ECMA31100 Introduction to Empirical Analysis II

Winter 2022, Week 6: Discussion Session

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Today's theme: partial identification

Law of decreasing credibility (Manski, 2003)

The credibility of inference decreases with the strength of the assumptions maintained

Outline

- Review of missing data example (Prof. Hardwick talked about this in lecture 2)
- Monotone instrumental variables
- Empirical example on the effect of parental schooling on their children's schooling

Further readings

- Recent survey papers: Tamer (2010) and Molinari (2020)

Contents

Partial identification

Motivating example (Manski, 1989)

Set-up

- Consider a random sample of $\{(x_i, y_i, z_i)\}_{i=1}^{n}$, where
  - x_i is a vector of covariates
  - z_i is a binary variable
  - y_i is a variable that is in the range of $[\underline{y}, \bar{y}]$ and is observed if $z_i = 1$
- We are interested in learning about $\mathbb{E}[y_i|x_i]$

Identification?

- We do not observe y_i if $z_i = 0 \implies \mathbb{E}[y_i|x_i, z_i = 0]$ is not observed
- $\mathbb{E}[y_i|x_i]$ is not point identified without further assumptions

Partial identification

Motivating example (Manski, 1989)

Identification region

- We can rewrite $\mathbb{E}[y_i|x_i]$ as follows:

$$\mathbb{E}[y_i|x_i] = \mathbb{E}[y_i|x_i, z_i = 1]\mathbb{P}[z_i = 1|x_i] + \mathbb{E}[y_i|x_i, z_i = 0]\mathbb{P}[z_i = 0|x_i]$$

  - $\mathbb{E}[y_i|x_i, z_i = 1]$, $\mathbb{P}[z_i = 1|x_i]$, and $\mathbb{P}[z_i = 0|x_i]$ can be observed from data
  - We do not observe $\mathbb{E}[y_i|x_i, z_i = 0]$ but assumed $y_i \in [\underline{y}, \bar{y}] \implies \mathbb{E}[y_i|x_i, z_i = 0] \in [\underline{y}, \bar{y}]$

- Hence, the identification region of $\mathbb{E}[y_i|x_i]$ is

$$\left[\mathbb{E}[y_i|x_i, z_i = 1]\mathbb{P}[z_i = 1|x_i] + \underline{y}\mathbb{P}[z_i = 0|x_i], \mathbb{E}[y_i|x_i, z_i = 1]\mathbb{P}[z_i = 1|x_i] + \bar{y}\mathbb{P}[z_i = 0|x_i]\right]$$

Special case

- Consider the case where y_i is binary so that $y \in \{0, 1\}$
- The region becomes: $[\mathbb{E}[y_i|x_i, z_i = 1]\mathbb{P}[z_i = 1|x_i], \mathbb{E}[y_i|x_i, z_i = 1]\mathbb{P}[z_i = 1|x_i] + \mathbb{P}[z_i = 0|x_i]]$

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Monotone instrumental variables

Overview

Set-up

- This section mainly follows Manski (1997) and Manski and Pepper (2000)
- $y_i(\cdot) : \mathcal{T} \longrightarrow \mathcal{Y}$ is a response function where $\mathcal{T} \equiv \text{supp}(t)$ and $\mathcal{Y} \equiv \text{supp}(y) = [\underline{y}, \overline{y}]$
- Each agent i has realized treatment $z_i \in \mathcal{T}$ and a realized outcome $y_i \equiv y_i(z_i)$
- Assume $\mathcal{T}$ to be an ordered set and drop the index i for notational simplicity
- Denote $\mathcal{H}(\tau)$ as the sharp identification region of the target parameter τ

Plan

- We will start by looking at $\mathcal{H}(\mathbb{E}[y(t)])$ under various assumptions
- Then, we study $\mathcal{H}(\Delta(s,t))$, where $\Delta(s,t) \equiv \mathbb{E}[y(t) - y(s)]$ is the ATE of $s \longrightarrow t$

Monotone instrumental variables

Instrumental variables (IV)

- Assumption: $\mathbb{E}[y(t)|w, v = v_1] = \mathbb{E}[y(t)|w, v = v_2]$ for any w and $v_1, v_2 \in \text{supp}(v) \equiv \mathcal{V}$
- Example: Agents with different ability $v \in \mathcal{V}$ have the same mean wage functions

Monotone instrumental variables (MIV)

Proof

- Assumption: $v_1 \geq v_2 \implies \mathbb{E}[y(t)|w, v = v_1] \geq \mathbb{E}[y(t)|w, v = v_2]$ if $\mathcal{V}$ is ordered
- Example: People with higher ability has weakly higher mean wage functions
- $\mathcal{H}_{\text{MIV}}(\mathbb{E}[y(t)])$:

$$\Bigg[\sum_{u\in\mathcal{V}} \mathbb{P}[v = u] \sup_{u_1 \leq u} \Big\{\mathbb{E}[y|v = u_1, z = t]\mathbb{P}[z = t|v = u_1] + \underline{y}\mathbb{P}[z \neq t|v = u_1]\Big\},$$

$$\sum_{u\in\mathcal{V}} \mathbb{P}[v = u] \inf_{u_2 \geq u} \Big\{\mathbb{E}[y|v = u_2, z = t]\mathbb{P}[z = t|v = u_2] + \overline{y}\mathbb{P}[z \neq t|v = u_2]\Big\}\Bigg]$$

