Problem Set 5 - Question 1

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Question, I

Last week we used an Edgeworth box to work out the effect of a (non-uniform) fossil fuel tax on aggregate fossil fuel consumption, show conditions under which levying the tax increases consumption.

Question, II

- *a) A world aggregate fossil-fuel demand curve can be traced out by varying the fuel-quantity dimension of the Edgeworth box and plotting that against the corresponding equilibrium relative factor price of fossil fuels. How is the slope of the aggregate fossil-fuel demand curve related to Marshall's Law of derived demand?*
- *b) Ignoring any externality for the moment, can the area under the fossil-fuel demand curve be interpreted as the value to consumers of fossil fuels?*

Question, III

- *c) Last week we also posited a fossil-fuel supply curve that increases with the relative factor price. Combining that with the demand curve yields a supply-demand diagram for the fossil-fuel market. Can that diagram represent last week's result regarding (preference) shifts in the relative demand for the local economy's output?*
- *d) Can the supply-demand diagram for the fossil-fuel market represent the possibility that the local fossil fuel tax increases aggregate fuel consumption? Does it require a fossil-fuel demand curve that slopes up?*
- *e) True, False, or Uncertain: As a practical matter, the possibility that the local tax increases global fossil fuel usage can be ignored as long as the local economy is sufficiently small relative to the rest of the world.*

Model, Recap

- A general equilibrium model.
- Assume free trade and free mobility of production factors.
- Production:
  - Two factors.
    - Fossil fuels (g) with exogenous supply $\bar{g}$.
    - A composite input (k) with exogenous supply $\bar{k}$.
  - Competitive firms in two sectors with CRS production technology.
    - Local sector produces goods x with $F(g, k)$.
    - ROW sector produces y with $G(g, k)$.
- Consumers:
  - Homothetic preferences $u(x, y)$ over two outputs.
    - Homotheticity: $u(x, y) = f(v(x, y))$ for some strictly increasing f and v homogenous of degree one.
  - Firms are owned by consumers.
  - Endowment $g_1^e, k_1^e, g_2^e, k_2^e$ are also owned by consumers.

General Equilibrium, Recap I

- What is an equilibrium?
- A set of prices and allocations satisfying the following conditions:
  - Consumers maximize utility subject to budget.
  - Firms maximize profits subject to technology.
  - All markets clear.

General Equilibrium, Recap II

- Focus on relative quantities only:
  - Relative prices: w, r, p_y.
  - Relative quantities: $g_1/k_1, g_2/k_2, x/y$.
  - Composite input employed for fuel extraction: k_g.
- Equilibrium conditions:

$$\frac{w}{r} = \frac{1}{H'(k_g)};$$

$$\frac{v_x(x/y, 1)}{v_y(x/y, 1)} = \frac{1}{p_y}; \frac{x}{y} = \frac{wg_1 + rk_1}{wg_2 + rk_2} p_y;$$

$$\frac{F_g(g_1/k_1, 1)}{F_k(g_1/k_1, 1)} = \frac{w}{r}; \frac{G_g(g_2/k_2, 1)}{G_k(g_2/k_2, 1)} = \frac{w}{r}$$

$$\frac{F_g(g_1/k_1, 1)}{G_g(g_2/k_2, 1)} = \frac{1}{p_y}; \frac{F_k(g_1/k_1, 1)}{G_k(g_2/k_2, 1)} = \frac{1}{p_y}$$

General Equilibrium, Recap III

- Suppose there is a tax τ levied on locally employed fossil fuel g_1.
  - Equilibrate after-tax factor wages $(1-\tau)w_1 = w_2 := w$.
  - Government revenue transferred back to local consumers.
- Equilibrium conditions:

$$\frac{w}{r} = \frac{1}{H'(k_g)};$$

$$\frac{v_x(x/y,1)}{v_y(x/y,1)} = \frac{1}{p_y}; \frac{x}{y} = \frac{wg_1/(1-\tau) + rk_1}{wg_2 + rk_2} p_y;$$

$$\frac{F_g(g_1/k_1,1)}{F_k(g_1/k_1,1)} = \frac{w}{(1-\tau)r}; \frac{G_g(g_2/k_2,1)}{G_k(g_2/k_2,1)} = \frac{w}{r}$$

$$\frac{F_g(g_1/k_1,1)}{G_g(g_2/k_2,1)} = \frac{1}{(1-\tau)p_y}; \frac{F_k(g_1/k_1,1)}{G_k(g_2/k_2,1)} = \frac{1}{p_y}$$

Goal

- Redo PSet 4 Question 1 with supply and demand.
  - Supply and demand diagrams are partial equilibrium objects.
- Think about general equilibrium effect.

