Price Theory I: Problem Set 7 Question 1

Grant Vaska

University of Chicago, Department of Economics

November 2021

Problem Set 7 Question 1

"Pfizer has invented an antiviral treatment for COVID-19, known as Paxlovid. When administered within three days of symptom onset, the survival rate in patients has been 100 percent (although some were hospitalized). Prior to Paxlovid, the majority of the population was coping with the disease by engaging in prevention behaviors such as vaccinations or other behaviors including mask wearing, quarantining health persons due to their contact infected patients, and staying at home."

Model

- There is a unit mass of individuals who are expected cost minimizers, where costs come from death and prevention/treatment.
  - Cost of death $= c_d$, cost of vaccine $= p_v$, cost of other preventions $= p_o$, cost of treatment (Paxlovid) $= p_t$.
  - Supply of prevention/treatment is perfectly elastic and prices are set competitively (and may include non-pecuniary costs to individuals).
- Individuals vary by a parameter $\theta_i \in [0, 1]$, distributed in the population as $G(\theta_i)$, which scales an individual's probability of death, conditional on being infected.
  - Otherwise, individuals are identical.
  - Where necessary, assume $g(\theta_i)$ is continuous and differentiable and the distribution has full support over $[0, 1]$.
- Paxlovid eliminates the risk of death (unrealistically, it does not affect hospitalizations), vaccines and other preventions reduce the individual risk of infection by fractions γ_v and $\gamma_o \in [0, 1]$, respectively.

Model

- Assume WLOG that vaccines are cheaper for reducing infections than other preventions ($\frac{p_v}{\gamma_v} < \frac{p_o}{\gamma_o}$).
  - In practice, few unvaccinated people voluntarily take other precautions.
  - These parameters are the same for everyone. A more complex model could include varying preferences (for example, maybe some people are deathly afraid of needles).
- Additionally assume that all prevention methods would be active for at least some values of s_h. (This rules out some uninteresting corner solutions.)
- Denote an individual's binary decision to undertake vaccination or other preventions d_{vi} and d_{oi}, respectively. Additionally, denote the share of people undertaking only vaccination as s_v and both vaccination and other preventions as s_o (abusing notation a little).

Model

- The base infection rate (share of people infected with no prevention) is $\eta \in (0, 1)$. The actual share of people infected, s_I, is impacted by the shares of people who undertake preventions.
- For simplicity, assume 100% of people infected are hospitalized, so $s_h = s_I$.
- An individual with parameter $\theta_i = 1$ has a death probability of $\pi_d(s_h) \in [0, 1]$, which is strictly increasing in s_h (higher share of hospitalizations increases the death rate). For simplicity, let $\pi_d(s_h) = s_h$.

Model

- The model is one period with three stages:
  - Stage 1:
    - ★ Individuals decide which preventative measures to take - none, vaccine only, or vaccine and other preventions. (An individual would never just take other preventions since a vaccine is cheaper.)
    - ★ Taking the (future) share hospitalized as given, consumers choose the prevention regime which minimizes their expected costs.
    - ★ Problem: $\min\{\mathbb{E}[c_{hi}(s_h)]\eta, \mathbb{E}[c_{hi}(s_h)](1-\gamma_v)\eta + p_v, \mathbb{E}[c_{hi}(s_h)](1-\gamma_v)(1-\gamma_o)\eta + p_v + p_o\}$
    - ★ $\mathbb{E}[c_{hi}(s_h)]$ is the expected cost of hospitalization given s_h for individual i.

Model

- (cont.)
  - Stage 2:
    - ★ Individuals are infected randomly at their personal infection rate $\pi_{Ii} = (1 - \gamma_v d_{vi})(1 - \gamma_o d_{oi})\eta$.
    - ★ The overall share infected (and hospitalized) is $s_h = s_I = [(1 - \gamma_v)s_v + (1 - \gamma_v)(1 - \gamma_o)s_o + (1 - s_v - s_o)]\eta$.
    - ★ If an individual is infected/hospitalized, they proceed to the next stage. If not, nothing happens to them in the third stage (they incur no further costs).

