Problem Set 2 - Question 2

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Question, I

Here we look at potentially costly cooperation between mothers and fathers for the purpose of investing in children. q denotes the quantity of human capital investments, with price normalized to one. Each parent perceives a value of $u(q)$ in units of money, regardless of how the investment is financed between the parents. That is, q is a household-level public good. u is increasing and strictly concave.

Question, II

- *a) What would be the equilibrium q if each parent acted independently, taking as given the resources the other parent puts toward investment?*
- *b) How does your previous answer compare to the Pareto-optimal quantity? [Let's not consider the child as a third party; the child's welfare is already reflected as part of u].*

Question, III

- *c) How does the gap g between one parent's marginal cost of q and the same parent's marginal benefit vary with q. You can focus on the two values of q you found in (a) and (b)?*
- *d) Suppose that, in addition to the resource costs of q, a couple that coordinates their contributions does so at a cost c(g), where c() is strictly convex. If c(0) were zero, how is that related to the nudge hypothesis from Chapter 5 of CPT?*
- *e) Does your answer from (c) suggest that c'(g) $>$ 0 for g $>$ 0?*

- *f) A coordination equilibrium is a quantity q and gap g that maximizes the total surplus, taking into account the coordination costs. Assuming that c'(g) $>$ 0 for g $>$ 0, how does the coordination equilibrium value of q compare to the two values you found in (a) and (b)?*
- *g) Under a positive externality, we observe the market quantity q that is less than the Pareto-efficient quantity q^P. True, false or uncertain: the difference u(q^P)- u(q) measures the opportunity cost of the market failure.*
- *h) It is sometimes claimed that living as a two-parent family can be stressful and otherwise more costly than a single-parent household. Others claim that the two-parent family arrangement increases the human capital of children. What does the model say about the mutual consistency, or lack thereof, of these two claims?*

Goal

- Think about externalities and public goods.
- Formalize the idea of costly coordination Casey discussed last week.
- Apply price theory to non-pecuniary settings.

Setup

- Let e_1 and e_2 be the effort invested by Parents 1 and 2, respectively.
- Normalize e_1 and e_2 so that the costs of effort are e_1 and e_2.
  - That is, you pay cost e_i for exerting e_i units of effort $(i = 1, 2)$.
  - This is a change of unit without loss. It will greatly simplify notations.
- Let $u(q)$ be the utility (in dollaras) to one parent.
  - Assume $u' > 0, u'' < 0$ and $u'(0) = +\infty$.
- Human capital investments are financed by parents: $q = e_1 + e_2$.
  - Investments are "financed" by parents: Hence additivity makes sense.
  - Buried in the functional form:
    - Constant marginal returns of effort.
    - Efforts are (perfect) substitutes in production.

Assumptions and Functional Forms?

- You can use a general production function $F(e_1, e_2)$.
  - As long as you are clear about your assumptions, it is fine.
- You can assume the following:
  - Assume F admits the Inada conditions (to rule out corner solutions), i.e., $F(0,0) = 0, F_1(0, e_2) = +\infty, F_2(e_1, 0) = +\infty$.
  - Assume monotonicity and diminishing returns, i.e., $F_i > 0, F_{ii} < 0, i = 1, 2$.
    - You may also assume F is first convex then concave. But I'll stick to diminishing returns to keep things simple.
  - Cross partials unsigned to allow for complementarity / substitutability
  - There can also be productivity differentials.
- But the question is not about these. So I simply assume $q = e_1 + e_2$.

Part (a): Act Independently

a) What would be the equilibrium q if each parent acted independently, taking as given the resources the other parent puts toward investment?

- If acting independently, Parent i solves (fixing e_{-i} constant)

$$\max_{e_i} \; u(e_1 + e_2) - e_i$$

- FOCs

$$\begin{aligned} u'(e_1 + e_2) &= 1 \text{ for Parent 1, fixing } e_2 \\ u'(e_1 + e_2) &= 1 \text{ for Parent 2, fixing } e_1 \end{aligned}$$

- Existence guaranteed by Inada.
- The equilibrium effort levels satisfy $u'\left(e_1^I + e_2^I\right) = 1$ and the human capital investment is q^I where $u'\left(q^I\right) = 1$.
  - The split of e_1^I and e_2^I is not determined by the model, but q^I is.
  - Supscript I stands for "independently".