Part (a): Fuel Demand Curve

- Trace out the demand curve using the MRTS equation
  - Define

$$MRTS_F(g/k) = \frac{F_g(g/k, 1)}{F_k(g/k, 1)};\ MRTS_G(g/k) = \frac{G_g(g/k, 1)}{G_k(g/k, 1)}$$

  - Demand curve given by

$$MRTS_F\left(\frac{g_1}{k_1}\right) = \frac{w}{r} = MRTS_G\left(\frac{\bar{g} - g_1}{\bar{k} - k_1}\right)$$

  - Two equations and three unknowns.
  - Left with one more degree of freedom: One-dimensional demand curve!
- Fixing $\bar{k}, F, G$, trace out the relation between w/r and $\bar{g}$.
- Downward sloping: $\bar{g} \uparrow \Rightarrow g_2/k_2 \uparrow$ and $g_1/k_1 \uparrow \Rightarrow w/r \downarrow$.

Part (a): Marshall's Law

- Can do Marshall's law, but the equations can get messy.
  - Do not have inelastically supplied k.
  - Multiple objects moving at the same time.
- Focus on ROW (notation: $\Delta x = dx/x$, drop subscript)

$$\begin{aligned}
\Delta p_y &= s_g \Delta w + s_k \Delta r \\
\Delta y &= \epsilon^D \Delta p_y \\
\Delta g - \Delta k &= -\sigma (\Delta w - \Delta r) \\
\Delta y &= s_g \Delta g + s_k \Delta k
\end{aligned}$$

  - σ determined by $MRTS_G$.
  - ϵ^D now has to incorporate demand shift due to price.
  - Express Δg in terms of price changes

$$\begin{aligned}
\Delta g &= \epsilon^D (s_g \Delta w + s_k \Delta r) - s_k \sigma (\Delta w - \Delta r) \\
&= \left(\epsilon^D s_g - s_k \sigma\right) \Delta w + \left(\epsilon^D + \sigma\right) s_k \Delta r
\end{aligned}$$

Part (b): Area under the Demand Curve

- This is the value to producers (output) in units of w/r.
- What about for consumers?
  - Yes.
  - Firms earn zero profits.
  - Their revenue all paid back to consumers in factor rents.

