The L-functions and Modular Forms DataBase (LMFDB)

John Voight
Dartmouth College

Simons Collaboration on Arithmetic Geometry,
Number Theory and Computation
Annual Meeting
Simons Foundation, NYC
10 January 2019

Outline of this talk

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1. (Re)Introduce the LMFDB;
2. New classical modular forms functionality (private screening/world premiere!);
3. Future plans.

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It is now managed like a journal, with managing editors (John Cremona, John Jones, Drew Sutherland, and me) and a team of associate editors.

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- Data should have statistics, to notice patterns in aggregate.

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Big picture

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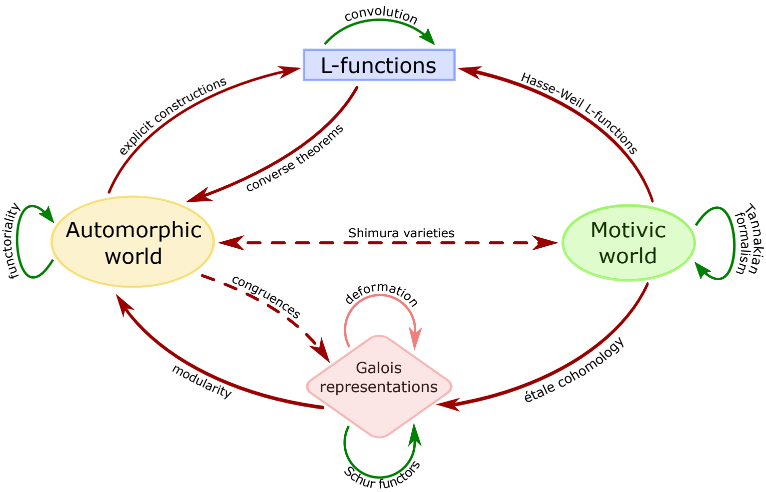

LMFDB Universe

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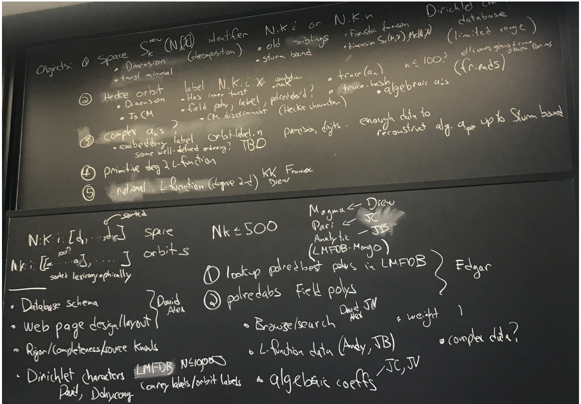

History: Wada 1971

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269	?
277	$(x^{11}-21x^9+152x^7-13x^6-445x^5+102x^4+450x^3-147x^2-56x+4)^2$
281	$x^{22}+50x^{20}+1081x^{18}+13285x^{16}+102611x^{14}+519821x^{12}+1748638x^{10}+3865180x^8+5420534x^6+4487135x^4+1884424x^2+265883$
293	?
313	$(x^{12}-21x^{10}-5x^9+158x^8+63x^7-509x^6-241x^5+648x^4+272x^3-250x^2-81x-4)^2$
317	?
337	$(x^{13}-x^{12}-22x^{11}+16x^{10}+182x^9-91x^8-697x^7+221x^6+1217x^5-215x^4-808x^3+61x^2+175x)^2$
349	?
353	$x^{28}+57x^{26}+1429x^{24}+20814x^{22}+196024x^{20}+1256811x^{18}+5621908x^{16}+17671372x^{14}+38764478x^{12}+58122566x^{10}+57447036x^8+35475661x^6+12722068x^4+2334034x^2+160344$
373	?
389	?
397	?
401	$x^{32}+71x^{30}+2276x^{28}+43639x^{26}+558857x^{24}+5055254x^{22}+33307004x^{20}+162429517x^{18}+589806266x^{16}+1590734165x^{14}+3152518983x^{12}+4498023798x^{10}+4468102725x^8+2926806000x^6+1151600000x^4+225600000x^2+12800000$
409	$(x^{16}+x^{15}-27x^{14}-27x^{13}+284x^{12}+272x^{11}-1487x^{10}-1296x^9+4094x^8+2998x^7-5757x^6-3006x^5+3774x^4+907x^3-964x^2+32)^2$
421	?

