

Winkelfunktionen

$$\begin{array}{l|l} \sin(x+y) = \sin x \cos y + \cos x \sin y & \sin(z \pm \pi) = -\sin z \\ \sin(x-y) = \sin x \cos y - \cos x \sin y & \cos(z \pm \pi) = -\cos z \\ \cos(x+y) = \cos x \cos y - \sin x \sin y & \sin(\pi/2 - x) = \cos x \\ \cos(x-y) = \cos x \cos y + \sin x \sin y & \cos(\pi/2 - x) = \sin x \\ \sin(nx) = 2 \cos x \sin((n-1)x) - \sin((n-2)x) & \sin(-z) = -\sin z \\ \cos(nx) = 2 \cos x \cos((n-1)x) - \cos((n-2)x) & \cos(-z) = \cos z \\ \cos^2 z + \sin^2 z = 1 & \cosh^2 z - \sinh^2 z = 1 \\ e^{iz} = \cos z + i \sin z & e^z = \cosh z + \sinh z \\ 2 \cos z = e^{iz} + e^{-iz} & 2 \cosh z = e^z + e^{-z} \\ 2i \sin z = e^{iz} - e^{-iz} & 2 \sinh z = e^z - e^{-z} \\ \end{array} \quad \begin{array}{l} \cosh(iz) = \cos z \\ \sinh(iz) = i \sin z \\ \cos(iz) = \cosh z \\ \sin(iz) = i \sinh z \end{array}$$

Reihen

$$\begin{array}{l} e^z = \sum_{k=0}^{\infty} \frac{z^k}{k!} \quad \left| \quad e^z = \lim_{n \rightarrow \infty} \left(1 + \frac{z}{n}\right)^n \quad \left| \quad \ln z = \lim_{h \rightarrow 0} \frac{z^h - 1}{h} \right. \\ \sin z = \sum_{k=0}^{\infty} (-1)^k \frac{z^{2k+1}}{(2k+1)!} \quad \left| \quad \sinh z = \sum_{k=0}^{\infty} \frac{z^{2k+1}}{(2k+1)!} \right. \\ \cos z = \sum_{k=0}^{\infty} (-1)^k \frac{z^{2k}}{(2k)!} \quad \left| \quad \cosh z = \sum_{k=0}^{\infty} \frac{z^{2k}}{(2k)!} \right. \\ \frac{z}{e^z - 1} = \sum_{k=0}^{\infty} \bar{B}_k \frac{z^k}{k!} \quad \left| \quad \ln(1-x) = (-1) \sum_{k=1}^{\infty} \frac{x^k}{k} \quad (-1 \leq x < 1) \right. \\ \frac{z}{1 - e^{-z}} = \sum_{k=0}^{\infty} B_k \frac{z^k}{k!} \quad \left| \quad (z+1)^a = \sum_{k=0}^{\infty} \binom{a}{k} z^k \quad (a \in \mathbb{C}, |z| < 1) \right. \\ \frac{1}{1-z} = \sum_{k=0}^{\infty} z^k \quad \left| \quad \frac{1}{(1-z)^n} = \sum_{k=0}^{\infty} \binom{n+k-1}{k} z^k \quad (|z| < 1) \right. \\ f[a](z) := e^{(z-a)D} f(a) = \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!} (z-a)^k \end{array}$$

