ECMA 31100: Intro to Empirical Analysis II
RDD

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Introduction

- In week 1 we discussed estimation of ATE,ATT and ATU under unconfoundedness:

$$y(0), y(1) \perp D|X.$$

- Saw several representations of the ATE which led to different estimators with different properties.
- For example, also assuming $\mathrm{E}(y(d)|X) = \alpha_d + X'\beta_d$ allowed us to write

$$\mathrm{E}(Y|D,X) = \alpha_0 + \beta_0 X + (\alpha_1 - \alpha_0) D + D \cdot X'(\beta_1 - \beta_0),$$

which gave

$$ATE = (\alpha_1 - \alpha_0) + \mathrm{E}(X)'(\beta_1 - \beta_0).$$

Introduction

- In general identified ATE as

$$ATE = \mathrm{E}(y_1 - y_0) = \mathrm{E}\left[\mathrm{E}(Y|D=1,x) - \mathrm{E}(Y|D=0,x)\right].$$

- Implicitly assumed overlap condition:
  $0 < \mathrm{P}(D=1|x=x') < 1$ for all x'.
- If for some x', $\mathrm{P}(D=1|x') = 0$, then there are no observations of Y for which $D=1$ and $x=x'$, so $\mathrm{E}(Y|D=1, x=x')$ is undefined.
- Constructed non-parametric estimates of the ATE assuming overlap.

Introduction

- Now consider a case where X is scalar and $D = \mathbf{1}(X \geq c)$.
- Treatment is a deterministic function of the running variable X.
- c is called the cutoff/threshold.
- A regression discontinuity design identifies the effect of the treatment at the cutoff c, and relies on an assumption that the untreated potential outcome mean is continuous at c.

Sharp RDD

- If $D = \mathbf{1}(X \geq c)$, design is called <u>sharp</u> because X determines D.
- Unconfoundedness holds when conditioning on X because of this:

  $$y_0, y_1 \perp D|X$$

  because D is deterministic after conditioning on X.
- Overlap fails because we never see y_0 and y_1 for the same covariate value x:

  $$\mathrm{P}(D = 1|X = x) = \begin{cases} 1 & x \geq c, \\ 0 & x < c. \end{cases}$$

Sharp RDD

- Assuming linear conditional means allowed us to circumvent the overlap condition in identifying the ATE, provided we could estimate the parameters α_d, β_d.
- This amounts to extrapolation: Estimate α_0, β_0 with i: $X_i \le c$ and α_1, β_1 with $i : X_i > c$.
- More flexible parametric model could be used.. but doesn't avoid issue of extrapolation.
- With this assumption the ATE is identified because

$$ATE = (\alpha_1 - \alpha_0) + \mathrm{E}(X)' (\beta_1 - \beta_0).$$

Example: Regression Kink + Discontinuity

- Suppose y is credit limit and x is credit score.
- Suppose only two categories of credit: 'Good' and 'Excellent'.
- Features:
  - Sharp discontinuity in credit limit as soon as individual crosses 'Excellent' boundary (Discontinuity)
  - Each additional unit of credit score improves credit limit more for individuals with excellent credit (Kink)
- Let

$$d_i = \begin{cases} 1 & \text{if individual } x_i \geq 0.5 \\ 0 & \text{if individual } x_i < 0.5 \end{cases}$$

Example: Regression Kink + Discontinuity

- Model:

$$
\begin{aligned}
Y_i &= \alpha_0 + \beta_0 X_i + D_i\left(\gamma + \delta X_i\right) + \epsilon_i \\
&= \alpha_0 + \beta_0 X_i + \gamma D_i + \delta D_i X_i + \epsilon_i .
\end{aligned}
$$

- When $x < c : \mathrm{E}\left(Y|X = x\right) = \alpha_0 + \beta_0 x$.
- When $x_i \geq c : \mathrm{E}\left(Y|X = x\right) = \left(\alpha_0 + \gamma\right) + \left(\beta_1 + \delta\right) x$.
- Here $\gamma = \alpha_1 - \alpha_0$, $\delta = \beta_1 - \beta_0$.
- δ represents the additional return to credit score for individuals with excellent credit.
- Discontinuity:

$$
\mathrm{E}\left(Y|X = c\right) - \lim_{x \uparrow c} \mathrm{E}\left(Y|X = c\right) = \gamma + \delta \cdot 0.5.
$$

