Price Theory I: Problem Set 2 Question 1

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Housekeeping Items

- Your PS1 grades along with comments are on Canvas.
  - Some high-level suggestions:
    - Setting up and solving pset questions is a great way to practice building models.
    - So, when you set up a model at the beginning, try to solve your model and answer questions with formal argument.
    - Writing psets is also a great way to practice writing academic papers.
    - In academia, a big part of our job is to convince people (particularly skeptics). So good writing goes a long way.
    - Psets in this class cover a wide range of topics. Pay attention to what may interest you. Write down your ideas, which you can develop later!
    - Do not worry too much about your grades. Try to focus on learning instead (this also goes for your other classes).

Housekeeping Items

- Although the pset questions are open-ended, and there are many ways to set up your model, it is good to keep in mind the pedagogical goals of this class.
  - Use the psets to improve your modeling skills and deepen your understanding of price theory.
  - You will make the most out of the class if you can apply what you have learned from lectures and readings to solving problem set questions.
  - From time to time, we will show how some other models can be done in a price theory framework and generate the same results.
- Make use of your TAs (in particular, TA office hours and TA sessions).
  - If you have questions about the class or psets, feel free to reach out to us via email.
  - If you have more substantive questions that require lengthy discussions, please see us during TA office hours or after TA sessions, or email us to arrange a meeting.

Marshallian and Hicksian Demand

- Marshallian demand follows from utility maximization subject to budget constraint:

$$\max_{X_1,\ldots,X_N} U(X_1, \ldots, X_N) \quad \text{s.t.} \quad \sum_{i=1}^{N} X_i P_i \leq M$$

  Write Marshallian demand: $X_i^M = X_i^M(P_1, \ldots, P_N, M)$ holding income constant.

- Hicksian demand follows from cost minimization subject to a given level of utility:

$$\min_{X_1,\ldots,X_N} \sum_{i=1}^{N} X_i P_i \quad \text{s.t.} \quad U(X_1, \ldots, X_N) = \overline{U}$$

  Write Hicksian demand: $X_i^H = X_i^H(P_1, \ldots, P_N, \overline{U})$ holding utility constant.

Marshallian and Hicksian Demand

- Relationship between Marshallian and Hicksian demand:
  - Slutsky correspondence:

$$X_i^H(P_1, ..., P_N, \overline{U}) = X_i^M\Big(P_1, ..., P_N, \underbrace{C(P_1, ..., P_N, \overline{U})}_{\text{cost function}}\Big)$$

    where cost function $C(P_1, ..., P_N, \overline{U}) = \min \sum_{i=1}^{N} X_i P_i$ s.t. $U(X_1, ..., X_N) = \overline{U}$.
  - Hicksian demand focuses on substitution effect. Marshallian demand also includes income effect. Slutsky equation:

$$\frac{\partial X_i^H}{\partial P_j} = \frac{\partial X_i^M}{\partial P_j} + \frac{\partial X_i^M}{\partial M} X_j$$

$$\epsilon_{ij}^M = \epsilon_{ij}^H - s_j \eta_i \qquad \text{(expressed in elasticities)}$$

$$\sum \epsilon_{ij}^M = \sum \epsilon_{ij}^H - \sum s_j \eta_i = \sum \epsilon_{ij}^H - 1 \qquad \text{(aggregation)}$$

Marshallian Adding Up, Homogeneity and Symmetry

Chicago Price Theory talks about Hicksian adding up, homogeneity and symmetry conditions in details in Chapter 2. Here we focus on Marshallian adding up, homogeneity and symmetry conditions.

- Adding Up:
  - Straight from binding budget constraint. Does not have anything to do with rationality.
  - To see this, take budget constraint $\sum_{i=1}^{N} X_i P_i = M$ and differentiate w.r.t. P_j:

$$\sum_{i=1}^{N} \frac{\partial X_i}{\partial P_j} P_i + X_j = 0 \iff \sum_{i=1}^{N} \frac{\partial X_i / X_i}{\partial P_j / P_j} \frac{X_i P_i}{M} + \frac{X_j P_j}{M} = 0 \iff \sum_{i=1}^{N} s_i \epsilon_{ij}^{M} + s_j = 0$$

  - Adding up says that an increase in the price of good j may increase the Marshallian demand for j (Giffen); but it must, weighted by prices, reduce the demand for other goods even more.

