

$$\begin{array}{l|l} \sin(x+y) = \sin x \cos y + \cos x \sin y & \sin(z \pm \pi) = -\sin z \\ \sin(x-y) = \sin x \cos y - \cos x \sin y & \cos(z \pm \pi) = -\cos z \\ \cos(x+y) = \cos x \cos y - \sin x \sin y & \cos z = \cos(-z) \\ \cos(x-y) = \cos x \cos y + \sin x \sin y & \sin(-z) = -\sin z \end{array}$$

$$\begin{array}{l|l|l} \cos^2 z + \sin^2 z = 1 & \cosh^2 z - \sinh^2 z = 1 & \cosh(iz) = \cos z \\ e^{iz} = \cos z + i \sin z & e^z = \cosh z + \sinh z & \sinh(iz) = i \sin z \\ 2 \cos z = e^{iz} + e^{-iz} & 2 \cosh z = e^z + e^{-z} & \cos(iz) = \cosh z \\ 2i \sin z = e^{iz} - e^{-iz} & 2 \sinh z = e^z - e^{-z} & \sin(iz) = i \sinh z \end{array}$$

$$e^z = \sum_{k=0}^{\infty} \frac{z^k}{k!} \quad \left| \quad e^z = \lim_{n \rightarrow \infty} \left(1 + \frac{z}{n}\right)^n \quad \left| \quad \ln z = \lim_{h \rightarrow 0} \frac{z^h - 1}{h}\right.$$

$$\begin{array}{l|l} \sin z = \sum_{k=0}^{\infty} (-1)^k \frac{z^{2k+1}}{(2k+1)!} & \sinh z = \sum_{k=0}^{\infty} \frac{z^{2k+1}}{(2k+1)!} \\ \cos z = \sum_{k=0}^{\infty} (-1)^k \frac{z^{2k}}{(2k)!} & \cosh z = \sum_{k=0}^{\infty} \frac{z^{2k}}{(2k)!} \end{array}$$

$$\begin{array}{l|l} \sum_{k=m}^{n-1} q^k = \frac{q^m - q^n}{q - 1}, & \sum_{k=m}^{n-1} k^p q^k = \left(q \frac{d}{dq}\right)^p \frac{q^m - q^n}{q - 1} \\ \sum_{k=1}^n k = (n/2)(n+1) & \sum_{k=m}^{n-1} (\Delta a)_k = a_n - a_m \\ \sum_{k=1}^n k^2 = (n/6)(n+1)(2n+1) & (\Delta a)_k := a_{k+1} - a_k \\ \sum_{k=1}^n k^3 = (n/2)^2(n+1)^2 & \end{array}$$

$$(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k, \quad \binom{n}{k} := \frac{1}{k!} n^{\underline{k}} = \frac{n!}{k!(n-k)!}$$

$$\frac{z}{e^z - 1} = \sum_{k=0}^{\infty} \bar{B}_k \frac{x^k}{k!}, \quad \frac{z}{1 - e^{-z}} = \sum_{k=0}^{\infty} B_k \frac{x^k}{k!}$$

$$f[a](z) := e^{(z-a)D} f(a) = \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!} (z-a)^k$$

$$f(x) = f(2a-x) \quad (\text{Achsensymmetrie})$$

$$f(x) = 2b - f(2a-x) \quad (\text{Punktsymmetrie})$$

$$\begin{array}{l|l} (e^x)' = e^x & (fg)' = f'g + g'f \\ \ln' x = 1/x & (f/g)' = (f'g - g'f)/g^2 \\ (a^x)' = a^x \ln a & (g \circ f)' = (g' \circ f)f' \\ (x^n)' = nx^{n-1} & (f^{-1})' = 1/(f' \circ f^{-1}) \end{array}$$

$$\begin{array}{l|l} \sin' x = \cos x & \tan' x = 1 + \tan^2 x = 1/\cos^2 x \\ \cos' x = -\sin x & \cot' x = -1 - \cot^2 x = -1/\sin^2 x \\ \sinh' x = \cosh x & \tanh' x = 1 - \tanh^2 x = 1/\cosh^2 x \\ \cosh' x = \sinh x & \coth' x = 1 - \coth^2 x = -1/\sinh^2 x \end{array}$$

$$\begin{array}{l|l} \arcsin' x = 1/\sqrt{1-x^2} & \arctan' x = 1/(1+x^2) \\ \arccos' x = -1/\sqrt{1-x^2} & \operatorname{arccot}' x = -1/(1+x^2) \\ \operatorname{arsinh}' x = 1/\sqrt{x^2+1} & \operatorname{artanh}' x = 1/(1-x^2) \\ \operatorname{arcosh}' x = 1/\sqrt{x^2-1} & \operatorname{arcoth}' x = 1/(1-x^2) \end{array}$$