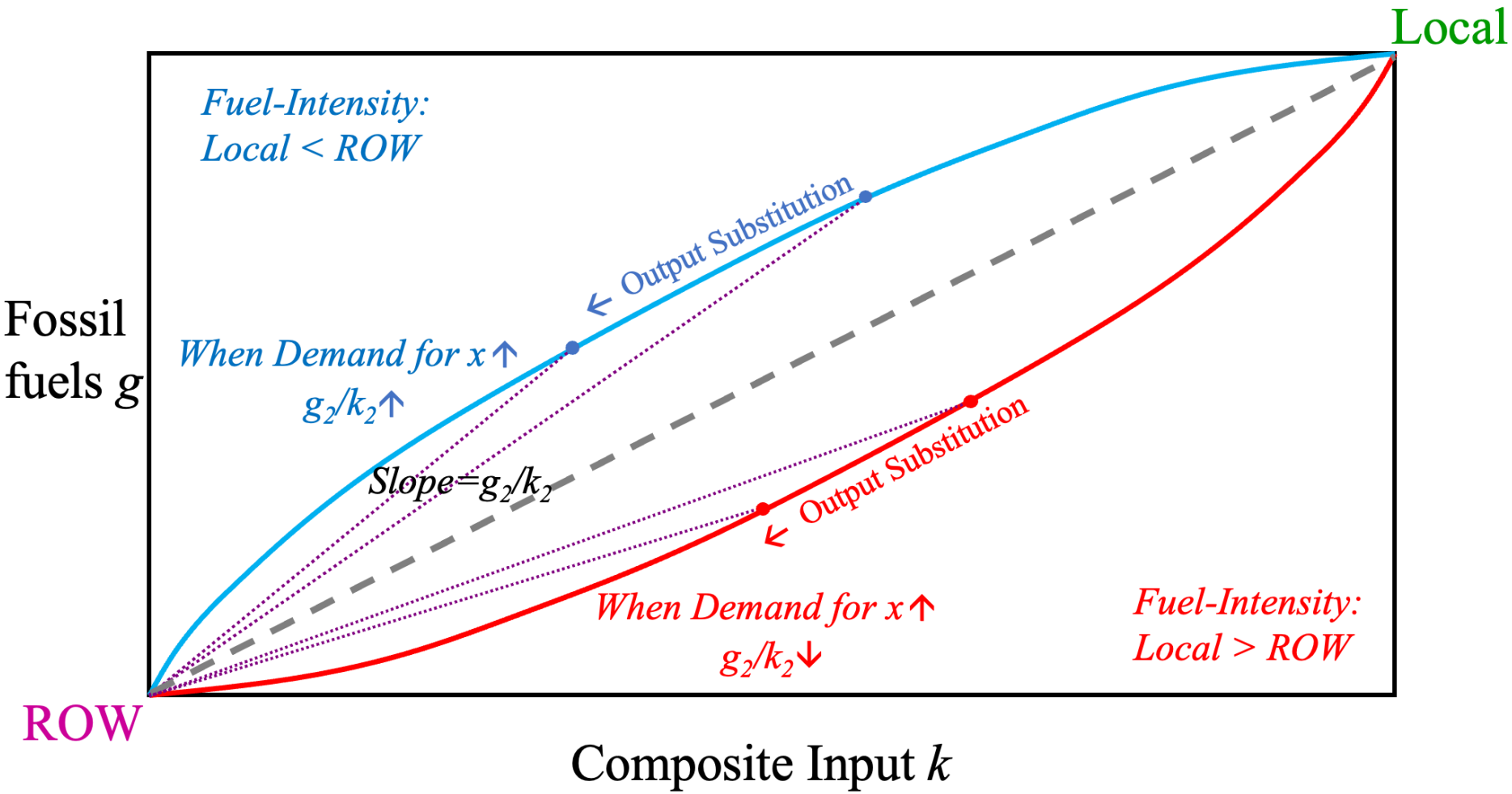
Part (c): Fuel Supply Curve

- Fossil fuel supply curve traced out by

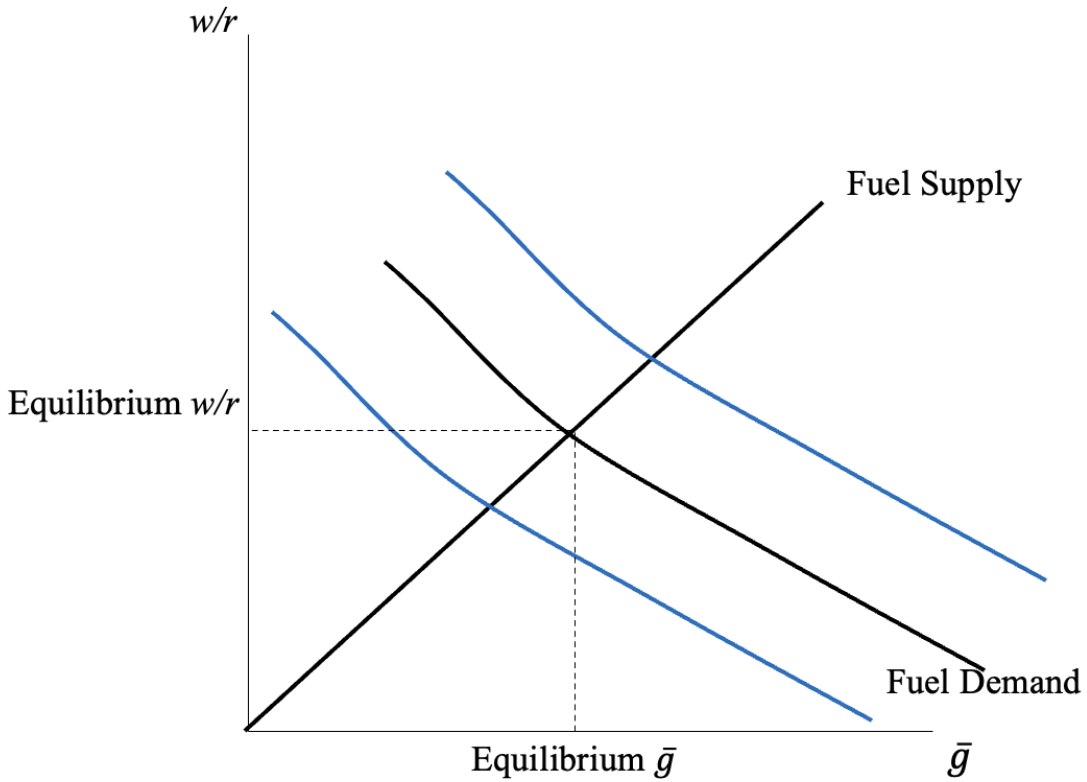
$$\frac{w}{r} = \frac{1}{H'(k_g)}$$

- Preference shifts $\Rightarrow$ ambiguous $\Delta w/r$
  - When consumers like x better, demand curve for x goes up.
  - Depending on the relative intensity, it is not necessary the case that demand for fuel goes up.