Differentialrechnung

$$\begin{array}{l|l} f'(x) := \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} & T(x) = f(x_0) + f'(x_0)(x - x_0) \\ & N(x) = f(x_0) - \frac{1}{f'(x_0)}(x - x_0) \\ (e^x)' = e^x & (fg)' = f'g + g'f \\ \ln' x = 1/x & (f/g)' = (f'g - g'f)/g^2 \\ (a^x)' = a^x \ln a & (g \circ f)' = (g' \circ f)f' \\ (x^n)' = nx^{n-1} & (f^{-1})' = 1/(f' \circ f^{-1}) \\ \sin' x = \cos x & \tan' x = 1 + \tan^2 x = 1/\cos^2 x \\ \cos' x = -\sin x & \cot' x = -1 - \cot^2 x = -1/\sin^2 x \\ \sinh' x = \cosh x & \tanh' x = 1 - \tanh^2 x = 1/\cosh^2 x \\ \cosh' x = \sinh x & \coth' x = 1 - \coth^2 x = -1/\sinh^2 x \\ \arcsin' x = 1/\sqrt{1-x^2} & \arctan' x = 1/(1+x^2) \\ \arccos' x = -1/\sqrt{1-x^2} & \operatorname{arccot}' x = -1/(1+x^2) \\ \operatorname{arsinh}' x = 1/\sqrt{x^2+1} & \operatorname{artanh}' x = 1/(1-x^2) \\ \operatorname{arcosh}' x = 1/\sqrt{x^2-1} & \operatorname{arcoth}' x = 1/(1-x^2) \end{array}$$

Integralrechnung

$$\begin{array}{l} \int_a^b f(x) dx := \lim_{n \rightarrow \infty} \sum_{k=1}^n f(a + k \frac{b-a}{n}) \frac{b-a}{n} \quad (f \in C[a, b]) \\ \int_a^b f'(x) dx = [f(x)]_a^b := f(b) - f(a) \quad (f \in C^1[a, b]) \\ \int_a^b f(g(x)) g'(x) dx = \int_{g(a)}^{g(b)} f(u) du \quad \left(\begin{array}{l} f \in C(I, \mathbb{R}), \\ g \in C^1([a, b], I) \end{array} \right) \\ \int_a^b f'(x) g(x) dx = [f(x) g(x)]_a^b - \int_a^b f(x) g'(x) dx \quad (f, g \in C^1) \\ t = \tan(\frac{x}{2}), \quad \sin x = \frac{2t}{1+t^2}, \quad \cos x = \frac{1-t^2}{1+t^2}, \quad dx = \frac{2dt}{1+t^2} \\ L\{f(t)\} := \int_0^{\infty} f(t) e^{-pt} dt, \quad F\{f(t)\} := \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \\ \frac{d}{dx} \int_{a(x)}^{b(x)} f(x, t) dt = g(x) + \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x, t) dt \\ \text{wobei } g(x) = f(x, b(x))b'(x) - f(x, a(x))a'(x) \end{array}$$

Komplexe Zahlen

$$\begin{array}{l|l} z = re^{i\varphi} = a + bi & |z| = r = \sqrt{a^2 + b^2} \\ \bar{z} = re^{-i\varphi} = a - bi & \arg(z) = \varphi = \operatorname{sgn}(b) \arccos(a/r) \\ \operatorname{Re} z = a = r \cos \varphi & z_1 + z_2 = (a_1 + a_2) + (b_1 + b_2)i \\ \operatorname{Im} z = b = r \sin \varphi & z_2 - z_1 = (a_1 - a_2) + (b_1 - b_2)i \\ z_1 z_2 = r_1 r_2 e^{i(\varphi_1 + \varphi_2)} = (a_1 a_2 - b_1 b_2) + (a_1 b_2 + a_2 b_1)i \\ \frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\varphi_1 - \varphi_2)} = \frac{a_1 a_2 + b_1 b_2}{a_2^2 + b_2^2} + \frac{a_2 b_1 - a_1 b_2}{a_2^2 + b_2^2} i \\ \frac{1}{z} = \frac{1}{r} e^{-i\varphi} = \frac{a}{a^2 + b^2} - \frac{b}{a^2 + b^2} i \end{array}$$

Algebra

$$\begin{array}{l} x^2 + px + q = 0 \Leftrightarrow 2x = -p \pm \sqrt{p^2 - 4q} \\ f(x) = f(2a - x) \quad (\text{Achsensymmetrie}) \\ f(x) = 2b - f(2a - x) \quad (\text{Punktsymmetrie}) \end{array}$$