Marshallian Adding Up, Homogeneity and Symmetry

- Homogeneity:
  - Increasing income and all prices by the same proportion has no effect on choices.
  - Intuitively, "real" income and prices remain the same.

$$\sum_{j=1}^{N} \epsilon_{ij}^{M} + \eta_i = 0$$

- Symmetry:
  - Derived from Hicksian symmetry using Slutsky equation.
  - Hicksian symmetry $\frac{\partial X_i^H}{\partial P_j} = \frac{\partial X_j^H}{\partial P_i}$. Derivation on page 42 of *Chicago Price Theory*:

$$\frac{\partial X_i^M}{\partial P_j} = \frac{\partial X_j^M}{\partial P_i} + \frac{X_i X_j}{M}(\eta_j - \eta_i)$$

$$s_i \epsilon_{ij}^M = s_j \epsilon_{ji}^M + s_i s_j (\eta_j - \eta_i) \qquad \text{(expressed in elasticities)}$$

Marshallian Adding Up, Homogeneity and Symmetry

- Symmetry and Homogeneity imply Adding up

Proof.

From symmetry, $s_i \epsilon_{ij}^M = s_j \epsilon_{ji}^M + s_i s_j (\eta_j - \eta_i)$, sum over j:

$$s_i \sum_{j=1}^{N} \epsilon_{ij}^M = \sum_{j=1}^{N} s_j \epsilon_{ji}^M + s_i \sum_{j=1}^{N} s_j \eta_j - s_i \eta_i \sum_{j=1}^{N} s_j = \sum_{j=1}^{N} s_j \epsilon_{ji}^M + s_i - s_i \eta_i$$

$$\iff s_i \underbrace{\left[\sum_{j=1}^{N} \epsilon_{ij}^M + \eta_i \right]}_{=0 \text{ by homogeneity}} = \sum_{j=1}^{N} s_j \epsilon_{ji}^M + s_i$$

□

Marshallian Adding Up, Homogeneity and Symmetry

- Symmetry and Adding Up imply Homogeneity

Proof.

From symmetry, $s_i\epsilon_{ij}^M = s_j\epsilon_{ji}^M + s_is_j(\eta_j - \eta_i)$, sum over j:

$$s_i \sum_{j=1}^{N} \epsilon_{ij}^M = \sum_{j=1}^{N} s_j\epsilon_{ji}^M + s_i \sum_{j=1}^{N} s_j\eta_j - s_i\eta_i \sum_{j=1}^{N} s_j = \sum_{j=1}^{N} s_j\epsilon_{ji}^M + s_i - s_i\eta_i$$

$$\iff s_i\left[\sum_{j=1}^{N} \epsilon_{ij}^M + \eta_i\right] = \underbrace{\sum_{j=1}^{N} s_j\epsilon_{ji}^M + s_i}_{=0 \text{ by adding up}}$$

$\square$

Marshallian Adding Up, Homogeneity and Symmetry

- In two-good case, Adding Up and Symmetry imply Homogeneity

Proof.

Adding Up: $s_1\epsilon_{11}^M + s_2\epsilon_{21}^M + s_1 = 0$

Homogeneity: $\epsilon_{11}^M + \epsilon_{12}^M + \eta_1 = 0$

Symmetry: $s_2\epsilon_{21}^M = s_1\epsilon_{12}^M + s_1 s_2(\eta_1 - \eta_2)$

Substitute Symmetry into Adding Up:

$$
\begin{aligned}
& s_1\epsilon_{11}^M + s_1\epsilon_{12}^M + s_1 s_2(\eta_1 - \eta_2) + s_1 = 0 \\
\Longleftrightarrow\ & \epsilon_{11}^M + \epsilon_{12}^M + s_2\eta_1 \underbrace{- s_2\eta_2 + 1}_{= s_1\eta_1} = 0 \\
\Longleftrightarrow\ & \epsilon_{11}^M + \epsilon_{12}^M + \eta_1 = 0
\end{aligned}
$$

□

The Question

"Here we consider the 'recreational' demand for opioids. The applicability of rational choice to the demand for addictive drugs is a matter of vigorous debate. The argument against notes that drugs cause 'persistent changes in the brain structures and functions known to be involved in the motivation of behavior' and that frequently 'the addict expresses a desire not to consume drugs prior to, after, or even during the drug intake.' We therefore do not model individual choice as a utility-maximization problem.

We begin by treating all opioids as a homogeneous commodity with a single price p. Therefore, consumers with income y face a budget constraint $y = rc + pq$, where q is the quantity of opioids, c is the quantity of all other goods, and r is their price."

This problem is very similar to the underage drinking problem we solved last week. However, in this problem, we assume consumers are "irrational" and do not rely on utility maximization.

Basic Setup

- **Consumers:**
  - There is a continuum of consumers. Normalize the measure of consumers to one (WLOG).
  - Each consumer makes consumption choice between opioids (assumed to be homogeneous) and a consumption bundle of all other goods. However, they do not solve utility maximization problems and are "irrational".
  - Assume consumer likes to consume more of something, so choice is on budget line.
  - Consumers are independent.
  - Price of consumption bundle is r.
- **Opioids Production and Sales:**
  - Opioids are produced competitively at constant marginal cost. Thus, $p = MC$.
  - For now, we ignore any fixed cost that may be incurred to obtain opioids.

