# Price Theory I TA Session 1
# Sample Problems

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# How to Approach a Problem

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- To begin with, why do we need a model?
- Set up the model so that the problem is no longer “open-ended”.
  - What are the essential modeling components?
  - Are the assumptions reasonable?
  - What unimportant details you should abstract from?
- Be transparent about the assumption-to-conclusion mapping.
  - Where do you use an assumption?
  - Parametric assumptions: tractability vs. generality.
- Blend mathematical formality and economic intuition.
- Derive economic insights, not just equations.
- The answer may be “ambiguous” sometimes, don’t stop there.
  - If an effect cannot be signed, explain the countervailing forces.

# Sample Question: Non-Compete Clause and Innovation

# Sample Question 1 Non-Compete Clause, I

- *An inventor creates a new product, manufactured with a linear technology, anticipating that for one period he will be the only one knowing how to produce it.*
- *Each consumer of the product during that period is able to also reverse engineer it and produce $n \geq 1$ units in the second period with no obligation toward the inventor.*
- *After the second period, the product is neither produced nor consumed.*
- *The market demand for the product's services is stable over time, with inverse denoted $v(Q_t)$ where $Q_t$ is aggregate consumption in period $t$.*

# Sample Question 1 Non-Compete Clause, II

a) *What will the equilibrium purchase price of the product in each period?*

b) *Is the inventor harmed by a larger value for n? Would your answer be different if the product were also produced in a third period, fourth period, etc., with the same copying technology (n)?*

c) *Are consumers harmed by a larger value for n?*

# Sample Question 1 Non-Compete Clause, III

- *Suppose instead that consumers cannot reverse engineer the product, but that employees engaged in production can obtain that knowledge.*
- *In the second period, any former employee can start his own production operation with capacity n, with no obligation toward their former employer.*
    - d) *What would be the equilibrium purchase price of the product in each period?*
    - e) *Would the inventor want to hire employees under "non-compete" clauses that prohibit them from producing or selling the product after they leave the inventor's employment?*
    - f) *What factors would determine whether the inventor hires employees with noncompete or with agreements to license production in the future?*
    - g) *How would a legal prohibition of non-compete clauses affect wages paid by the inventor? The number of employees he hires?*

# Model Setup, I

Set up the model so that the problem is no longer “open-ended”.

- Decision makers and their choice variables.
- Technology, preferences, and information.
- Solution concepts / equilibrium notion.

Envronment

- Note that many assumptions are already in the question.
- Consider a two period model with an inventor and many consumers.
- Inventor maximizes payoff by producing $y_1$ and $y_2$ in Periods 1 and 2.
  - Inventor understands that consumers become competitors in Period 2.
- Consumers make purchase (and production) decisions.
  - Consumers know if purchase in Period 1, they can get Period 2 profits.
- Common discount $\beta \in (0, 1]$.
- Ignore integer constraints.

# Model Setup, II

Inventor

- Chooses quantities $y_1, y_2$ to maximize profits across both periods.
- Linear production tech $y = Al$, where $A$ is productivity and $l$ is labor.
- Assume wage rate $w$ is exogenous and constant across time.
  - Constant marginal cost $MC = \frac{w}{A}$.

Consumers

- Decides on purchase and production (if purchase in Period 1).
- Many consumers, unit demand each period, quasilinear preferences.
- Suppose consumer $i$ has value $v^i$ for the product, constant over time.
  - Order $v^i$ to obtain inverse demand $v(Q_t)$ where $Q_t$ is quantity at $t$.
  - Assuming a linear demand may help with tractability, but unnecessary.
- Price-takers when purchase, may have market power when produce.
- Assume it is costless for consumers to reverse engineer the product, and they will have the same linear production technology as Inventor.

# Consumer's Problem

- Period 1
  - Consumers compare price to value plus Period 2 profits.
  - WTP in period 1 is $v^i + \beta \Pi_2^C$, where $\Pi_2^C$ is profits from Period 2.
  - Focus on symmetric equilibrium so that same profits $\Pi_2^C$ in Period 2.
- Period 2
  - Period 1 consumers choose production $q^i (0 \leq q_i \leq n)$.
  - All consumers then decide whether to purchase given the price $v(Q_2)$ and their willingness to pay $v^i$.

# Inventor's Problem

Solve by backward induction.