  - Demand for fuel can go down.
- Takeaway: In a general equilibrium model,
  - Tax is more than a wedge in the supply and demand diagram.
  - It may also shift the supply / demand curve (demand curve here).

Part (c): Illustration, Edgeworth Box



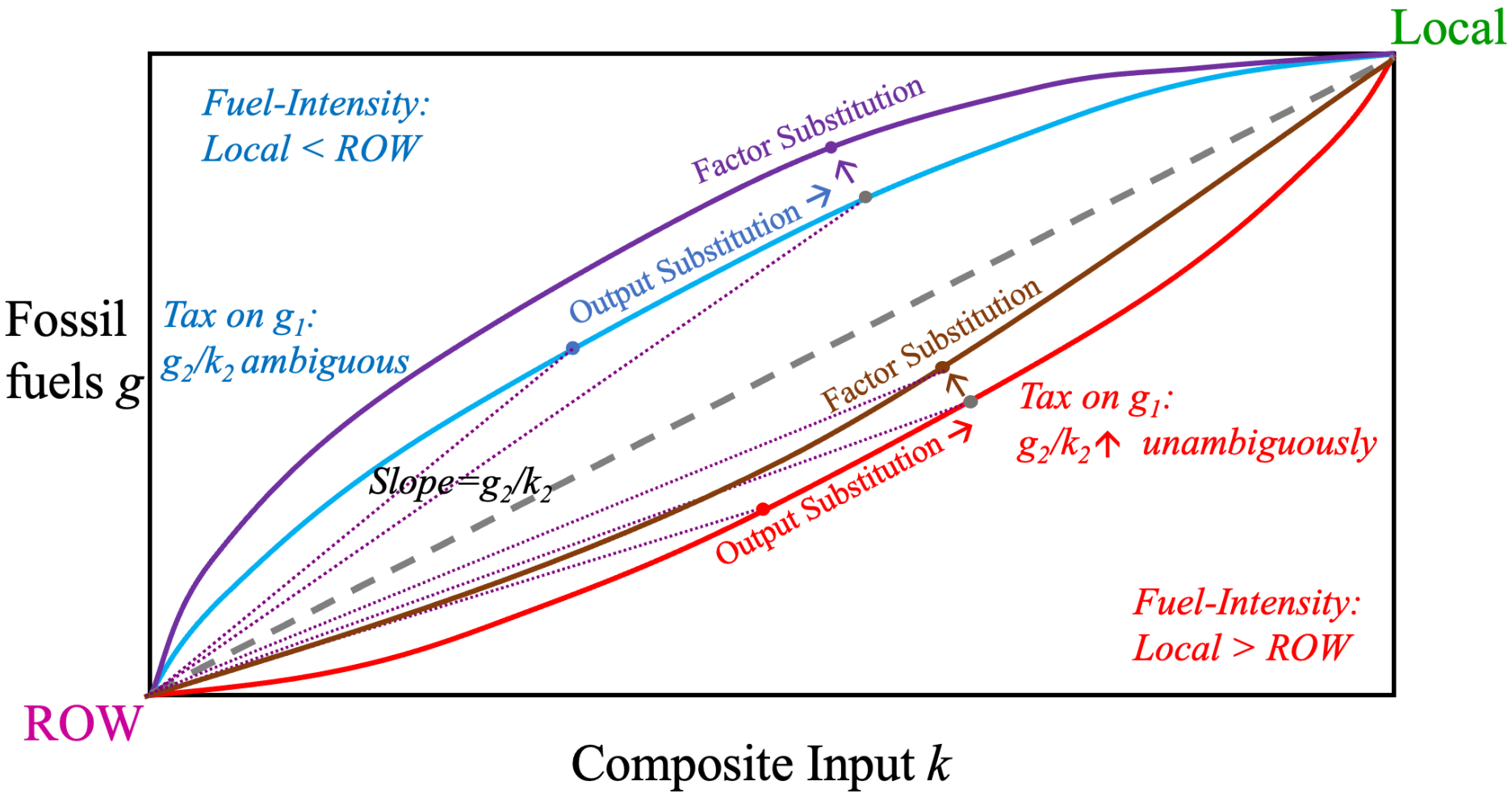
Part (c): Illustration, Supply and Demand



