

Price Theory I: Problem Set 4 Question 1

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October 2021

Problem Set 4 Question 1

"An inventor creates a new product, manufactured with a linear technology, anticipating that for one period he will be the only one knowing how to produce it. Each consumer of the product during that period is able to also reverse engineer it and produce $n \geq 1$ units in the second period with no obligation toward the inventor. After the second period, the product is neither produced nor consumed. The market demand for the product's services is stable over time, with inverse denoted $v(c_t)$ where c_t is aggregate consumption in period t ."

Model

- The inventor (and possibly the representative consumer in period 2) produces the product with the linear technology $c_{it} = A\ell_{it}$, where ℓ_{it} is labor hired by the inventor or representative consumer in period t .
- Ignore discounting and assume labor's wage is exogenous.
- The product is consumable (not durable) and it can be consumed and still be reproduced in the next period.

Model

- Assume the inventor can commit to future production.
 - ▶ A somewhat consequential assumption.
- There are no income effects on the consumer's utility for the product.
 - ▶ This is sort of given by the question, with stable demand for services.
 - ▶ Equivalent to quasi-linear utility.
- Assume the representative consumer is a price taker (in the second period) and have rational expectations.
- The market demand for the product's **services** is $v(c_t)$ in each period. However the market demand for the product itself may be higher in period 1 since the consumer can potentially produce it for profit in period 2.
 - ▶ Assume $v(c_t)$ is continuously differentiable, decreasing, and is such that the solution to a static, one-period monopoly problem exists and FOC are sufficient to find it.

Model

- The inventor solves:

$$\max_{0 \leq c_{i1}, c_{i2}} (p_1(c_{i1}, c_{i2}) - \frac{w}{A})c_{i1} + (p_2(c_{i1}, c_{i2}) - \frac{w}{A})c_{i2}$$

- As a price taker, the consumer produces:

$$c_{c2} \in \begin{cases} nc_{i1} & \text{if } p_2 > \frac{w}{A} \\ [0, nc_{i1}] & \text{if } p_2 = \frac{w}{A} \\ 0 & \text{if } p_2 < \frac{w}{A} \end{cases}$$

- There are two cases, $p_2 > \frac{w}{A}$ and $p_2 = \frac{w}{A}$.

Part (a) - Prices

“What will the equilibrium purchase price of the product in each period?”

Part (a) - Prices

- First let's consider a static, one-period monopoly (it will be helpful to compare later).
The inventor's problem is:

$$\max_{0 \leq c} \left(v(c) - \frac{w}{A} \right) c$$

$$\text{FOC: } v'(c)c + v(c) - \frac{w}{A} = 0$$

- We've assumed $v(c)$ is such that the problem is well-defined and FOC is sufficient, so $c^* > 0$.

Part (a) - Prices

- Back to our two-period problem. Assume we are in the $p_2 > \frac{w}{A}$ case. Then $c_{i2} = nc_{i1}$ (consumers produce as much as possible). The inventor's problem becomes:

$$\max_{0 \leq c_{i1}, c_{i2}} (v(c_{i1}) + n(v(c_{i2} + nc_{i1}) - \frac{w}{A}) - \frac{w}{A})c_{i1} + (v(c_{i2} + nc_{i1}) - \frac{w}{A})c_{i2}$$

- Where the $n(v(c_{i2} + nc_{i1}) - \frac{w}{A})$ term is from the consumer's additional willingness to pay in the first period since they can make a profit in the second period.

Part (a) - Prices

- Take FOCs:

$$[c_{i1}] : \nu'(c_{i1})c_{i1} + \nu(c_{i1}) + n^2\nu'(c_{i2} + nc_{i1})c_{i1} + nv(c_{i2} + nc_{i1}) - (n+1)\frac{w}{A} \\ + nv(c_{i2} + nc_{i1})c_{i2} = 0$$

$$\nu'(c_{i1})c_{i1} + \nu(c_{i1}) + nv'(c_{i2} + nc_{i1})(c_{i2} + nc_{i1}) + nv(c_{i2} + nc_{i1}) - (n+1)\frac{w}{A} = 0$$

$$[c_{i2}] : nv'(c_{i2} + nc_{i1})c_{i1} + \nu'(c_{i2} + nc_{i1})c_{i2} + \nu(c_{i2} + nc_{i1}) - \frac{w}{A} = 0$$

$$\nu'(c_{i2} + nc_{i1})(c_{i2} + nc_{i1}) + \nu(c_{i2} + nc_{i1}) - \frac{w}{A} = 0$$