Lineare Algebra

$$\begin{array}{l|l} \det(\lambda A) = \lambda^n \det(A), \quad \det(A^{-1}) = \frac{1}{\det A} & \\ \langle Av, w \rangle = \langle v, A^H w \rangle & (AB)^H = B^H A^H \\ \langle v, w \rangle = |v||w| \cos \varphi & (AB)^{-1} = B^{-1} A^{-1} \\ |v \times w| = |v||w| \sin \varphi & \det(AB) = \det(A) \det(B) \\ \operatorname{proj}[w](v) = \frac{\langle v, w \rangle}{\langle w, w \rangle} w, \quad w_k := v_k - \sum_{i=1}^{k-1} \operatorname{proj}[w_i](v_k) & \\ \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix} & \\ \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}, \quad R(\varphi) = \begin{pmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{pmatrix} & \end{array}$$

Polarkoordinaten

$$\begin{array}{l} x = r \cos \varphi \\ y = r \sin \varphi \\ \varphi \in (-\pi, \pi] \\ \det J = r \end{array}$$

Zylinderkoordinaten

$$\begin{array}{l} x = r_{xy} \cos \varphi \\ y = r_{xy} \sin \varphi \\ z = z \\ \det J = r_{xy} \end{array}$$

Kugelkoordinaten

$$\begin{array}{l} x = r \sin \theta \cos \varphi \\ y = r \sin \theta \sin \varphi \\ z = r \cos \theta \\ \varphi \in (-\pi, \pi], \quad \theta \in [0, \pi] \\ \det J = r^2 \sin \theta \\ \theta = \beta - \pi/2 \\ \beta \in [-\pi/2, \pi/2] \\ \cos \theta = \sin \beta \\ \sin \theta = \cos \beta \end{array}$$

Vektoranalysis

$$\begin{array}{l|l} \nabla(|x|^2) = 2x & \nabla(fg) = g \nabla f + f \nabla g \\ \nabla|x| = x/|x| & \nabla \langle f, g \rangle = (Df)^T g + (Dg)^T f \\ \nabla(\frac{1}{g}) = -\frac{\nabla g}{g^2} & \nabla(f/g) = (g \nabla f - f \nabla g)/g^2 \\ \nabla \times \nabla f = 0 & \langle \nabla, f \mathbf{v} \rangle = \langle \nabla f, \mathbf{v} \rangle + f \langle \nabla, \mathbf{v} \rangle \\ \langle \nabla, \nabla \times \mathbf{v} \rangle = 0 & \nabla \times (f \mathbf{v}) = f(\nabla \times \mathbf{v}) - \mathbf{v} \times \nabla f \\ \nabla \times \nabla \times \mathbf{v} = \nabla \langle \nabla, \mathbf{v} \rangle - \Delta \mathbf{v} & \\ \langle \nabla, v \times w \rangle = \langle w, \nabla \times v \rangle - \langle v, \nabla \times w \rangle & \\ \int_Y f ds := \int_a^b f(t) |Y'(t)| dt, \quad \int_Y \langle F, dx \rangle := \int_a^b \langle F(x(t)), x'(t) \rangle dt & \\ \int_{\varphi(U)} f(x) dx = \int_U f(\varphi(u)) |\det D\varphi(u)| du & \end{array}$$

Extremwerte

$$\begin{array}{l} f(x) = \text{extrem} \Rightarrow f'(x) = 0, \quad f(p) = \text{extrem} \Rightarrow df_p = 0 \\ f(x, y) = \text{extrem unter } g(x, y) = 0 \Rightarrow df = \lambda dg \\ J[x] := \int_a^b L(t, x(t), x'(t)) dt = \text{extrem} \Rightarrow \frac{\partial L}{\partial x_k} = \frac{d}{dt} \frac{\partial L}{\partial x'_k} \end{array}$$

Interpolation

$$\begin{array}{l} \text{Linear: } p(x) = y_1 + \frac{y_2 - y_1}{x_2 - x_1} (x - x_1) \\ \text{Quadratisch: } p(x) = y_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) \\ a_1 = \frac{y_1 - y_0}{x_1 - x_0}, \quad a_2 = \frac{1}{x_2 - x_1} \left(\frac{y_2 - y_0}{x_2 - x_0} - a_1 \right) \end{array}$$

