#### Winkelfunktionen

 $\sin(x + y) = \sin x \cos y + \cos x \sin y \quad | \sin(z \pm \pi) = -\sin z$  $\sin(x - y) = \sin x \cos y - \cos x \sin y$  $\cos(z \pm \pi) = -\cos z$  $\cos(x+y) = \cos x \cos y - \sin x \sin y$  $\sin(\pi/2 - x) = \cos x$  $\cos(x - y) = \cos x \cos y + \sin x \sin y \mid \cos(\pi/2 - x) = \sin x$  $\sin(nx) = 2\cos x \sin((n-1)x) - \sin((n-2)x) \quad |\sin(-z)| = -\sin z$  $\cos(nx) = 2\cos x \cos((n-1)x) - \cos((n-2)x) \left| \cos(-z) = \cos z \right|$  $\cos^2 z + \sin^2 z = 1$  |  $\cosh^2 z - \sinh^2 z = 1$  |  $\cosh(iz) = \cos z$  $e^{iz} = \cos z + i \sin z$  $e^z = \cosh z + \sinh z$ sinh(iz) = i sin z $2\cos z = e^{iz} + e^{-iz}$  $2\cosh z = e^z + e^{-z}$  $\cos(iz) = \cosh z$  $2i \sin z = e^{iz} - e^{-iz}$   $2 \sinh z = e^{z} - e^{-z}$  $\sin(iz) = i \sinh z$ 

#### Reihen

$$\begin{split} \mathrm{e}^{z} &= \sum_{k=0}^{\infty} \frac{z^{k}}{k!} \quad | \quad \mathrm{e}^{z} &= \lim_{n \to \infty} \left( 1 + \frac{z}{n} \right)^{n} \quad | \quad \ln z = \lim_{k \to 0} \frac{z^{h} - 1}{k} \\ \sin z &= \sum_{k=0}^{\infty} (-1)^{k} \frac{z^{2k+1}}{(2k+1)!} \quad | \quad \sinh z = \sum_{k=0}^{\infty} \frac{z^{2k+1}}{(2k+1)!} \\ \cos z &= \sum_{k=0}^{\infty} (-1)^{k} \frac{z^{2k}}{(2k)!} \quad | \quad \cosh z = \sum_{k=0}^{\infty} \frac{z^{2k+1}}{(2k)!} \\ \frac{z}{\mathrm{e}^{z} - 1} &= \sum_{k=0}^{\infty} \overline{B}_{k} \frac{z^{k}}{k!} \quad | \quad \ln(1 - x) = (-1) \sum_{k=1}^{\infty} \frac{x^{k}}{k} \quad (-1 \le x < 1) \\ \frac{z}{1 - \mathrm{e}^{-z}} &= \sum_{k=0}^{\infty} B_{k} \frac{z^{k}}{k!} \quad | \quad (z + 1)^{a} &= \sum_{k=0}^{\infty} \binom{a}{k} z^{k} \quad (a \in \mathbb{C}, |z| < 1) \\ \frac{1}{1 - z} &= \sum_{k=0}^{\infty} z^{k} \quad | \quad \frac{1}{(1 - z)^{n}} &= \sum_{k=0}^{\infty} \binom{n + k - 1}{k} z^{k} \quad (|z| < 1) \\ f[a](z) &:= \mathrm{e}^{(z - a)D} f(a) &= \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!} (z - a)^{k} \end{split}$$