History: Antwerp IV 1975

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SOURCES AND RELIABILITY OF THE TABLES.

1. Tables 2-6.

Table 2 has been prepared for this volume by N.M. Stephens and James Davenport using the ATLAS at the Chilton laboratory. No errors are likely, but it is possible that in one or two cases a multiple of a generator has been listed rather than a generator.

Table 3 was prepared by J. Vélu; except for the eigenvalues of the W_q, the calculations have been performed independently by Stephens and Vélu, and no discrepancy has been observed; there are also strong requirements of consistency. The W_q-eigenvalues were calculated separately by D.J. Tingley, by operating on the one-dimensional homology; his results were incorporated in Vélu's tables by reading in cards. Consistency requirements make any errors in these particular calculations of Tingley hardly conceivable; copying errors are possible but unlikely. The identifying letters of the curves in this table have been added in manuscript; I hope they have never been jumbled.

Table 4 has been extracted from the unpublished 1966 Manchester thesis of F.B. Coghlan. Unless some mathematical error has remained undetected, the table should be complete.

The third column of Table 5 comes from a much larger table supplied by A.O.L. Atkin; the final column was supplied by D.J. Tingley, who has completely computed the action of the Hecke algebra on the 1-dimensional homology for N ⩽ 300. His complete tables of eigenvalues will be published elsewhere. Table 6, too, is copied from a table of Atkin's; this table is new, note that the much shorter table of Deuring contains errors. Tables 5 and 6 were produced by typewriter rather than line-printer; it was very difficult to avoid slips.

11 = 11

A	0	-1	1	0	0	-	1	I1	
B	0	-1	1	-10	-20	-	5	I5	
C	0	-1	1	-7820	-263580	-	1	I1	

14 = 2.7

A	1	0	1	-1	0	-	2, 1	I2,	I
B	1	0	1	-11	12	+	1, 2	I1,	I
C	1	0	1	4	-6	-	6, 3	I6,	I
D	1	0	1	-36	-70	+	3, 6	I3,	I
E	1	0	1	-171	-874	-	18, 1	I18,	I
F	1	0	1	-2731	-55146	+	9, 2	I9,	I

15 = 3.5

A	1	1	1	0	0	-	1, 1	I1,	I
B	1	1	1	-5	2	+	2, 2	I2,	I
C	1	1	1	-10	-10	+	4, 4	I4,	I
D	1	1	1	-80	242	+	1, 1	I1,	I1
E	1	1	1	-135	-660	+	8, 2	I8,	I
F	1	1	1	35	-28	-	2, 8	I2,	I
G	1	1	1	-110	-880	-	16, 1	I16,	I1
H	1	1	1	-2160	-39540	+	4, 1	I4,	I1

17 = 17

A	1	-1	1	-1	0	+	1	I1	
B	1	-1	1	-6	-4	+	2	I2	
C	1	-1	1	-1	-14	-	4	I4	
D	1	-1	1	-91	-310	+	1	I1	

19 = 19

A	0	1	1	1	0	-	1	I1	
B	0	1	1	-9	-15	-	3	I3	
C	0	1	1	-769	-8470	-	1	I1	

20 = 2.2.5

A	C	1	0	-1	0	+	4, 1	IV,	I1
B	0	1	0	4	4	-	8, 2	IV*,	I2
C	0	1	0	-41	-116	+	4, 3	IV,	I3
D	0	1	0	-36	-140	-	8, 6	IV*,	I6

Stein 2000s

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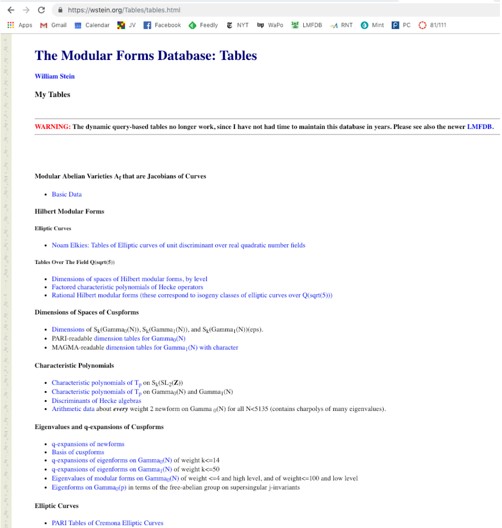

