Price Theory I: Problem Set 1 Question 2

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The Question

"Here we consider two state-level policies intended to discourage youth ('teen') drinking of alcoholic beverages ('alcohol'). One is setting a legal drinking age, thereby prohibiting sales to persons under that age. Another is an excise tax on all sales of alcoholic beverages.

Both the drinking age and the local excise tax can be avoided by paying a fixed travel cost to a state with different alcohol taxes and regulations. The drinking age can also be avoided with other fixed costs such as fake IDs, searching out weak points in sellers' enforcement protocols, or establishing a relationship with a surrogate purchasers of legal age."

The Question

What's this question about?

- Nonlinear budget constraints.
- Consumers facing different prices.
- Heterogeneous consumers in terms of preferences.
- Unintended policy consequences.

The Setup

- To set the stage, suppose there are two states, Illinois and Indiana.
  - The two states are ex-ante identical.
  - Illinois ("treated") is evaluating an underage drinking regulation - either a legal drinking age or an excise tax. Indiana ("control") does not intend to impose any such regulation.
- **Consumers:**
  - There are N teens in Illinois.
  - Each consumer makes consumption choice between alcohol and a general numeraire consumption good (e.g., a consumption bundle). Quasi-linear utility:

$$U = u(q) + c$$

    where q is alcohol consumption and c is consumption of numeraire good.
  - Price of numeraire good is normalized to one.

The Setup

- **Alcohol Production and Sales:**
  - Alcohol producers produce and sell alcohol to consumers in each state.
  - Alcohol prices are determined separately in each state. In both states, alcohol production is perfectly competitive with constant marginal cost.
    - ⋆ In each state: $p_s = MC_s$, where s indexes state. $s \in \{0, 1\}$ with 0 indicating Indiana and 1 indicating Illinois.
    - ⋆ Constant marginal cost is consistent with empirical observations.
    - ⋆ Alternatively, you can model with increasing marginal cost. For simplicity, you can assume that the two states are ex-ante identical, and the marginal cost function is the same in both states, such that $p_s = MC(Q_s)$ where Q_s is aggregate demand in state s.

The Setup

- **Ways to Avoid Regulation:**
  - If a drinking age is established in Illinois:
    - Teens can incur a fixed cost f to obtain alcohol in Illinois. This fixed cost involves fake IDs, searching out weak points in sellers' enforcement protocols, establishing relationships with surrogate purchasers etc.
    - Alternatively, teens can incur a fixed travel cost t to purchase alcohol in Indiana instead.
    - Suppose $t < f$ ($f > t$ case is analogous).
  - If an excise tax is imposed in Illinois:
    - Teens can incur a fixed travel cost t to purchase alcohol in Indiana.
    - Otherwise, they must pay $p + \tau$ for alcohol in Illinois. $\tau > 0$ is excise tax on alcohol.

The Setup

- Some Notes:
  - Prices in the two states may differ.
  - Adults can also incur a fixed travel cost t and drive across state border to purchase alcohol, if they so choose.
  - Assume if legal drinking age is established in Illinois, Indiana teens cannot obtain alcohol in Illinois (i.e., fixed costs infinity).
  - By assuming constant marginal cost, we can focus on Illinois teens' responses without modeling other agents.
  - Consumers only incur a fixed cost if they consume $q > 0$.
  - Assume consumers cannot "pool" with others to share the fixed costs.

Some Comments

- Setting up the problem is perhaps the most important (and most difficult) part of building a model.
  - In your own research, you should think about what key economic elements you are trying to capture.
  - A good model is simple but captures the main economic intuition. It should not try to replicate all the moving pieces in the real world.
  - Price Theory I trains you to set up models. Subsequent courses (Price Theory II and III etc.) teach you some existing models and teach you how to solve what you've already set up.
- Knowing how to distill a complex problem into a few key model ingredients is very powerful, for both theorists and empiricists.
- Price Theory I is about learning by doing.
  - As a PhD student, you should no longer be a passive receiver of knowledge. Research is about discovering and answering difficult and seemingly "open-ended" questions.