Part (a)

"Suppose that each consumer chooses a point on his budget set randomly. Each point is equally likely to be chosen. What is market-level opioid demand as a function of p and the distribution of income? Could an increase in p, holding incomes constant, increase aggregate opioid consumption?"

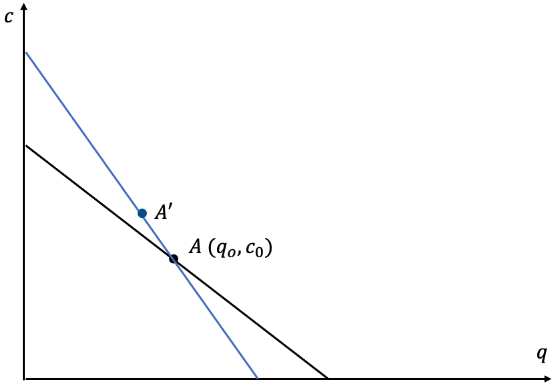
Part (a) - Market Demand (Impulsive Consumers)

- Suppose that each consumer chooses a point on his budget set randomly. Each point is equally likely to be chosen.
  - See Becker (1962)
  - Consider the simple case that consumers have the same income y.
  - For consumer i, his opioid consumption $q^i \sim U\left[0, \frac{y}{p}\right]$.
  - Aggregate demand:

$$Q = \mathbb{E}[q^i] = \frac{y}{2p}$$

  - When p increases, holding income constant, the aggregate opioid consumption Q decreases.
  - Thus, we have generated downward-sloping market demand even with "irrational" consumers.

Part (a) - Market Demand (Impulsive Consumers)



Part (a) - Market Demand (Impulsive Consumers)

- More generally, assume that consumers have heterogeneous income. Each consumer's income y^i is an i.i.d. draw from a distribution with cdf $F(\cdot)$ (with pdf $f(\cdot)$).
  - $q^i = \left(\frac{q^i}{y^i}\right) y^i$, where $\frac{q^i}{y^i} \sim U\left[0, \frac{1}{p}\right]$ and $y^i \sim F$.
  - $\frac{q^i}{y^i}$ and y^i are independent.
  - Use the bivariate transformation method to derive pdf of q^i: $g(q) = p \int_0^{\frac{1}{p}} \frac{1}{x} f\left(\frac{q}{x}\right) dx$ Derivation

$$\mathbb{E}[q^i] = \int_0^{\infty} q g(q) dq = p \int_0^{\infty} \int_0^{\frac{1}{p}} \frac{q}{x} f\left(\frac{q}{x}\right) dx dq = p \int_0^{\frac{1}{p}} x \underbrace{\left[\int_0^{\infty} \frac{q}{x} f\left(\frac{q}{x}\right) d\frac{q}{x}\right]}_{= \mathbb{E}[y^i]} dx = \frac{\mathbb{E}[y^i]}{2p}$$

  - Aggregate demand: $Q = \mathbb{E}[q^i] = \frac{\mathbb{E}[y^i]}{2p}$, which is decreasing in p.

Part (b)

"Does market-level demand satisfy symmetry, homogeneity, or adding up?"

Part (b) - Market Demand (Impulsive Consumers)

- Since for each consumer i, $c^i = \frac{y^i - pq^i}{r}$. Aggregate demand for other goods:

$$C = \frac{\mathbb{E}[y^i]}{r} - \frac{p}{r}\frac{\mathbb{E}[y^i]}{2p} = \frac{\mathbb{E}[y^i]}{2r}$$

- Let's calculate elasticities.
  - Price elasticities: $\epsilon_{qc} = \epsilon_{cq} = 0$, $\epsilon_q = \frac{\partial Q}{\partial p}\frac{p}{Q} = -1$, $\epsilon_c = \frac{\partial C}{\partial r}\frac{r}{C} = -1$
  - Income elasticities: $\eta_q = \frac{\partial Q}{\partial \mathbb{E}[y^i]}\frac{\mathbb{E}[y^i]}{Q} = 1$, $\eta_c = \frac{\partial C}{\partial \mathbb{E}[y^i]}\frac{\mathbb{E}[y^i]}{C} = 1$
- Check symmetry, homogeneity and adding up.
  - ✓ Symmetry: $s_c\epsilon_{cq} = s_q\epsilon_{qc} + s_1 s_2(\eta_q - \eta_c)$
  - ✓ Homogeneity: $\epsilon_q + \epsilon_{qc} + \eta_q = 0$
  - ✓ Adding up: $s_q\epsilon_q + s_2\epsilon_{cq} + s_q = 0$

Part (c)

"Suppose instead that most consumers set $q = 0$, but that a random subset of them spend all of their disposable income on opioids. [Here disposable income refers to income after spending on necessities such as food.] Does market-level demand satisfy symmetry, homogeneity, or adding up?"