# Differential rechnung $|x_{n+1} = x_n - f(x_n)/f'(x_n)$

$$f'(x) := \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \left| \begin{array}{l} T(x) = f(x_0) + f'(x_0)(x - x_0) \\ N(x) = f(x_0) - \frac{1}{f'(x_0)}(x - x_0) \end{array} \right|$$

$$(e^x)' = e^x \quad \left| \begin{array}{l} (fg)' = f'g + g'f \\ (f'g)' = (f'g - g'f)/g^2 \\ (a^x)' = a^x \ln a \\ (x^n)' = nx^{n-1} \end{array} \right| \quad (f^{-1})' = 1/(f' \circ f^{-1})$$

$$\sin' x = \cos x \quad \left| \begin{array}{l} \tan' x = 1 + \tan^2 x = 1/\cos^2 x \\ \cot' x = -1 - \cot^2 x = -1/\sin^2 x \\ \cosh' x = \sinh x \quad \coth' x = 1 - \tanh^2 x = 1/\cosh^2 x \\ \arcsin' x = 1/\sqrt{1-x^2} \quad \arctan' x = 1/(1+x^2) \\ \arcsin' x = 1/\sqrt{1-x^2} \quad \arctan' x = 1/(1+x^2) \\ \arcsin' x = 1/\sqrt{x^2+1} \quad \arctan' x = 1/(1-x^2) \\ \arcsin' x = 1/\sqrt{x^2-1} \quad \arctan' x = 1/(1-x^2) \\ \arctan' x = 1/(1-x^2) \quad \arctan' x = 1/(1-x^2) \\ \arctan' x = 1/(1-x^2) \quad \arctan' x = 1/(1-x^2) \\ \arctan' x = 1/(1-x^2) \quad \arctan' x = 1/(1-x^2) \\ \arctan' x = 1/(1-x^2) \quad \arctan' x = 1/(1-x^2) \\ \arctan' x = 1/(1-x^2) \quad \arctan' x = 1/(1-x^2) \\ \arctan' x = 1/(1-x^2) \quad \arctan' x = 1/(1-x^2) \\ \arctan' x = 1/(1-x^2) \quad \arctan' x = 1/(1-x^2) \\ \arctan' x = 1/(1-x^2) \quad \arctan' x = 1/(1-x^2) \\ \arctan' x = 1/(1-x^2) \quad \arctan' x = 1/(1-x^2) \\ \arctan' x = 1/(1-x^2) \quad \arctan' x = 1/(1-x^2) \\ \arctan' x = 1/(1-x^2) \quad \arctan' x = 1/(1-x^2) \\ \arctan' x = 1/(1-x^2) \quad \arctan' x = 1/(1-x^2) \\ \arctan' x = 1/$$

# Integralrechnung

$$\int_{a}^{b} f(x) \, \mathrm{d}x \coloneqq \lim_{n \to \infty} \sum_{k=1}^{n} f(a + k \frac{b-a}{n}) \frac{b-a}{n} \quad (f \in C[a,b])$$

$$\int_{a}^{b} f'(x) \, \mathrm{d}x = [f(x)]_{a}^{b} \coloneqq f(b) - f(a) \quad (f \in C^{1}[a,b])$$

$$\int_{a}^{b} f(g(x)) g'(x) \, \mathrm{d}x = \int_{g(a)}^{g(b)} f(u) \, \mathrm{d}u \quad \left( \frac{f \in C(I,\mathbb{R})}{g \in C^{1}([a,b],I)} \right)$$

$$\int_{a}^{b} f'(x) g(x) \, \mathrm{d}x = [f(x)g(x)]_{a}^{b} - \int_{a}^{b} f(x)g'(x) \, \mathrm{d}x \quad (f,g \in C^{1})$$

$$t = \tan(\frac{x}{2}), \quad \sin x = \frac{2t}{1+t^{2}}, \quad \cos x = \frac{1-t^{2}}{1+t^{2}}, \quad \mathrm{d}x = \frac{2\mathrm{d}t}{1+t^{2}}$$

$$L\{f(t)\} \coloneqq \int_{0}^{\infty} f(t) \mathrm{e}^{-pt} \, \mathrm{d}t, \quad F\{f(t)\} \coloneqq \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) \mathrm{e}^{-\mathrm{i}\omega t} \, \mathrm{d}t$$

$$\frac{\mathrm{d}}{\mathrm{d}x} \int_{a(x)}^{b(x)} f(x,t) \, \mathrm{d}t = \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x,t) \, \mathrm{d}t + g(x)$$

