Economics 301

Problem Set due October 20, 2023 Upload your answers to canvas

1) Consider the production function $F(L,K_1,K_2,\ldots K_N)$ which gives output as a function of labor and N different capital inputs, $K_1,K_2 \ldots K_N$. Typically we desire to aggregate these capital inputs into a single capital stock measure, K.

a) If the production function can be written as $F(L,G(K_1,\ldots.K_N))$, where G is homothetic, define a capital aggregate, K and an associated capital input rental rate, R, that will allow us to deal with a single capital good.

b) Suppose further that each capital rental rate reflects a constant rate tax on the input amount. Each input has its own tax rate. What do your findings from (a) suggest about defining an aggregate tax rate on capital inputs?

c) If the relative rental rates of the capital goods are fixed and the production function has the general form $F(L,K_1,K_2,\ldots K_N)$ define a capital aggregate and a capital rental rate measure that will allow us to deal with a single capital good.

d) Under what conditions, if any, would the aggregate capital rental rate measure put negative weight on the rental rate of one of the capital inputs?

e) Under what conditions, if any, would the owner of the firm, who rents the various inputs and sells the output, be better off when one of the capital inputs became more expensive?

f) Can you find a single empirical strategy that would work if you knew that either the conditions in part (a) or part (c) were true but did not know which was correct (i.e. is there a general theory of constructing a capital stock measure)?

2) What is the relationship between the Slutsky equation and Marshall's Laws of Derived demand? Recall that Marshall's Laws are often analyzed with an equation (hereafter, "the ML equation") that, like Slutsky's, decomposes a price response into a "scale effect" and a "substitution effect." See Chapter 11 of Chicago Price Theory for a discussion of Marshall's Laws.

a) First discuss and analyze the relationship assuming the mapping from consumption amounts to utility (a.k.a., output) exhibits constant returns to scale.

b) Cite an application where the Slutsky equation would be your preferred analytical tool. Cite another where the ML equation would be preferred.

c) Where in your derivation does the partial elasticity of substitution (in output) between two quantities appear? If all pairs of quantities had the same elasticity of substitution, what form would the utility function take? The cost function?

d) What does this approach say about inferior inputs?

e) Now drop the constant returns assumption and show the revised Slutsky and ML equations. [*Hint 1*: revisit part (a) with a supply-demand diagram and think about how diminishing returns changes it. *Hint 2*: Define "returns to scale" as the ratio of average cost to marginal cost and use that as a summary statistic.]