Problem Set 1 - Question 2

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October 6, 2023

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Question, I

Princeton professors Case and Deaton write in their latest book "If people are withdrawing their labor, wages should rise; but in the late part of the twentieth century and into the twenty-first, wages fell along with employment, a clear indication that the problem [reduced quantity of labor] lies with falling demand, not falling supply." Here we accept for the sake of argument that wages did fall with employment ("the data")

Question, II

- *a. Using comparative statics of the supply-demand model of the labor market d[w,a] = q = s[w,b], express the data as an algebraic expression [Hint: it is not complicated!]. w denotes the wage and q the quantity of labor. a summarizes shifters of the labor demand curve. b represents shifters of the labor supply curve.*
- *b. How can Case and Deaton's conclusion about the quantity of labor be expressed algebraically? [Hint: you may want to contrast actual quantity (the data) with a counterfactual quantity – where something was different]*
- *c. What is the algebraic representation of the opposite of their conclusion – that the quantity of labor fell primarily due to shifts in supply?*
- *d. Is, together with the data, downward-sloping labor demand and upward sloping supply sufficient to reach their conclusion? If not, what else must be assumed?*

Question, III

- *e. In the labor supply-demand diagram, what is the economic interpretation of the area under the labor demand curve?*
- *f. Can labor's share of national income be calculated just from a labor supply-demand diagram?*

Goal

- Interpret supply and demand diagram.
- Translate English into precise statements in mathematics.

Setup

Consider a simple supply and demand model.

- Let w denote wage.
- Labor supply $s(w, b)$, labor demand $d(w, a)$.
  - Assumption: Supply slopes up, and demand slopes down.

$$\frac{\partial s(w,b)}{\partial w} > 0; \ \frac{\partial d(w,a)}{\partial w} < 0.$$

- a and b are aggregate shifters of demand and supply, respectively.
  - Without loss, we can normalize the shifters so that

$$\frac{\partial s(w,b)}{\partial b} = \frac{\partial d(w,a)}{\partial a} = 1.$$

  - This normalization can be achieved by changing the unit.
- Equilibrium $d(w, a) = q = s(w, b)$.

Part (a): Data

a. Using comparative statics of the supply-demand model of the labor market, express the data as an algebraic expression.

- "Data" in English: "[W]ages fell along with employment".
- Comparative statics: Compare the equilibrium outcomes before and after some change in some underlying parameter.
- Suppose the equilibrium was (q_0, w_0) where

$$d(w_0, a_0) = q_0 = s(w_0, b_0).$$

- Now something (i.e., shifters) has changed, new equilibrium (q_1, w_1)

$$d(w_1, a_1) = q_1 = s(w_1, b_1).$$

- "Data" in math: We observe falling wages and falling employment

$$\Delta w^{\text{act}} := w_1 - w_0 < 0; \ \Delta q^{\text{act}} := q_1 - q_0 < 0.$$

Part (b): Hypothesis

b. How can Case and Deaton's conclusion about the quantity of labor be expressed algebraically?

- "Hypothesis" in English: "[Reduced quantity of labor] lies with falling demand, not falling supply".
- Decrease in labor should mostly be attributed to a demand shift.
- Consider a hypothetical equilibrium $(\tilde{q}, \tilde{w})$ with no change in supply,

$$d(\tilde{w}, a_1) = \tilde{q} = s(\tilde{w}, b_0).$$

- Let $\Delta q^{\text{dmd}} := \tilde{q} - q_0, \Delta q^{\text{sup}} := q_1 - \tilde{q}$.
- "Hypothesis" in math:

$$\frac{\Delta q^{\text{dmd}}}{\Delta q^{\text{act}}} > \frac{1}{2}, \text{ or equivalently, } \Delta q^{\text{dmd}} < \Delta q^{\text{sup}} \,(< 0)$$

  - Depends on how you interpret "mostly".

Part (c): Alternative Hypothesis

c. *What is the algebraic representation of the opposite of their conclusion – that the quantity of labor fell primarily due to shifts in supply?*

- Alternative hypothesis:

$$\frac{\Delta q^{\text{dmd}}}{\Delta q^{\text{act}}} < \frac{1}{2}, \text{ or equivalently, } \Delta q^{\text{sup}} < \Delta q^{\text{dmd}} \, (< 0)$$

Part (d): Deduction

d. Is, together with the data, downward-sloping labor demand and upward sloping supply sufficient to reach their conclusion? If not, what else must be assumed?