$$g(x) = f(x,b(x))b'(x) - f(x,a(x))a'(x)$$

#### Komplexe Zahlen

$$z = re^{i\varphi} = a + bi$$

$$\overline{z} = re^{-i\varphi} = a - bi$$

$$Re z = a = r \cos \varphi$$

$$Im z = b = r \sin \varphi$$

$$z_1 z_2 = r_1 r_2 e^{i(\varphi_1 - \varphi_2)} = \frac{a_1 a_2 + b_1 b_2}{a_2^2 + b_2^2} + \frac{a_2 b_1 - a_1 b_2}{a_2^2 + b_2^2} i$$

$$\frac{1}{z} = \frac{1}{r} e^{-i\varphi} = \frac{a}{a^2 + b^2} - \frac{b}{a^2 + b^2} i$$

#### Algebra

$$x^2 + px + q = 0 \Leftrightarrow 2x = -p \pm \sqrt{p^2 - 4q}$$
  
 $f(x) = f(2a - x)$  (Achsensymmetrie)  
 $f(x) = 2b - f(2a - x)$  (Punktsymmetrie)

$$\begin{array}{l} \textbf{Lineare Algebra} \mid \det(\lambda A) = \lambda^n \det(A), \ \det(A^{-1}) = \frac{1}{\det A} \\ \langle Av, w \rangle = \langle v, A^H w \rangle & (AB)^H = B^H A^H \\ \langle v, w \rangle = |v| |w| \cos \varphi & (AB)^{-1} = B^{-1} A^{-1} \\ |v \times w| = |v| |w| \sin \varphi & \det(AB) = \det(A) \det(B) \\ \text{proj}[w](v) = \frac{\langle v, w \rangle}{\langle w, w \rangle} w, \quad w_k := v_k - \sum_{i=1}^{k-1} \operatorname{proj}[w_i](v_k) \\ \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix} \\ \begin{pmatrix} a \\ b \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d \\ -c \end{pmatrix}, \quad R(\varphi) = \begin{pmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{pmatrix}$$

Kugelkoordinaten

# Polarkoordinaten

$x = r \cos \varphi$	$x = r\sin\theta\cos\varphi$
$y = r \sin \varphi$	$y = r \sin \theta \sin \varphi$
$\varphi \in (-\pi, \pi]$	$z = r \cos \theta$
$\det J = r$	$\varphi \in (-\pi, \pi], \ \theta \in [0, \pi]$
Zylinderkoordinaten	$\det J = r^2 \sin \theta$
$x = r_{xy} \cos \varphi$	$\theta = \beta - \pi/2$
$y = r_{xy} \sin \varphi$	$\beta \in [-\pi/2, \pi/2]$
J X 9 - 1	$\cos \theta = \sin \beta$

# Vektoranalysis

 $\det J = r_{xy}$ 

$$\begin{array}{l|l} \nabla(|\mathbf{x}|^2) = 2\mathbf{x} & \nabla(fg) = g\nabla f + f\nabla g \\ \nabla|\mathbf{x}| = \mathbf{x}/|\mathbf{x}| & \nabla\langle f,g\rangle = (Df)^T g + (Dg)^T f \\ \nabla(\frac{1}{g}) = -\frac{\nabla g}{g^2} & \nabla\langle f/g\rangle = (g\nabla f - f\nabla g)/g^2 \\ \nabla \times \nabla f = 0 & \langle \nabla, f\mathbf{v}\rangle = \langle \nabla f, \mathbf{v}\rangle + f\langle \nabla, \mathbf{v}\rangle \\ \langle \nabla, \nabla \times \mathbf{v}\rangle = 0 & \nabla \times (f\mathbf{v}) = f(\nabla \times \mathbf{v}) - \mathbf{v} \times \nabla f \\ \nabla \times \nabla \times \mathbf{v} = \nabla\langle \nabla, \mathbf{v}\rangle - \Delta \mathbf{v} \\ \langle \nabla, v \times w\rangle = \langle \mathbf{w}, \nabla \times \mathbf{v}\rangle - \langle \mathbf{v}, \nabla \times \mathbf{w}\rangle \\ \int_{\gamma} f \, \mathrm{d} \mathbf{s} := \int_{a}^{b} f(t) |\gamma'(t)| \, \mathrm{d} t, \ \int_{\gamma} \langle \mathbf{F}, \mathbf{d} \mathbf{x}\rangle := \int_{a}^{b} \langle \mathbf{F}(\mathbf{x}(t)), \mathbf{x}'(t)\rangle \, \mathrm{d} t \\ \int_{\sigma(U)} f(\mathbf{x}) \, \mathrm{d} \mathbf{x} = \int_{U} f(\varphi(\mathbf{u})) |\det D\varphi(\mathbf{u})| \, \mathrm{d} \mathbf{u} \end{array}$$

