $\det(AB) = \det(A)\det(B)$

$$\mathrm{proj}[w](v) = \tfrac{\langle v,w\rangle}{\langle w,w\rangle}w, \quad w_k := v_k - \textstyle\sum_{i=1}^{k-1}\mathrm{proj}[w_i](v_k)$$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}, \quad R(\varphi) = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}$$

Kombinatorik

$$\sum_{k=m}^{n-1} q^k = \frac{q^m - q^n}{q-1}, \qquad \sum_{k=m}^{n-1} k^p q^k = \left(q\frac{\mathrm{d}}{\mathrm{d}q}\right)^p \frac{q^m - q^n}{q-1}$$

$\sum_{k=1}^{n} k = (n/2)(n+1)$ | $\sum_{k=m}^{n-1}(\Delta a)_k = a_n - a_m$
$\sum_{k=1}^{n} k^2 = (n/6)(n+1)(2n+1)$ | $(\Delta a)_k := a_{k+1} - a_k$
$\sum_{k=1}^{n} k^3 = (n/2)^2(n+1)^2$

$$(a+b)^n = \sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k, \qquad \binom{n}{k} := \frac{1}{k!}n^{\underline{k}} = \frac{n!}{k!(n-k)!}$$

$$n! = n\cdot(n-1)!, \quad n! = \Gamma(n+1), \quad \Gamma(z+1) = z\Gamma(z)$$

$$\tbinom{n+1}{k+1} = \tbinom{n}{k} + \tbinom{n+1}{k}, \quad \tbinom{n}{k} = \tbinom{n}{n-k}, \quad \tbinom{n}{0} = 1$$

$$n! \approx \sqrt{2\pi n}\, n^n \exp\left(\tfrac{1}{12n} - n\right) \approx \sqrt{2\pi n}\,(n/e)^n$$

Regression

$$y = \overline{y} + \frac{s_{xy}}{s_x}(x - \overline{x}), \ \ s_x = \sum_{k=1}^{n}(x_k - \overline{x})^2, \ \ s_{xy} = \sum_{k=1}^{n}(x_k - \overline{x})(y_k - \overline{y})$$

A	B	$A \wedge B$	$A \vee B$	$A \rightarrow B$	$A \leftrightarrow B$	$A \oplus B$	$A \uparrow B$
0	0	0	0	1	1	0	1
0	1	0	1	1	0	1	1
1	0	0	1	0	0	1	1
1	1	1	1	1	1	0	0

Disjunktion	**Konjunktion**	**Bezeichnung**
$A \vee A \equiv A$	$A \wedge A \equiv A$	Idempotenzgesetze
$A \vee 0 \equiv A$	$A \wedge 1 \equiv A$	Neutralitätsgesetze
$A \vee 1 \equiv 1$	$A \wedge 0 \equiv 0$	Extremalgesetze
$A \vee \overline{A} \equiv 1$	$A \wedge \overline{A} \equiv 0$	Komplementärgesetze
$A \vee B \equiv B \vee A$	$A \wedge B \equiv B \wedge A$	Kommutativgesetze
$(A \vee B) \vee C \equiv A \vee (B \vee C)$	$(A \wedge B) \wedge C \equiv A \wedge (B \wedge C)$	Assoziativgesetze
$\overline{A \vee B} \equiv \overline{A} \wedge \overline{B}$	$\overline{A \wedge B} \equiv \overline{A} \vee \overline{B}$	De Morgansche Regeln
$A \vee (A \wedge B) \equiv A$	$A \wedge (A \vee B) \equiv A$	Absorptionsgesetze

$(A \rightarrow B) \equiv \overline{A} \vee B$ | $(A \leftrightarrow B) \equiv (A \rightarrow B) \wedge (B \rightarrow A)$

$(A \rightarrow B) \equiv (\overline{B} \rightarrow \overline{A})$ | $(A \leftrightarrow B) \equiv (\overline{A} \vee B) \wedge (\overline{B} \vee A)$

$A \vee \forall_x P_x \equiv \forall_x (A \vee P_x)$ | $\forall_x (P_x \wedge Q_x) \equiv \forall_x P_x \wedge \forall_x Q_x$

$A \wedge \exists_x P_x \equiv \exists_x (A \wedge P_x)$ | $\exists_x (P_x \vee Q_x) \equiv \exists_x P_x \vee \exists_x Q_x$

$(I \models M) :\Leftrightarrow \forall \varphi \in M\colon I(\varphi)$

$(\models \varphi) :\Leftrightarrow \forall I\colon I(\varphi)$ | $(M \models \varphi) :\Leftrightarrow \forall I\colon ((I \models M) \Rightarrow I(\varphi))$

$\mathrm{erf}(\varphi) :\Leftrightarrow \exists I\colon I(\varphi)$ | $\mathrm{erf}(M) :\Leftrightarrow \exists I\colon (I \models M)$

$\mathrm{erf}(\{\varphi_1, \ldots, \varphi_n\}) \Leftrightarrow \mathrm{erf}(\varphi_1 \wedge \ldots \wedge \varphi_n)$

$\mathrm{erf}(\varphi_1 \vee \ldots \vee \varphi_n) \Leftrightarrow \mathrm{erf}(\varphi_1) \vee \ldots \vee \mathrm{erf}(\varphi_n)$

$(M \vdash \varphi) \Rightarrow (M \models \varphi)$ (Korrektheit)

$(M \models \varphi) \Rightarrow (M \vdash \varphi)$ (Vollständigkeit)

$(M \cup \{\varphi\} \vdash \psi) \Leftrightarrow (M \vdash \varphi \rightarrow \psi)$

$(M \cup \{\varphi\} \models \psi) \Leftrightarrow (M \models \varphi \rightarrow \psi)$

$A \cap B := \{x \mid x \in A \wedge x \in B\}$ | $A \subseteq B :\Leftrightarrow \forall_x (x \in A \Rightarrow x \in B)$

$A \cup B := \{x \mid x \in A \vee x \in B\}$ | $A = B :\Leftrightarrow \forall_x (x \in A \Leftrightarrow x \in B)$

$A \setminus B := \{x \mid x \in A \wedge x \notin B\}$ | $A = B :\Leftrightarrow A \subseteq B \wedge B \subseteq A$

$\bigcap_{i \in I} A_i := \{x \mid \forall i \in I\colon x \in A_i\}$ | $f(M) := \{y \mid \exists x \in M\colon y = f(x)\}$

$\bigcup_{i \in I} A_i := \{x \mid \exists i \in I\colon x \in A_i\}$ | $f^{-1}(N) := \{x \mid f(x) \in N\}$

$A \times B := \{t \mid \exists x \in A\colon \exists y \in B\colon t = (x, y)\}$

$A \subseteq B \Leftrightarrow A \cup B = B \Leftrightarrow A \cap B = A$

$f(M \cup N) = f(M) \cup f(N)$ | $f^{-1}(M \cup N) = f^{-1}(M) \cup f^{-1}(N)$

$f(M \cap N) \subseteq f(M) \cap f(N)$ | $f^{-1}(M \cap N) = f^{-1}(M) \cap f^{-1}(N)$

$M \subseteq N \Rightarrow f(M) \subseteq f(N)$ | $M \subseteq N \Rightarrow f^{-1}(M) \subseteq f^{-1}(N)$

$(g \circ f)(M) = g(f(M))$ | $(g \circ f)^{-1}(M) = f^{-1}(g^{-1}(M))$