Problem Set 2 - Question 1

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Question

Households live and consume in each of two periods. During their youth, they can invest in a local project or in a distant project, or both, but distant investment requires a travel cost to make the investment and later recoup the (deterministic) returns. Neither project's rate of return (gross of travel costs) depends on the amount invested. Households differ from each other in terms of the endowment they have to consume or invest during youth. Neither period's consumption is an inferior good.

Goals

1. Work with “increasing returns”–declining marginal cost, useful in many settings
   - Buying a fuel efficient vehicle
   - Subscription business models–Uber One
   - Human capital and wages
   - Prescription vs ilicit opioids
2. Set up a basic demand framework and map between data and predictions

Setup

- Households consume c_1 (youth) and c_2 (adulthood)
- Households split youth income between consumption and investment I in one of two projects.
- Normalize prices to be 1.
- Local investment earns returns r_l, far investment earns r_f –e.g. for 10% rate of return, $r = 1.1$
- Households must pay a fixed cost, τ to get access to far investment
- $r_f > r_l$ or else the problem is uninteresting

Setup

Household, i problem:

$$\max_{c_1, c_2, I_l, I_f} u(c_1, c_2)$$

$$c_1 + I_l + I_f + \tau 1\{I_f > 0\} = m_i$$

$$c_2 = r_l I_l + r_f I_f$$

$$c_1, c_2, I_l, I_f \geq 0$$

Investment Decisions

- Suppose someone wants to consume c in adulthood, would they ever invest in both projects?
- Returns are deterministic so no–low levels of adult consumption will be cheapest using local project.
- Can simplify problem with two stages:
  1. Compute minimum expenditure needed to achieve each c_2 in adulthood
  2. Choose optimal c_2

Minimal Expenditures–1st Stage

- Least youth consumption to achieve c_2 in adulthood

$$e(c_2) = \min\{\frac{c_2}{r_l}, \tau + \frac{c_2}{r_f}\}$$

- Implies cutoff $\tilde{c}$ for investment in far project:

$$\frac{\tilde{c}}{r_l} = \tau + \frac{\tilde{c}}{r_f}$$

$$\implies \tilde{c} = \frac{r_f r_l \tau}{r_f - r_l}$$

- $e(c_2)$ is piecewise linear with slope $1/r_l$ until $\tilde{c}$ when the slope changes to $1/r_f$

2nd Stage

Household problem is now just:

$$\max_{c_2} u(m_i - e(c_2), c_2)$$

No income effects on future consumption suggests something like quasilinear utility (with a discount factor for illustration purposes):

$$\max_{c_2} m - e(c_2) + \beta\theta(c_2)$$

$\theta(\cdot)$ is strictly concave.
Yields FOC:

$$\beta\theta'(c_2) = e'(c_2)$$

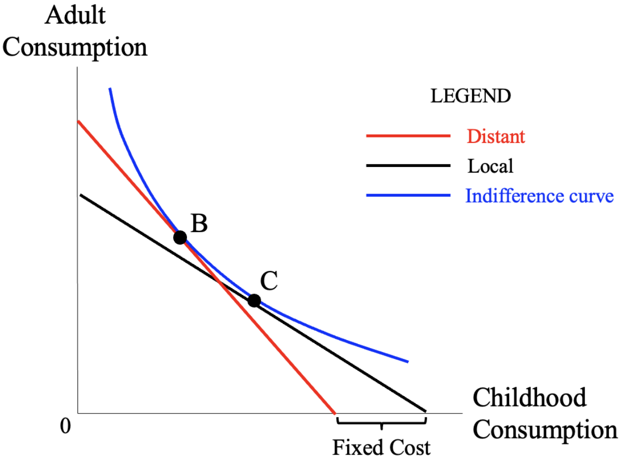
Two cases:

$$\beta\theta'(c_2) = \frac{1}{r_l}$$

$$\beta\theta'(c_2) = \frac{1}{r_f}$$

From FOC, no unique solution

Indifference Between High/Low Adult Consumption



Part A

Ignore for the moment income effects on future consumption. Is it possible that a uniform increase in the two gross rates of return would increase the amount consumed during youth? Relate your answer to the household's Marshallian or Hicksian demand functions.