 $\sin \theta = \cos \beta$ 

# Extremwerte

$$f(x) = \text{extrem} \Rightarrow f'(x) = 0, \ f(p) = \text{extrem} \Rightarrow df_p = 0$$
  
 $f(x,y) = \text{extrem unter } g(x,y) = 0 \Rightarrow df = \lambda dg$   
 $J[\mathbf{x}] := \int_a^b L(t,\mathbf{x}(t),\mathbf{x}'(t)) dt = \text{extrem} \Rightarrow \frac{\partial L}{\partial x_k} = \frac{d}{dt} \frac{\partial L}{\partial x_k'}$ 

## Interpolation

Linear: 
$$p(x) = y_1 + \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$
  
Quadratisch:  $p(x) = y_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1)$   
 $a_1 = \frac{y_1 - y_0}{x_1 - x_0}, \quad a_2 = \frac{1}{x_2 - x_1} \left(\frac{y_2 - y_0}{x_2 - x_0} - a_1\right)$ 

#### Regression

$$y = \overline{y} + \frac{s_{xy}}{s_{xx}}(x - \overline{x}), \ s_{xx} = \sum_{k=1}^{n} (x_k - \overline{x})^2, \ s_{xy} = \sum_{k=1}^{n} (x_k - \overline{x})(y_k - \overline{y})$$

#### Logik

$\boldsymbol{A}$	B	$A \wedge B$	$A \vee B$	$A \rightarrow B$	$A \leftrightarrow B$	$A \oplus B$	$A \uparrow B$
0	0	0	0	1	1	0	1
0	1	0	1	1	0	1	1
1	0	0	1	0	0	1	1
1	1	1	1	1	1	0	0

Disjunktion	Konjunktion	Bezeichnung
$A \lor A \equiv A$	$A \wedge A \equiv A$	Idempotenzgesetze
$A \lor 0 \equiv A$	$A \wedge 1 \equiv A$	Neutralitätsgesetze
$A \lor 1 \equiv 1$	$A \wedge 0 \equiv 0$	Extremalgesetze
$A \vee \overline{A} \equiv 1$	$A \wedge \overline{A} \equiv 0$	Komplementärgesetze
$A \vee B \equiv B \vee A$	$A \wedge B \equiv B \wedge A$	Kommutativgesetze
$(A \lor B) \lor C \equiv A \lor (B \lor C)$	$(A \land B) \land C \equiv A \land (B \land C)$	Assoziativgesetze
$\overline{A \vee B} \equiv \overline{A} \wedge \overline{B}$	$\overline{A \wedge B} \equiv \overline{A} \vee \overline{B}$	De Morgansche Regelr
$A\vee (A\wedge B)\equiv A$	$A \wedge (A \vee B) \equiv A$	Absorptionsgesetze

$$(A \rightarrow B) \equiv \overline{A} \lor B \qquad | (A \leftrightarrow B) \equiv (A \rightarrow B) \land (B \rightarrow A)$$

$$(A \rightarrow B) \equiv (\overline{B} \rightarrow \overline{A}) \qquad | (A \leftrightarrow B) \equiv (\overline{A} \lor B) \land (\overline{B} \lor A)$$