Part (d): Effects of a Fuel Tax

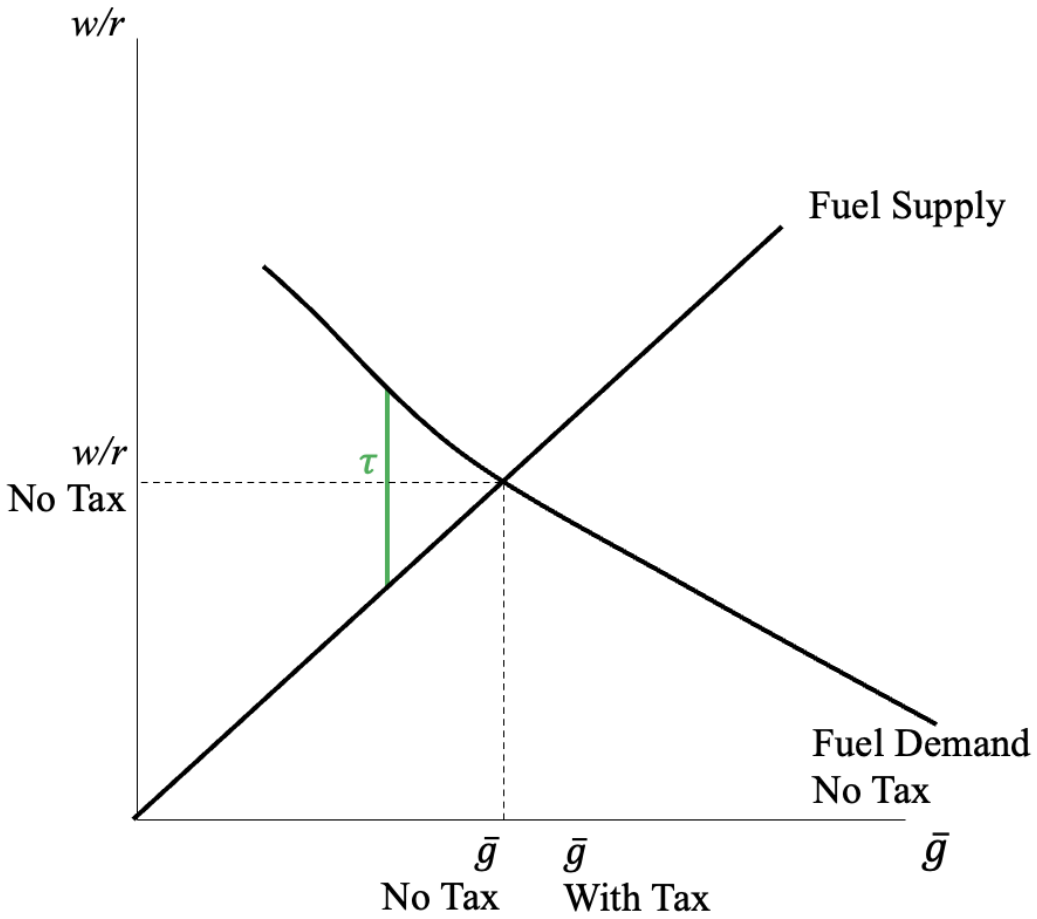
- No, we do not require an upward-sloping demand.
- Again, demand for fuel can go either up or down with the tax.