Part (b): Pareto Optimality

b) How does your previous answer compare to the Pareto-optimal quantity?

- Pareto optimal choices maximize total surplus,

$$\max_{e_1, e_2} 2u(e_1 + e_2) - e_1 - e_2$$

- FOC pins down Pareto optimal effort $e_1^P + e_2^P$ by

$$u'\left(e_1^P + e_2^P\right) = \frac{1}{2}$$

- Compare this to Part (a), optimal human capital $q^P = e_1^P + e_2^P > q^I$.
    - Marginal family benefit > marginal personal benefit.

c) How does the gap g between one parent's marginal cost of q and the same parent's marginal benefit vary with q?

- Let $g(q)$ be the gap between one parent's MC of q and their MB of q.

$$g(q) = 1 - u'(q)$$

- If you use some production function $F(e_1, e_2)$, you will get

$$g(q) = \frac{1}{F_i(e,e)} - u'(F(e,e)) \text{ where } q = F(e,e)$$

  - You need to make sure your definition compiles.
  - E.g., assume symmetry, $F_1(e,e) = F_2(e,e)$. Same gap for both.
    - Without symmetry, you need to keep track of g's for two parents.
  - We can invert $q = F(e,e)$ to obtain a unique e due to monotonicity.
  - Therefore, $g(q)$ is indeed a function of q.

Part (c): MC−MB, II

- In (a), there is no such gap.

$$g(q^I) = 1 - u'\left(q^I\right) = 0$$

- In (b), the gap is positive since $q^P > q^I$

$$g(q^P) = 1 - u'\left(q^P\right) > 0$$

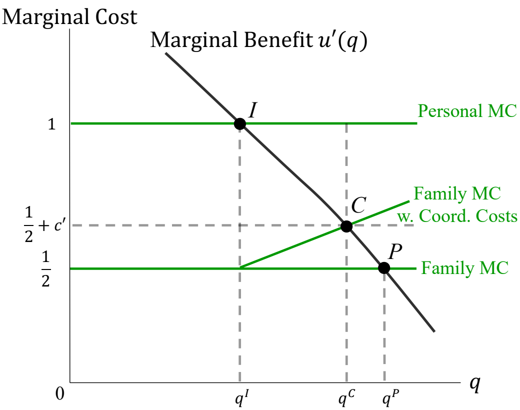
  - On average, the couple is better off.
  - At the margin, there is an incentive to shirk.
- More generally, $g'(q) = -u''(q) > 0$.

Part (d): Nudging

d) Suppose that, in addition to the resource costs of q, a couple that coordinates their contributions does so at a cost c(g), where c() is strictly convex. If c'(0) were zero, how is that related to the nudge hypothesis from Chapter 5 of CPT?

- It is usually easy to nudge people if they are near the optimal choice.
- Consider a consumer demand figure

Part (d): $c'(0) = 0$



Marginal Cost
Marginal Benefit $u'(q)$
I
Personal MC
1
C
Family MC
w. Coord. Costs
$\frac{1}{2} + c'$
P
$\frac{1}{2}$
Family MC
0
q^I
q^C
q^P
q

Part (e)

e) Does your answer from (c) suggest that c'(g) $>$ 0 for g $>$ 0?

- Yes.
- At the margin, there is always an incentive to shirk.
  - The higher g is, the more attractive shirking becomes.
  - The couple needs something to stop that.
  - Therefore, c should be increasing in g.

Part (f): Coordination Equilibrium, I

f) A coordination equilibrium is a quantity q and gap g that maximizes the total surplus, taking into account the coordination costs. Assuming that c'(g) $>$ 0 for g $>$ 0, how does the coordination equilibrium value of q compare to the two values you found in (a) and (b)?

- In a coordination equilibrium, the couple maximizes

$$\max_{e_1, e_2} 2u\left(e_1 + e_2\right) - e_1 - e_2 - c\left(g\left(e_1 + e_2\right)\right)$$

- FOC becomes

$$u'\left(q\right) = \frac{1}{2} + \frac{1}{2}c'\left(g\left(q\right)\right)g'\left(q\right)$$

where

$$c'(\cdot) > 0;\ g'(\cdot) > 0$$

- This pins down the output q^C in a coordination equilibrium.