$$A \lor \forall_x P_x \equiv \forall_x (A \lor P_x) \qquad | \forall_x (P_x \land Q_x) \equiv \forall_x P_x \land \forall_x Q_x$$

$$A \land \exists_x P_x \equiv \exists_x (A \land P_x) \qquad | \exists_x (P_x \lor Q_x) \equiv \exists_x P_x \lor \exists_x Q_x$$

$$(I \models M) :\Leftrightarrow \forall \varphi \in M: I(\varphi)$$

$$(\models \varphi) :\Leftrightarrow \forall I: I(\varphi) \qquad | (M \models \varphi) :\Leftrightarrow \forall I: ((I \models M) \Rightarrow I(\varphi))$$

$$\operatorname{erf}(\varphi) :\Leftrightarrow \exists I: I(\varphi) \qquad | \operatorname{erf}(M) :\Leftrightarrow \exists I: (I \models M)$$

$$\operatorname{erf}(\{\varphi_1, \dots, \varphi_n\}) \Leftrightarrow \operatorname{erf}(\varphi_1 \land \dots \land \varphi_n)$$

$$\operatorname{erf}(\varphi_1 \lor \dots \lor \varphi_n) \Leftrightarrow \operatorname{erf}(\varphi_1) \lor \dots \lor \operatorname{erf}(\varphi_n)$$

$$(M \vdash \varphi) \Rightarrow (M \vdash \varphi) \qquad (\text{Korrektheit})$$

$$(M \models \varphi) \Rightarrow (M \vdash \varphi) \qquad (\text{Vollständigkeit})$$

$$(M \cup \{\varphi\} \vdash \psi) \Leftrightarrow (M \vdash \varphi \rightarrow \psi)$$

$$(M \cup \{\varphi\} \models \psi) \Leftrightarrow (M \models \varphi \rightarrow \psi)$$

 $(M \models \varphi_1) \land (M \models \varphi_2) \land (\{\varphi_1, \varphi_2\} \models \psi) \Rightarrow (M \models \psi)$ 

# Mengenlehre

$$A \cap B := \{x \mid x \in A \land x \in B\} \\ A \cup B := \{x \mid x \in A \land x \in B\} \\ A \setminus B := \{x \mid x \in A \land x \notin B\} \\ \bigcap_{i \in I} A_i := \{x \mid \forall i \in I : x \in A_i\} \\ \bigcup_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i := \{x \mid \exists i \in I : x \in A_i\} \\ \bigcap_{i \in I} A_i :=$$

$$\frac{1}{B} \sum_{k=m}^{n-1} q^k = \frac{q^n - q^m}{q - 1}, \quad \sum_{k=m}^{n-1} k^p q^k = \left(q \frac{\mathrm{d}}{\mathrm{d}q}\right)^p \frac{q^n - q^m}{q - 1}$$

$$\sum_{k=1}^n k = (n/2)(n+1)$$

$$\sum_{k=1}^n k^2 = (n/6)(n+1)(2n+1)$$

$$\sum_{k=1}^n k^3 = (n/2)^2(n+1)^2$$

$$(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k, \quad \binom{n}{k} := \frac{1}{k!} n^k = \frac{n!}{k!(n-k)!}$$

$$n! = \Gamma(n+1), \quad \Gamma(z+1) = z\Gamma(z), \quad \Gamma(z)\Gamma(1-z) = \frac{\pi}{\sin(\pi z)}$$

$$e \sum_{k=0}^{n+1} \binom{n}{k} + \binom{n+1}{k}, \quad \binom{n}{k} = \binom{n}{n-k}, \quad \binom{n}{0} = \binom{n}{n} = 1$$

$$n! \approx \sqrt{2\pi n} n^n \exp\left(\frac{1}{12n} - n\right) \approx \sqrt{2\pi n} (n/e)^n$$

