Sage[1]: free, open-source mathematical software that supports research and teaching

John Cremona
University of Warwick, UK

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Introduction

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- ▶ Why am I talking about Sage? Number theorists have always used computers in their research. Sage is a relatively new tool, and deserves to be more widely known. It is not just about number theory!

What is Sage

- Well over 300,000 lines of new Python/Cython code
- A Distribution of mathematical software (nearly 100 third-party packages); builds from source without dependency (over 5 million lines of code)
- Exact and numerical linear algebra, optimization (numpy, scipy, R, and gsl all included)
- Group theory, number theory, combinatorics, graph theory
- Symbolic calculus
- Coding theory, cryptography and cryptanalysis
- 2d and 3d plotting
- Statistics (Sage includes R)
- Overall range of functionality rivals that of Maple, Matlab, and Mathematica and is growing very rapidly
- Sage is huge! (The reference manual is over 4800 pages.)

Where did Sage come from, and who is behind Sage?

- William Stein (U. Washington, Seattle) started Sage at Harvard in January 2005.
- He reckoned that no existing math software (free or commercial) was good enough.
- Sage-1.0 released February 2006 at Sage Days 1 (San Diego).
- Sage Days Workshops 1, 2, ..., 26, at many locations, including Bristol (SD6, November 2007).
- Over 170 accounts on `http://trac.sagemath.org/sage_trac`
- The last release (4.3.2 on 2010-02-07) had contributions from 41 people, including two first-timers.
- supported by UW, NSF, DoD, Microsoft, Google, Sun, etc.

The Sage community

- `sage-support@googlegroups.com`: 1600+ members, several hundred messages per month. Users usually get a response very quickly.

- `sage-support@googlegroups.com` for Sage developers: 1123 members, 1–2000 messages per month. Where most design decisions are made (democratically!).

- 2 IRC channels at `irc.freenode.net` (sage-devel for development issues, sage-support for support questions)

- trac server for tracking bugs, suggested enhancements etc. All new code is peer-reviewed before acceptance.

- Over 80% of Sage library code functions have documentation (from which the reference manual is automatically created) and examples ("doctests") which are tested on many platforms before each release.

Sage worldwide

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http://www.sagemath.org/development-map.html

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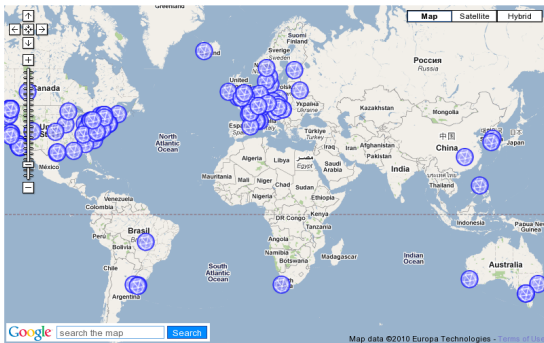
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Sage developers around the world

This is a map of all contributors to the Sage project. There are currently 181 contributors in 117 different places from all around the world.

Map Zoom: Earth - USA (UW, West, East) - Europe - Asia - S. America - Australia

William Stein. Tim Abbott. Michael Abshoff. Antti Ajanki. Martin Albrecht. Nick Alexander. Bill Allombert. Ivan Andrus. Benjamin Antieau. Maite Aranes. Jennifer Balakrishnan. Jason Bandlow. Gregory Bard. Sébastien Barthélemy. Rob Beezer. Karim Belabas. Arnaud Bergeron. Luis Berlioz. Francois Bissey. Jonathan Bober. Tom Boothby. Nicolas Borie. Robert Bradshaw. Michael Brickenstein. Nils Bruin. Stanislav Bulygin. Dan Bump. Iftikhar Burhanuddin. Paul Butler. Ondrej Certik. Wilson Cheung. Dan Christensen. Craig Citro. Anders Claesson. Francis Clarke. Timothy Clemans. Alex Clemesha. Nathann Cohen. Jenny Cooley. John Cremona. Karl-Dieter Crisman. Fidel Barrera Cruz. Doug Cutrell. Alyson Deines. Vincent Delecroix. Tom Denton. Didier Deshommes. Ryan Dingman. Dan Drake. Tom Draper. Alexander Dreyer. Tim Dumol. Nathan Dunfield. Gabriel Ebner. Burcin Erocal. Ron Evans. Richard J. Fateman. Lars Fischer. Evan Fosmark. Gary Furnish. Alex Ghitza. Andrzej Giniewicz. Amy Glen. Daniel Gordon. Chris Gorecki. Jan Groenewald. Rob Gross. Jason Grout. Carlo Hamalainen. Marshall Hampton. Jon Hanke. David Møller Hansen. Mike Hansen.