Part (a) - No Legal Drinking Age (Benchmark)

- Case 1: $p_0 \geq p_1$
  - Illinois teens purchase alcohol in Illinois. An Illinois teen solves: (assume interior solution)

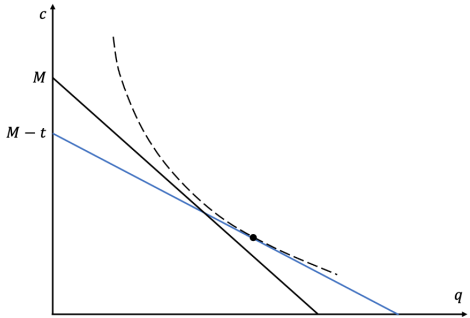
$$\max_{q,c} \quad U = u(q) + c \quad \text{s.t.} \quad p_1 q + c = M \quad \Longrightarrow \text{FOC:} \quad MU(q) = p_1$$

- Case 2: $p_0 < p_1$
  - Illinois teen either consume locally at price p_1 or incur a fixed cost t to purchase alcohol in Indiana at price p_0. An Illinois teen's problem:

$$\begin{aligned} \max_{q,c} \quad & U = u(q) + c \\ \text{s.t.} \quad & p_1 q + c \leq M \quad \text{if consume in Illinois} \\ & \begin{cases} p_0 q + c \leq M - t & \text{if } q > 0 \text{ in Indiana} \\ c \leq M & \text{if } q = 0 \text{ in Indiana} \end{cases} \end{aligned}$$

Part (a) - No Legal Drinking Age (Benchmark)

- Solve Case 2



  - High volume consumers choose to drive to Indiana for cheaper alcohol.
  - Solve problem: $\max_{q,c} u(q) + c \quad \text{s.t.} \quad p_1 q + c \leq M$
    - ★ FOC: $MU(q) = p_1$. Thus, $q = V(p_1)$ (where $V(p)$ is inverse demand curve).

Part (a) - No Legal Drinking Age (Benchmark)

- Solve problem:

$$\max_{q,c} \quad u(q) + c \quad \text{s.t.} \quad \begin{cases} p_0 q + c \leq M - t & \text{if } q > 0 \\ c \leq M & \text{if } q = 0 \end{cases}$$

  - ★ FOC to reduced problem: $\max_{q,c} u(q) + c \quad \text{s.t.} \quad p_0 q + c \leq M - t$: $q = V(p_0)$.
  - ★ It makes sense for Illinois teen to travel if:

$$\underbrace{u(V(p_0)) - p_0 V(p_0)}_{\equiv H(p_0)} > t \iff H(p_0) > t$$

  $H(p)$ is decreasing in p Proof. Let $p^*(t)$ be such that $H(p^*) = t$.

Part (a) - No Legal Drinking Age (Benchmark)

- ⋆ Consumer adheres to a threshold rule (note $p^*(t)$ depends on individual preferences):

$$q = \begin{cases} V(p_0) & \text{if} \quad p_0 < p^*(t) \\ 0 & \text{if} \quad p_0 \geq p^*(t) \end{cases}$$

- A teen travels to Indiana for alcohol if $p_0 < p^*(t)$ and

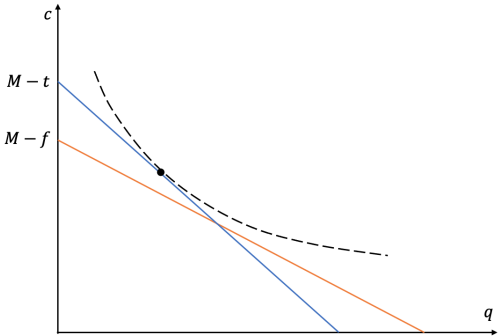
$$\underbrace{u(V(p_1)) - p_1 V(p_1)}_{\equiv H(p_1)} + M < \underbrace{u(V(p_0)) - p_0 V(p_0)}_{\equiv H(p_0)} + M - t \iff H(p_0) - H(p_1) > t$$