Example: Regression Kink + Discontinuity

- Can recenter X to get δ to represent discontinuity:

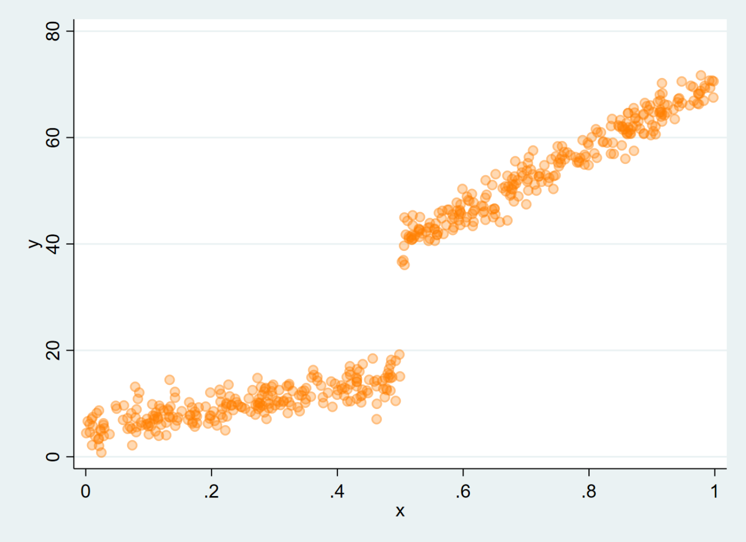
$$Y_i = \alpha_0 + \beta_0 (X_i - c) + \gamma D_i + \delta D_i (X_i - c) + \epsilon_i.$$

- Now

$$\mathrm{E}(Y|X = c) - \lim_{x \uparrow c} \mathrm{E}(Y|X = c) = \gamma.$$

Example

- Credit card limits offered to new customers as a function of credit score:



Sharp RDD

- We can identify the treatment effect at the cutoff with a much weaker assumption:

$$\mathrm{E}\left(y_0|X=x\right) \text{ is continuous at } c.$$

- Then

$$\begin{aligned}\mathrm{E}\left(y_1-y_0|X=c\right) &= \mathrm{E}\left(y_1|X=c\right)-\lim_{x\uparrow c}\mathrm{E}\left(y_0|X=c\right)\\ &= \mathrm{E}\left(Y|X=c\right)-\lim_{x\uparrow c}\mathrm{E}\left(Y|X=c\right).\end{aligned}$$

Sharp RDD

- Since we don't want to assume $\mathrm{E}\left(Y|X=c\right)$ is continuous at c, we estimate this mean in a neighborhood of c.
- Common approach: Choose a bandwidth h and solve

$$\min_{\alpha_0,\beta_0,\gamma,\delta} \sum_{i=1}^{n} \mathbf{1}\left(|X_i - c| \leq h\right)\left(y_i - \alpha_0 - \beta_0 x_i - \gamma d_i - \delta d_i x_i\right)^2.$$

- Called Local Linear Regression. Gives an approximation to conditional mean in a neigborhood of cutoff.

Sharp RDD

- Equivalent to two separate linear regressions on subsamples $\{i : X_i \in [c, c+h]\}, \{i : X_i \in [c-h, c)\}$.
- Split up the OLS minimization problem into two sums:

$$\min_{\alpha_0,\beta_0,\gamma,\delta} \sum_{i=1}^{n} \mathbf{1}\left(|X_i - c| \leq h\right)\left(y_i - \alpha_0 - \beta_0 x_i - \gamma d_i - \delta d_i x_i\right)^2$$

$$\equiv \min_{\alpha_0,\beta_0,\gamma,\delta} \left[\sum_{i:D_i=0}^{n} \mathbf{1}\left(|X_i - c| \leq h\right)\left(y_i - \alpha_0 - \beta_0 x_i\right)^2 \right.$$

$$\left. + \sum_{i:D_i=1}^{n} \mathbf{1}\left(|X_i - c| \leq h\right)\left(y_i - (\alpha_0 + \gamma) - (\beta_0 + \delta)\, x_i\right)^2 \right]$$

- Can minimize each sum independently since fixing α_0, β_0 leaves γ, δ free.