Part (a) - Prices

- Combine FOCs:

$$\begin{aligned} & \sqrt{(c_{i1})} c_{i1} + v(c_{i1}) + n\sqrt{(c_{i2} + nc_{i1})(c_{i2} + nc_{i1})} + nv(c_{i2} + nc_{i1}) \\ & - (n+1)\frac{w}{A} = 0 \end{aligned}$$

c_{i1} FOC

$$\begin{aligned} & \sqrt{(c_{i1})} c_{i1} + v(c_{i1}) + n(-v(c_{i2} + nc_{i1}) + \frac{w}{A}) + nv(c_{i2} + nc_{i1}) \\ & - (n+1)\frac{w}{A} = 0 \end{aligned}$$

Sub in c_{i2} FOC

$$\sqrt{(c_{i1})} c_{i1} + v(c_{i1}) - \frac{w}{A} = 0$$

Cancel terms

- This is the same as in the static monopoly problem. $c_{i1}^* = c^*$

Part (a) - Prices

- Now find c_{i2} and check constraints. c_{i2} FOC is:

$$\nu'(c_{i2} + nc_{i1})(c_{i2} + nc_{i1}) + \nu(c_{i2} + nc_{i1}) - \frac{w}{A} = 0$$

- This is also the same as the static monopoly problem, however quantity includes the amount consumers are producing and we have already found c_{i1} .
 - If $n = 1$ then this is also satisfied if $c_{i2} = 0$.
 - If $n > 1$ then it must be that $c_{i2} < 0$, which violates our constraints. So consider the potential corner solution where $c_{i2} = 0$ instead.

Part (a) - Prices

- When $c_{i2} = 0$ (and $p_2 > \frac{w}{A}$), the inventor's problem becomes:

$$\begin{aligned}& \max_{0 \leq c_{i1}} \left(v(c_{i1}) + n(v(nc_{i1}) - \frac{w}{A}) - \frac{w}{A} \right) c_{i1} \\& \Leftrightarrow \max_{0 \leq c_{i1}} \left(v(c_{i1}) + nv(nc_{i1}) - (n+1)\frac{w}{A} \right) c_{i1} \\& \text{FOC: } v'(c_{i1})c_{i1} + n^2v'(nc_{i1})c_{i1} + v(c_{i1}) + nv(nc_{i1}) - (n+1)\frac{w}{A} = 0 \\& \Leftrightarrow (v'(c_{i1}) + n^2v'(nc_{i1}))c_{i1} + v(c_{i1}) + nv(nc_{i1}) - (n+1)\frac{w}{A} = 0\end{aligned}$$

- Clearly this is not the monopoly solution, but note that $p_1 > p_2$, $p_1 \geq v(c^*)$, and $v(c^*) \geq p_2$ (where c^* is the static monopoly quantity).
 - $p_1 > p_2$ because more will be produced in period 2, so prices are lower.
 - $p_1 \geq v(c^*)$ because if $p_1 < v(c^*)$ the inventor could get closer to the two period monopoly profit by reducing c_{i1} towards the static monopoly amount (nc_{i1} too).

Part (a) - Prices

- We can't say much more about the previous expression without assuming a form for $v(c_t)$, since we don't know what $v'(c)$ looks like (other than it's negative).
- Let's assume a linear demand function $v(c) = \alpha - \beta c$ for simplicity ($\alpha > \frac{w}{A}$).
- Then the FOC becomes:

$$\begin{aligned}(-\beta - n^2\beta)c_{i1} + \alpha - \beta c_{i1} + n(\alpha - \beta c_{i1}) - (n+1)\frac{w}{A} &= 0 \\ \Leftrightarrow -(n^2 + 1)\beta c_{i1} + (n+1)(\alpha - \beta c_{i1}) - (n+1)\frac{w}{A} &= 0 \\ \Leftrightarrow -(n^2 + n + 2)\beta c_{i1} + (n+1)(\alpha - \frac{w}{A}) &= 0\end{aligned}$$