$$\begin{array}{l|l} \nabla(|\mathbf{x}|^2) = 2\mathbf{x} & \nabla(fg) = g\nabla f + f\nabla g \\ \nabla|\mathbf{x}| = \mathbf{x}/|\mathbf{x}| & \nabla\langle f, g \rangle = (Df)^T g + (Dg)^T f \\ \nabla\left(\frac{1}{g}\right) = -\frac{\nabla g}{g^2} & \nabla(f/g) = (g\nabla f - f\nabla g)/g^2 \end{array}$$

$$t = \tan\left(\frac{x}{2}\right), \quad \sin x = \frac{2t}{1+t^2}, \quad \cos x = \frac{1-t^2}{1+t^2}, \quad dx = \frac{2dt}{1+t^2}$$

$$L\{f(t)\} := \int_0^{\infty} f(t)e^{-pt} dt, \quad F\{f(t)\} := \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t)e^{-i\omega t} dt$$

$$\int_Y f ds := \int_a^b f(t) |Y'(t)| dt$$

$$\begin{array}{l|l} z = re^{i\varphi} = a + bi & |z| = r = \sqrt{a^2 + b^2} \\ \bar{z} = re^{-i\varphi} = a - bi & \arg(z) = \varphi = \operatorname{sgn}(b) \arccos(a/r) \\ \operatorname{Re} z = a = r \cos \varphi & z_1 + z_2 = (a_1 + a_2) + (b_1 + b_2)i \\ \operatorname{Im} z = b = r \sin \varphi & z_2 - z_1 = (a_1 - a_2) + (b_1 - b_2)i \end{array}$$

$$\begin{array}{l} z_1 z_2 = r_1 r_2 e^{i(\varphi_1 + \varphi_2)} = (a_1 a_2 - b_1 b_2) + (a_1 b_2 + a_2 b_1)i \\ \frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\varphi_1 - \varphi_2)} = \frac{a_1 a_2 + b_1 b_2}{a_2^2 + b_2^2} + \frac{a_2 b_1 - a_1 b_2}{a_2^2 + b_2^2} i \end{array}$$

$$\frac{1}{z} = \frac{1}{r} e^{-i\varphi} = \frac{a}{a^2 + b^2} - \frac{b}{a^2 + b^2} i$$

$$x^2 + px + q = 0: \quad x = -\frac{p}{2} \pm \frac{1}{2} \sqrt{p^2 - 4q}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \quad R(\varphi) = \begin{bmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{bmatrix}$$

Polarkoordinaten

$$x = r \cos \varphi$$

$$y = r \sin \varphi$$

$$\varphi \in (-\pi, \pi]$$

$$\det J = r$$

Zylinderkoordinaten

$$x = r_{xy} \cos \varphi$$

$$y = r_{xy} \sin \varphi$$

$$z = z$$

$$\det J = r_{xy}$$

Kugelkoordinaten

$$x = r \sin \theta \cos \varphi$$

$$y = r \sin \theta \sin \varphi$$

$$z = r \cos \theta$$

$$\varphi \in (-\pi, \pi], \quad \theta \in [0, \pi]$$

$$\det J = r^2 \sin \theta$$

$$\theta = \beta - \pi/2$$

$$\beta \in [-\pi/2, \pi/2]$$

$$\cos \theta = \sin \beta$$

$$\sin \theta = \cos \beta$$

$$\langle Av, w \rangle = \langle v, A^H w \rangle$$

$$\langle v, w \rangle = |v||w| \cos \varphi$$

$$|v \times w| = |v||w| \sin \varphi$$

$$\operatorname{proj}[w](v) = \frac{\langle v, w \rangle}{\langle w, w \rangle} w, \quad w_k := v_k - \sum_{i=1}^{k-1} \operatorname{proj}[w_i](v_k)$$

$$y = \bar{y} + \frac{s_{xy}}{s_x} (x - \bar{x}), \quad s_x = \sum_{k=1}^n (x_k - \bar{x})^2, \quad s_{xy} = \sum_{k=1}^n (x_k - \bar{x})(y_k - \bar{y})$$

$$n! = n \cdot (n-1)!, \quad \Gamma(z+1) = z\Gamma(z)$$

A	B	$A \wedge B$	$A \vee B$	$A \rightarrow B$	$A \leftrightarrow B$	$A \oplus B$	$A \uparrow B$
0	0	0	0	1	1	0	1
0	1	0	1	1	0	1	1
1	0	0	1	0	0	1	1
1	1	1	1	1	1	0	0

Disjunktion	Konjunktion	Bezeichnung
$A \vee A \equiv A$	$A \wedge A \equiv A$	Idempotenzgesetze
$A \vee 0 \equiv A$	$A \wedge 1 \equiv A$	Neutralitätsgesetze
$A \vee 1 \equiv 1$	$A \wedge 0 \equiv 0$	Extremalgesetze
$A \vee \bar{A} \equiv 1$	$A \wedge \bar{A} \equiv 0$	Komplementärgesetze