Part (c) - Market Demand (Some Drug Abusers)

- Suppose instead that most consumers set $q = 0$. However, a random fraction λ of consumers spend all of their disposable income on opioids.
- Suppose that consumption of necessities (e.g., food), c_0, is the same across individuals. Treat necessities as numeraire with price r_0. Spending on necessities is thus $r_0 c_0$.
- Similar to what we have done in Part (a), market demand for opioids can be derived:
  - Focus on the measure λ of consumers who spend all of their disposable income on opioids, since the total demand for opioids by the other $1 - \lambda$ consumers is zero.
  - Among the consumers who spend all of their disposable income on opioids, consumer i's opioid consumption is $q^i = \frac{y^i - r_0 c_0}{p}$.
    - ★ Here we implicitly assume $y^i \geq r_0 c_0$, as consumers need to consume c_0 to survive, so consumers with $y^i < r_0 c_0$ have died.
    - ★ In real life, many government assistance programs such as SNAP.

Part (c) - Market Demand (Some Drug Abusers)

- Aggregate demand for opioids:

$$Q = \frac{\lambda}{p}\int_{r_0 c_0}^{\hat{y}} (y^i - r_0 c_0) dF(y^i) = \frac{\lambda}{p}\int_{r_0 c_0}^{\hat{y}} y^i dF(y^i) - \frac{\lambda}{p} r_0 c_0 \int_{r_0 c_0}^{\hat{y}} dF(y^i) = \frac{\lambda}{p}(\mathbb{E}[y^i] - r_0 c_0)$$

since $y^i \in [r_0 c_0, \hat{y})$.

- Aggregate demand for other goods:

$$C = \frac{1}{r}\left[\mathbb{E}[y^i] - pQ\right] = \frac{1}{r}\left\{(1-\lambda)\mathbb{E}[y^i] + \lambda r_0 c_0\right\}$$

Part (c) - Market Demand (Some Drug Abusers)

- Calculate elasticities:
  - Price elasticities: $\epsilon_{qc} = \epsilon_{cq} = 0$, $\epsilon_q = \epsilon_c = -1$
  - Income elasticities:

$$\eta_q = \frac{\partial Q}{\partial \mathbb{E}[y^i]} \frac{\mathbb{E}[y^i]}{Q} = \frac{\mathbb{E}[y^i]}{\mathbb{E}[y^i] - r_0 c_0}$$

$$\eta_c = \frac{\partial C}{\partial \mathbb{E}[y^i]} \frac{\mathbb{E}[y^i]}{C} = \frac{(1-\lambda)\mathbb{E}[y^i]}{(1-\lambda)\mathbb{E}[y^i] + \lambda r_0 c_0}$$

    Thus, $\eta_q \neq \eta_c$ unless $r_0 c_0 = 0$.

  - Check symmetry, homogeneity and adding up:
    - ★ X Symmetry: $s_c \epsilon_{cq} = s_q \epsilon_{qc} + s_1 s_2 (\eta_q - \eta_c)$
    - ★ X Homogeneity: $\epsilon_q + \epsilon_{qc} + \eta_q = 0$
    - ★ ✓ Adding up: $s_q \epsilon_q + s_2 \epsilon_{cq} + s_q = 0$

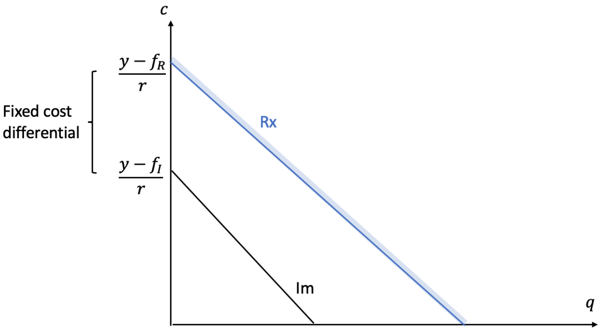
Extended Setup

- The extended setup follows Mulligan (2021).
- **Opioids:**
  - Two broad categories of opioids: prescriptions (Rx) and illicitly manufactured (Im).
    - Im opioids (such as heroin and fentanyl) are more potent than Rx opioids.
  - Rx opioids can be obtained by prescription at fixed cost f_R. Im opioids can be obtained in illegal markets at fixed cost f_I. Suppose $f_I > f_R$ ($f_I \leq f_R$ case is analogous).
  - Continue to assume that opioids are produced competitively at constant marginal costs. Let p_R denote marginal price of Rx opioids and p_I denote marginal price of Im opioids.
  - Let q_R and q_I denote quantities of Rx and Im opioids respectively, measured in morphine gram equivalents (MGE).
  - Opioid consumption $q = q_R + q_I$.