Part (a) - Prices

- Solve for c_{i1} :

$$(n^2 + n + 2)\beta c_{i1} = (n + 1)(\alpha - \frac{w}{A})$$
$$c_{i1}^* = \frac{(n + 1)(\alpha - \frac{w}{A})}{(n^2 + n + 2)\beta}$$
$$c_{i1}^* = \frac{(n + 1)(\alpha - \frac{w}{A})}{(n(n + 1) + 2)\beta}$$

- And:

$$c_2^* = nc_{i1}^* = \frac{n(n + 1)(\alpha - \frac{w}{A})}{(n(n + 1) + 2)\beta}$$

Part (a) - Prices

- For comparison, the solution to the static monopoly problem with linear demand:

$$-\beta c + (\alpha - \beta c) - \frac{w}{A} = 0 \quad \text{FOC}$$

$$2\beta c = \alpha - \frac{w}{A}$$

$$c^* = \frac{\alpha - \frac{w}{A}}{2\beta}$$

- When $n = 1$, $c_{i1}^* = c_2^* = nc_{i1}^* = c^*$ (the static monopoly solution), as we've already seen.
- As n increases, c_{i1}^* falls, asymptoting to zero, while $c_2^* = nc_{i1}^*$ rises, asymptoting to $2c^*$ (which in this case is also the competitive quantity).

Part (a) - Prices

- We've so far ignored the possibility that $p_2 = \frac{w}{A}$.
- If $p_2 = \frac{w}{A}$ then the inventor's problem becomes:

$$\begin{aligned} & \max_{0 \leq c_{i1}, c_{i2}} \left(v(c_{i1}) - \frac{w}{A} \right) c_{i1} \\ & \text{s.t. } c_{i2} + c_{c2}(c_{i1}, c_{i2}) = \bar{c}, \text{ where } v(\bar{c}) = \frac{w}{A} \end{aligned}$$

- So it's just a static monopoly problem in the first period. The inventor is indifferent to the level of c_{i2} .
- However, there is an additional constraint that $c_{i2} + c_{c2} = \bar{c}$.
 - ▶ If $nc^* \leq \bar{c}$, then any $c_{i2} \in [\bar{c} - nc^*, \bar{c}]$ will fulfill the condition, with the consumers providing the remaining supply.
 - ▶ If $nc^* > \bar{c}$, then any $c_{i2} \in [0, \bar{c}]$ will fulfill the condition, with the consumers providing the remaining supply.

Part (a) - Prices

- Note that the inventor can always choose this $p_2 = \frac{w}{A}$ equilibrium since they can commit to future production.
- So the single period static monopoly profit is the lower bound for inventor's profit.
 - ▶ This would not be the case if the inventor could not commit to future production.
- For the linear case, profit asymptotes to zero with increasing n if you assume $p_2 > \frac{w}{A}$, so above some value of n the inventor just implements the first period monopoly + second period competitive solution.

Part (a) - Prices

● Answer:

- ▶ If $n = 1$, then prices are the monopoly price in both periods ($p_1 = v(c^*)$, where c^* solves $v'(c^*)c^* + v(c^*) - \frac{w}{A} = 0$).
- ▶ In general, if $n > 1$ then $p_1 > p_2$, $p_1 \geq v(c^*)$, and $v(c^*) \geq p_2$.
 - ★ We solved for the linear demand case explicitly. If n is high enough $p_1 = v(c^*)$ and $p_2 = \frac{w}{A}$.

Part (b) - Inventor Profits

"Is the inventor harmed by a larger value for n ? Would your answer be different if the product were also produced in a third period, fourth period, etc., with the same copying technology (n)?"

Part (b) - Inventor Profits

- Assume the market is in the $p_2 > \frac{w}{A}$ case (in the $p_2 = \frac{w}{A}$ case the inventor isn't harmed since they implement a single period monopoly).
- Suppose the initial solution is c_{i1}^* (so $c_2^* = nc_{i1}^*$). If n increases, the inventor can choose c_{i1}^{**} so $c_{i1}^{**} \leq c_{i1}^*$ and $c_2^* \leq c_2^{**}$ (with at least one strict) or so both quantities are strictly less than or greater than the previous quantities.
- In the first case, at least one quantity is getting further away from the monopoly quantity, $c_{i1}^* \leq c^* \leq c_2^*$ (and the other is at best the same distance), so profits are lower.
- In the second case, one quantity is getting closer to the monopoly quantity (the other further away), but that was an option before and the second quantity would have been closer. Since the inventor didn't chose this, it must be lower profit (both before and after the change in n).
- You can also show this with an envelope theorem argument.