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What is Sage for, and who uses it?

- ▶ Mission statement: "Creating a viable free open source alternative to Magma, Maple, Mathematica, and Matlab."

- ▶ Mathematical research: Sage's founders are researchers in number theory, who wanted a tool they could use, and extend. Sage is now used by many researchers in mathematics (not just number theory: combinatorics, algebra, graph theory, and more).

- ▶ Algorithm development: Sage is a good platform for developing new algorithms. Once tested and peer-reviewed, they can (easily) be incorporated into Sage and used by others.

- ▶ Teaching: many people have used sage as an adjunct to traditional courses, or as the basis of an entire course, in subject ranging from calculus and linear algebra to cryptography and number theory.

What is in Sage?

- ▶ Sage is built out of nearly 100 open-source packages and features a unified interface.
- ▶ Sage can be used to study elementary and advanced, pure and applied mathematics.
- ▶ This includes a huge range of mathematics, including basic algebra, calculus, elementary to very advanced number theory, cryptography, numerical computation, commutative algebra, group theory, combinatorics, graph theory, exact linear algebra and much more.
- ▶ Sage combines various software packages and seamlessly integrates their functionality into a common experience.

Components of Sage

- ATLAS: Automatically Tuned Linear Algebra Software
- BLAS: Basic Fortan 77 linear algebra routines
- Bzip2: High-quality data compressor
- Cddlib: Double Description Method of Motzkin
- Common Lisp: Multiparadigm and general-purpose programming language
- CVXOPT: Convex optimization, linear programming, least squares, etc.
- Cython: C-Extensions for Python
- Docutils: an open-source text processing system for processing plaintext documentation into useful formats, such as HTML or LaTeX. It includes reStructuredText, the easy to read, easy to use, what-you-see-is-what-you-get plaintext markup language.
- mwrank: mwrank is a program for computing Mordell-Weil groups of elliptic curves over Q via 2-descent. Since November 2007 mwrank has formed part of the eclib package which is included in Sage.
- F2c: Converts Fortran 77 to C code
- Flint: Fast Library for Number Theory
- FpLLL: Euclidean lattice reduction
- FreeType: A Free, High-Quality, and Portable Font Engine
- G95: Open source Fortran 95 compiler
- GAP: Groups, Algorithms, Programming
- GD: Dynamic graphics generation tool
- Genus2reduction: Curve data computation
- Gfan: Grbner fans and tropical varieties
- Givaro: C++ library for arithmetic and algebra
- GMP-ECM: Elliptic Curve Method for Integer Factorization
- GNU TLS: Secure networking
- GSL: Gnu Scientific Library
- Jinja: state of the art, general purpose template engine
- JsMath: JavaScript implementation of LaTeX
- IML: Integer Matrix Library
- IPython: Interactive Python shell

Components of Sage (continued)

- LAPACK: Fortan 77 linear algebra library
- Lcalc: L-functions calculator
- Libgcrypt: General purpose cryptographic library
- Libgpg-error: Common error values for GnuPG components
- libpng: Bitmap image support
- Linbox: C++ linear algebra library
- M4RI: Linear Algebra over GF(2)
- Matplotlib: Python plotting library
- Maxima: computer algebra system
- Mercurial: Revision control system
- MoinMoin Wiki
- MPFI: Multiple Precision Floating-point Interval library
- MPFR: C library for multiple-precision floating-point computations with correct rounding
- MPIR: Multiple Precision Integers and Rationals
- ECLib:Cremona's Programs for Elliptic curves
- NetworkX: Graph theory
- NTL: Number theory C++ library
- Numpy: Numerical linear algebra
- OpenCDK: Open Crypto Development Kit
- OpenOpt: Integrates solvers for numerical optimization into a single common Python-based framework.
- PALP: A Package for Analyzing Lattice Polytopes
- PARI/GP: Number theory calculator
- Pexpect: Pseudo-tty control for Python
- PolyBoRi: Polynomials Over Boolean Rings
- PyCrypto: Python Cryptography Toolkit
- Python: Interpreted language
- Pynac: Symbolic manipulation with Python objects (based on GiNaC)
- Qd: Quad-double/Double-double Computation Package
- R: Statistical Computing