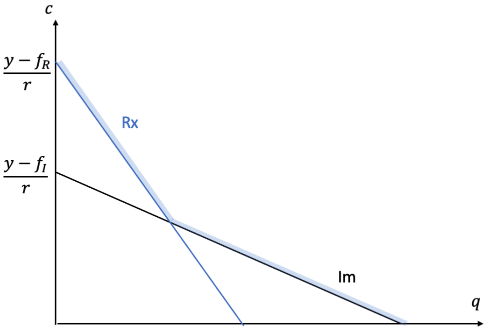
Part (d)

“Draw an individual’s budget set in the $[q, c]$ plane. What points in the set are dominated by at least one other point in the set?”

Part (d) - Budget Constraint



Case 1: Im marginal prices are high



Case 2: Im marginal prices are low

Part (d) - Budget Constraint

- Note that:
  - Im marginal prices could be higher or lower than Rx marginal prices.
    - Rx opioids were indeed much cheaper than Im opioids in the past, and were known as "poor man's heroin". However, heroin is now cheaper and easier to get than prescription opioids. (Source: Mulligan (2021) and National Institute on Drug Abuse)
  - Case 1: if Im prices are higher than Rx prices, consuming Rx opioids dominates (since we have assumed $f_I > f_R$).
  - Case 2: if Im prices are lower than Rx prices, we generate a nonlinear budget constraint with a kink. For low volume users, consuming Rx dominates; for high volume users, consuming Im dominates.

Part (e)

"If an individual spends all disposable income on opioids, but never chooses a dominated point from the budget set, how does his total demand vary with the price of prescription opioids? Does market-level demand satisfy symmetry, homogeneity, or adding up?"

Part (e) - Consumer Demand

- Consider a consumer who spends all disposable income $y - r_0 c_0$ on opioids.
  - Case 1: Im prices are high
    - ★ The consumer only consumes Rx opioids. His opioid demand is

$$q = \begin{cases} \frac{y - f_R - r_0 c_0}{p_R} & \text{if} \quad y \geq f_R + r_0 c_0 \\ 0 & \text{if} \quad y < f_R + r_0 c_0 \end{cases}$$

  - Case 2: Im prices are low
    - ★ Low volume consumer purchases Rx opioids; high volume consumer purchases Im opioids.

$$q = \begin{cases} 0 & \text{if} \quad y < f_R + r_0 c_0 \\ \frac{y - f_R - r_0 c_0}{p_R} & \text{if} \quad f_R + r_0 c_0 \leq y < f_I + r_0 c_0 \\ \frac{y - f_R - r_0 c_0}{p_R} & \text{if} \quad q < \frac{f_I - f_R}{p_R - p_I} \quad \text{and} \quad y \geq f_I + r_0 c_0 \\ \frac{y - f_I - r_0 c_0}{p_I} & \text{if} \quad q \geq \frac{f_I - f_R}{p_R - p_I} \quad \text{and} \quad y \geq f_I + r_0 c_0 \end{cases}$$

Part (e) - Consumer Demand

- Now consider the effect of an increase in p_R on demand.
  - Case 1: Im prices are high. The consumer's total opioid demand q is decreasing in p_R.
  - Case 2: Im prices are low. Rewrite the expression for q:

$$q = \begin{cases} 0 & \text{if} \quad y < f_R + r_0 c_0 \\ \frac{y - f_R - r_0 c_0}{p_R} & \text{if} \quad f_R + r_0 c_0 \leq y < \frac{p_R}{p_R - p_I} f_I - \frac{p_I}{p_R - p_I} f_R + r_0 c_0 \\ \frac{y - f_I - r_0 c_0}{p_I} & \text{if} \quad y \geq \frac{p_R}{p_R - p_I} f_I - \frac{p_I}{p_R - p_I} f_R + r_0 c_0 \end{cases}$$

    - ★ Since $\frac{p_R}{p_R - p_I} f_I - \frac{p_I}{p_R - p_I} f_R + r_0 c_0$ is decreasing in p_R, as p_R increases, the threshold decreases. As a result, the consumer is more likely to be in the Im segment of the budget constraint.
    - ★ Since Im prices are lower than Rx, he would end up consuming more.