Part (a) - Introducing Legal Drinking Age

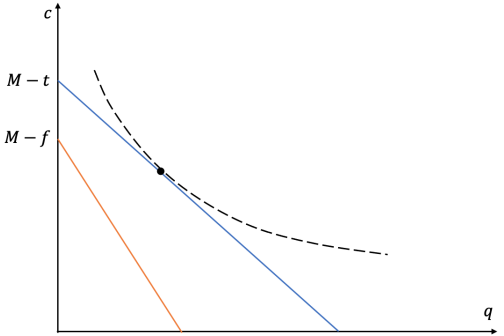
- Illinois teens can either incur fixed cost f and purchase alcohol locally at price p_1, or incur fixed travel cost t and obtain alcohol in Indiana at price p_0.
- An Illinois teen solves:

$$\begin{aligned} \max_{q,c} \quad & U = u(q) + c \\ \text{s.t.} \quad & \text{if} \quad q > 0 : \begin{cases} p_1 q + c \leq M - f \\ p_0 q + c \leq M - t \end{cases} \\ & \text{if} \quad q = 0 : \quad c \leq M \end{aligned}$$

Part (a) - Introducing Legal Drinking Age



Case 1: $p_0 \geq p_1$



Case 2: $p_0 < p_1$

Part (a) - Introducing Legal Drinking Age

- Solve the following problems separately and find the option that yields the highest utility:

$$\max_{q,c} \quad u(q) + c \quad \text{s.t.} \quad \begin{cases} p_1 q + c = M - f & \text{if} \quad q > 0 \\ c = M & \text{if} \quad q = 0 \end{cases}$$

$$\max_{q,c} \quad u(q) + c \quad \text{s.t.} \quad \begin{cases} p_0 q + c = M - t & \text{if} \quad q > 0 \\ c = M & \text{if} \quad q = 0 \end{cases}$$

- Solution to the first problem (note $p^*(f)$ depends on individual preferences):

$$q = \begin{cases} V(p_1) & \text{if} \quad p_1 < p^*(f) \\ 0 & \text{if} \quad p_1 \geq p^*(f) \end{cases}$$

Part (a) - Introducing Legal Drinking Age

- Solution to the second problem (note $p^*(t)$ depends on individual preferences):

$$q = \begin{cases} V(p_0) & \text{if} \quad p_0 < p^*(t) \\ 0 & \text{if} \quad p_0 \geq p^*(t) \end{cases}$$

- Note p^* is decreasing in fixed costs Why?. So $p^*(t) > p^*(f)$.
- This teen drives to Indiana iff:

$$u\Big(V(p_0)\Big) - p_0 V(p_0) - t > u\Big(V(p_1)\Big) - p_1 V(p_1) - f$$
$$\iff \quad H(p_1) - H(p_0) < f - t$$

Part (a) - Introducing Legal Drinking Age

- Case 1: $p_0 \geq p_1$
  - Benchmark: Illinois teens consume locally at price p_1.
  - With legal drinking age:
    - ★ For teens with $H(p_1) - H(p_0) < f - t$ and $p^*(t) > p_0$: travel to Indiana, pay price p_0.
    - ★ For teens with $H(p_1) - H(p_0) \geq f - t$ and $p^*(f) > p_1$: consume locally, pay price p_1.
    - ★ For others: demand "choked" off.
- Case 2: $p_0 < p_1$
  - Benchmark:
    - ★ For teens with $H(p_0) - H(p_1) > t$ and $p^*(t) > p_0$: travel to Indiana, pay price p_0.
    - ★ For others: consume locally, pay price p_1.
  - With legal drinking age: note $H(p_1) - H(p_0) < 0 < f - t$, and $p^*(f) > p_1 \Rightarrow p^*(t) > p_0$, so any teen who would consume locally would prefer to travel to Indiana. Some teens' demand "choked" off.

