INTRODUCTION TO HOMOLOGICAL ALGEBRA

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0. MATH 697 Homework Zero.Two

AM 2.1: Show that $(\mathbb{Z}/m\mathbb{Z}) \otimes (\mathbb{Z}/n\mathbb{Z}) = 0$ if m and n are coprime.

Proof. Observe that since m and n are relatively prime, there exists $s, t \in \mathbb{Z}$ such that $ms + nt = 1$. Now let $a \otimes b \in (\mathbb{Z}/m\mathbb{Z}) \otimes (\mathbb{Z}/n\mathbb{Z})$. Observe that

$$\begin{aligned}
a \otimes b &= a \cdot 1 \otimes b \cdot 1 \\
&= a(ms + nt) \otimes b(ms + nt) \\
&= (ams + ant) \otimes (bms + bnt) \\
&= ams \otimes (bms + bnt) + ant \otimes (bms + bnt) \\
&= ams \otimes bms + ams \otimes bnt + ant \otimes bms + ant \otimes bnt \\
&= 0 \otimes bms + ams \otimes 0 + ant \otimes bms + ant \otimes 0 \\
&= ant \otimes bms \\
&= atn \otimes bms \\
&= at \otimes nbms \\
&= at \otimes 0 \\
&= 0
\end{aligned}$$

Hence we have shown that any simple tensor is zero, so any finite linear combination of simple tensors is zero.

Conclude that $(\mathbb{Z}/m\mathbb{Z}) \otimes (\mathbb{Z}/n\mathbb{Z}) = 0$ holds. □

AM 2.2: Let A be a ring, a an ideal, M an A-module. Show that $(A/a) \otimes_A M$ is isomorphic to M/aM. [Tensor the exact sequence $0 \longrightarrow a \longrightarrow A \longrightarrow A/a \longrightarrow 0$ with M.]

Proof. □

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