Model

- (cont.)
  - Stage 3:
    - ★ Hospitalized individuals decide whether or not to take treatment (Paxlovid).
    - ★ Problem: $\mathbb{E}[c_{hi}(s_h)] = \min\{\pi_d(s_h)\theta_i c_d, p_t\} = \min\{s_h\theta_i c_d, p_t\}$
    - ★ If treatment is unavailable, set $p_t = \infty$.

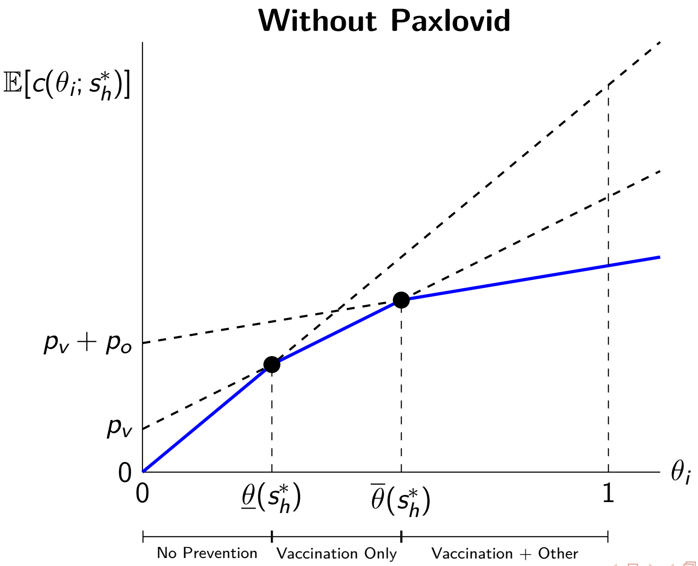
Model

- First, consider the model without Paxlovid.
- Each individual chooses no prevention, vaccination only, or vaccination with other preventions.
- Individuals with low θ_i choose no prevention, those with high θ_i choose both preventions, those with medium θ_i choose vaccination only.
  - This follows from the assumption all prevention regimes are active for some s_h.

Model

- In equilibrium, individual choices are consistent with the share hospitalized.
  - Note that the benefit of preventions increases in s_h since it increases the risk of death.
  - An equilibrium always exists.
    - If $s_h = \eta$, the maximum (no one undertakes any preventions), the minimum cost curve would imply $s_h < \eta$ (some fraction of individuals undertake prevention).
    - If $s_h = (1-\gamma_v)(1-\gamma_o)\eta$, the minimum (everyone undertakes both vaccination and other preventions), the minimum cost curve would imply $s_h > (1-\gamma_v)(1-\gamma_o)\eta$ since some low θ_i never take any preventions.
    - There must be some s_h^* in between those extremes for which the minimum cost curve and the prevention behaviors implied by it are consistent.

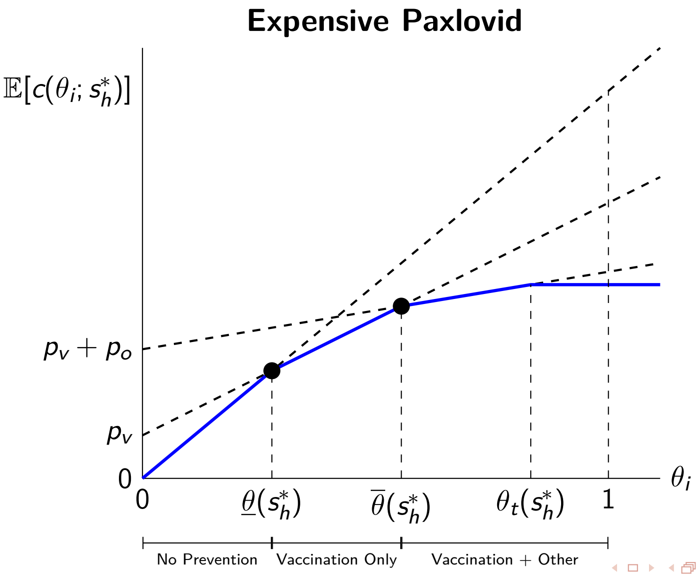
Model


Without Paxlovid
$\mathbb{E}[c(\theta_i; s_h^*)]$
$p_v + p_o$
p_v
0
0
$\underline{\theta}(s_h^*)$
$\overline{\theta}(s_h^*)$
1
θ_i
No Prevention
Vaccination Only
Vaccination + Other