**Twelvefold way.** Fächer: n := |N|, Karten: k := |K|

	$f: K \rightarrow N$	$f \in \mathrm{Inj}(K, N)$	$f \in \operatorname{Sur}(K, N)$
f	$n^k$	$n^{\underline{k}}$	$n!\binom{k}{n}$
$f \circ S_k$	$\binom{n+k-1}{k}$	$\binom{n}{k}$	$\binom{k-1}{k-n}$
$S_n \circ f$	$\sum_{i=0}^{n} {k \brace i}$	$[k \leq n]$	${k \choose n}$
$S_n \circ f \circ S_k$	$p_n(n+k)$	$[k \leq n]$	$p_n(k)$

 $S_k$ : Karten nicht unterscheidbar | Inj: max. 1 Karte pro Fach  $S_n$ : Fächer nicht unterscheidbar Sur: mind. 1 Karte pro Fach

$$\begin{cases}
 \binom{n}{k} = k \binom{n-1}{k} + \binom{n-1}{k-1} \\
 \binom{n}{k} = (n-1) \binom{n-1}{k} + \binom{n-1}{k-1} \\
 x^n = \sum_{k=0}^n \binom{n}{k} x^{\underline{k}} \\
 \end{cases}$$

$$\begin{cases}
 \binom{n}{0} = \binom{n}{0} = [n=0] \\
 \binom{n}{1} = [n>0] \\
 \binom{n}{1} = [n>0] \\
 \binom{n}{2} = (2^{n-1} - 1)[n>0]$$

## Wahrscheinlichkeitsrechnung

$$\begin{split} &P(A \cup B) = P(A) + P(B) - P(A \cap B) \\ &P(A \cap B) = P(A)P(B \mid A) \\ &A \text{ unabhängig zu } B :\Leftrightarrow P(A \cap B) = P(A)P(B) \\ &P(A) = \sum_{k=1}^n P(A \mid B_k)P(B_k) \text{ für Zerlegung } (B_k) \text{ von } \Omega \\ &P(A) = \int_{-\infty}^\infty P(A \mid X = x) \, \mathrm{d}F_X(x) \\ &E(X) = \sum_{\omega \in \Omega} X(\omega)P(\{\omega\}) = \sum_{x \in \Omega'} xP(X = x) \\ &E(X) = \int_{-\infty}^\infty xf(x) \, \mathrm{d}x = \int_0^\infty (1 - F(x)) \, \mathrm{d}x - \int_{-\infty}^0 F(x) \, \mathrm{d}x \\ &E(g(X)) = \int_{-\infty}^\infty g(x)f(x) \, \mathrm{d}x \quad \middle| P(A) = E(1_A) \\ &E(X \mid A) = E(1_A X)/P(A) \quad \middle| P(A \mid B) = E(1_A \mid B) \\ &P(X \leq x) = F(x) = \int_{-\infty}^x f(t) \, \mathrm{d}t \\ &P(a \leq X \leq b) = F(b) - F(a) = \int_a^b f(t) \, \mathrm{d}t \end{split}$$

Normalverteilung
$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2} dt = \frac{1}{2} + \frac{1}{2} \operatorname{erf}(\frac{x}{\sqrt{2}})$$

$$F(x) = \Phi(\frac{x-\mu}{\sigma})$$

# Mechanik

$\mathbf{v} = \mathbf{x}'(t)$	$\omega =$	$\varphi'(t)$	
$\mathbf{a} = \mathbf{v}'(t)$	$\alpha =$	$\omega'(t)$	
$\mathbf{F} = \mathbf{p}'(t)$	M =	$\mathbf{L}'(t)$	
$\mathbf{p} = m\mathbf{v}$	L =	$J\omega$	
$\mathbf{F} = m\mathbf{a}$	M =	: <i>J</i> α	
$P = \langle \mathbf{F}, \mathbf{v} \rangle$	P =	$\langle M, \boldsymbol{\omega} \rangle$	
$E_{\rm kin} = \frac{1}{2}m $	$ \mathbf{v} ^2 \mid E_{\text{rot}}$	$=\frac{1}{2}J\omega^2$	
$s = \varphi r \mid \mathbf{N}$	$\mathbf{M} = \mathbf{r} \times \mathbf{F}$	$E_{\text{pot}} = m$	gh
$v = \omega r \mid \mathbf{L}$	$= \mathbf{r} \times \mathbf{p}$	$E_{\rm kin} + E_{\rm p}$	$p_{\text{ot}} = \text{const.}$
$a = \alpha r \mid \mathbf{v}$	$r = \omega \times r$	F = Ds	(Feder)