- Inventor can produce nothing in Period 1 and earn monopoly profits in Period 2 if profits turn out to be higher this way.
- We ignore this case since it is possible but uninteresting.

Period 2:

- $y_1$ consumers (who purchase in Period 1) and Inventor compete.
- Assume the competition is Cournot with $y_1 + 1$ producers.
- Obtain Period 2 profits $\Pi_2^C$ and $\Pi_2^I$, and quantity $y_2$ as functions of $y_1$.

Period 1:

- Consumers are willing to pay up to $v^i + \beta\Pi_2^C(y_1)$.
- Inventor faces demand $v(y_1) + \beta\Pi_2^C(y_1)$
    - Demand for the product's services plus Period 2 profits.
- All Period 2 profits captured by Inventor through Period 1 pricing.

# Inventor's Problem, details I

Period 2: Consider whether the consumers' capacity constraint binds.

- Suppose consumers' capacity constraint does not bind in Period 2: Producer $i$ chooses quantity $q^i$ to maximize his Period 2 profits, given others choices $q^{-i}$,

$$\max_{q^i} \Pi_2^I = \Pi_2^C = q^i \left[ v \left( \sum_{-i} q^j + q^i \right) - MC \right].$$

- Focus on symmetric equilibrium $q^i = q$, and the FOC is

$$qv'\left((y_1+1)q\right) + v\left((y_1+1)q\right) = MC$$

- This pins down $q^*(y_1)$ and Period 2 profits

$$\Pi_2^I = \Pi_2^C = q^*(y_1)\left[v\left((y_1+1)q^*(y_1)\right) - MC\right]$$

- Inventor and first period consumer each produce $q^*(y_1)$.
  - Assume the existence (and uniqueness) of $q^*(y_1)$.

# Inventor's Problem, details II

- Suppose now that consumers' capacity constraint binds in Period 2: They will produce quantity $n$ each in Period 2 and Inventor chooses the optimal $y_2$,

$$\max_{y_2} \Pi_2^I = y_2 \left[ v \left( n y_1 + y_2 \right) - MC \right]$$
$$\text{FOC: } MC = y_2 v'(n y_1 + y_2) + v(n y_1 + y_2)$$

- This pins down $y_2^*(y_1)$ and Inventor's Period 2 profits

$$\Pi_2^I = y_2^*(y_1) \left[ v \left( n y_1 + y_2^*(y_1) \right) - MC \right].$$

- Each consumer produces $n$ and earns profits $\Pi_2^C = n \left[ v \left( n y_1 + y_2^*(y_1) \right) - MC \right]$.

# Inventor's Problem, details III

Period 1: Inventor maximizes total profits by choosing $y_1$,

$$\max_{y_1} \Pi^I = \Pi_1^I + \Pi_2^I = y_1 \left[ v(y_1) + \beta \Pi_2^C(y_1) - MC \right] + \beta \Pi_2^I(y_1).$$

- $\Pi_2^C(y_1)$ and $\Pi_2^C(y_1)$ kink at $n = q^*(y_1)$ due to capacity constraint.
- Inventor takes the FOCs in both regions $n \geq q^*(y_1)$ and $n < q^*(y_1)$,

  $$y_1 \left[ v'(y_1) + \beta \Pi_2^{C\prime}(y_1) \right] + v(y_1) + \beta \Pi_2^C(y_1) - MC + \beta \Pi_2^{I\prime}(y_1) = 0,$$

  and picks $y_1^*$ that generates the highest profits.
- We will not solve this explicitly (tedious even with linear demand!). Instead, we will use comparative statics to answer the questions.

# Part (a): Equilibrium Prices

*a) What will the equilibrium purchase price of the product in each period?*

- Our model setup has already answered this.
- Perior 1 price $p_1^* = v(y_1^*) + \beta \Pi_2^C(y_1^*)$.
  - That is, price is the marginal consumer's value at $y_1^*$ plus consumers' discounted profits from period 2.
- Period 2 price $p_2^* = v\left(y_2^*(y_1^*) + y_1^* q^*(y_1^*)\right)$. That is, price is the marginal consumer's value at quantity $y_2^* + y_1^* q^*$.
- Key insight: Consumer's potential profits in Period 2 will be completely captured by Inventor.