Regression

$$y = \bar{y} + \frac{s_{xy}}{s_{xx}} (x - \bar{x}), \quad s_{xx} = \sum_{k=1}^n (x_k - \bar{x})^2, \quad s_{xy} = \sum_{k=1}^n (x_k - \bar{x})(y_k - \bar{y})$$

Logik

A	B	$A \wedge B$	$A \vee B$	$A \rightarrow B$	$A \leftrightarrow B$	$A \oplus B$	$A \uparrow B$
0	0	0	0	1	1	0	1
0	1	0	1	1	0	1	1
1	0	0	1	0	0	1	1
1	1	1	1	1	1	0	0

Disjunktion	Konjunktion	Bezeichnung
$A \vee A \equiv A$	$A \wedge A \equiv A$	Idempotenzgesetze
$A \vee 0 \equiv A$	$A \wedge 1 \equiv A$	Neutralitätsgesetze
$A \vee 1 \equiv 1$	$A \wedge 0 \equiv 0$	Extremalgesetze
$A \vee \bar{A} \equiv 1$	$A \wedge \bar{A} \equiv 0$	Komplementärsgesetze
$A \vee B \equiv B \vee A$	$A \wedge B \equiv B \wedge A$	Kommutativgesetze
$(A \vee B) \vee C \equiv A \vee (B \vee C)$	$(A \wedge B) \wedge C \equiv A \wedge (B \wedge C)$	Assoziativgesetze
$A \vee \bar{B} \equiv \bar{A} \wedge \bar{B}$	$A \wedge \bar{B} \equiv \bar{A} \vee \bar{B}$	De Morgansche Regeln
$A \vee (A \wedge B) \equiv A$	$A \wedge (A \vee B) \equiv A$	Absorptionsgesetze

$(A \rightarrow B) \equiv \bar{A} \vee B$	$(A \leftrightarrow B) \equiv (A \rightarrow B) \wedge (B \rightarrow A)$
$(A \rightarrow B) \equiv (\bar{B} \rightarrow \bar{A})$	$(A \leftrightarrow B) \equiv (\bar{A} \vee B) \wedge (\bar{B} \vee A)$
$A \vee \forall x P_x \equiv \forall x (A \vee P_x)$	$\forall x (P_x \wedge Q_x) \equiv \forall x P_x \wedge \forall x Q_x$
$A \wedge \exists x P_x \equiv \exists x (A \wedge P_x)$	$\exists x (P_x \vee Q_x) \equiv \exists x P_x \vee \exists x Q_x$
$(I \models M) :\Leftrightarrow \forall \varphi \in M: I(\varphi)$	
$(\models \varphi) :\Leftrightarrow \forall I: I(\varphi)$	$(M \models \varphi) :\Leftrightarrow \forall I: ((I \models M) \Rightarrow I(\varphi))$
$\text{erf}(\varphi) :\Leftrightarrow \exists I: I(\varphi)$	$\text{erf}(M) :\Leftrightarrow \exists I: (I \models M)$
$\text{erf}(\{\varphi_1, \dots, \varphi_n\}) \Leftrightarrow \text{erf}(\varphi_1 \wedge \dots \wedge \varphi_n)$	
$\text{erf}(\varphi_1 \vee \dots \vee \varphi_n) \Leftrightarrow \text{erf}(\varphi_1) \vee \dots \vee \text{erf}(\varphi_n)$	
$(M \vdash \varphi) \Rightarrow (M \models \varphi)$ (Korrektheit)	
$(M \models \varphi) \Rightarrow (M \vdash \varphi)$ (Vollständigkeit)	
$(M \cup \{\varphi\} \vdash \psi) \Leftrightarrow (M \vdash \varphi \rightarrow \psi)$	
$(M \cup \{\varphi\} \models \psi) \Leftrightarrow (M \models \varphi \rightarrow \psi)$	
$(M \models \varphi_1) \wedge (M \models \varphi_2) \wedge (\{\varphi_1, \varphi_2\} \models \psi) \Rightarrow (M \models \psi)$	