Part (e) - Market Demand

- Next, consider market-level demand for opioids.
  - Case 1: Im prices are high. Market opioid demand:

$$Q = \lambda \int_{f_R + r_0 c_0}^{\infty} \frac{y - f_R - r_0 c_0}{p_R} dF(y) = \frac{\lambda}{p_R} \int_{f_R + r_0 c_0}^{\infty} y dF(y) - \frac{\lambda}{p_R} [1 - F(f_R + r_0 c_0)](f_R + r_0 c_0)$$

  - Case 2: Im prices are low. Market opioid demand: (let $H \equiv \frac{p_R}{p_R - p_I} f_I - \frac{p_I}{p_R - p_I} f_R + r_0 c_0$)

$$\begin{aligned} Q &= \lambda \Big\{ \int_{f_R + r_0 c_0}^{H} \frac{y - f_R - r_0 c_0}{p_R} dF(y) + \int_{H}^{\infty} \frac{y - f_I - r_0 c_0}{p_I} dF(y) \Big\} \\ &= \frac{\lambda}{p_R} \int_{f_R + r_0 c_0}^{H} y dF(y) + \frac{\lambda}{p_I} \int_{H}^{\infty} y dF(y) - \frac{\lambda}{p_R} (f_R + r_0 c_0)[F(H) - F(f_R + r_0 c_0)] \\ &\quad - \frac{\lambda}{p_I} (f_I + r_0 c_0)[1 - F(H)] \end{aligned}$$

Part (e) - Market Demand

- Market-level demand for other goods:
  - Case 1: Im prices are high.

$$C = \frac{1}{r}\left[\mathbb{E}[y] - p_R Q\right] = \frac{1}{r}\left[\mathbb{E}[y] - \lambda \int_{f_R + r_0 c_0}^{\infty} y dF(y) + \lambda[1 - F(f_R + r_0 c_0)](f_R + r_0 c_0)\right]$$

  - Case 2: Im prices are low.

$$\begin{aligned} C &= \frac{1}{r}\left[\mathbb{E}[y] - \text{spending on opioids}\right] \\ &= \frac{1}{r}\left\{\mathbb{E}[y] - \lambda \int_{f_R + r_0 c_0}^{H} (y - f_R - r_0 c_0) dF(y) - \lambda \int_{H}^{\infty} (y - f_I - r_0 c_0) dF(y)\right\} \end{aligned}$$

- Verify whether market demand satsify symmetry, homogeneity, and adding up.

Part (f)

"If instead an individual spends a fixed budget share on opioids (including variable and fixed costs), but never chooses a dominated point from the budget set, how does his total demand vary with the price of prescription opioids? Does market-level demand satisfy symmetry, homogeneity, or adding up?"

Part (f) - Consumer Demand (Fixed Budget Share)

- Suppose a consumer spends a fixed budget share k on opioids (variable and fixed costs). For simplicity, ignore spending on necessities (so disposable income $= y$).
  - Case 1: Im prices are high
    - ★ The consumer only consumes Rx opioids. His opioid demand is

$$q = \begin{cases} \frac{ky - f_R}{p_R} & \text{if} \quad y \geq \frac{f_R}{k} \\ 0 & \text{if} \quad \text{otherwise} \end{cases}$$

  - Case 2: Im prices are low
    - ★ Low volume consumer purchases Rx; high volume consumer purchases Im.

$$q = \begin{cases} 0 & \text{if} \quad y < \frac{f_R}{k} \\ \frac{ky - f_R}{p_R} & \text{if} \quad q < \frac{f_I - f_R}{p_R - p_I} \quad \text{and} \quad y \geq \frac{f_R}{k} \\ \frac{ky - f_I}{p_I} & \text{if} \quad q \geq \frac{f_I - f_R}{p_R - p_I} \quad \text{and} \quad y \geq \frac{f_R}{k} \end{cases}$$

Part (f) - Consumer Demand (Fixed Budget Share)

- Now consider the effect of an increase in p_R on demand.
  - Case 1: Im prices are high. q is decreasing in p_R.
  - Case 2: Im prices are low. Rewrite the expression for q:

$$q = \begin{cases} 0 & \text{if} \quad y < \frac{f_R}{k} \\ \frac{ky - f_R}{p_R} & \text{if} \quad \frac{f_R}{k} \leq y < \frac{p_R f_I - p_I f_R}{k(p_R - p_I)} \\ \frac{ky - f_I}{p_I} & \text{if} \quad y \geq \frac{p_R f_I - p_I f_R}{k(p_R - p_I)} \end{cases}$$

    - ★ Since $\frac{p_R f_I - p_I f_R}{k(p_R - p_I)}$ is decreasing in p_R, as p_R increases, the threshold decreases. The consumer thus is more likely to be in the Im segment of the budget constraint.
    - ★ Since Im prices are lower than Rx, he would consume more.