Model

- Now, consider the model with Paxlovid, priced low enough so some buy it.
- This essentially puts a cap on the expected cost of hospitalization. High θ_i types above threshold $\theta_t(p_t, s_h) = \frac{p_t}{s_h c_d}$ will take Paxlovid if they are hospitalized.
- Let s_h^* be the equilibrium hospitalized share in the absence of Paxlovid.
  - If $\theta_t(p_t, s_h^*) \geq \overline{\theta}(s_h^*)$, then high enough types have lower costs since they can take Paxlovid instead of risking death, but no one's prevention behavior changes ($s_h^{*\prime} = s_h^*$).
  - If $\underline{\theta}(s_h^*) \leq \theta_t(p_t, s_h^*) < \overline{\theta}(s_h^*)$, then no one finds it worthwhile to take other prevention actions beyond vaccination ($s_h^{*\prime} > s_h^*$).
  - If $\theta_t(p_t, s_h^*) < \underline{\theta}(s_h^*)$, then no one finds it worthwhile to take any prevention actions at all ($s_h^{*\prime} = \eta > s_h^*$).

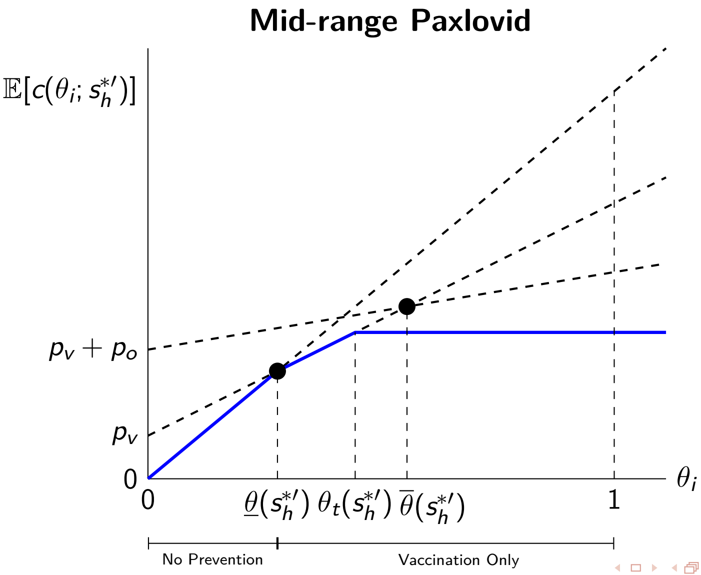
Model


Expensive Paxlovid
$\mathbb{E}[c(\theta_i; s_h^*)]$
$p_v + p_o$
p_v
0
θ_i
0
$\underline{\theta}(s_h^*)$
$\overline{\theta}(s_h^*)$
$\theta_t(s_h^*)$
1
No Prevention
Vaccination Only
Vaccination + Other

Model

- Suppose we're in the $\underline{\theta}(s_h^*) \leq \theta_t(p_t, s_h^*) < \overline{\theta}(s_h^*)$ case. How does $s_h^{*\prime}$ (and prevention behaviors) change?
  - Note that the slope of the cost curves for each prevention regime is scaled by s_h and so their intersections shift left (θ thresholds decrease) as s_h increases.
  - Additionally, $\theta_t(p_t, s_h)$ decreases as s_h increases.
  - It can be shown $\underline{\theta}(s_h) \leq \theta_t(p_t, s_h) < \overline{\theta}(s_h)$ for any s_h, including the new equilibrium. Appendix
  - It cannot be that $s_h^{*\prime} < s_h^*$, because the cost curve would imply reduced prevention behaviors, which contradict a reduction in s_h.
  - The share hospitalized will rise to some level $s_h^{*\prime} > s_h^*$ where the new minimum cost curve and implied prevention behavior are again consistent.
    - This new equilibrium exists due to an argument similar to the no Paxlovid case.