Bandwidth Choice

- Without linearity of conditional means, h should shrink as $n \to \infty$, otherwise local linear estimator will not necessarily reflect behaviour of $\mathrm{E}\left(Y|X=x\right)$ near $x=c$.
- 'Optimal' bandwidth chooses h to minimize an approximation to
$$MSE\left(h\right)=\mathrm{E}\left(\hat{\delta}\left(h\right)-\delta\right)^2.$$
Turns out that $h=C\cdot n^{-1/5}$ works best.
- This leads to an asymptotic bias term in the asymptotic distribution.

Bandwidth Choice

- Intuitively, larger h leads to higher bias (using observations far from cutoff) but lower variance (larger sample).
- Imbens and Kalyanaraman (2012), Calonico, Cattaneo and Titiunik (2014) propose two different estimates of C.
- CCT account for asymptotic bias term by estimating this bias and incorporating the estimation error in the variance of the resulting estimator of $\mathrm{E}(Y|X = c)$.
- Alternative <u>undersmoothing</u> approach sets $n^{1/5}h \to 0$ which eliminates asymptotic bias.

Failure of Identification

- Manipulation at the cutoff: If some individuals can choose treatment status by ensuring their X value is just above/below cutoff, then individuals to the right and left may not be the same, which might suggest a violation of continuity.
- Several treatments: If there are multiple treatments at the boundary, cannot identify which of them caused the discontinuity unless we essentially assume the second has no effect on average.
- McCrary (2008) proposes a test of a discontinuity in the density of X at c - may suggest manipulation at cutoff. See also Bugni and Canay (2021).

Covariates

- Confoundedness holds with or without additional covariates W, since D is a function of X.
- Discontinuities in the distribution of pre-determined covariates at the cutoff may also be suggestive of a violation of the continuity assumption, since the change in covariate value might be causing the change in outcome rather than the treatment.
- RDD with outcome replaced by W should show no effect if W responds continuously to X. Null is that $\mathrm{E}(W|X = x)$ is continuous at c.
- Since intuition is about entire distribution of X, Canay and Kamat (2018) propose a more powerful test based on the entire distribution of $W|X$.

Fuzzy Design

- Previously discussed case $\mathrm{P}\left(D=1|X=x\right)=\mathbf{1}\left(x\geq c\right)$.
- Now assume only that $\mathrm{P}\left(D=1|X=x\right)$ is discontinuous at c.
- Now $Z=\mathbf{1}\left(X\geq c\right)$ is an instrument for D.
- Exogeneity satisfied: $y_0, y_1\perp Z|X$ because Z is a function of X.
- Relevance conditional on $X=x$ requires $Cov\left(D,Z|X=c\right)\neq 0$. True since

$$\lim_{x\downarrow c}\mathrm{P}\left(D=1|Z=1,X=x\right)\neq \mathrm{P}\left(D=1|Z=0,X=x\right).$$

Fuzzy Design

- Now assume no defiers: $\mathrm{P}[D_1 \geq D_0] = 1$. Discontinuity at c is therefore a positive jump.
- Assume $\mathrm{E}(y_1 - y_0 | X = x, \text{complier})$ is continuous at $x = c$. Same for always and never-takers.
- Assume $\mathrm{P}(\text{complier} | X = x)$ continuous at $X = c$. Same for always and never-takers.
- Linear IV regression in a neighborhood of c will essentially produce a LATE:

$$
\begin{aligned}
& \mathrm{E}(y_1 - y_0 | X = c, \text{complier}) \\
= & \frac{\lim_{x \downarrow c} \mathrm{E}(Y | X = x) - \lim_{x \uparrow c} \mathrm{E}(Y | X = x)}{\lim_{x \downarrow c} \mathrm{E}(D | X = x) - \lim_{x \uparrow c} \mathrm{E}(D | X = x)}.
\end{aligned}
$$

Implementation

- Can estimate these conditional means with local linear estimators either side of cutoff.
- Turns out to be equivalent to doing 2SLS with:
- First Stage:

$$D = \pi_0 + \pi_1 Z + \pi_2 (X - c) + \pi_3 Z (X - c) + v$$

- Second Stage:

$$Y = \beta_0 + \beta_1 D + \beta_2 (X - c) + \beta_3 Z (X - c) + u$$

on the subsample $\{i : |X_i - c| \leq h\}$.