Part (b) - Excise Tax

- With excise tax, an Illinois teen solves the following two problems and choose the option that yields the highest utility:
  - Option 1: purchasing alcohol locally

$$\max_{q,c} \quad u(q) + c \quad \text{s.t.} \quad (p_1 + \tau)q + c = M$$

    FOC: $MU(q) = p_1 + \tau$.
  - Option 2: traveling to Indiana for alcohol

$$\max_{q,c} u(q) + c \quad \text{s.t.} \quad \begin{cases} p_0 q + c = M - t & \text{if} \quad q > 0 \\ c = M & \text{if} \quad q = 0 \end{cases}$$

    The teen consumes positive amount when $p_0 < p^*(t)$ and $MU(q) = p_0$.
- The Illinois teen chooses to travel to Indiana if $p_0 < p^*(t)$ and $H(p_0) - H(p_1 + \tau) > t$.

Part (b) - Rationality

- The empirical observation that excise tax decreases alcohol consumption by teens has nothing to do with "rationality".
  - See Becker (1962).
  - In class, we learned Hicks' generalized law of demand: good-by-good, quantities are non-increasing in their prices.
    - ⋆ We only assumed scarcity and that consumers like more of something. The key intuition is that cost minimization dictates that consumers would buy more of a good that is cheap.
  - To the extent that heterogeneous consumers have different "entry" points, the aggregate market demand is naturally downward-sloping.
  - Teens who derive high utility from alcohol consumption are more likely to travel for alcohol. If researchers are simply measuring teen alcohol consumption within Illinois, we would expect a reduction in consumption simply because some teens drive to Indiana for alcohol.

Part (c) - Teen Spending on Alcohol Consumption

- Total amount that Illinois teens spend on alcohol consumption includes actual expenditures paying for alcohol and fixed costs incurred.
- Case 1: $p_0 \geq p_1$
  - Without legal drinking age, Illinois teen i purchases alcohol locally. Spending:

$$p_1 \sum_{i=1}^{N} V^i(p_1)$$

  - With legal drinking age: Illinois teen i:
    - ★ travels to Indiana for alcohol if $\Delta H^i \equiv H(p_1) - H(p_0) < f - t$ and $p_i^*(t) > p_0$
    - ★ purchases alcohol locally if $\Delta H^i \geq f - t$ and $p_i^*(f) > p_1$
    - ★ does not purchase alcohol if otherwise

Part (c) - Teen Spending on Alcohol Consumption

$$\underbrace{p_1 \sum_{i:\{\Delta H^i \geq f-t;\, p_i^*(f) > p_1\}} V^i(p_1) + f \sum_{i=1}^{N} \mathbf{1}_{\{\Delta H^i \geq f-t;\, p_i^*(f) > p_1\}}}_{\text{spending by Illinois teens in Illinois}}$$

$$+ \underbrace{p_0 \sum_{i:\{\Delta H^i < f-t;\, p_i^*(t) > p_0\}} V^i(p_0) + t \sum_{i=1}^{N} \mathbf{1}_{\{\Delta H^i < f-t;\, p_i^*(t) > p_0\}}}_{\text{spending by Illinois teens in Indiana}}$$

- Imposing legal drinking age "chokes" off some teen demand, which tends to decrease teen spending on alcohol consumption.
- However, higher volume drinkers pay up fixed costs and some also buy alcohol in Indiana at higher prices, which tend to increase spending.

