

Price Theory I: Problem Set 1 Question 1

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Problem Set 1 Question 1

“Here we look at the determinants of inflation using the three-good version of the basic consumer theory. The goods are leisure, food, and “money.” To limit the substitution possibilities, we assume that the non-satiable convex preferences for these three goods can be represented by an additively separable utility function.

Money is in fixed supply and everyone is endowed with an equal share of the total. Normalize its equilibrium price to one. Consumers also have a fixed time endowment which they can use for leisure or supply to firms that produce food from labor with a linear technology.”

Common Confusion

- “How are firms paying workers? Do they need money?”
- “Can you buy goods with money?”
- I think this confusion came from “money” being in the utility function.
- A couple ways to (hopefully) clarify this:
 - ▶ Label “money” as something else, for example, cars (which are in fixed supply and the numeraire for some reason).
 - ▶ Think about firms paying workers “in kind”. Workers make food, firm takes the food, turns around, and gives it right back as their (equilibrium) wage.

Setup - Consumers

- Representative consumer maximizes utility
- Endowments
 - ▶ Money endowment: M
 - ▶ Time endowment: T
- Preferences
 - ▶ “non-satiable, convex preferences [...] additively separable utility function”
 - ▶ $U(f, \ell, m) = u_f(f) + u_\ell(\ell) + u_m(m)$ Notation
 - ▶ All three subutility functions are strictly increasing, concave, and continuously differentiable.
 - ▶ Additionally assume food and leisure subutility functions have the properties $\lim_{x \rightarrow 0} u'_x(x) = \infty$, $\lim_{x \rightarrow \infty} u'_x(x) = 0$ (this allows for quasi-linear utility in money).
- “Technology”
 - ▶ The consumer can use their time endowment for either work (h) or leisure. $T = \ell + h$

Setup - Firms

- “firms that produce food from labor with a linear technology”
- Many firms in a competitive market with a common linear production function.
- $F = AH$ where F is aggregate food supply, $A \geq 0$ is (labor) productivity, and H is aggregate work time.
- Firms pay workers a wage w . (It's probably obvious what that wage is.)

Setup - Equilibrium

- A market equilibrium is a set of prices and quantities such that the following hold:
 - ▶ Consumer Optimality
 - ★ Taking prices as given, the representative consumer maximizes utility with their chosen consumption bundle.
 - ▶ Firm Optimality
 - ★ Taking prices as given, firms maximize profits with their chosen inputs and outputs.
 - ▶ Market Clearing
 - ★ Supply = demand for food, leisure/work, and money.

Part (a) - Inferior Goods

“In this model, can leisure, food, or money be an inferior good? Can more than one of them be an inferior good?”

Part (a) - Inferior Goods

- Recall: An **inferior good** is a good with negative income elasticity (if income goes up, demand for the good goes down). In math (for food):

$$\eta_f = \frac{I}{f} \frac{\partial f^M(p_f, p_\ell, p_m, I)}{\partial I} < 0$$

- Where η_f is the income elasticity of food, I is income, p_f , p_ℓ , p_m are prices, and $f^M(p_f, p_\ell, p_m, I)$ is Marshallian demand for food.
- Since $\frac{I}{f} > 0$, we just need to know the sign of $\frac{\partial f^M(p_f, p_\ell, p_m, I)}{\partial I}$.

Part (a) - Inferior Goods

● Intuition:

- ▶ Food, leisure, and money are all "goods" (as opposed to "bads") and the consumption level of a good has no affect on the marginal utility from consuming the other goods (additively separable utility function).
- ▶ At the optimal consumption bundle for any prices and income, marginal utility per dollar for the three goods are equal.
- ▶ Focus on the non-quasi-linear case. If income increases (and prices are unchanged), the consumer will buy more of at least one good (since they are "goods"). But if they buy less of one of the goods, it will not be the case that marginal utilities per dollar are equal! (Due to declining marginal utility and additive separability.)
- ▶ So, with an increase in income the consumer demands more of all three goods, implying none can be inferior. (In the quasi-linear case, the consumer demands more money and the same amount of the others.)