Natürliches Schließen

$\frac{\Gamma \vdash A \wedge B}{\Gamma \vdash A}$	$\frac{\Gamma \vdash A}{\Gamma \vdash A \vee B}$	$\frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \wedge B}$	$\frac{}{\Gamma, A \vdash A}$	$\frac{}{\Gamma \vdash \top}$
$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \rightarrow B}$	$\frac{\Gamma \vdash A \vee B \quad \Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma \vdash C}$	$\frac{\Gamma, A \vdash \perp}{\Gamma \vdash \neg A}$		
$\frac{\Gamma \vdash A \rightarrow B \quad \Gamma \vdash A}{\Gamma \vdash B}$	$\frac{\Gamma \vdash \neg A \quad \Gamma \vdash A}{\Gamma \vdash \perp}$	$\frac{\Gamma \vdash \perp}{\Gamma \vdash A}$	$\left(\frac{\Gamma \vdash \neg \neg A}{\Gamma \vdash A} \right)$	
$\frac{\Gamma \vdash A}{\Gamma \vdash \forall x: A} (x \notin \text{FV}(\Gamma))$	$\frac{\Gamma \vdash \forall x: A}{\Gamma \vdash A[x := t]}$	$\frac{\Gamma \vdash A}{\Gamma, \Gamma' \vdash A}$		
$\frac{\Gamma \vdash A[x := t]}{\Gamma \vdash \exists x: A}$	$\frac{\Gamma \vdash \exists x: A \quad \Gamma, A \vdash B}{\Gamma \vdash B} (x \notin \text{FV}(\Gamma, B))$			

Mengenlehre

$A \cap B := \{x \mid x \in A \wedge x \in B\}$	$A \subseteq B :\Leftrightarrow \forall x (x \in A \Rightarrow x \in B)$
$A \cup B := \{x \mid x \in A \vee x \in B\}$	$A = B :\Leftrightarrow \forall x (x \in A \Leftrightarrow x \in B)$
$A \setminus B := \{x \mid x \in A \wedge x \notin B\}$	$A = B :\Leftrightarrow A \subseteq B \wedge B \subseteq A$
$\bigcap_{i \in I} A_i := \{x \mid \forall i \in I: x \in A_i\}$	$f(A) := \{y \mid \exists x \in A: y = f(x)\}$
$\bigcup_{i \in I} A_i := \{x \mid \exists i \in I: x \in A_i\}$	$f^{-1}(B) := \{x \mid f(x) \in B\}$
$A \times B := \{t \mid \exists x \in A: \exists y \in B: t = (x, y)\}$	
$A \subseteq B \Leftrightarrow A \cup B = B \Leftrightarrow A \cap B = A \Leftrightarrow A \setminus B = \emptyset$	
$f(A \cup B) = f(A) \cup f(B)$	$f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$
$f(A \cap B) \subseteq f(A) \cap f(B)$	$f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$
$A \subseteq B \Rightarrow f(A) \subseteq f(B)$	$A \subseteq B \Rightarrow f^{-1}(A) \subseteq f^{-1}(B)$
$(g \circ f)(A) = g(f(A))$	$(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$
$A \subseteq f^{-1}(f(A))$	$f(f^{-1}(B)) \subseteq B$
$f \text{ inj.} \Leftrightarrow A = f^{-1}(f(A))$	$f \text{ sur.} \Leftrightarrow B = f(f^{-1}(B))$

Kombinatorik

$$\sum_{k=m}^{n-1} q^k = \frac{q^n - q^m}{q - 1}, \quad \sum_{k=m}^{n-1} k^p q^k = \left(q \frac{d}{dq} \right)^p \frac{q^n - q^m}{q - 1}$$

$$\sum_{k=1}^n k = (n/2)(n+1) \quad \left| \quad \sum_{k=m}^{n-1} (\Delta a)_k = a_n - a_m \right.$$

$$\sum_{k=1}^n k^2 = (n/6)(n+1)(2n+1) \quad \left| \quad (\Delta a)_k := a_{k+1} - a_k \right.$$

$$\sum_{k=1}^n k^3 = (n/2)^2 (n+1)^2 \quad \left| \quad n! = n \cdot (n-1)! \right.$$