Part (d): Illustration, Edgeworth Box



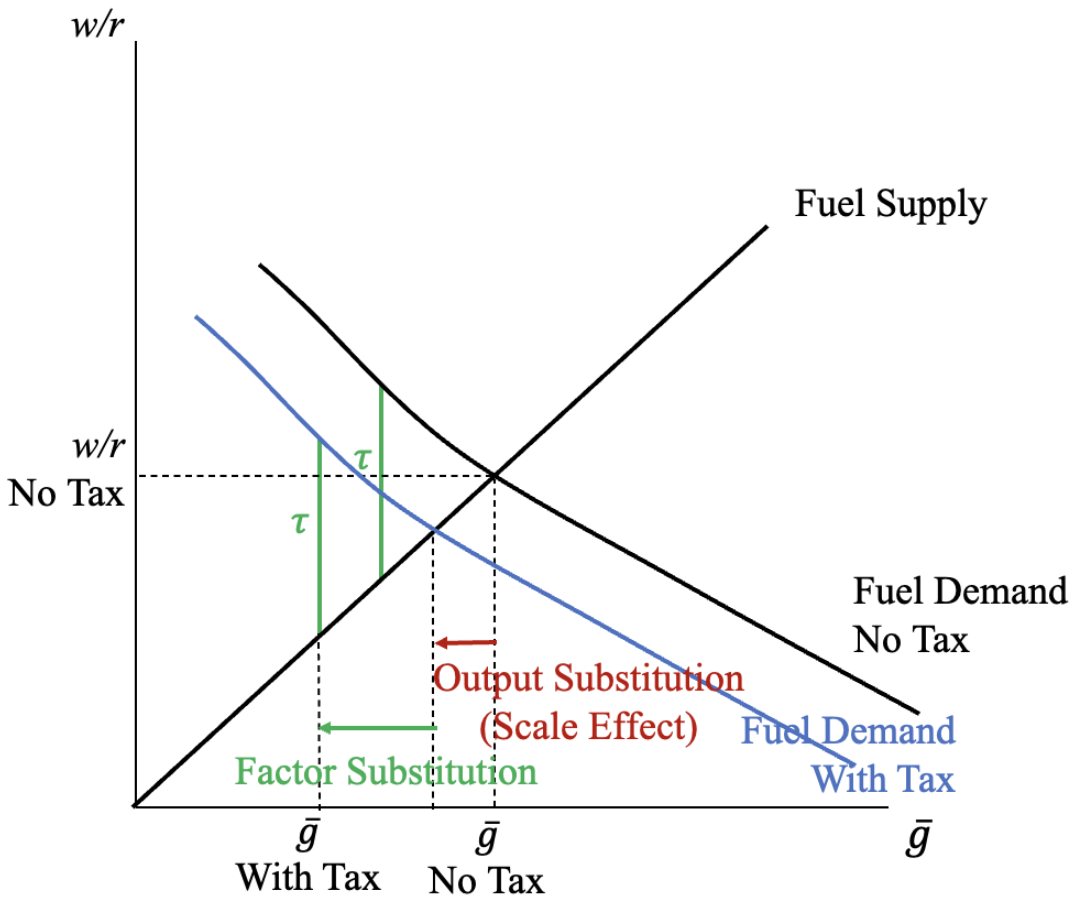
Part (d): Illustration, Supply and Demand, I

- Partial equilibrium:
  - Supply and demand curves may shift due to reasons other than prices.
  - Price changes are movements along the curve(s)