Part (f) - Market Demand (Fixed Budget Share)

- Next, consider market-level demand for opioids and other goods.
  - Assume that a fraction $1-\rho$ of consumers do not consume opioids.
  - Case 1: Im prices are high. Market demand:

$$Q = \rho \int_{\frac{f_R}{k}}^{\infty} \frac{ky - f_R}{p_R} dF(y) = \frac{\rho}{p_R}\left\{ k \int_{\frac{f_R}{k}}^{\infty} y dF(y) - f_R\left[1 - F\left(\frac{f_R}{k}\right)\right]\right\}$$

$$C = \frac{1}{r}\mathbb{E}[y] - \frac{\rho}{r}\left\{ k \int_{\frac{f_R}{k}}^{\infty} y dF(y) - f_R\left[1 - F\left(\frac{f_R}{k}\right)\right]\right\}$$

  - Case 1: Im prices are high. Market demand:

$$Q = \frac{\rho}{p_R} \int_{\frac{f_R}{k}}^{\frac{p_R f_I - p_I f_R}{k(p_R - p_I)}} (ky - f_R) dF(y) + \frac{\rho}{p_I} \int_{\frac{p_R f_I - p_I f_R}{k(p_R - p_I)}}^{\infty} (ky - f_I) dF(y)$$

$$C = \frac{1}{r}\mathbb{E}[y] - \frac{\rho}{r} \int_{\frac{f_R}{k}}^{\frac{p_R f_I - p_I f_R}{k(p_R - p_I)}} (ky - f_R) dF(y) - \frac{\rho}{r} \int_{\frac{p_R f_I - p_I f_R}{k(p_R - p_I)}}^{\infty} (ky - f_I) dF(y)$$

Part (g)

"How would the demand curve from (f) be differed if individuals maximized a Cobb-Douglas utility function over c and q?"

Part (g) - Cobb-Douglas Utility

- In the previous sections, we consider "irrational" consumer behavior. Now consider a utility-maximizing consumer with a Cobb-Douglas utility function:

$$u(q, c) = q^k c^{1-k}$$

- Case 1: Im prices are high. The consumer's problem:

$$\max_{q,c} \quad q^k c^{1-k} \quad \text{s.t.} \quad \begin{cases} p_R q + rc \leq y - f_R & \text{if} \quad q > 0 \\ rc \leq y & \text{if} \quad q = 0 \end{cases}$$

  - Note that if $q = 0$, utility is zero. Thus, as long as the consumer's income can cover fixed cost, $y \geq f_R$, he would like to spend $q > 0$ on opioids.
  - Solve reduced problem $\max_{q,c} q^k c^{1-k} \quad \text{s.t.} \quad p_R q + rc \leq y - f_R$, FOCs: $\frac{p_R q}{rc} = \frac{k}{1-k}$ Proof. This is equivalent to spending fixed budget share k on opioids.

Part (g) - Cobb-Douglas Utility

- Case 2: Im prices are low. The consumer solves two problems and choose the option that yields the highest utility:

$$\max_{q,c} \quad q^k c^{1-k} \quad \text{s.t.} \quad \begin{cases} p_R q + rc \leq y - f_R & \text{if} \quad q > 0 \\ rc \leq y & \text{if} \quad q = 0 \end{cases}$$

$$\max_{q,c} \quad q^k c^{1-k} \quad \text{s.t.} \quad \begin{cases} p_I q + rc \leq y - f_I & \text{if} \quad q > 0 \\ rc \leq y & \text{if} \quad q = 0 \end{cases}$$

  - If $y < f_R$, the consumer cannot consume opioids, i.e., $q = 0$.
  - If $f_R \leq y < f_I$, the consumer can only consume Rx. The consumer maximizes utility subject to $p_R q + rc \leq y - f_R$. FOCs: $\frac{p_R q}{rc} = \frac{k}{1-k}$.
  - If $y \geq f_I$, consumer chooses Im when $p_R \geq \left(\frac{y-f_R}{y-f_I}\right)^{\frac{1}{k}} p_I$ Proof. FOCs: $\frac{p_I q}{rc} = \frac{k}{1-k}$.

Part (g) - Cobb-Douglas Utility

- Now focus on Case 2, and compare the consumer's demand to that of an irrational consumer in Part f.
  - In Part f, consumer switches from Rx to Im when

$$y \geq \frac{p_R f_I - p_I f_R}{k(p_R - p_I)} \iff p_R \geq \frac{ky - f_R}{ky - f_I} p_I$$

  - Here, consumer switches from Rx to Im when

$$p_R \geq \left(\frac{y - f_R}{y - f_I} \right)^{\frac{1}{k}} p_I$$

  - We can show that

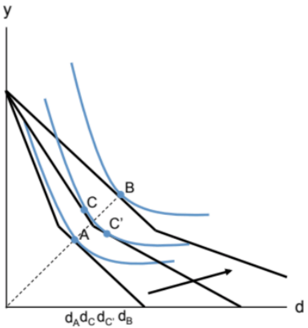
$$\left(\frac{y - f_R}{y - f_I} \right)^{\frac{1}{k}} p_I < \frac{ky - f_R}{ky - f_I} p_I$$