$$(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k, \quad \binom{n}{k} := \frac{1}{k!} n^{\underline{k}} = \frac{n!}{k!(n-k)!}$$

$$n! = \Gamma(n+1), \quad \Gamma(z+1) = z\Gamma(z), \quad \Gamma(z)\Gamma(1-z) = \frac{\pi}{\sin(\pi z)}$$

$$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}, \quad \binom{n}{k} = \binom{n}{n-k}, \quad \binom{n}{0} = \binom{n}{n} = 1$$

$$n! \approx \sqrt{2\pi n} n^n \exp\left(-\frac{1}{12n} - n\right) \approx \sqrt{2\pi n} (n/e)^n$$

$$(a_k) * (b_k) = \left(\sum_{i=0}^k a_i b_{k-i} \right) = \left(\sum_{i=0}^k \sum_{j=0}^k a_i b_j [k=i+j] \right)$$

$$y = \lfloor x \rfloor \Leftrightarrow y \in \mathbb{Z} \wedge 0 \leq x - y < 1$$

$$y = \lceil x \rceil \Leftrightarrow y \in \mathbb{Z} \wedge 0 \leq y - x < 1$$

Twelfold way. Fächer: $n := |N|$, Karten: $k := |K|$

	$f: K \rightarrow N$	$f \in \text{Inj}(K, N)$	$f \in \text{Sur}(K, N)$
f	n^k	$n^{\underline{k}}$	$n! \binom{k}{n}$
$f \circ S_k$	$\binom{n+k-1}{k}$	$\binom{n}{k}$	$\binom{k-1}{k-n}$
$S_n \circ f$	$\sum_{i=0}^n \binom{k}{i}$	$[k \leq n]$	$\binom{k}{n}$
$S_n \circ f \circ S_k$	$p_n(n+k)$	$[k \leq n]$	$p_n(k)$

S_k : Karten nicht unterscheidbar

S_n : Fächer nicht unterscheidbar

Inj: max. 1 Karte pro Fach

Sur: mind. 1 Karte pro Fach

$$\binom{n}{k} = k \binom{n-1}{k-1} + \binom{n-1}{k}$$

$$\binom{n}{k} = (n-1) \binom{n-1}{k} + \binom{n-1}{k-1}$$

$$x^n = \sum_{k=0}^n \binom{n}{k} x^{\underline{k}}$$

Wahrscheinlichkeitsrechnung

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B \mid A)$$

$$A \text{ unabhängig zu } B :\Leftrightarrow P(A \cap B) = P(A)P(B)$$

$$P(A) = \sum_{k=1}^n P(A \mid B_k)P(B_k) \text{ für Zerlegung } (B_k) \text{ von } \Omega$$

$$P(A) = \int_{-\infty}^{\infty} P(A \mid X = x) dF_X(x)$$

$$P(g(X) = y) = \sum_{x \in g^{-1}(\{y\})} P(X = x)$$

$$P(g(X, Y) = z) = \sum_{(x, y) \in g^{-1}(\{z\})} P(X = x, Y = y)$$

$$E(X) = \sum_{\omega \in \Omega} X(\omega)P(\{\omega\}) = \sum_x xP(X = x)$$

$$E(g(X)) = \sum_{\omega \in \Omega} g(X(\omega))P(\{\omega\}) = \sum_x g(x)P(X = x)$$

$$E(g(X, Y)) = \sum_{(x, y)} g(x, y)P(X = x, Y = y)$$

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx = \int_0^{\infty} (1 - F(x)) dx - \int_{-\infty}^0 F(x) dx$$

$$E(g(X)) = \int_{-\infty}^{\infty} g(x) f(x) dx \quad \left| \quad P(A) = E(1_A) \right.$$

$$E(X \mid A) = E(1_A X) / P(A) \quad \left| \quad P(A \mid B) = E(1_A \mid B) \right.$$

$$P(X \leq x) = F(x) = \int_{-\infty}^x f(t) dt$$

$$P(a \leq X \leq b) = F(b) - F(a) = \int_a^b f(t) dt$$

Normalverteilung

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-t^2/2} dt = \frac{1}{2} + \frac{1}{2} \text{erf}\left(\frac{x}{\sqrt{2}}\right)$$