Monotone instrumental variables

Monotone treatment selection (MTS)

▸ Proof

- Assumption: $z_1 \geq z_2 \implies \mathbb{E}[y(t)|z = z_1] \geq \mathbb{E}[y(t)|z = z_2]$
- Example: People with higher level of schooling have weakly higher mean wage
- $\mathcal{H}_{\text{MTS}}(\mathbb{E}[y(t)])$: $\left[\underline{y}\mathbb{P}[z < t] + \mathbb{E}[Y(t)|z = t], \overline{y}\mathbb{P}[z > t] + \mathbb{E}[Y(t)|z = t]\mathbb{P}[z \leq t]\right]$

Monotone treatment response (MTR)

▸ Proof

- Assumption: $t_1 \geq t_2 \implies y_i(t_1) \geq y_i(t_2)$
- Example: People's wage function is weakly increasing in the years of schooling
- $\mathcal{H}_{\text{MTR}}(\mathbb{E}[y(t)])$: $\left[\mathbb{E}[y|t \geq z]\mathbb{P}[z \leq t] + \underline{y}\mathbb{P}[z > t], \mathbb{E}[y|t \leq z]\mathbb{P}[z \geq t] + \overline{y}\mathbb{P}[z < t]\right]$

Monotone instrumental variables

MTS-MTR

Proof

- Combines both the MTS and MTR assumptions
- Implication: $u_1 \leq u_2 \implies \underbrace{\mathbb{E}[y(u_1)|z = u_1]}_{=\mathbb{E}[y|z=u_1]} \leq \mathbb{E}[y(u_2)|z = u_1] \leq \underbrace{\mathbb{E}[y(u_2)|z = u_2]}_{=\mathbb{E}[y|z=u_2]}$
- $\mathcal{H}_{\text{MTS-MTR}}(\mathbb{E}[y(t)])$:

$$\left[\sum_{u<t} \mathbb{E}[y|z = u]\mathbb{P}[z = u] + \mathbb{E}[y|z = t]\mathbb{P}[z \geq t], \sum_{u<t} \mathbb{E}[y|z = u]\mathbb{P}[z = u] + \mathbb{E}[y|z = t]\mathbb{P}[z \leq t]\right]$$

Monotone instrumental variables

Average treatment effect (ATE)

▸ Proof

- Define the ATE for moving from s to t with $s < t$ as $\Delta(s,t) \equiv \mathbb{E}[y(t) - y(s)]$
- To find the identified region, we use $\mathcal{H}_{\text{MTS-MTR}}(\mathbb{E}[y(s)])$ and $\mathcal{H}_{\text{MTS-MTR}}(\mathbb{E}[y(t)])$:
  - Upper bound $=$ Upper bound of $\mathcal{H}_{\text{MTS-MTR}}(\mathbb{E}[y(t)])$ $-$ Lower bound of $\mathcal{H}_{\text{MTS-MTR}}(\mathbb{E}[y(s)])$
  - Lower bound $=$ Lower bound of $\mathcal{H}_{\text{MTS-MTR}}(\mathbb{E}[y(t)])$ $-$ Upper bound of $\mathcal{H}_{\text{MTS-MTR}}(\mathbb{E}[y(s)])$
- $\mathcal{H}_{\text{MTS-MTR}}(\Delta(s,t))$: Denote $P_x \equiv \mathbb{P}[z = x]$ and $F_x \equiv \mathbb{P}[z \leq x]$

$$\Bigg[\sum_{u<t}\{\mathbb{E}[y|z=u]-\mathbb{E}[y|z=s]\}P_u+\{\mathbb{E}[y|z=s]-\mathbb{E}[y|z=t]\}(F_t-F_s)+\sum_{u>s}\{\mathbb{E}[y|z=u]-\mathbb{E}[y|z=t]\}P_u,$$

$$\sum_{u<s}\{\mathbb{E}[y|z=t]-\mathbb{E}[y|z=u]\}P_u+\{\mathbb{E}[y|z=t]-\mathbb{E}[y|z=s]\}(F_t-F_s)+\sum_{u>t}\{\mathbb{E}[y|z=u]-\mathbb{E}[y|z=s]\}P_u\Bigg]$$

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Next class

de Haan (2011)

- Look at the effect of parents' schooling on the schooling of their child
- Used a sample from the Wisconsin Longitudinal Study

de Haan and Leuven (2020)

- Study the effect of Head Start on education and wage income
- Incorporate monotone instrumental variables motivated by stochastic dominance

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