# Gleichstrom

$$U = RI$$
 |  $Q = It$  |  $GR = 1$   
 $I = GU$  |  $W = Pt$   
 $P = UI$  |  $W = QU$ 

### Wechselstrom

$$\begin{array}{c|cccccc} \overline{U} &= \overline{ZI} & \overline{Z} = R + jX & Z^2 = R^2 + X^2 \\ \overline{I} &= \underline{YU} & \underline{Y} = G + jQ & R = Z\cos\varphi \\ \overline{S} &= \overline{UI} & \overline{S} = P + jB & X = Z\sin\varphi \\ \hline \underline{Z} &= R & \text{Widerstand} & \omega = 2\pi f \\ \overline{Z} &= jX_C & \text{Kondensator} & X_C = -1/(\omega C) \\ \overline{Z} &= jX_L & \text{Spule} & X_L = \omega L \\ \hline u_s &= \sqrt{2}\,U_{\text{eff}} & u = u_s\sin(\omega t + \varphi_0) \\ i_s &= \sqrt{2}\,I_{\text{eff}} & i = i_s\sin(\omega t + \varphi_0) \end{array}$$

## Allgemeine Gleichungen

$$u = Ri$$
  
 $i = Cu'(t)$   
 $u = Li'(t)$ 
 $p = ui$ 

# **Elektrostatisches Feld**

$$\begin{split} F &= \frac{1}{4\pi\varepsilon} \frac{Q_1 Q_2}{r^2} \; \middle| \; \mathbf{F}_1 = \frac{1}{4\pi\varepsilon} Q_1 Q_2 \frac{\mathbf{x}_1 - \mathbf{x}_2}{|\mathbf{x}_1 - \mathbf{x}_2|^3} \\ \mathbf{F} &= q \mathbf{E} \; \middle| \; Q = C U \; \middle| \; U = \varphi(B) - \varphi(A) \\ \mathbf{D} &= \varepsilon \mathbf{E} \; \middle| \; \varepsilon = \varepsilon_0 \varepsilon_r \; \middle| \; W = Q U \\ \mathbf{E} &= -\nabla \varphi \\ \varepsilon_0 E^2 &= 2 w_e \end{split}$$

# **Plattenkondensator**

$$U = Ed \mid C = \varepsilon A/d$$

# Homogenes Feld in der Spule

$$Hl = NI \mid Bl = \mu NI \mid \Theta = NI$$

# **Magnetostatisches Feld**

$$\begin{aligned} \mathbf{F} &= q\mathbf{v} \times \mathbf{B} & \Phi &= BA \\ F &= qvB & \mathbf{B} &= \mu\mathbf{H} \\ F &= BIl & \mu &= \mu_0\mu_r \\ H &= I/(2\pi r) & \text{(Feld um einen geraden Leiter)} \\ B^2 &= 2\mu_0 w_m \end{aligned}$$