Part (a) - Inferior Goods

- Formally:

- ▶ Let $p_m = 1$. Now find Marshallian demand.

$$\max_{f, \ell, m} U(f, \ell, m)$$

$$\text{s.t. } p_f f + p_\ell \ell + m \leq I$$

$$\iff \max_{f, \ell, m} u_f(f) + u_\ell(\ell) + u_m(m)$$

$$\text{s.t. } p_f f + p_\ell \ell + m \leq I$$

- ▶ Lagrangian:

$$\mathcal{L}(f, \ell, m, \lambda) = u_f(f) + u_\ell(\ell) + u_m(m) + \lambda(I - p_f f - p_\ell \ell - m)$$

Part (a) - Inferior Goods

- FOC:

$$u'_f(f) = \lambda p_f$$

$$u'_\ell(\ell) = \lambda p_\ell$$

$$u'_m(m) = \lambda$$

$$I = p_f f + p_\ell \ell + m$$

- Solve for λ (sort of):

$$I = p_f (u'_f)^{-1}(\lambda p_f) + p_\ell (u'_\ell)^{-1}(\lambda p_\ell) + (u'_m)^{-1}(\lambda)$$

- ★ Note since $u'_f(\cdot)$ (etc) is a strictly decreasing function so is $(u'_f)^{-1}(\cdot)$. Therefore $\frac{\partial \lambda}{\partial I} < 0$.
- ★ This is for the non-quasi-linear case, in that case $u'_m(m) = \lambda$ pins down λ , which doesn't change with income.

Part (a) - Inferior Goods

- ▶ Marshallian demand:

$$f^M(p_f, p_\ell, I) = (u'_f)^{-1}(p_f \lambda(p_f, p_\ell, I))$$

- ★ And similarly for leisure and (non-quasi-linear) money.
- ★ Since $f^M(\cdot)$ decreases with λ and λ (weakly) decreases with I , demand for food (and leisure, (non-quasi-linear) money) (weakly) increases in income. (Money with quasi-linear utility will "soak up" extra income.)
- **Answer:** None of the three goods can be inferior. (Money is normal.)

Part (b) - Time Endowment Effects

“Do the equilibrium amounts of leisure or food depend on the size of the time endowment? Do their prices? If so, how?”

Part (b) - Time Endowment Effects

● Intuition:

- ▶ The first part of the question is probably obvious. Money is in fixed supply (the consumer can't transform food or leisure into money, in aggregate) and the time endowment can either be used for leisure or work (making food), so clearly the time endowment will affect equilibrium amounts.
- ▶ Given we know the size of the time endowment affects equilibrium amounts, we also know that prices must change.
 - ★ Suppose prices didn't change. Income is effectively higher since the consumer has more time to work (or not).
 - ★ We learned in (a) that money is normal, so demand for money would go up with increased income.
 - ★ But money supply is fixed, so this cannot be an equilibrium! There is a shortage of money.
 - ★ Therefore it must be prices of food and leisure fall so money demand = money supply.

Part (b) - Time Endowment Effects

- **Formally:**

- ▶ First, note that the “price” of leisure is the wage w .
 - ★ By using time for leisure instead of work, the consumer forgoes the wage.
- ▶ Additionally, due to linear technology and competitive markets the price of food is the wage divided by productivity, $p_f = \frac{w}{A}$.
- ▶ Therefore, we have the following relation between the price of leisure and the price of food:
 $p_\ell = Ap_f$.