Proof

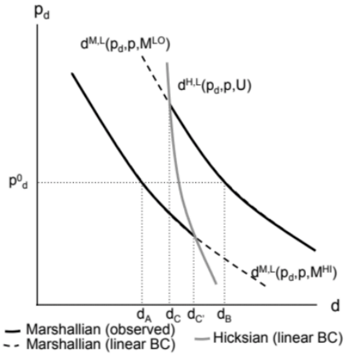
Part (g) - Cobb-Douglas Utility

- Thus, the "irrational" demand curve (Part f) and the rational demand curve for a consumer with Cobb-Douglas preferences (Part g) are similar, in the sense that the consumer spends a fixed budget share on opioids.
- An important difference is the switching point from Rx to Im. In particular, the rational consumer switches from Rx to Im before Rx is strictly dominated. The irrational consumer holds out a bit longer.

Part (g)



(a) Maximization Problem



(b) Demands

References

Becker, Gary S., "Irrational Behavior and Economic Theory," *Journal of Political Economy*, 1962, *70* (1), 1–13.

Mulligan, Casey B., "Prices and Policies in Opioid Markets," Working Paper 2021.

Appendix - Bivariate Transformation Method

- $X \sim f_X(x)$ and $Y \sim f_Y(y)$ are independent. What is the pdf of $Z \equiv XY$? Let

$$\begin{cases} Z = XY \\ W = X \end{cases} \Longrightarrow \begin{cases} X = W \\ Y = \frac{Z}{W} \end{cases}$$

- Jacobian:

$$J = \begin{vmatrix} \frac{\partial x}{\partial z} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial z} & \frac{\partial y}{\partial w} \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ \frac{1}{w} & -\frac{z}{w^2} \end{vmatrix} = -\frac{1}{w}$$

- Joint pdf of Z and W

$$f_{Z,W}(z,w) = \frac{1}{|w|} f_X(x) f_Y\left(\frac{z}{w}\right)$$

- Take integral over w to get $f_Z(z)$

$$f_Z(z) = \int_\Omega \frac{1}{|x|} f_X(x) f_Y\left(\frac{z}{x}\right) dx$$

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Appendix - Cobb-Douglas Utility I

- Consider problem $\max_{q,c} \quad q^k c^{1-k} \quad \text{s.t.} \quad p_R q + rc = y - f_R$.
- Lagrangian: $\mathcal{L} = q^k c^{1-k} + \mu(y - f_R - p_R q - rc)$
- FOCs:

$$[q]: \quad kq^{k-1}c^{1-k} = \mu p_R$$

$$[c]: \quad (1-k)q^k c^{-k} = \mu r \iff \mu = \frac{(1-k)q^k c^{-k}}{r}$$

Thus,

$$kq^{k-1}c^{1-k} = \frac{(1-k)q^k c^{-k}}{r} p_R \iff \frac{p_R q}{rc} = \frac{k}{1-k}$$

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Appendix - Cobb-Douglas Utility II

- Solution to the first problem is: $\frac{p_R q}{rc} = \frac{k}{1-k}$. Since $p_R q + rc = y - f_R$,

$$p_R q = k(y - f_R) \iff q = \frac{k}{p_R}(y - f_R) \quad rc = (1-k)(y - f_R) \iff c = \frac{1-k}{r}(y - f_R)$$

  Solution to the second problem is: $\frac{p_I q}{rc} = \frac{k}{1-k}$. Since $p_I q + rc = y - f_I$,

$$p_I q = k(y - f_I) \iff q = \frac{k}{p_I}(y - f_I) \quad rc = (1-k)(y - f_R) \iff c = \frac{1-k}{r}(y - f_I)$$

- Substitute into Cobb-Douglas utility, purchasing Im dominates when

$$\left(\frac{k}{p_R}\right)^k \left(\frac{1-k}{r}\right)^{1-k} (y - f_R) \leq \left(\frac{k}{p_I}\right)^k \left(\frac{1-k}{r}\right)^{1-k} (y - f_I) \iff \left(\frac{p_R}{p_I}\right)^k \geq \frac{y - f_R}{y - f_I}$$

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Appendix - Cobb-Douglas Utility III

- We only need to show that

$$\left(\frac{ky - f_R}{ky - f_I}\right)^k > \frac{y - f_R}{y - f_I}$$

- Let $\phi(k) \equiv \left(\frac{ky-f_R}{ky-f_I}\right)^k$, take derivative with respect to k:

$$\frac{\partial \phi(k)}{\partial k} = k\left(\frac{ky - f_R}{ky - f_I}\right)^{k-1} \frac{y(ky - f_I) - y(ky - f_R)}{(ky - f_I)^2} < 0$$

- Thus, since $k < 1$, $\phi(k) > \phi(1)$.

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