$$F(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$$

Mechanik

$$\begin{array}{l|l} \mathbf{v} = \mathbf{x}'(t) & \omega = \varphi'(t) \\ \mathbf{a} = \mathbf{v}'(t) & \alpha = \omega'(t) \\ \mathbf{F} = \mathbf{p}'(t) & \mathbf{M} = \mathbf{L}'(t) \\ \mathbf{p} = m\mathbf{v} & L = J\omega \\ \mathbf{F} = m\mathbf{a} & M = J\alpha \\ P = \langle \mathbf{F}, \mathbf{v} \rangle & P = \langle \mathbf{M}, \boldsymbol{\omega} \rangle \\ E_{\text{kin}} = \frac{1}{2}m|\mathbf{v}|^2 & E_{\text{rot}} = \frac{1}{2}J\omega^2 \\ s = \varphi r & \mathbf{M} = \mathbf{r} \times \mathbf{F} & E_{\text{pot}} = mgh \\ v = \omega r & \mathbf{L} = \mathbf{r} \times \mathbf{p} & E_{\text{kin}} + E_{\text{pot}} = \text{const.} \\ a = \alpha r & \mathbf{v} = \boldsymbol{\omega} \times \mathbf{r} & F = Ds \quad (\text{Feder}) \end{array}$$

Gleichstrom

$$\begin{array}{l|l|l} U = RI & Q = It & GR = 1 \\ I = GU & W = Pt & \\ P = UI & W = QU & \end{array}$$

Wechselstrom

$$\begin{array}{l|l|l} \underline{U} = \underline{ZI} & \underline{Z} = R + jX & Z^2 = R^2 + X^2 \\ \underline{I} = \underline{YU} & \underline{Y} = G + jQ & R = Z \cos \varphi \\ \underline{S} = \underline{UI} & \underline{S} = P + jB & X = Z \sin \varphi \\ \underline{Z} = R & \text{Widerstand} & \omega = 2\pi f \\ \underline{Z} = jX_C & \text{Kondensator} & X_C = -1/(\omega C) \\ \underline{Z} = jX_L & \text{Spule} & X_L = \omega L \end{array}$$

$$\begin{array}{l|l} u_s = \sqrt{2} U_{\text{eff}} & u = u_s \sin(\omega t + \varphi_0) \\ i_s = \sqrt{2} I_{\text{eff}} & i = i_s \sin(\omega t + \varphi_0) \end{array}$$

Allgemeine Gleichungen

$$\begin{array}{l|l} u = Ri & p = ui \\ i = Cu'(t) & \\ u = Li'(t) & \end{array}$$

Elektrostatisches Feld

$$\begin{array}{l|l} F = \frac{1}{4\pi\epsilon} \frac{Q_1 Q_2}{r^2} & \mathbf{F}_1 = \frac{1}{4\pi\epsilon} Q_1 Q_2 \frac{\mathbf{x}_1 - \mathbf{x}_2}{|\mathbf{x}_1 - \mathbf{x}_2|^3} \\ \mathbf{F} = q\mathbf{E} & Q = CU & U = \varphi(B) - \varphi(A) \\ \mathbf{D} = \epsilon\mathbf{E} & \epsilon = \epsilon_0 \epsilon_r & W = QU \\ \mathbf{E} = -\nabla\varphi & \\ \epsilon_0 E^2 = 2w_e & \end{array}$$

Plattenkondensator

$$U = Ed \mid C = \epsilon A/d$$

Homogenes Feld in der Spule

$$Hl = NI \mid Bl = \mu NI \mid \Theta = NI$$

Magnetostatisches Feld

$$\begin{array}{l|l} \mathbf{F} = q\mathbf{v} \times \mathbf{B} & \Phi = BA \\ F = qvB & \mathbf{B} = \mu\mathbf{H} \\ F = BI l & \mu = \mu_0 \mu_r \\ H = I/(2\pi r) & (\text{Feld um einen geraden Leiter}) \\ B^2 = 2\mu_0 w_m & \end{array}$$