Model



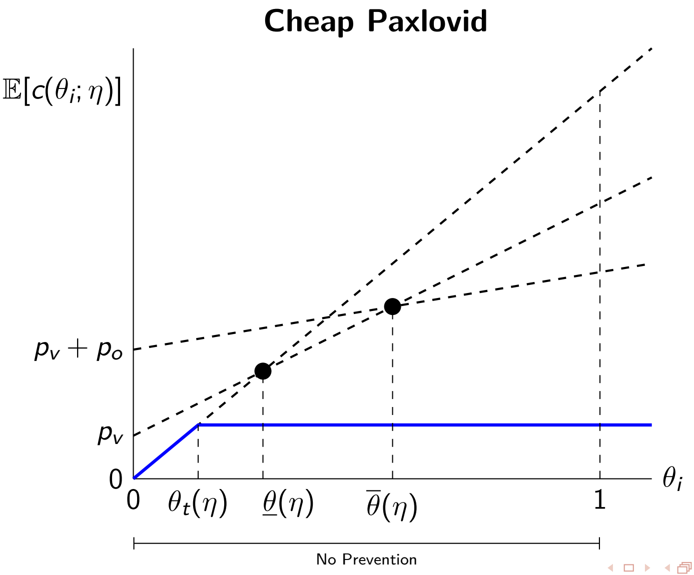
Mid-range Paxlovid
$\mathbb{E}[c(\theta_i; s_h^{*\prime})]$
$p_v + p_o$
p_v
0
θ_i
0
$\underline{\theta}(s_h^{*\prime})$ $\theta_t(s_h^{*\prime})$ $\overline{\theta}(s_h^{*\prime})$
1
No Prevention
Vaccination Only

Model

- The $\theta_t(p_t, s_h^*) < \underline{\theta}(s_h^*)$ case is similar to the previous case.

- Instead of eliminating only other prevention behavior and increasing $s_h^{*\prime}$ somewhat, all prevention behavior, including vaccines, is eliminated and $s_h^{*\prime}$ increases to the maximum, η.

Model



Cheap Paxlovid
$\mathbb{E}[c(\theta_i;\eta)]$
$p_v + p_o$
p_v
0
0
$\theta_t(\eta)$
$\underline{\theta}(\eta)$
$\overline{\theta}(\eta)$
1
θ_i
No Prevention

Model

- Which case is likely to prevail in practice (if we take this model seriously)?
- In the U.S., a Paxlovid treatment will cost something like \$700. Not very expensive compared to the risk of dying with Covid, even for young people.
- Suggests a low price case is more likely.

Part (a) - Effect on Deaths

"Is it possible that making Paxlovid available to the general public would increase deaths from COVID-19?"

Part (a) - Effect on Deaths

- We've seen that the introduction of Paxlovid increases the share of people who are hospitalized.
- Although Paxlovid itself eliminates deaths among those who take it, among those who don't deaths will increase due to higher hospitalizations.
- Whether deaths actually increase in aggregate depends on parameters.