Part (b) - Inventor Profits

- For more than two periods, assume the copying technology only operates on previous period consumption, not cumulative previous consumption.
 - This allows the intuition that if $n = 1$ the inventor can implement a static monopoly in all periods to go through.
- The argument is the same, but there are more cases to consider.
- The general intuition is higher n forces the inventor to disperse the chosen period quantities more, so the quantities are further from the static monopoly quantity on average.

Part (b) - Inventor Profits

- **Answer:** Assuming the market is in the $p_2 > \frac{w}{A}$ case, the inventor is harmed by larger n .
 - ▶ If $p_2 = \frac{w}{A}$ the inventor isn't harmed.
 - ▶ This result carries through to more periods.

Part (c) - Consumer Welfare

“Are consumers harmed by a larger value for n ? ”

Part (c) - Consumer Welfare

- Note that the inventor captures any consumer profits in period 2 by charging a higher price in period 1, so we only have to consider consumer surplus from the product's services.
- By the same argument in (b), c_2^* (weakly) increases with n but c_1^* (weakly) decreases with n (at least one strict if we're in the $p_2 > \frac{w}{A}$ case).
- Since these are opposing movements, the answer is not obvious in the general case (we don't know what $v(c_t)$ looks like, nor the magnitude of quantity changes).
- Assume $v(c_t) = \alpha - \beta c_t$ again. Then $u(c_t) = \int v(c_t) = \alpha c_t - \frac{\beta}{2} c_t^2$ (set the constant to zero) and $CS(c_t) = u(c_t) - v(c_t)c = \frac{\beta}{2} c_t^2$.

Part (c) - Consumer Welfare

- So total consumer surplus in both periods is:

$$\begin{aligned}CS(c_{i1}^*) + CS(nc_{i1}^*) &= \frac{\beta}{2} \left(\left(\frac{(n+1)(\alpha - \frac{w}{A})}{(n(n+1)+2)\beta} \right)^2 + \left(\frac{n(n+1)(\alpha - \frac{w}{A})}{(n(n+1)+2)\beta} \right)^2 \right) \\&= \frac{\beta}{2} \left(\frac{\alpha - \frac{w}{A}}{\beta} \right)^2 \left(\left(\frac{(n+1)}{(n(n+1)+2)} \right)^2 + \left(\frac{n(n+1)}{(n(n+1)+2)} \right)^2 \right) \\&= \frac{\beta}{2} \left(\frac{\alpha - \frac{w}{A}}{\beta} \right)^2 \frac{(n^2+1)(n+1)^2}{(n(n+1)+2)^2}\end{aligned}$$

- You can show this increases in n and asymptotes to a single period of competitive consumer surplus.
- However, for high enough n the inventor just implements the monopoly-then-competition equilibrium, so then increasing n has no affect on consumer welfare.

Part (c) - Consumer Welfare

- **Answer:** This is unclear in the general case, but in the linear demand case consumers benefit with higher n as long as $p_2 > \frac{w}{A}$.

Part (d) - Employee Production

“Suppose instead that consumers cannot reverse engineer the product, but that employees engaged in production can obtain that knowledge. In the second period, any former employee can start his own production operation with capacity n , with no obligation toward their former employer. What would be the equilibrium purchase price of the product in each period?”

Part (d) - Employee Production

- Now instead of consumers being able to reproduce the product, employees can.
- Note that the inventor may be able to pay lower wages in the first period since the employees may be able to make a profit by selling the product in the second period.
- Note that $\ell_{i1} = \frac{c_{i1}}{A}$.
- The inventor's problem (assuming $p_2 > \frac{w}{A}$) is now:

$$\max_{0 \leq c_{i1}, c_{i2}} \left(v(c_{i1}) - \frac{w - n(v(c_{i2} + n\frac{c_{i1}}{A}) - \frac{w}{A})}{A} \right) c_{i1} + \left(v(c_{i2} + n\frac{c_{i1}}{A}) - \frac{w}{A} \right) c_{i2}$$