Part (b) - Time Endowment Effects

- From the FOCs in (a):

$$\frac{u'_f(f)}{p_f} = \frac{u'_\ell(\ell)}{p_\ell} = \frac{u'_m(m)}{p_m}$$
$$\frac{u'_f(f)}{p_f} = \frac{u'_\ell(\ell)}{Ap_f} = u'_m(M)$$

- Further:

$$A = \frac{u'_\ell(\ell)}{u'_f(f)}$$

- ★ The ratio of leisure and food marginal utilities is constant.
- ★ Therefore with, for example, increased T , both food and leisure increase, since $T = \frac{f}{A} + \ell$ and marginal utility is decreasing for both.

Part (b) - Time Endowment Effects

- ▶ Regarding prices:

$$\frac{u'_f(f)}{p_f} = u'_m(M)$$

- ★ Since $u'_m(M)$ is fixed and f increases with increased T , it must be that p_f decreases.
- ★ p_ℓ also decreases since it is proportional to p_f .
- **Answer:** If the time endowment increases, the equilibrium amounts of food and leisure both increase and the equilibrium prices of both decrease.

Part (c) - Money Supply Effects

“Do the equilibrium amounts of leisure or food depend on the supply of money? Do their prices? If so, how?”

Part (c) - Money Supply Effects

● Intuition:

- ▶ Money cannot be transformed into food or leisure and the supply has no effect on their marginal utilities, so equilibrium amounts of food and leisure are unchanged with a change in money supply.
- ▶ (Non-quasi-linear.) Marginal utility for money is lower with higher supply while marginal utilities are the same for food and leisure, so prices must rise so there isn't a surplus or money and a shortage of food and leisure. (For quasi-linear, prices are unchanged.)
- ▶ Nominal changes may have nominal, but not real, effects.

Part (c) - Money Supply Effects

- **Formally:**

- ▶ From (b):

$$A = \frac{u'_\ell(\ell)}{u'_f(f)}$$
$$T = \frac{f}{A} + \ell$$

- ★ These equations solve for equilibrium food and leisure and clearly do not depend on money supply M at all.
- ▶ Also from (b):

$$\frac{u'_f(f)}{p_f} = u'_m(M)$$

- ★ If M increases, the RHS (weakly) decreases. Since f is unchanged, p_f must (weakly) increase (and p_ℓ will too since $p_\ell = Ap_f$).

Part (c) - Money Supply Effects

- **Answer:** If the money supply increases, the equilibrium amounts of food and leisure are both unchanged and the equilibrium prices of both (weakly) increase (strictly if non-quasi-linear).

Part (d) - Minimum Wage (Modeling)

“Now we want to consider regulations that set the money wage rate above the equilibrium level. Is this model adequate? If not, how could you modify it?”

Part (d) - Minimum Wage (Modeling)

- **Intuition:**

- ▶ Economics: From Econ 101, we know a binding price control causes shortages/surpluses (here workers want to work more than possible). To complete the model there must be some mechanism which leads to $\text{supply} = \text{demand}$.
 - ★ Possible mechanisms include: job search costs (time or other costs), changes in non-wage components of compensation, job lottery, flexible money supply...
- ▶ Math: Adding a binding minimum wage adds an equation to our system of equations without adjusting the number of variables we need to solve for (quantities and prices). Therefore we need to add some variable or remove some equation from our system.

- **Answer:** The model is not adequate. You could modify it in several possible ways.

Part (d) - Minimum Wage (Modeling)

- I introduce job search costs.
 - ▶ Motivation: Before the minimum wage anyone who wanted a job could get one, now there are too few to go around, so workers need to expend some effort searching for a job.
 - ▶ For simplicity, this is represented as a price per hour worked.
 - ▶ This is a cost to workers, but not revenue to firms.
 - ▶ The search cost/price is determined in equilibrium like other prices, think of search costs growing with larger job shortages.
- New/altered equations:

$$\underline{w} = Ap_f$$

$$p_\ell = \underline{w} - p_s$$

- ▶ \underline{w} is the (binding) minimum wage, p_s is search cost/price.