Part (a) - Effect on Deaths

- A simple discrete distribution example:
  - Suppose there are only two types, $\theta_L = 0.1$ and $\theta_H = 1.0$, present in equal proportion.
  - Further, the vaccine is 100% effective ($\gamma_v = 1.0$) and costs $p_v = 40$, while other preventions have no effect ($\gamma_o = 0.0$).
  - Finally, the base infection rate is 100% ($\eta = 1$) and the cost of death is $c_d = 100$.
  - Without Paxlovid, the high type takes the vaccine since a high type individual's cost would be $0.5 * 1.0 * 100 = 50 > 40 = p_v$ without it. The low type doesn't take the vaccine since their expected cost is only $0.5 * 0.1 * 100 = 5 < 30 = p_v$. (Half are hospitalized.)
    - ★ Total death rate: $0.05 * 0.5 = 0.025$
  - Suppose Paxlovid is introduced at $p_t = 30$.
  - With Paxlovid, the high type takes Paxlovid instead of the vaccine. The low type doesn't take Paxlovid because it isn't worth it, even with a doubled death rate ($1.0 * 0.1 * 100 = 10 < 30 = p_t < 40 = p_v$). (All are hospitalized.)
    - ★ Total death rate: $0.1 * 0.5 = 0.05 > 0.025$

Part (a) - Effect on Deaths

- (cont.)
  - Note that the introduction of Paxlovid in this example is still beneficial on average even though deaths increase.
    - ★ Total costs are $0.5 * 40 + 0.5 * 5 = 22.5$ without Paxlovid and $0.5 * 30 + 0.5 * 10 = 20$ with Paxlovid.
    - ★ This goes back to Casey's excess burden lecture–the benefit of Paxlovid is greater than the change in deaths (which in this case is negative).
    - ★ It could be that introducing Paxlovid increases cost (consider $p_t = 39$), but still the total "benefit" (cost) will be greater (less negative) than when just considering the change in deaths.

Part (a) - Effect on Deaths

- Essentially, in this example Paxlovid is a perfect substitute for the vaccine at the individual level and cheaper too. However, individuals do not internalize their effect on the overall death rate, so the previous half-vaccination equilibrium unravels and low types die at a higher rate.
- This would occur in more general cases whenever the indirect effect of the increase in hospitalizations on deaths outweighs the direct effect of Paxlovid preventing deaths.
- **Answer:** Paxlovid could increase deaths from COVID-19.

Part (b) - Effect on Prevention

"What will Paxlovid do to prevention behaviors? Will the vaccine be obsolete? As part of your answers, contrast the behavior of young people, who can expect little health cost from the disease even without prevention or treatment, and old people."

Part (b) - Effect on Prevention

- Suppose θ_i represents, or is at least positively correlated with, age.
- Before Paxlovid is introduced, the young undertake no prevention behaviors, the middle-aged get vaccinated but do not undertake other prevention behaviors, and the old both get vaccinated and undertake other prevention behaviors.
- How the introduction of Paxlovid affect prevention behaviors depends on its price (suppose it is low enough that some choose to take it).
  - If Paxlovid is expensive, old individuals benefit from capped expected costs, but it doesn't change their behavior. It doesn't change the behavior of the middle aged or young either.
  - If the price of Paxlovid is mid-range, old individuals no longer undertake other prevention behaviors. Additionally, on the margin between young and middle-aged individuals more get vaccinated since the share hospitalized increases.
  - If Paxlovid is cheap, no one gets vaccinated or undertakes other prevention behaviors. (As a result, the share hospitalized increases to the maximum possible amount.)

Part (b) - Effect on Prevention

- **Answer:** The effect of Paxlovid depends on its price. It may have no effect, may eliminate other prevention behaviors but increase vaccination, or may eliminate all prevention behaviors entirely.
  - The vaccine only becomes obsolete if the price of Paxlovid is low.
  - Older people always undertake (weakly) more prevention behaviors, but Paxlovid may change the thresholds for those behaviors.
  - Since the share hospitalized (weakly) rises, we know efficacy-weighted average prevention behaviors (weakly) fall.

Part (c) - Vaccinated vs Unvaccinated Behaviors

"To the extent that vaccines and Paxlovid coexist in the market, how will other prevention behaviors be different between the vaccinated and the unvaccinated?"