Part (c) - Teen Spending on Alcohol Consumption

- Case 2: $p_0 < p_1$
  - Without legal drinking age, Illinois teen i:
    - ★ travels to Indiana for alcohol if $\Delta H^i < -t$ and $p^*(t) > p_0$
    - ★ purchases alcohol locally if otherwise

$$\underbrace{p_1 \sum_{i:\Delta H^i \geq -t \text{ or } p_i^*(t) \leq p_0} V^i(p_1)}_{\text{spending in Illinois}} + \underbrace{p_0 \sum_{i:\Delta H^i < -t; p_i^*(t) > p_0} V^i(p_0) + t \sum_{i=1}^{N} \mathbf{1}_{\{\Delta H^i < -t; p_i^*(t) > p_0\}}}_{\text{spending in Indiana}}$$

  - With legal drinking age, Illinois teen i:
    - ★ travels to Indiana for alcohol if $p_i^*(t) > p_0$
    - ★ does not consume if otherwise

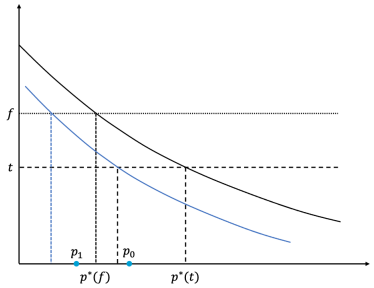
$$p_0 \sum_{p_i^*(t) \leq p_0} V^i(p_0) + t \sum_{i=1}^{N} \mathbf{1}_{p_i^*(t) > p_0}$$

Part (d) - Labor in Alcohol Sales and Distribution

- The amount of labor employed in the sales and distribution of alcohol include both fixed costs incurred by consumers to obtain alcohol and costs incurred by alcohol producers to discourage or screen out underage purchasers.
  - The focus of this question is on how the fixed costs incurred by teens respond to the establishment of legal drinking age.
  - Case 1: $p_0 \geq p_1$
    - Teens' fixed costs in benchmark case without legal drinking age: zero
    - Teens' fixed costs with legal drinking age: $f \sum_{i=1}^{N} \mathbf{1}_{\{\Delta H^i \geq f - t; p_i^*(f) > p_1\}} + t \sum_{i=1}^{N} \mathbf{1}_{\{\Delta H^i < f - t; p_i^*(t) > p_0\}}$
  - Case 2: $p_0 < p_1$
    - Teens' fixed costs in benchmark case without legal drinking age: $t \sum_{i=1}^{N} \mathbf{1}_{\Delta H^i < -t; p_i^*(t) > p_0}$
    - Teens' fixed costs with legal drinking age: $t \sum_{i=1}^{N} \mathbf{1}_{p_i^*(t) > p_0}$
  - Teens' fixed costs increase with the establishment of legal drinking age.

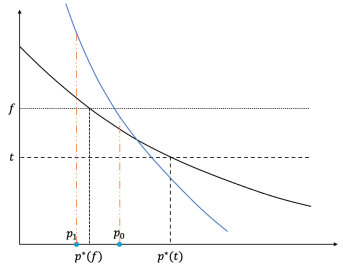
Part (e) - Legal Drinking Age

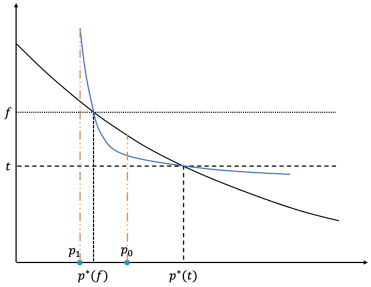
- From Part (a), Illinois teen i:
  - travels to Indiana for alcohol if $\Delta H^i < f - t$ and $p_i^*(t) > p_0$
  - purchases alcohol locally if $\Delta H^i \geq f - t$ and $p_i^*(f) > p_1$
  - does not consume alcohol if otherwise
- If a teen derives very low utility from alcohol consumption, the thresholds $p_i^*(f)$ and $p_i^*(t)$ are low. Thus, the teen's demand will be "choked" off.