$A \vee B \equiv B \vee A$	$A \wedge B \equiv B \wedge A$	Kommutativgesetze
$(A \vee B) \vee C \equiv A \vee (B \vee C)$	$(A \wedge B) \wedge C \equiv A \wedge (B \wedge C)$	Assoziativgesetze
$\overline{A \vee B} \equiv \bar{A} \wedge \bar{B}$	$\overline{A \wedge B} \equiv \bar{A} \vee \bar{B}$	De Morgansche Regeln
$A \vee (A \wedge B) \equiv A$	$A \wedge (A \vee B) \equiv A$	Absorptionsgesetze

$$(A \rightarrow B) \equiv \bar{A} \vee B \quad \left| \quad (A \leftrightarrow B) \equiv (A \rightarrow B) \wedge (B \rightarrow A) \right.$$

$$(A \rightarrow B) \equiv (\bar{B} \rightarrow \bar{A}) \quad \left| \quad (A \leftrightarrow B) \equiv (\bar{A} \vee B) \wedge (\bar{B} \vee A) \right.$$

$$A \vee \forall_x P_x \equiv \forall_x (A \vee P_x) \quad \left| \quad \forall_x (P_x \wedge Q_x) \equiv \forall_x P_x \wedge \forall_x Q_x \right.$$

$$A \wedge \exists_x P_x \equiv \exists_x (A \wedge P_x) \quad \left| \quad \exists_x (P_x \vee Q_x) \equiv \exists_x P_x \vee \exists_x Q_x \right.$$

$$(I \models M) :\Leftrightarrow \forall \varphi \in M: I(\varphi)$$

$$(\models \varphi) :\Leftrightarrow \forall I: I(\varphi) \quad \left| \quad (M \models \varphi) :\Leftrightarrow \forall I: ((I \models M) \Rightarrow I(\varphi)) \right.$$

$$\text{erf}(\varphi) :\Leftrightarrow \exists I: I(\varphi) \quad \left| \quad \text{erf}(M) :\Leftrightarrow \exists I: (I \models M) \right.$$

$$\text{erf}(\{\varphi_1, \dots, \varphi_n\}) \Leftrightarrow \text{erf}(\varphi_1 \wedge \dots \wedge \varphi_n)$$

$$\text{erf}(\varphi_1 \vee \dots \vee \varphi_n) \Leftrightarrow \text{erf}(\varphi_1) \vee \dots \vee \text{erf}(\varphi_n)$$

$$(M \vdash \varphi) \Rightarrow (M \models \varphi) \quad (\text{Korrektheit})$$

$$(M \models \varphi) \Rightarrow (M \vdash \varphi) \quad (\text{Vollständigkeit})$$

$$(M \cup \{\varphi\} \vdash \psi) \Leftrightarrow (M \vdash \varphi \rightarrow \psi)$$

$$(M \cup \{\varphi\} \models \psi) \Leftrightarrow (M \models \varphi \rightarrow \psi)$$

$$A \cap B := \{x \mid x \in A \wedge x \in B\} \quad \left| \quad A \subseteq B :\Leftrightarrow \forall_x (x \in A \Rightarrow x \in B) \right.$$

$$A \cup B := \{x \mid x \in A \vee x \in B\} \quad \left| \quad A = B :\Leftrightarrow \forall_x (x \in A \Leftrightarrow x \in B) \right.$$

$$A \setminus B := \{x \mid x \in A \wedge x \notin B\} \quad \left| \quad A = B :\Leftrightarrow A \subseteq B \wedge B \subseteq A \right.$$

$$\bigcap_{i \in I} A_i := \{x \mid \forall i \in I: x \in A_i\} \quad \left| \quad f(M) := \{y \mid \exists x \in M: y = f(x)\} \right.$$

$$\bigcup_{i \in I} A_i := \{x \mid \exists i \in I: x \in A_i\} \quad \left| \quad f^{-1}(N) := \{x \mid f(x) \in N\} \right.$$

$$A \times B := \{t \mid \exists x \in A: \exists y \in B: t = (x, y)\}$$

$$A \subseteq B \Leftrightarrow A \cup B = B \Leftrightarrow A \cap B = A$$

$$f(M \cup N) = f(M) \cup f(N) \quad \left| \quad f^{-1}(M \cup N) = f^{-1}(M) \cup f^{-1}(N) \right.$$

$$f(M \cap N) \subseteq f(M) \cap f(N) \quad \left| \quad f^{-1}(M \cap N) = f^{-1}(M) \cap f^{-1}(N) \right.$$

$$M \subseteq N \Rightarrow f(M) \subseteq f(N) \quad \left| \quad M \subseteq N \Rightarrow f^{-1}(M) \subseteq f^{-1}(N) \right.$$

$$(g \circ f)(M) = g(f(M)) \quad \left| \quad (g \circ f)^{-1}(M) = f^{-1}(g^{-1}(M)) \right.$$