Part (c) - Vaccinated vs Unvaccinated Behaviors

- My model has one-dimensional heterogeneity, where “older” individuals have a higher willingness to pay for protection from death.
  - As a result, older people always undertake weakly more prevention behaviors. This could differ with other model set ups (for example, multiple dimensions of heterogeneity).
- Suppose Paxlovid and vaccines coexist. Then the price of Paxlovid is at least mid-range.
- The unvaccinated never take other prevention behaviors.
- If the price of Paxlovid is mid-range, no one among the vaccinated undertake other prevention behaviors.
- Paxlovid is expensive, some (but not all) among the vaccinated undertake other prevention behaviors.
- **Answer:** The unvaccinated never undertake other prevention behaviors. Some of the vaccinated will do so only if Paxlovid is expensive.

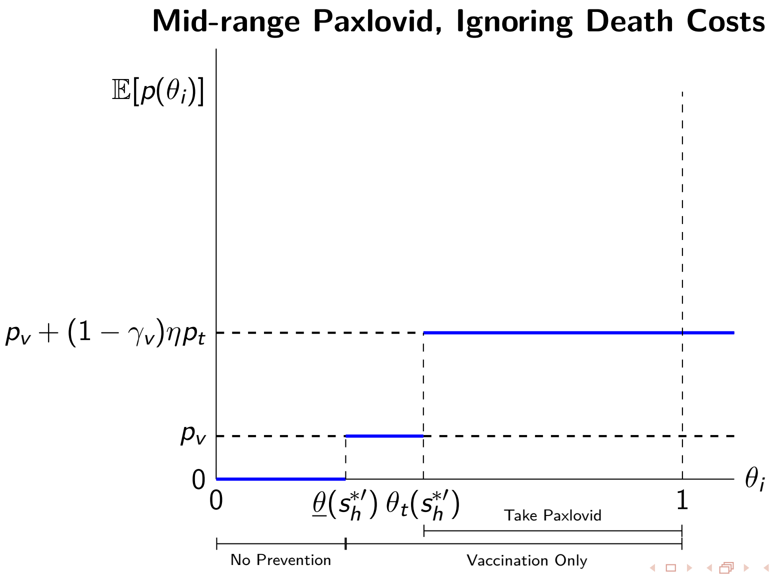
Part (d) - Disease Cost

"Will the expected cost of the disease be different for the unvaccinated and the vaccinated? What about the composition of disease costs?"

Part (d) - Disease Cost

- Suppose that at least some people get vaccinated.
- Clearly the expected cost differs for the unvaccinated and the vaccinated, whether considering just financial costs or including the "cost" of death.
- The vaccinated have higher costs under either definition–just look at the minimum cost graph (ignoring death costs is like removing costs which come from the slope).
- The vaccinated have higher costs because their higher chance of death causes them to undertake more (costly) prevention/treatment.
    - This doesn't push their cost below the unvaccinateds' since then the unvaccinated would expect benefits from getting vaccinated.
- The unvaccinated face zero financial costs, so their only costs are from death.

Model



Mid-range Paxlovid, Ignoring Death Costs
$\mathbb{E}[p(\theta_i)]$
$p_v + (1-\gamma_v)\eta p_t$
p_v
0
θ_i
0
$\underline{\theta}(s_h^{*\prime})$
$\theta_t(s_h^{*\prime})$
1
Take Paxlovid
No Prevention
Vaccination Only

Part (d) - Disease Cost

- **Answer:** Both the expected and composition of disease costs differ between the unvaccinated and vaccinated. The vaccinated have higher costs and undertake some costly interventions, while the unvaccinated do not.

Part (e) - Price Elasticity

"What factors determine the price elasticity of demand for Paxlovid?"

Part (e) - Price Elasticity

- We can think about this in terms of Marshall's law.
- Vaccination, other preventions, and treatment are all "inputs" which produce an "output" of reduced mortality risk.
- The price elasticity of demand is decomposed into a scale effect and a substitution effect.
- As we'll see, within each case there is no substitution effects between treatment and preventions.
- So only the scale effect is present, roughly speaking it depends on the mass of individuals at the threshold of taking treatment.