Elektrodynamik

$$\begin{array}{l} \mathbf{E} = -\nabla\varphi \\ \epsilon\Delta\varphi = -\rho(x) \\ \epsilon_0 E^2 = 2w_e \\ B^2 = 2\mu_0 w_m \end{array}$$

Maxwell-Gleichungen

$$\begin{array}{l|l} \langle \nabla, \mathbf{D} \rangle = \rho_f(x) & \langle \nabla, \epsilon_0 \mathbf{E} \rangle = \rho(x) \\ \langle \nabla, \mathbf{B} \rangle = 0 & \langle \nabla, \mathbf{B} \rangle = 0 \\ \nabla \times \mathbf{E} = -\partial_t \mathbf{B} & \nabla \times \mathbf{E} = -\partial_t \mathbf{B} \\ \nabla \times \mathbf{H} = \mathbf{J}_f + \partial_t \mathbf{D} & \nabla \times \mathbf{B} = \mu_0 (\mathbf{J} + \epsilon_0 \partial_t \mathbf{E}) \end{array}$$

Spezielle Relativitätstheorie

$$\begin{array}{l} \gamma = \frac{1}{\sqrt{1-\beta^2}}, \quad \beta = v/c \\ \gamma = \cosh \varphi, \quad \beta\gamma = \sinh \varphi, \quad \beta = \tanh \varphi \\ ct' = \gamma(ct - \beta x), \quad x' = \gamma(x - \beta ct), \quad (y, z)' = (y, z) \\ t = \gamma\tau & E_{\text{kin}} = E - E_0 \\ p = \gamma mv & E_{\text{kin}} = \gamma mc^2 - mc^2 \\ E = \gamma mc^2 & E^2 = (pc)^2 + (mc^2)^2 \\ \Lambda_v = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ g = \text{diag}(1, -1, -1, -1) \\ (\partial_\mu) = (\partial_{ct}, \partial_x, \partial_y, \partial_z) \\ (\partial^\mu) = (\partial_{ct}, -\partial_x, -\partial_y, -\partial_z) \end{array}$$

Optik

$$\begin{array}{l} \frac{1}{f} = \frac{1}{g} + \frac{1}{b}, \quad A = \frac{B}{G} = \frac{b}{g} \\ n_1 \sin(\varphi_1) = n_2 \sin(\varphi_2) \\ c_0 = nc \end{array}$$

Thermodynamik

$$\begin{array}{l|l|l} R = N_A k_B & m = nM & V = nV_m \\ R = R_s M & m = Nm_T & N = nN_A \\ pV = nRT & \\ \frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} & \\ Q = mc\Delta T & \end{array}$$

Konstanten

$$\begin{array}{l} \epsilon_0 = 8.8542 \times 10^{-12} \text{ C/(V m)} \\ \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \\ c_0 = 2.9979 \times 10^8 \text{ m/s} \\ e = 1.6022 \times 10^{-19} \text{ C} \\ G = 6.674 \times 10^{-11} \text{ m}^3/(\text{kg s}^2) \\ N_A = 6.0221 \times 10^{23} \text{ mol}^{-1} \\ k_B = 1.3806 \times 10^{-23} \text{ J/K} \\ R = 8.3145 \text{ J/(mol K)} \\ 0 \text{ K} = -273.15^\circ \text{C} \\ u = 1.6605 \times 10^{-27} \text{ kg} \\ h = 6.6261 \times 10^{-34} \text{ Js} \\ \hbar = 1.0546 \times 10^{-34} \text{ Js} \\ \sigma = 5.6704 \times 10^{-8} \text{ W/(m}^2 \text{K}^4) \end{array}$$

$$\begin{array}{l} m_e = 9.1094 \times 10^{-31} \text{ kg} \\ m_p = 1.6726 \times 10^{-27} \text{ kg} \\ m_n = 1.6749 \times 10^{-27} \text{ kg} \\ m_\alpha = 6.6447 \times 10^{-27} \text{ kg} \end{array}$$