# Part (b): Who's Harmed by Larger $n$

*b) Is the inventor harmed by a larger value for n? Would your answer be different if the product were also produced in a third period, fourth period, etc., with the same copying technology (n)?*

- Inventor is (weakly) worse off under a larger $n$.
- Inventor's payoff can be written as

$$\Pi^I = y_1 \left[ v(y_1) - MC \right] + \beta \left[ \Pi_2^C(y_1) + \Pi_2^I(y_1) \right]$$

  - The first term is consumer surplus extracted from Period 1.
  - The second term is the total oligopoly profits from Period 2.
- Larger $n \Rightarrow$ more output in Period 2, lower oligopoly profits.
  - If capacity constraint binds at $n$, there will be strictly more output.
  - If capacity constraint does not bind, output stays the same.
- Inventor also has a behavioral response by changing $y_1^*$.
  - The effect on profits is second-order by the envelope theorem.
  - Nevertheless, the second-order effect is in favor of our conclusion: In fear of competition, Inventor further cut $y_1$, which is already below the monopoly quantity.
- More periods: notationally heavy but qualitatively the same.

# Part (c): Who's Harmed by Larger $n$

*c) Are consumers harmed by a larger value for n?*

- Period 2 profits fully extracted by Inventor.
- Sufficient to consider consumer surplus from the product's service, which increases in total market output.
- Overall effect is ambiguous, but
  - Period 2: More competition $\Rightarrow$ larger output and benefits consumers.
  - Period 1: Fear of competition $\Rightarrow$ smaller $y_1$ and harms consumers.
- If you assume linear demand, maybe you can sign the overall effect.
  - But such conclusion hinges completely on linear demand.

# Parts (d)-(g): Questions

- *Suppose instead that consumers cannot reverse engineer the product, but that employees engaged in production can obtain that knowledge.*
- *In the second period, any former employee can start his own production operation with capacity n, with no obligation toward their former employer.*
    - d) *What would be the equilibrium purchase price of the product in each period?*
    - e) *Would the inventor want to hire employees under "non-compete" clauses that prohibit them from producing or selling the product after they leave the inventor's employment?*
    - f) *What factors would determine whether the inventor hires employees with noncompete or with agreements to license production in the future?*
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# Additional Setup: Employee Production

We will be brief here.
Your answer is still expected to have detailed argument (as we did above).
Employee:

- Decides whether to take the job given wages and Period 2 profits.
- Every employee has a unit of labor to supply.
- Normalize their outside option to zero (e.g., disutility from working).

# Investor's Problem, Employee Production

No production restriction:

- Period 2: Employees and Inventor engage in Cournot competition.
- Period 1: Investor can pay a below market wage to extract all rent.

Non-compete clause:

- Investor can be monopoly in both periods.
- But she has to pay the market wage $w$.

# Parts (d): Answer

*d) What would be the eqm purchase price of the product in each period?*

- Similar to (a), but no rent-extraction term in Period 1 price.

# Parts (e) and (f): Answer

*e) Would the inventor want to hire employees under "non-compete" clauses that prohibit them from producing or selling the product after they leave the inventor's employment?*
*f) What factors would determine whether the inventor hires employees with noncompete or with agreements to license production in the future?*

- Non-compete clause is better in our model.
- Without non-compete, Investor profits given by

$$\Pi^I = y_1 \left[ v(y_1) - MC \right] + \beta \left[ \Pi_2^C(y_1) + \Pi_2^I(y_1) \right]$$

- Under non-compete, Inventor acts as monopoly in both periods.
  - Inventor can separately decide on output for both periods.
  - Monopoly profits in Period 1 $>$ the first term.
  - Monopoly profits in Period 2 $>$ the second term (oligopoly profits).
- At first glimpse, unclear whether "non-compete" benefits Inventor.
  - Non-compete clause means higher wages but less competition.
  - Our model provides an answer.
  - What can potentially go wrong with our model?

# Part (g): Answer

*g) How would a legal prohibition of non-compete clauses affect wages paid by the inventor? The number of employees he hires?*

- Wages:
  - Period 1: Increases to market level.
  - Period 2: Unchanged.
- Employment:
  - Period 1: Decreases due to less output.
  - Period 2: Increases due to employee production.

# Extensions

- Implications on incentives for innovation?
  - We assumed that Inventor has sunk the cost to invent the product.
  - The model can endogenize Inventor's decision to invent something.
  - Production limit $n$ can have implications on what would be invented.
  - Non-compete clauses can also affect innovation and employment.