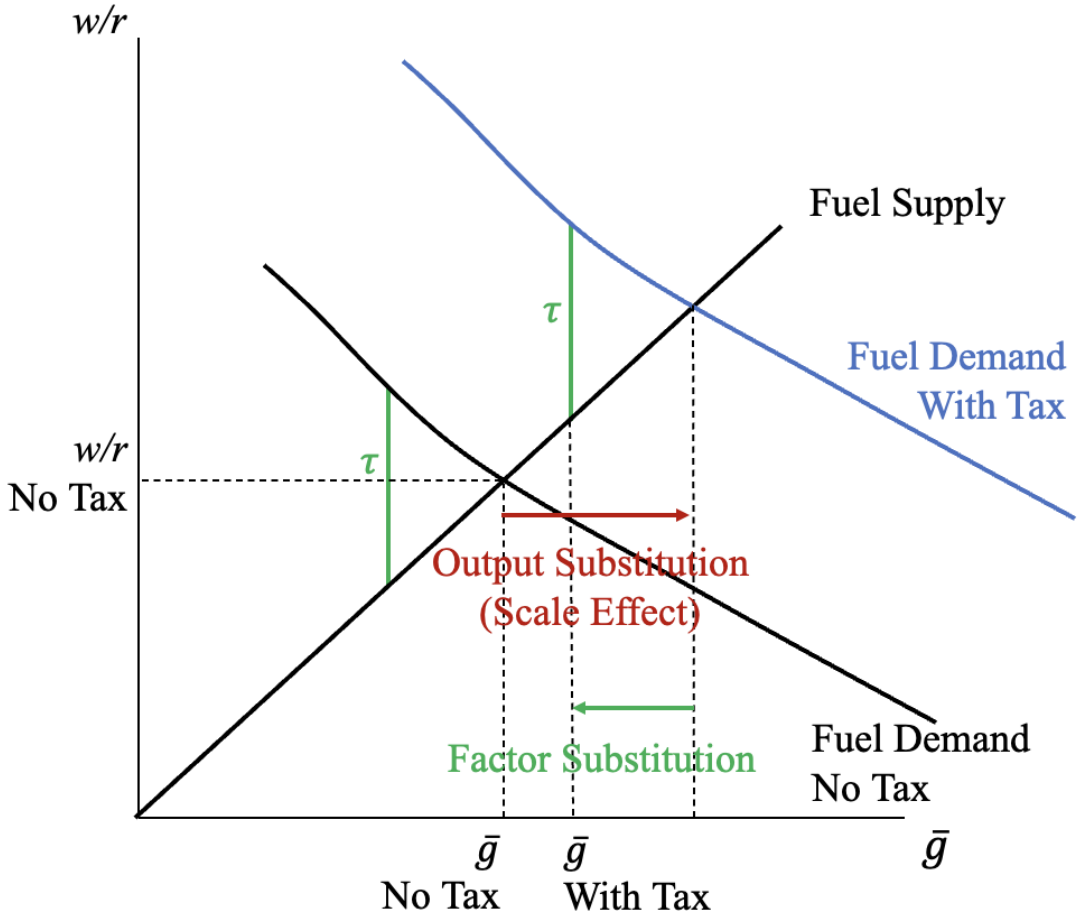
Part (d): Illustration, Supply and Demand, II

- General equilibrium:
  - Price changes can also shift the curves (via a scale effect in this case).
  - When local is more fuel-intensive, scale effect always in our favor.



Part (d): Illustration, Supply and Demand, III

- General equilibrium:
  - Price changes can also shift the curves (via a scale effect in this case).
  - When local is less fuel-intensive, scale effect can work against us.