Part (e) - Legal Drinking Age

- Case 1: $p_0 \geq p_1$
  - If a teen derives high marginal utility from alcohol, and is a high volume drinker, the teen is more likely to consume locally.
  - If a teen's demand elasticity is high, the teen is more likely to consume alcohol locally (paying a lower price but incurring a higher fixed cost).
  - For these teens, their consumption is $q^i = V^i(p_1)$, the same as before.



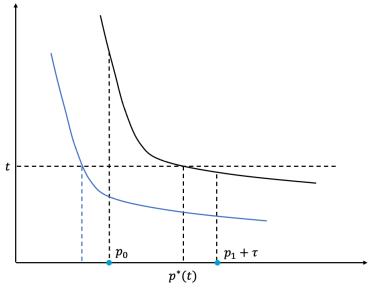


Part (e) - Legal Drinking Age

- Case 2: $p_0 < p_1$
  - $\Delta H^i < 0$. Teens with $p_i^*(t) > p_0$ travel to Indiana for alcohol.
  - In benchmark case without legal drinking age, Teens for whom $\Delta H^i < -t$ and $p_i^*(t) > p_0$ travel to Indiana.
  - Thus, more teens travel to Indiana for alcohol if a legal drinking age is established. **These teens consume more alcohol**, since $q^i = V^i(p_0) > V^i(p_1)$.
  - If $p_i^*(t) \leq p_0$, then $p_i^*(f) < p_i^*(t) \leq p_0 < p_1$. Thus, these teens simply do not consume alcohol as their demand is "choked" off.

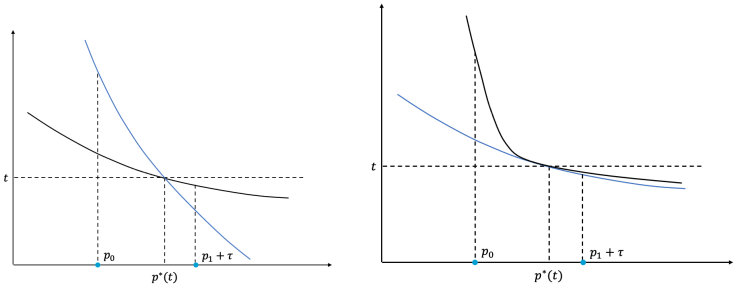
Part (e) - Excise Tax

- From Part (b), Illinois teen i:
  - travels to Indiana for alcohol if $H^i(p_0) - H^i(p_1 + \tau) > t$ and $p_i^*(t) > p_0$
  - consumes locally if otherwise
- If a teen derives very low utility from alcohol consumption, the threshold $p_{i,t}^*(t)$ is low. Thus, the teen will not travel for alcohol.



Part (e) - Excise Tax

- Case 1: $p_0 > p_1$
  - Without excise tax, Illinois teens consume locally.
  - If τ is very small, such that $p_0 \geq p_1 + \tau$, Illinois teens still purchase alcohol locally.
  - If τ is high enough, such that $p_0 < p_1 + \tau$, a teen travels to Indiana for alcohol if $H^i(p_0) - H^i(p_1 + \tau) > t$ and $p_i^*(t) > p_0$.
    - ★ Thus, a teen is more likely to travel if the teen derives high marginal utility from alcohol and is a high volume drinker, or if the teen's demand is elastic.



Part (e) - Excise Tax

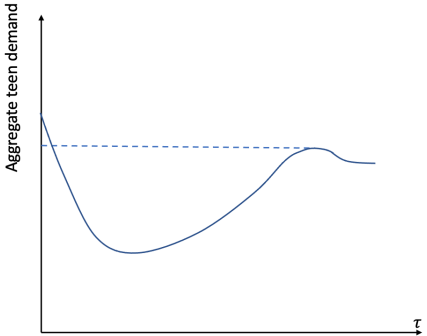
- Case 2: $p_0 \leq p_1$
  - Thus, $p_0 < p_1 + \tau$.
  - Without excise tax, Illinois teens for whom $\Delta H^i < -t$ and $p_i^*(t) > p_0$ travel to Indiana.
  - Since $H^i(p_1 + \tau) - H^i(p_0) < \Delta H^i$, more Illinois teens would travel to Indiana when an excise tax is imposed. **These teens consume more.**
    - ⋆ Thus, the answer to Part (f) is yes. An excise tax may increase drinking by some teens.