Part (d) - Minimum Wage (Modeling)

- Note that now:

$$A > \frac{p_\ell}{p_f} = \frac{\underline{w} - p_s}{\underline{w}/A} = \frac{u'_\ell(\ell)}{u'_f(f)}$$

- ▶ So consumers consume less food and more leisure compared to the no-policy equilibrium.

Part (d) - Minimum Wage (Modeling)

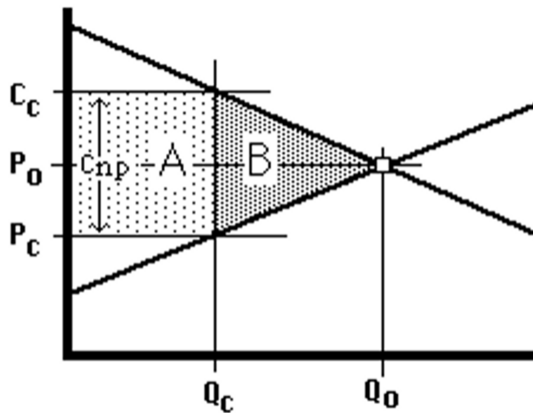


Figure 17-4

- Words.

Part (e) - Minimum Wage (Food Price)

“Do you expect the minimum wage to affect the price of food?”

Part (e) - Minimum Wage (Food Price)

- **Intuition:**

- ▶ Competitive firms price at marginal cost and the marginal cost here is the wage divided by productivity. Since the minimum wage is binding, wages are higher than without the policy so food prices are too.

- This is immediate from the equation:

$$\underline{w} = Ap_f$$

- **Answer:** Introducing a (binding) minimum wage increases the price of food proportionally.

Part (f) - Minimum Wage (Money Supply Effects)

“With the minimum wage in place, do the equilibrium amounts of leisure or food depend on the supply of money? If so, how?”

Part (f) - Minimum Wage (Money Supply Effects)

● Intuition:

- ▶ Compared to the no-policy equilibrium, with a minimum wage in place consumers consume less food and more leisure (the relative price of leisure compared to food is lower due to positive search costs).
- ▶ The size of this distortion depends on the size of the difference between the minimum wage and the no-policy equilibrium wage.
- ▶ (Focus on non-quasi-linear case.) The minimum wage is set in **nominal** terms and we know from (c) that in the no-policy model increased money supply increases prices (including the wage).
- ▶ So if money supply increases, the minimum wage is closer to the no-policy equilibrium wage and the distortion in food/leisure quantities is smaller (more food, less leisure consumed).

Part (f) - Minimum Wage (Money Supply Effects)

- **Formally:**

- ▶ Assume the minimum wage is always binding.
- ▶ From the FOCs in (a) (which still hold):

$$\frac{u'_f(f)}{p_f} = \frac{u'_\ell(\ell)}{p_\ell} = u'_m(M)$$

- ▶ Substituting in equations for prices:

$$\frac{u'_f(f)}{\underline{w}/A} = \frac{u'_\ell(\ell)}{\underline{w} - p_s} = u'_m(M)$$

- ▶ Solves for f , given M :

$$\frac{u'_f(f)}{\underline{w}/A} = u'_m(M)$$

★ If money supply M increases, then food consumed f must increase as well.

Part (f) - Minimum Wage (Money Supply Effects)

- ▶ Solves for ℓ , given f :

$$T = \frac{f}{A} + \ell$$

★ If food consumption f increases, then leisure ℓ must decrease.

- ▶ Solves for p_s , given ℓ and M :

$$\frac{u'_\ell(\ell)}{\underline{w} - p_s} = u'_m(M)$$

★ Additionally, if M increases and ℓ decreases, then search cost p_s must decrease.

- **Answer** Assuming the minimum wage is binding and utility is not quasi-linear, an increase in the money supply increases food consumption and decreases leisure (both towards the no-policy quantity). (In the quasi-linear case, the money supply has no effect on equilibrium food/leisure amounts.)