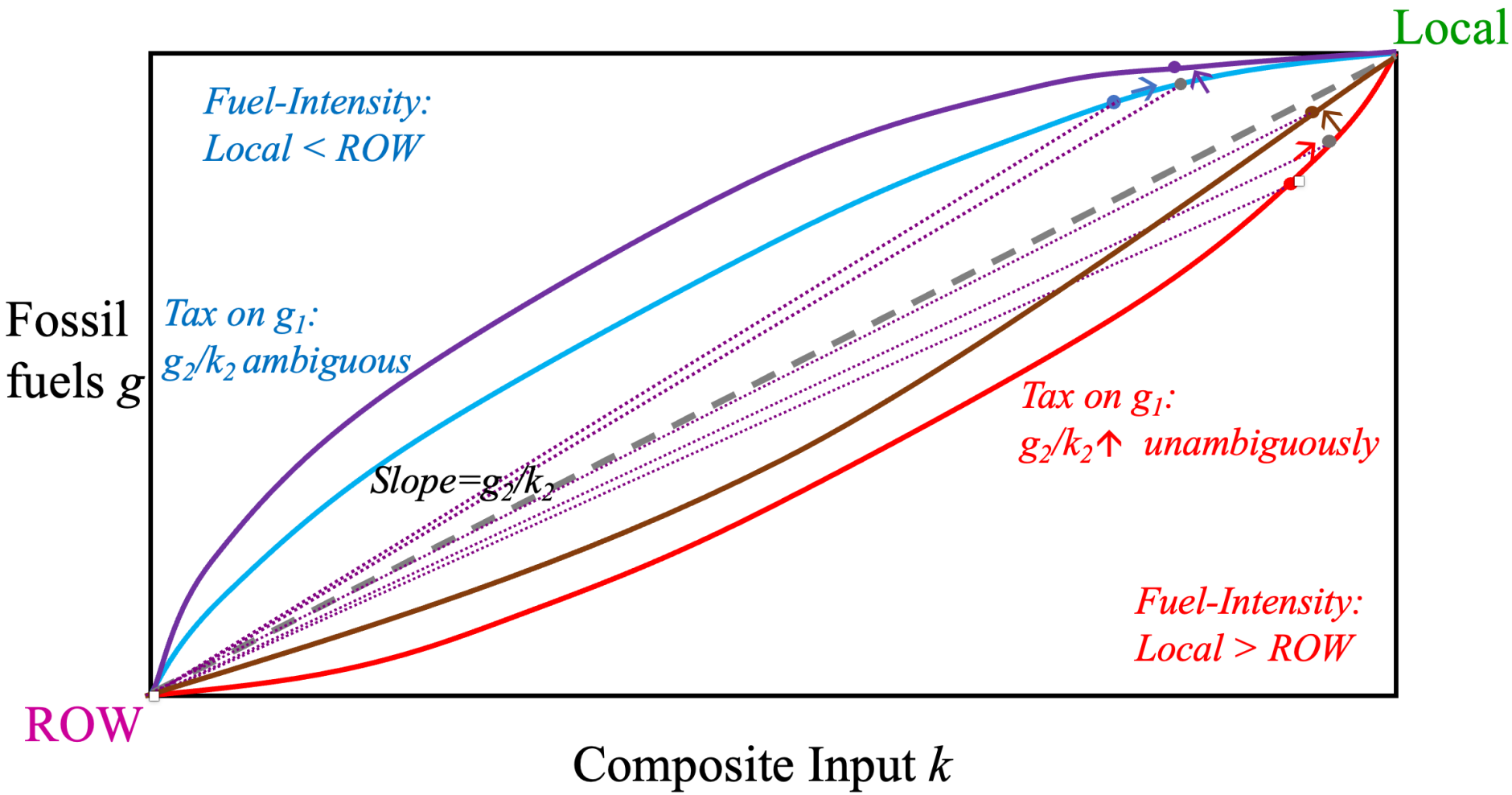


Part (e): A Small Economy, I

- Tax on g_1: $\Delta w/r$ ambiguous if ROW more fuel-intensive.
  - Factor substitution: $g_1/k_1 \downarrow \Rightarrow g_2/k_2 \uparrow$.
  - Output substitution (scale effect): tax$\Rightarrow p_x \uparrow \Rightarrow$output $x \downarrow \Rightarrow g_2/k_2$?
- For the factor substitution to dominate, we need
  - Either the local economy is more fuel-intensive;
  - Or output substitution is sufficiently small.
- Focus on whether a sufficiently small local economy implies sufficiently small output substitution effect.
  - Local consumption may be small: Little consumed locally.
  - Local production may be negligible: Little x produced.

Part (e): A Small Economy, II

- Despite a small local economy, the scale effect can still be against us.



Part (e): A Small Economy, III

- The statement is Uncertain / False.
- Whether local consumption is small matters little.
- If local production is small
  - Output substitution may still outweigh factor substitution.
  - The magnitude may be smaller but the possibility exists.
- The extreme case: Local production negligible, no longer two sectors. Local tax does nothing. (But it clearly would not increase fuel usage.)