Part (e) - Price Elasticity

- Let π_{It} be the infection rate for the marginal Paxlovid taking individual. Then demand for Paxlovid takes the following form for all three Paxlovid price cases:

$$D(p_t) = \pi_{It}(1 - G(\theta_t(p_t, s_h^*(p_t))))$$

- Where the second term is the fraction of the population that will take Paxlovid if they get infected.

- Take the derivative of demand:

$$\frac{dD(p_t)}{dp_t} = -\pi_{It}\left[\frac{\partial g\theta_t(p_t, s_h^*(p_t))}{\partial p_t} + \frac{\partial s_h^*(p_t)}{\partial p_t}\frac{\partial g\theta_t(p_t, s_h^*(p_t))}{\partial s_h^*}\right] g(\theta_t(p_t, s_h^*(p_t)))$$

Part (e) - Price Elasticity

- Substitute in known forms:

$$\frac{dD(p_t)}{dp_t} = -\pi_{It}\left[\frac{1}{c_d s_h^*(p_t)} - \frac{\partial s_h^*(p_t)}{\partial p_t}\frac{p_t}{c_d (s_h^*)^2(p_t)}\right] g(\frac{p_t}{c_d s_h^*(p_t)})$$

- Form the elasticity:

$$\frac{dD(p_t)}{dp_t}\frac{p_t}{D(p_t)} = \frac{-\pi_{It}\left[\frac{1}{c_d s_h^*(p_t)} - \frac{\partial s_h^*(p_t)}{\partial p_t}\frac{p_t}{c_d (s_h^*)^2(p_t)}\right] g(\frac{p_t}{c_d s_h^*(p_t)})p_t}{\pi_{It}(1 - G(\frac{p_t}{c_d s_h^*(p_t)}))}$$

Part (e) - Price Elasticity

- Simplify:

$$\frac{dD(p_t)}{dp_t}\frac{p_t}{D(p_t)} = -\left[\frac{1}{c_d s_h^*(p_t)} - \frac{\partial s_h^*(p_t)}{\partial p_t}\frac{p_t}{c_d (s_h^*)^2(p_t)}\right]\frac{g(\frac{p_t}{c_d s_h^*(p_t)})}{1 - G(\frac{p_t}{c_d s_h^*(p_t)})} p_t$$

- Now, consider the cheap Paxlovid case. Then $s_h^*(p_t) = \eta$, which is unaffected by the price of Paxlovid. This simplifies the expression to:

$$\frac{dD(p_t)}{dp_t}\frac{p_t}{D(p_t)} = -\frac{g(\frac{p_t}{c_d \eta})}{1 - G(\frac{p_t}{c_d \eta})}\frac{p_t}{c_d \eta}$$

Part (e) - Price Elasticity

- The expensive Paxlovid case is similar, but now $s_h^*(p_t) = s_h^*(= s_h^{*\prime})$.
  - Still invariant to p_t, but implicitly determined in the no-Paxlovid equilibrium.
  - See the equation for share hospitalized and note that s_v^* and s_o^* are themselves functions of s_h^*.

$$\frac{dD(p_t)}{dp_t}\frac{p_t}{D(p_t)} = -\frac{g(\frac{p_t}{c_d s_h^*})}{1 - G(\frac{p_t}{c_d s_h^*})}\frac{p_t}{c_d s_h^*}$$

Part (e) - Price Elasticity

- Finally, the mid-range Paxlovid price case is similar to the expensive case, but $s_h^*(p_t) = s_h^{*\prime}$.
  - Again invariant to p_t, but now implicitly determined by the neither Paxlovid nor other prevention equilibrium.