# Elektrodynamik

$$\mathbf{E} = -\nabla \varphi$$

$$\varepsilon \Delta \varphi = -\rho(x)$$

$$\varepsilon_0 E^2 = 2w_e$$

$$B^2 = 2\mu_0 w_m$$

### Maxwell-Gleichungen

$$\begin{split} \langle \nabla, \mathbf{D} \rangle &= \rho_f(x) \\ \langle \nabla, \mathbf{B} \rangle &= 0 \\ \nabla \times \mathbf{E} &= -\partial_t \mathbf{B} \\ \nabla \times \mathbf{H} &= \mathbf{J}_f + \partial_t \mathbf{D} \end{split} \qquad \begin{split} \langle \nabla, \varepsilon_0 \mathbf{E} \rangle &= \rho(x) \\ \langle \nabla, \mathbf{B} \rangle &= 0 \\ \nabla \times \mathbf{E} &= -\partial_t \mathbf{B} \\ \nabla \times \mathbf{B} &= \mu_0 (\mathbf{J} + \varepsilon_0 \partial_t \mathbf{E}) \end{split}$$

# Spezielle Relativitätstheorie

$$\begin{split} \gamma &= \frac{1}{\sqrt{1-\beta^2}}, \quad \beta = v/c \\ \gamma &= \cosh \varphi, \quad \beta \gamma = \sinh \varphi, \quad \beta = \tanh \varphi \\ ct' &= \gamma(ct-\beta x), \quad x' = \gamma(x-vt), \quad (y,z)' = (y,z) \\ t &= \gamma \tau \qquad \left| \begin{array}{c} E_{\rm kin} = E-E_0 \\ E_{\rm kin} = \gamma mc^2 - mc^2 \\ E &= \gamma mc^2 \end{array} \right| E^2 &= (pc)^2 + (mc^2)^2 \\ \Lambda_v &= \begin{bmatrix} \gamma & -\beta \gamma & 0 & 0 \\ -\beta \gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ g &= {\rm diag}(1,-1,-1,-1) \\ (\partial_\mu) &= (\partial_{ct},\partial_x,\partial_y,\partial_z) \\ (\partial^\mu) &= (\partial_{ct},-\partial_x,-\partial_y,-\partial_z) \end{split}$$

$$\frac{1}{f} = \frac{1}{g} + \frac{1}{b}, \quad A = \frac{B}{G} = \frac{b}{g}$$

$$n_1 \sin(\varphi_1) = n_2 \sin(\varphi_2)$$

$$c_0 = nc$$

# Thermodynamik

$$R = N_A k_B \mid m = nM \mid V = nV_m$$

$$R = R_s M \mid m = Nm_T \mid N = nN_A$$

$$pV = nRT$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$Q = mc\Delta T$$

# Konstanten $\varepsilon_0 = 8.8542 \times 10^{-12} \,\mathrm{C/(V\,m)}$

$$\mu_0 = 4\pi \times 10^{-7} \,\mathrm{H/m}$$
 $c_0 = 2.9979 \times 10^8 \,\mathrm{m/s}$ 
 $e = 1.6022 \times 10^{-19} \,\mathrm{C}$ 
 $G = 6.674 \times 10^{-11} \,\mathrm{m}^3/(\mathrm{kg} \,\mathrm{s}^2)$ 
 $N_A = 6.0221 \times 10^{23} \,\mathrm{mol}^{-1}$ 
 $k_B = 1.3806 \times 10^{-23} \,\mathrm{J/K}$ 
 $R = 8.3145 \,\mathrm{J/(mol} \,\mathrm{K)}$ 
 $0 \,\mathrm{K} = -273.15 \,^{\circ}\mathrm{C}$ 
 $u = 1.6605 \times 10^{-27} \,\mathrm{kg}$ 
 $h = 6.6261 \times 10^{-34} \,\mathrm{Js}$ 
 $\hbar = 1.0546 \times 10^{-34} \,\mathrm{Js}$ 
 $\sigma = 5.6704 \times 10^{-8} \,\mathrm{W/(m^2 K^4)}$ 
 $m_e = 9.1094 \times 10^{-31} \,\mathrm{kg}$ 
 $m_p = 1.6726 \times 10^{-27} \,\mathrm{kg}$ 
 $m_n = 1.6749 \times 10^{-27} \,\mathrm{kg}$ 

 $m_{\alpha} = 6.6447 \times 10^{-27} \text{ kg}$