Part A

- No income effects on adult consumption implies all additional income goes to youth consumption–adding up
- Use the Slutsky Equation:

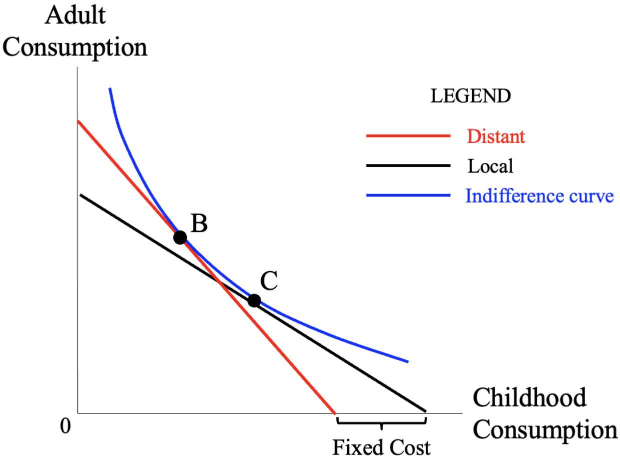
$$\frac{\partial c_1^M}{\partial p_2} = \frac{\partial c_1^H}{\partial p_2} - \frac{\partial c_1^M}{\partial M} C_2$$

- Standard setup: c_1 increases if income effect of rate of return gain outweighs substitution effect

Part A: Positive Substitution Effects

- Normally we think of higher rate of return leads to more adult consumption in a Hicksian sense
- Now consider someone indifferent to high/low adult consumption
- Increase of gross return to each project could cause swap to low adult consumption point–large increase in youth consumption

Part A: Positive Substitution Effects



Part B

In past generations, a flat-rate tax on local investment was implemented and it's effects on investment in both local and distant projects were carefully measured. How would those findings help predict the effect of a new tax on both projects on old-age consumption? Does it matter whether the data and predictions are individual-level or aggregate?

Aggregate Demand

- So far only focused on the household–thinking about aggregate demand will help with predictions
- Who chooses to pay the fixed costs? Even without income effects, it has to be high income types. Look at previous figure and vary the size of the fixed cost as a fraction of income.
- Implies an income cutoff $m^*(r_l, r_f)$ where every household above uses the far project.
- Let $F(m)$ be the CDF of the income distribution–share of people with income below m

Aggregate Demand

We can write average demand for adult consumption as:

$$\bar{D}(r_l, r_f) = F(m^*(r_l, r_f))H(r_l) + (1 - F(m^*(r_l, r_f)))H(r_f)$$

- $F(m^*(r_l, r_f))$ is the fraction of households using the local project
- $H(r_l)$ and $H(r_f)$ are Hicksian demand for adult consumption at corresponding rates of return–relies on no income effects
- To make predictions, we can totally differentiate this expression

Aggregate Demand–Total Differentiation

First compute partials:

$$\frac{\partial \bar{D}}{\partial r_l} = F'(m^*)\frac{\partial m^*}{\partial r_l}H(r_l) + F(m^*)H'(r_l) - F'(m^*)\frac{\partial m^*}{\partial r_l}H(r_f)$$

$$\frac{\partial \bar{D}}{\partial r_f} = F'(m^*)\frac{\partial m^*}{\partial r_f}H(r_l) - F'(m^*)\frac{\partial m^*}{\partial r_f}H(r_f) + (1 - F(m^*))H'(r_f)$$

Then combine to get total derivative:

$$d\bar{D} = F(m^*)H'(r_l)dr_l + (1 - F(m^*))H'(r_f)dr_f +$$

$$(H(r_l) - H(r_f))F'(m^*)\left(\frac{\partial m^*}{\partial r_l}dr_l + \frac{\partial m^*}{\partial r_f}dr_f\right)$$

Aggregate Demand–Total Differentiation

Total Derivative:

$$d\bar{D} = F(m^*)H'(r_l)dr_l + (1 - F(m^*))H'(r_f)dr_f +$$

$$(H(r_l) - H(r_f))F'(m^*)\left(\frac{\partial m^*}{\partial r_l}dr_l + \frac{\partial m^*}{\partial r_f}dr_f\right)$$

- First line is standard movement along demand curves
- Second line is the "switching effect"–changes in consumption due to people moving from one project to another

Effect of Local Tax

$$d\bar{D} = F(m^*)H'(r_l)dr_l + (1 - F(m^*))H'(r_f)dr_f +$$
$$(H(r_l) - H(r_f))F'(m^*)\left(\frac{\partial m^*}{\partial r_l}dr_l + \frac{\partial m^*}{\partial r_f}dr_f\right)$$

- Tax rate increase: $dr_l < 0$
- Even with aggregate data, can back out $H'(r_l)$–everything else is data

Effect of Joint Tax

$$d\bar{D} = F(m^*)H'(r_l)dr_l + (1 - F(m^*))H'(r_f)dr_f +$$
$$(H(r_l) - H(r_f))F'(m^*)\left(\frac{\partial m^*}{\partial r_l}dr_l + \frac{\partial m^*}{\partial r_f}dr_f\right)$$

- Tax rate increase: $dr_l = dr_f < 0$
- Key unknowns for predicting change in consumption: $H'(r_f)$ and $\frac{\partial m^*}{\partial r_l} + \frac{\partial m^*}{\partial r_f}$
- Switching effect will offset

Part C

Would income effects on future consumption change your answers to (a) or (b)?