Drinking intensity across situations (e.g., weekdays vs. weekends) is analogous. Suppose teens prefer to consume alcohol on weekends than during the week. Assuming no time discounting. Then, we an think of a teen on weekend and the teen on a weekday as two different teens. Our results above carry over.

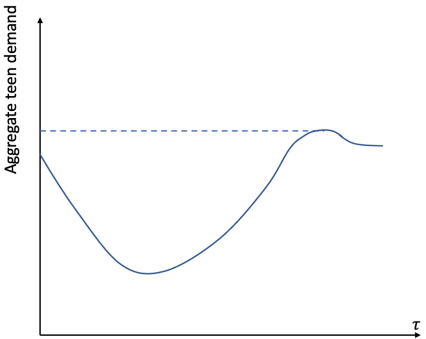
Part (g) - Aggregate Teen Demand

- Case 1: $p_0 > p_1$
  - When τ is small, such that $p_0 \geq p_1 + \tau$, Illinois teens still purchase alcohol locally.
    - ★ As τ increases, price facing teens increases, aggregate teen demand decreases.
  - When τ is high enough, such that $p_0 < p_1 + \tau$, some Illinois teens start to travel to Indiana for alcohol.
    - ★ For a teen who travels to Indiana, the teen's consumption $q^i = V^i(p_0) > V^i(p_1 + \tau)$
    - ★ As τ increases, more teens start driving to Indiana. Eventually, all teens with $p_i^*(t) > p_0$ consume in Indiana. Teens who do not travel face higher price in Illinois as τ further increases.
- Case 2: $p_0 \leq p_1$
  - At $\tau = 0$, teens with $\Delta H^i < -t$ and $p_i^*(t) > p_0$ already travel to Indiana.
  - As τ increases, teens who purchase alcohol locally face higher prices and thus consume less. However, more teens start traveling to Indiana for alcohol, and these teens consume more.
    - ★ Eventually, all teens with $p_i^*(t) > p_0$ consume in Indiana.

Part (g) - Aggregate Teen Demand



Case 1



Case 2

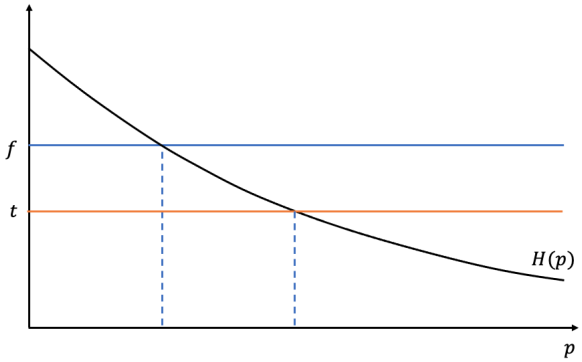
Appendix

Proof: Let $H(p) \equiv u\Big((MU)^{-1}(p)\Big) - p(MU)^{-1}(p)$, taking derivative with respect to p:

$$\begin{aligned}
\frac{\partial H}{\partial p} &= \frac{\partial u\Big((MU)^{-1}(p)\Big)}{\partial (MU)^{-1}(p)} \frac{\partial (MU)^{-1}(p)}{\partial p} - (MU)^{-1}(p) - p\frac{\partial (MU)^{-1}(p)}{\partial p} \\
&= MU\Big((MU)^{-1}(p)\Big) \frac{\partial (MU)^{-1}(p)}{\partial p} - (MU)^{-1}(p) - p\frac{\partial (MU)^{-1}(p)}{\partial p} \\
&= -(MU)^{-1}(p) = -q < 0
\end{aligned}$$

Back

Appendix



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