- Can we deduce the hypothesis from our assumptions and data. Formally, whether the following conditional is true:

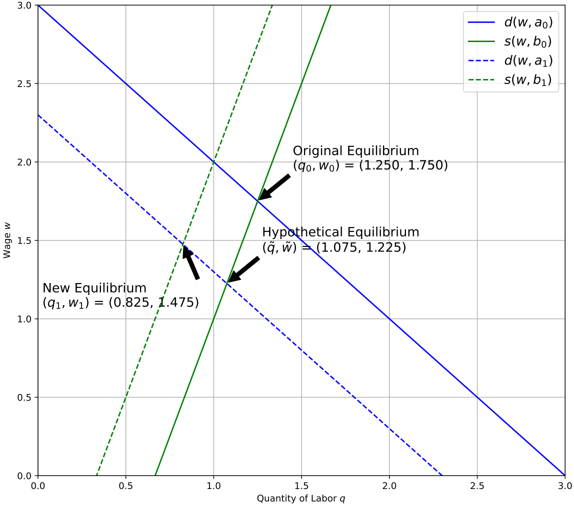
$$\frac{\partial d(w,a)}{\partial w} < 0, \frac{\partial s(w,b)}{\partial w} > 0, \Delta w^{\text{act}} < 0, \Delta q^{\text{act}} < 0$$

$$\Rightarrow$$

$$\frac{\Delta q^{\text{dmd}}}{\Delta q^{\text{act}}} > \frac{1}{2}$$

- The answer is no.

Part (d): Counterexample



- $\Delta q^{\text{act}} = -0.425$ but $\Delta q^{\text{dmd}} = -0.175$. $\dfrac{\Delta q^{\text{dmd}}}{\Delta q^{\text{act}}} < \dfrac{1}{2}$.

Part (d): Additional Assumptions?

- In fact, we can find examples so that $\dfrac{\Delta q^{\text{dmd}}}{\Delta q^{\text{act}}} = x$ for any $x \in (0, 1)$.
  - That is, given our assumptions and data, we are not able to conclude whether a supply or a demand shift causes more decrease in quantity.
- One sufficient condition for $\dfrac{\Delta q^{\text{dmd}}}{\Delta q^{\text{act}}} > \dfrac{1}{2}$ is that at (q_0, w_0), labor supply is more elastic than labor demand, or, formally,

$$\varepsilon_0^D + \varepsilon_0^S > 0$$

Part (d): Elasticities, I

- The wage elasticities of labor demand and labor supply at (q_0, w_0) are

$$\varepsilon_0^D = \frac{\partial d(w_0, a_0)}{\partial w} \frac{w_0}{q_0} < 0; \ \varepsilon_0^S = \frac{\partial s(w_0, b_0)}{\partial w} \frac{w_0}{q_0} > 0$$

- Assume all changes are small and first order approximation is valid.
  - This allows us to express Δq^{dmd} and Δq^{sup} in elasticities.

$$\frac{\Delta q^{\text{dmd}}}{q_0} = \varepsilon_0^S \cdot \frac{\Delta w^{\text{dmd}}}{w_0}; \ \frac{\Delta q^{\text{sup}}}{q_0} = \varepsilon_0^D \cdot \frac{\Delta w^{\text{sup}}}{w_0}$$

    - When fixing supply, equilibrium moves alone the demand curve.
    - When fixing demand, equilibrium moves alone the supply curve.
  - You can also use arc elasticities to avoid assuming small changes.
- Decompose Δq^{act} as

$$\begin{aligned}\Delta q^{\text{act}} &= \Delta q^{\text{dmd}} + \Delta q^{\text{sup}} \\ &= \left(\varepsilon_0^S \Delta w^{\text{dmd}} + \varepsilon_0^D \Delta w^{\text{sup}}\right) \frac{q_0}{w_0}\end{aligned}$$

Part (d): Elasticities, II

- Take the ratio

$$\frac{\Delta q^{\text{dmd}}}{\Delta q^{\text{act}}} = \frac{\varepsilon_0^S \Delta w^{\text{dmd}}}{\varepsilon_0^S \Delta w^{\text{dmd}} + \varepsilon_0^D \Delta w^{\text{sup}}} = \frac{1}{1 + \left(\varepsilon_0^D / \varepsilon_0^S\right) \left(\Delta w^{\text{sup}} / \Delta w^{\text{dmd}}\right)}$$

- Data says

$$\Delta w^{\text{act}} = \Delta w^{\text{dmd}} + \Delta w^{\text{sup}} < 0, \text{or equivalently, } \frac{\Delta w^{\text{sup}}}{\Delta w^{\text{dmd}}} \in (-1, 0).$$

- Note that

$$\varepsilon_0^D + \varepsilon_0^S > 0 \Rightarrow \frac{\varepsilon_0^D}{\varepsilon_0^S} \in (-1, 0) \Rightarrow \frac{\Delta q^{\text{dmd}}}{\Delta q^{\text{act}}} > \frac{1}{2}$$

Part (e): Interpret Labor Demand

e. In the labor supply-demand diagram, what is the economic interpretation of the area under the labor demand curve?

- Assume competitive market, the area under the labor demand curve is the total output in dollars.
    - Labor demand represents firms' WTP for each unit of labor.
    - Firms are willing to pay up to the total output to earn positive profits.
    - Under competitive market, labor demand curve coincides with the marginal product of labor curve.
- When firms have market power in the labor market, there can be a mark-down on wages.
    - If there is market power, labor demand is different from the marginal product of labor.

Part (f): Industry Model

f. Can labor's share of national income be calculated just from a labor supply-demand diagram?

- Yes.
  - Suppose the market is competitive.
  - Normalize the price of output to 1.
- Labor's share of national income is

$$\frac{wL}{Y}$$

  - The numerator is observed from the equilibrium point.
  - The denominator is the area under the labor demand curve.
  - The rectangular area over the trepozoid area.