Income Effects for Part A

- Adding up implies less of an income effect on present consumption–now splitting gain in income between the two goods

- Back to Slutsky:

$$\frac{\partial c_1^M}{\partial p_2} = \frac{\partial c_1^H}{\partial p_2} - \frac{\partial c_1^M}{\partial M} C_2$$

- Less likely for income effect to outweigh substitution effect in the normal sense

- However still possible for increase in rate of return to lead to switching to local project–increasing present income

Income Effects for Part B

- Total derivative becomes *much* more complicated
- Now can't use $H(r)$ and must track how income affects the Marshallian demand curve $D_2(r, m)$
- Changing the rate of return will have a direct income effect and an income effect operating through the change in m^*
- Can try and sign the effects:
  - Savings rate generally rises with income–suggests larger income effects on adult consumption for those with higher incomes
  - First order: increasing rates of return will have a larger direct effect on adult consumption through the income effects
  - Switching effect should be unaffected

Part D

How are the cross-household patterns of consumption and investment related to Marshallian and Hicksian demand functions?

Cross-Household Patterns

- What can we learn about the demand functions from a single cross-section of data?
- For future consumption, no income effects means Marshallian and Hicksian demand functions are equal: $H(r)$
- Data gives us two points on this curve: $H(r_l)$ and $H(r_f)$
- What else can we say about demand for c_2?

Cross-Household Patterns

- Remember FOC's:

$$\beta\theta'(c_2^*) = \frac{1}{r_l}$$

$$\beta\theta'(c_2^*) = \frac{1}{r_f}$$

- With some assumption for β–can back out two points on $\theta'(\cdot)$

- Taking derivative with respect to r_j yields:

$$\beta\theta''(c_2)\frac{\partial c_2}{\partial r_j} = -\frac{1}{r_j^2}$$

$$\implies \frac{\partial c_2}{\partial r_j} = -\frac{1}{\beta\theta''(c_2)r_j^2}$$

$$\implies \epsilon_{2j} = -\frac{1}{\beta\theta''(c_2)r_j c_2}$$

Cross-Household Patterns

$$\epsilon_{2j} = -\frac{1}{\beta\theta''(c_2) r_j c_2}$$

- We know two points on $\theta'(\cdot)$ so can try and estimate/bound $\theta''(\cdot)$
- Tells us demand elasticity as a function of prices and consumption–tends to 0 as you increase rates of return
- This relies heavily on no income effects! Unrealistic but useful math...

Part E

Now consider taste variation too, perhaps as simply as splitting the population into two groups, with common tastes within groups. The endowment distribution is the same for the two groups. Could taste variation be more important than endowment variation for learning about the demand functions?

Adding Taste

- We can parameterize tastes by having household specific β_i
- Still have two stage problem yielding FOC's

$$\frac{1}{r_f} = \beta_i \theta'(c_2)$$

$$\frac{1}{r_l} = \beta_i \theta'(c_2)$$

Adding Taste

- Moving the discount factor to the other side

$$\frac{1}{\beta_i r_f} = \theta'(c_2)$$

$$\frac{1}{\beta_i r_l} = \theta'(c_2)$$

- Heterogeneous tastes is isomorphic to households differing in their rates of return

- It's as if households who value the future more have higher rates of returns and larger relative differences between the two projects–both factors push toward paying the fixed cost

- Even though there are only two real rates of return, taste variation allows us to observe demand at a continuum of prices

Part F

What, if any, comparative statics in this model cause old-age consumption and its share coming from local projects to be positively correlated?

Comparative Statics

- Comparative Statics: how does changing the parameters of the problem change our endogenous variables?
- Parameters: r_f, r_l, τ
- Endogenous Variables: c_1, c_2, I_f, I_l
- Intuition: in general, more adulthood consumption is associated with higher share from *distant* projects

Comparative Statics

- Increasing r_f: Increases consumption in adulthood by substitution and switching effect–increases share from far projects $\implies$ negative correlation
- Increasing r_l: Increases consumption in adulthood by substitution but decreases from switching effect–increases share from local projects. If substitution$>$switching $\implies$ positive correlation
- Increasing τ: no change in adult consumption conditional on investment decision but reduces share using the far project and therefore also adult consumption $\implies$ negative correlation.