https://wstein.org/Tables/tables.html

Apps Gmail Calendar JV Facebook Feedly NYT WaPo LMFDB RNT Mint PC 81/111

The Modular Forms Database: Tables

William Stein

My Tables

WARNING: The dynamic query-based tables no longer work, since I have not had time to maintain this database in years. Please see also the newer LMFDB.

Modular Abelian Varieties A_f that are Jacobians of Curves

- Basic Data

Hilbert Modular Forms

Elliptic Curves

- Noam Elkies: Tables of Elliptic curves of unit discriminant over real quadratic number fields

Tables Over The Field Q(sqrt(5))

- Dimensions of spaces of Hilbert modular forms, by level
- Factored characteristic polynomials of Hecke operators
- Rational Hilbert modular forms (these correspond to isogeny classes of elliptic curves over Q(sqrt(5)))

Dimensions of Spaces of Cuspforms

- Dimensions of S_k(Gamma$_0$(N)), S_k(Gamma$_1$(N)), and S_k(Gamma$_1$(N))(eps).
- PARI-readable dimension tables for Gamma$_0$(N)
- MAGMA-readable dimension tables for Gamma$_1$(N) with character

Characteristic Polynomials

- Characteristic polynomials of T_p on $S_k(SL_2(\mathbf{Z}))$
- Characteristic polynomials of T_p on Gamma$_0$(N) and Gamma$_1$(N)
- Discriminants of Hecke algebras
- Arithmetic data about ***every*** weight 2 newform on Gamma $_0$(N) for all N<5135 (contains charpolys of many eigenvalues).

Eigenvalues and q-expansions of Cuspforms

- q-expansions of newforms
- Basis of cuspforms
- q-expansions of eigenforms on Gamma$_0$(N) of weight k<=14
- q-expansions of eigenforms on Gamma$_1$(N) of weight k<=50
- Eigenvalues of modular forms on Gamma$_0$(N) of weight <=4 and high level, and of weight<=100 and low level
- Eigenforms on Gamma$_0$(p) in terms of the free-abelian group on supersingular j-invariants

Elliptic Curves

- PARI Tables of Cremona Elliptic Curves

Stein tables

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Search page and 11.2.a.a (to show homepages)

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   Search page

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5. Can download the form to get more coefficients, reduce, ...

Item 6: Trace tables (matching classical modular forms)

Item 6: Trace tables (matching classical modular forms)

Dear John, please forgive a cold call.

With colleagues, I have been calculating local zeta functions for Calabi-Yau varieties in a family depending on t. In general this gives rise to a factor

$$1 + aT + bpT^2 + ap^3T^3 + p^6T^4.$$

For $t = 33 \pm 8\sqrt{17}$, the quartic factors in the form

$$(1 - \alpha pT + p^3T^2)(1 - \beta T + p^3T^2)$$

whenever 17 is a square mod p.

We have lists of data of the form $p, t, \{\alpha, \beta\}$ for when this happens, and we believe that the coefficients $\{\alpha, \beta\}$ are coefficients of a (Hilbert) modular form.

So finally I come to my question, which is: is there a practical way of extracting lists of coefficients of such modular forms from LMFDB with which to compare our lists?

We have a number of cases similar to the example above so we hope for a simple procedure that we can repeat as necessary.

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where f, g are classical modular forms of weights $2, 4$.
- We have a fair amount of eigenvalue data:

p	13	19	43	67	$\cdots$
$a_p(f)$	2	-4	-4	-4	$\cdots$
$a_p(g)$	-42	60	508	-676	$\cdots$

Let's search for the form!

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d. Dimension data for subspaces by projective image type. 1161.1.i.

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10. Provable analytic ranks (including non-self dual forms of analytic rank 1). We found a rational form (2.42.a.a) of weight 42 and a form (8.14.b.a) of weight 14 each having analytic rank 1.

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We hope the LMFDB will be a useful tool on your workbench!