Part (d) - Employee Production

- If you follow the same steps as before (assume $p_2 > \frac{w}{A}$, find FOC, solve for c_{j1}^* , check constraint, etc), you get essentially the same results.
- If $\frac{n}{A} \leq 1$, then the inventor can set the monopoly quantity in each period and extract the employees' profit with lower wages.
 - ▶ If $\frac{n}{A} < 1$, the inventor produces enough in addition to former employee production to get to the static monopoly quantity.
- If $\frac{n}{A} > 1$, then it's analogous to the model where consumers produce the product, price is (weakly) higher than the monopoly price in the first period and (weakly) lower in the second period.
 - ▶ Any difference between the two is due to labor being some multiple of quantity, so the comparable "n" in the consumer version is different.
 - ▶ The inventor extracts employee profit with lower wages instead of consumer profit with higher prices.

Part (d) - Employee Production

- **Answer:** Prices are essentially the same as in the consumer production case.
 - ▶ If $\frac{n}{A} \leq 1$, prices are the monopoly price in both periods.
 - ▶ If $\frac{n}{A} > 1$, first period price is higher than second period price, first period price is (weakly) higher than the monopoly price, and second period price is (weakly) lower than the monopoly price.

Part (e) - Non-Competes

“Would the inventor want to hire employees under “non-compete” clauses that prohibit them from producing or selling the product after they leave the inventor’s employment?”

Part (e) - Non-Competes

- If the inventor can hire employees under non-compete agreements, they can prevent employees from producing in the second period.
 - ▶ This essentially separates the periods for the inventor, what they choose in period 1 no longer has an effect in period 2.
 - ▶ They'll have to pay the full wage to employees, but it ends up being worth it since the wage reduction was just a way to capture employee profits (which can be less than monopoly profits).
- The inventor's problem with non-competes becomes:

$$\max_{0 \leq c_{i1}, c_{i2}} \left(v(c_{i1} - \frac{w}{A})c_{i1} + (v(c_{i2} - \frac{w}{A})c_{i2} \right)$$

- ▶ This is just a static monopoly problem in each period.
- **Answer:** If $\frac{n}{A} > 1$ then the inventor would want to use non-competes since they can get the static monopoly profit in both periods, which wasn't possible before.

Part (f) - Non-Competes vs Licensing

“What factors would determine whether the inventor hires employees with noncompete or with agreements to license production in the future?”

Part (f) - Non-Competes vs Licensing

- “Licensing” is what we’ve already been considering, employees are permitted to reproduce the product in the second period for some “payment” (wage reduction) to the inventor.
 - ▶ The payment is just the future profit from reproducing the product per unit labor, if it were lower than that the inventor should charge more, if more no employees would take the deal.
- **Answer:** The inventor wants to use non-competes if $\frac{n}{A} > 1$, otherwise they are indifferent between non-competes and licensing since they are already earning the double static monopoly profit.

Part (g) - Non-Compete Prohibition

"How would a legal prohibition of non-compete clauses affect wages paid by the inventor? The number of employees he hires?"

Part (g) - Non-Compete Prohibition

- Assume the inventor uses non-competes before prohibition.
- Prohibiting non-competes would reduce wages in the first period (second period wages unaffected).
 - ▶ The employees are still compensated the same for their labor, but some of it is in future profits instead of the wage.
- If $\frac{n}{A} \leq 1$ this has no affect on total employees hired since the quantity produced is unaffected (monopoly quantity in both periods). The inventor's first period labor is also unaffected since they are the only producer. However, it would reduce the inventor's hiring in the second period since now former employees are employing labor instead.
- If $\frac{n}{A} > 1$ Then total employees hired would fall in the first period and rise in the second (overall effect including both periods unclear). Inventor hiring falls in the first period and falls to zero in the second period.

Part (g) - Non-Compete Prohibition

- **Answer:** Assuming the inventor uses non-competes before prohibition, with the prohibition wages fall in the first period and inventor hiring falls in both periods.
 - ▶ If $\frac{n}{A} > 1$, inventor hiring in the second period falls to zero.

Additional

- For this question, we assumed the inventor had already invented something. What if they were considering inventing something and would have to pay a cost to do so?
- The inventor would invent if their profit from the product would be greater than the cost of invention.
- Their profit varies with n (and whether non-competes are permitted). They might choose to invent even without non-competes (which are sort of like patents).
- If n varies over possible inventions, innovation will be biased towards inventions with certain values of n , even though those don't necessarily correspond to inventions with the highest social benefit.