Part (f): Coordination Equilibrium, II

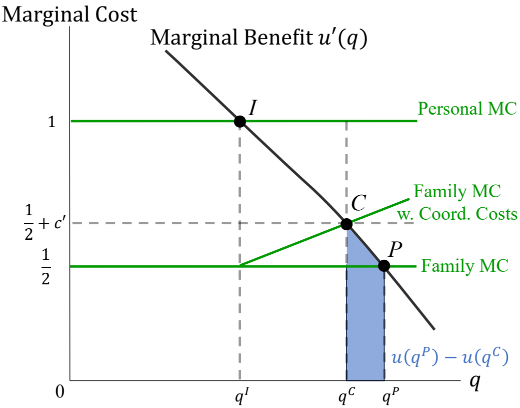
- Compare the above FOC to Part (a), $q^I < q^C$.
  - When parents act independently, $g = 0$ and $c'(0) = 0$.
  - However, if coordination involves a fixed cost, they may be better off in (a).
- Compare the above FOC to Part (b), $q^C < q^P$.
  - Higher marginal cost in a coordination equilibrium.

Part (g): Opportunity Cost

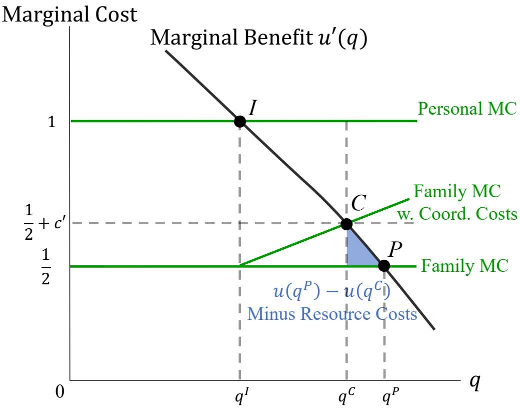
g) Under a positive externality, we observe the market quantity q that is less than the Pareto-efficient quantity q^P. True, false or uncertain: the difference $u(q^P)$ - $u(q)$ measures the opportunity cost of the market failure.

- False.
- Interpret the externality figure.

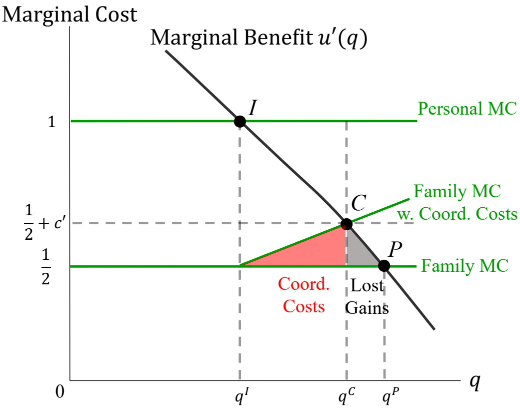
Part (g): $u\left(q^{P}\right)-u\left(q\right)$



Part (g): $u\left(q^P\right) - u\left(q\right)$ minus Resource Costs



Part (g): Cost of Market Failure

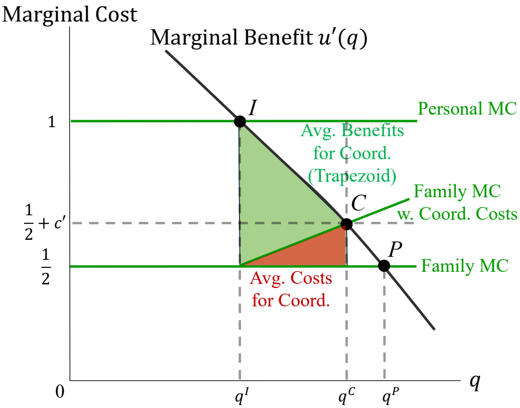


Part (h)

h) It is sometimes claimed that living as a two-parent family can be stressful and otherwise more costly than a single-parent household. Others claim that the two-parent family arrangement increases the human capital of children. What does the model say about the mutual consistency, or lack thereof, of these two claims?

- On average, coordination makes the couple better off.
  - Total gain is the larger triangular area.
- At the margin, coordination can still be costly.
  - People coordinate until the marginal cost equals the marginal benefit.

Part (h): Average Benefit vs. Average Cost



Part (h): Marginal Benefit vs. Marginal Cost