Part (g) - Unemployment Insurance

“Instead of a regulated wage, there is a program in which all citizens are required to devote part of their time endowment to produce food, which is given to people for the duration of time than anyone is not working. The amount of food given per unit time is set in terms of dollars. E.g., \$15 worth of food per hour out of work. How is this policy different from a wage regulation?”

Common Confusion

- “What exactly is going on here?”
- Gov policy sets an “unemployment/leisure subsidy” rate, e.g. \$15/hr, which is paid to consumers for the amount of time they are not working.
- Gov requires consumers to work enough to fund that subsidy at equilibrium food price and amount of leisure.
- It’s important that the policy sets the rate in the numeraire, not in terms of food (which would be equivalent to requiring a certain amount of work regardless of prices).
- Note that the government is not directly controlling any prices.

Part (g) - Unemployment Insurance

● Intuition:

- ▶ This is essentially unemployment insurance (or more accurately a leisure subsidy in this setup).
- ▶ Note that like the minimum wage, unemployment insurance is set in dollar (nominal) terms (the government directs people to work for it enough to fund the policy).
- ▶ In many ways this is analogous to the minimum wage policy (and model), but instead of job search costs reducing the effective price of leisure, a subsidy explicitly does so.
- ▶ (Focus on non-quasi-linear case.) So this unemployment insurance is not so different from a wage regulation, consumers will consume less food and more leisure, and an increased money supply will reduce the distortion by reducing the real value of the subsidy.
- ▶ Additionally, food prices increase with the policy since food consumed falls (marginal utility rises) and the price must rise so there isn't a surplus of money/shortage of food.

Part (g) - Unemployment Insurance

- **Formally:**

- ▶ Assume that in equilibrium workers choose to work more than the government requires them to.
- ▶ Now the price of leisure is:

$$p_\ell = w - b$$

- ★ Where b is the benefit rate. Note this is extremely similar to the minimum wage case but now b is fixed by policy and w is a flexible price.

Part (g) - Unemployment Insurance

- ▶ Now:

$$A > \frac{p_\ell}{p_f} = \frac{w - b}{p_f} = \frac{Ap_f - b}{p_f} = \frac{u'_\ell(\ell)}{u'_f(f)} \quad (1)$$

- ★ So, similarly to the minimum wage, consumers consume less food and more leisure compared to the no-policy equilibrium.

- ▶ Again:

$$\frac{u'_f(f)}{p_f} = u'_m(M)$$

- ★ Since the money supply/RHS is fixed and food consumption f decreases with the policy, the food price p_f must rise.

Part (g) - Unemployment Insurance

- Finally:

$$u'_m(M) = \frac{u'_f(f)}{p_f} = \frac{u'_\ell(\ell)}{Ap_f - b}$$

- ★ If money supply M increases, then the LHS decreases. Therefore at least one of p_f and f must increase, and similarly for p_f and ℓ .
 - ★ Obviously f and ℓ can't both increase simultaneously, so p_f must increase.
 - ★ Looking back at the previous slide, if p_f increases the LHS will increase towards A , so leisure decreases and food consumption increases towards their no-policy equilibrium quantities.
- **Answer:** The effects of unemployment insurance are essentially the same as those of a minimum wage, at least for the ones we've examined.
 - ▶ Nominal changes have real effects in both cases.
 - ▶ Increased money supply reduces the distortion in both cases, though the minimum wage will become non-binding after a certain point, while the unemployment insurance distortion only disappears asymptotically.

(Appendix) Setup - Notation

- f = food, ℓ = leisure, m = money.
- Capital letters (M) are generally aggregate amounts, while lowercase letters (m) are “individual” (demanded by the representative consumer) amounts.
 - ▶ In equilibrium, aggregate = individual (supply = demand), but we must entertain the possibility they aren't equal at non-equilibrium prices.
- $u_f(f)$ is the subutility function for food, **not** the partial derivative of $U(f, \ell, m)$ w.r.t. food (note lowercase u).
 - ▶ $u'_f(f)$ is the derivative of that subutility function.

Back