- Though the introduction of Paxlovid at a mid-range price increases the share hospitalized, changing its price (while still in that range) has no further effect on what measures people take or the share hospitalized.

$$\frac{dD(p_t)}{dp_t}\frac{p_t}{D(p_t)} = -\frac{g(\frac{p_t}{c_d s_h^{*\prime}})}{1 - G(\frac{p_t}{c_d s_h^{*\prime}})}\frac{p_t}{c_d s_h^{*\prime}}$$

Part (e) - Price Elasticity

- In summary, the price elasticity of demand depends on:
  - Price of Paxlovid p_t.
    - Both to determine which case to consider and the elasticity within that case.
  - Cost of death c_d.
  - Equilibrium share hospitalized (in the applicable case) s_h^*.
  - Inverse Mills ratio of the type distribution $G(\cdot)$.
    - Itself evaluated at the treatment type threshold, a function of the above variables.
- Additionally, the equilibrium share hospitalized depends on:
  - Baseline infection rate η
  - Efficacy of preventions γ_v, γ_o.
  - Type distribution $G(\cdot)$.

Part (e) - Price Elasticity

- If we consider $\frac{p_t}{c_d s_h}$ (for the appropriate s_h) as a sort of normalized price, the elasticity only depends on that and the type distribution.
- **Answer:** Price elasticity of demand for Paxlovid depends on most of the parameters of the model.
  - We can't say much about how it depends on them without knowing the type distribution.

Part (f) - Effect on Eradication

"Does the invention of Paxlovid affect the likelihood that COVID-19 will be eradicated [specifically that the human population would go more than 14 days without any new cases]?"

Part (f) - Effect on Eradication

- Let's ignore explicit dynamics and acquired immunity and assume that the infection rate is negatively correlated with the likelihood that COVID-19 is eradicated.
- As we have seen, if Paxlovid is cheap enough its introduction increases the infection rate, so it reduces the likelihood that the disease is eradicated.
- Intuitively, Paxlovid reduces the cost of getting sick, so on average people take fewer measures to avoid getting sick.

Part (f) - Effect on Eradication

- This is related to part (a), people substitute into using Paxlovid, which only has a personal benefit, instead of more stringent prevention behaviors, which also have positive spillover effects.
  - In some sense, eradication is a form of prevention, it makes sense that people substitute away from it like aggregate preventions.
- **Answer:** Paxlovid reduces the likelihood that COVID-19 is eradicated.

Part (g) - Vaccination Externalities

"Consider an individual that chooses not to be vaccinated. Does this choice impose a negative externality on others by increasing the likelihood that the individual will spend time in a hospital seeking treatment for COVID-19?"

Part (g) - Vaccination Externalities

- Externalities are possible when something isn't priced.
- Hospitalization isn't priced, people just show up and can't be turned away.
- There's a cost to hospitalization, and that cost is influenced by peoples' vaccination decisions, but it isn't a price, the costs of death are just lost, no one gets revenue from it.
- Each individual takes the future share hospitalized (which has an impact on death rates) as given when they decide on their actions.
- However, aggregate actions determine the share hospitalized, and individuals do not consider their impact on aggregate actions.
- An unvaccinated person is more likely to end up in the hospital, marginally pushing up the share hospitalized and death rates.
- **Answer:** Yes, choosing to forgo vaccination imposes a negative externality on others.
    - The part (a) and (f) results are related.

(Appendix) Threshold Order

- Solving for the relevant thresholds results in:

$$\underline{\theta}(s_h^{*\prime}) = \frac{p_v}{\gamma_v c_d \eta s_h^{*\prime}}$$
$$\theta_t(s_h^{*\prime}) = \frac{p_t}{s_h^{*\prime} c_d}$$
$$\overline{\theta}(s_h^{*\prime}) = \frac{p_o}{\gamma_o(1-\gamma_v) c_d \eta s_h^{*\prime}}$$

- Assuming the initial ordering at s_h^*, show this implies the same ordering for all s_h:

$$\frac{p_v}{\gamma_v c_d \eta s_h} < \frac{p_t}{s_h c_d} < \frac{p_o}{\gamma_o(1-\gamma_v) c_d \eta s_h}$$
$$\frac{p_v}{\gamma_v \eta} < p_t < \frac{p_o}{\gamma_o(1-\gamma_v)\eta}$$

- So the ordering doesn't depend on the value of s_h.

Back