# Sample Question: Value of Learning Loss During COVID

# Sample Question 2: Value of Learning Loss During COVID

- *Here we consider the opportunity costs of closing schools for a year during a pandemic. We assume that no net learning occurs during that year: students end the year with the same human capital they had at the beginning.*
- *Treat schooling as a production process with many inputs: especially effort and attention by students, teachers, and parents, at various ages.*
- *The ultimate output is human capital that enhances earnings and household production during the student's adult lives, the present value of which is in the millions of dollars per person.*

**Questions:**

- What forces determine how much effort goes into learning after schools reopen? Could effort decline?
- How would you value any decline to human capital resulting from closing schools?

# Start with Intuition

- **What makes this question important?**
  A policy maker needs to balance the health benefits of a lock-down with the costs of reduced schooling.
- **What would make your answer useful? What would the value be of formalizing the problem?**
  A model let's us map precisely from assumptions and data to the value of learning lost due to closing schools. Maybe we find something unexpected.
- **What intuitively should matter?**
  The more students make up for lost learning, the less costly it was to close schools. What factors determine how much students make up?

# Starting Simply

- **Justify simplifications briefly up top**
- **Need at least three periods**: COVID, post COVID schooling, Adulthood. Continuous time probably overkill as policy makers likely think in discrete school years! Unless you're great at continuous time methods!
- **Modelling students, parents, and teachers separately is costly.** Are key insights to the question derived from how these parties interact? For now, aggregate effort across students and parents.
- **See where you get with your simplifications–can always revise later.**

# Setup

A parent's maximization problem is the following:

$$\max_{e_1, e_2} V(F(e_1, e_2)) - c_1(e_1) - c_2(e_2)$$

- $e_1$ and $e_2$ is aggregated effort in each time period of the child's life prior to adulthood
- $F(e_1, e_2)$ is the production function mapping from effort levels to human capital
- $V(H)$ is the present discounted value of human capital $H$
- $c_t(e_t)$ is the cost for given effort at time period $t$

# Functional Forms?

$$\max_{e_1, e_2} V(F(e_1, e_2)) - c_1(e_1) - c_2(e_2)$$

- For now, no explicit functional form assumptions, but we can say something about our functions using intuition...
- $V(H)$ involves both lifetime income and any changes to quality of life from additional human capital. It is increasing–likely initially convex and eventually concave.
- $F(e_1, e_2)$ is increasing in each argument–but the cross-partial derivative is uncertain.
- $c_t(\cdot)$ is the cost of effort. It's safe to assume that it's increasing and because time and resources of parents and students are limited, it is convex.

# Solving the Model

We get FONC's:

$$V'(F(e_1, e_2))F_1(e_1, e_2) = c_1'(e_1)$$
$$V'(F(e_1, e_2))F_2(e_1, e_2) = c_2'(e_1)$$

*The marginal utility of effort in each period is equal to it's benefit.*

- From convexity of costs, we can rule out infinite effort
- Zero effort is possible but uninteresting so let $e_1^*, e_2^*$ be non-pandemic optimal effort levels (an interior solution)

# Part A: Could Students Reduce Effort?

- Assume no first period effort: $e_1 = 0$
- Consider what happens to our FONC when maintaining $e_2^*$

$$V'(F(0, e_2^*))F_2(0, e_2^*) = c_2'(e_1)$$

  - Human capital falls: $F(0, e_2^*) < F(e_1^*, e_2^*)$
  - $V'$ falls if we were on the convex part of the function, increases if on concave part of the function
  - The effect on $F_2(0, e_2^*)$ depends on how *complementary* effort is. If learning one year makes learning next year easier, $F_2(0, e_2^*)$ falls.
- *Insight:* Students with low human capital to begin with may *reduce* effort post pandemic if learning becomes harder–making closing schools more costly and possibly driving inequality.

# General Tips

- Always multiple (good) ways to setup an open-ended problem
- Start simply–then add complications if you need them. Don't end up with a model you can't solve!
- Remember the goal is to be insightful yet precise
- Grading is less predictable than other courses due to judgement calls. We will try to be transparent!
- PT1 exercises a different part of the brain than other first year coursework. Try to enjoy it!