Components of Sage (continued)

- Readline: Line-editing
- Rpy: Python interface to R
- Scipy: Python library for scientific computation
- Singular: fast commutative and noncommutative algebra
- Scons: Software construction tool
- Sphinx: Python Documentation Generator
- SQLAlchemy: The Python SQL Toolkit and Object Relational Mapper
- SQLite: Relation database
- Sympow: L-function calculator
- Symmetrica: Representation theory
- Sympy: Python library for symbolic computation o mpmath: Mpmath is a pure-Python library for multiprecision floating-point arithmetic.
- Tachyon: lightweight 3d ray tracer
- Termcap: Simplifies the process of writing portable text mode applications
- Twisted: Python networking library
- Weave: Tools for including C/C++ code within Python
- Zlib: Data compression library
- ZODB: Object-oriented database

Command line and notebook interfaces

Sage can be used in several ways. First, there is a command-line interface:

```
jec@selmer%sage
----------------------------------------------------------------
| Sage Version 4.3.2, Release Date: 2010-02-06
| Type notebook() for the GUI, and license() for information.
----------------------------------------------------------------
sage: 2+2
4
sage: (1+factorial(30)).factor()
31 * 12421 * 82561 * 1080941 * 7719068319927551
sage: Fp=GF(next_prime(2^100)); Fp
Finite Field of size 1267650600228229401496703205653
sage: time EllipticCurve(Fp,[123,456]).cardinality()
CPU times: user 0.00 s, sys: 0.00 s, total: 0.00 s
Wall time: 0.26 s
1267650600228229939829009573820
```
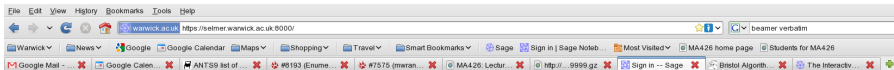
Notebook (via built-in web server)

Sage has a built-in webserver which enables users to connect to it either on a local machine (as on my laptop here) or on a remote server (such as the one at `http://www.sagenb.org/`). Departments can run their own Sage servers for their students, or students can download and run their own copy of Sage.

Welcome!

Sage is a different approach to mathematics software.

The Sage Notebook

With the Sage Notebook anyone can create, collaborate on, and publish interactive worksheets. In a worksheet, one can write code using Sage, Python, and other software included in Sage.

General and Advanced Pure and Applied Mathematics

Use Sage for studying calculus, elementary to very advanced number theory, cryptography, commutative algebra, group theory, graph theory, numerical and exact linear algebra, and more.

Use an Open Source Alternative

By using Sage you help to support a viable open source alternative to Magma, Maple, Mathematica, and MATLAB. Sage includes many high-quality open source math packages.

Use Most Mathematics Software from Within Sage

Sage makes it easy for you to use most mathematics software together. Sage includes GAP, GP/PARI, Maxima, and Singular, and dozens of other open packages.

Use a Mainstream Programming Language

You work with Sage using the highly regarded scripting language Python. You can write programs that combine serious mathematics with anything else.

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☐	isogenies char=3, j nonzero case	Kimi / cremona Share now	19 days ago by Kimi
☐	magma char 3 computation	cremona / Kimi Add or Delete	20 days ago by Kimi
☐	W2	cremona Share now	21 days ago by cremona
☐	Weierstrass	cremona Share now	23 days ago by cremona
☐	MA426_L2	cremona Share now	24 days ago by cremona
☐	isogenies_genus_0	cremona / Kimi Add or Delete	25 days ago by cremona
☐	ssec	cremona Share now	25 days ago by cremona
☐	polynomial rings	cremona Share now	28 days ago by cremona
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2+2
```
    4

```
(1+factorial(30)).factor()
```
    31 * 12421 * 82561 * 1080941 * 7719068319927551

```
Fp=GF(next_prime(2^100)); Fp
```
    Finite Field of size 1267650600228229401496703205653

```
E = EllipticCurve(Fp,[123,456]); E.cardinality()
```
    1267650600228229939829009573820

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