Summary of Notations from Logic & Discrete Mathematics

PROPOSITIONS: where P and Q are any such logical formulae

$\neg P$	logical negation: "not P"	
$P \vee Q$	logical disjunction: "P or Q"	
$P \wedge Q$	logical conjunction: "P and Q"	
$P \Rightarrow Q$	logical implication: "P implies Q"	$\widehat{\equiv} \neg P \vee Q$
$P \Leftrightarrow Q$	logical equivalence: "P iff Q"	$\widehat{\equiv} (P \Rightarrow Q) \wedge (Q \Rightarrow P)$
true	the proposition that *always* holds	
false	the proposition that *never* holds	$\widehat{\equiv} \neg \mathsf{true}$

PREDICATES: propositions over 'variables', where P is a predicate (on x)

$x = y$	equality of values, provided x and y have the same 'type'	
$x \neq y$	inequality of values (its negation)	$\widehat{\equiv} \neg(x = y)$
$\forall x \bullet P$	universal quantification: "P holds for *all* values x"	
$\exists x \bullet P$	existential quantification: "P holds for *some* value x"	

SETS: where S identifies some (defined) set, and P is a predicate (on x)

$\mathsf{fin}\ S$	the property that S is *finite*	
$\# S$	cardinality: number of elements in S, provided S is *finite*	
$x \in S$, $x \notin S$	membership: x is an element of S, and its negation	
$\forall x : S \bullet P$	universal quantification over S	$\widehat{\equiv} \forall x \bullet (x \in S \Rightarrow P)$
$\exists x : S \bullet P$	existential quantification over S	$\widehat{\equiv} \exists x \bullet (x \in S \wedge P)$

SUBSETS (of S)

$\wp S$	powerset: the set of all *subsets* of S	
$\mathsf{set}\ S$	the set of all *finite* subsets of S	$\widehat{=} \mathsf{fin} \cdot \wp S$
$\emptyset[S]$	the *empty* subset of S (where its 'type' $[S]$ is usually omitted)	
$\{x : S \bullet P\}$	comprehension: subset of S with elements such that P holds	
	assuming $x_1, \ldots x_n : S$	
$\{x_1\}$	the singleton subset of S, containing one element x_1	
$\{x_1, \ldots x_n\}$	enumerated subset of S, containing only elements $x_1, \ldots x_n$	
	assuming $X, Y : \wp S$	
$X \subseteq Y$	subset: X is included in Y	$\widehat{\equiv} \forall x : X \bullet x \in Y$
$X \subset Y$	subset: X *strictly* included in Y	$\widehat{\equiv} X \subseteq Y \wedge X \neq Y$
$X \not\subseteq Y$, $X \not\subset Y$	and their negations ...	
$X = Y$, $X \neq Y$	set equality, inequality	$\widehat{\equiv} X \subseteq Y \wedge Y \subseteq X, \ \ldots$
$X \cup Y$	union of X and Y	$\widehat{=} \{x : S \bullet x \in X \vee x \in Y\}$
$X \cap Y$	intersection of X and Y	$\widehat{=} \{x : S \bullet x \in X \wedge x \in Y\}$
$X \setminus Y$	set difference of X and Y	$\widehat{=} \{x : S \bullet x \in X \wedge x \notin Y\}$

$\mathsf{SETS\ of\ SUBSETS}$ (of S), e.g. $SS \mathrel{\widehat{=}} \{X_1, \ldots\ X_n\}$ for $X_1, \ldots\ X_n : \wp\, S$

$\bigcup SS$	union of subsets, $\widehat{=}\ \bigcup X_i : SS \bullet X_i$
$\bigcap SS$	intersection of subsets, $\widehat{=}\ \bigcap X_i : SS \bullet X_i$
$\Diamond\ SS$	pair-wise disjoint subsets, $\widehat{\equiv}\ \forall X_i, X_j : SS \bullet i \neq j \ \Rightarrow\ X_i \cap X_j = \emptyset$
$\mathsf{part}\ S$	the set of all *partitions* of S, $\widehat{=}\ \{\, SS : \wp\,(\wp\, S) \bullet \bigcup SS = S \wedge \Diamond\, SS \,\}$

$\mathsf{PRE\text{-}DEFINED\ SETS}$: types, operators and constants (*others are also provided*)

BOOL	boolean values	$\widehat{=}\ \{\, \mathsf{false}, \mathsf{true} \,\}$
NAT	the natural numbers	$\widehat{=}\ \{\, 0,\, 1,\, \ldots \}$
POS	positive natural numbers	$\widehat{=}\ \mathsf{NAT} \setminus \{0\}$

with the usual (numeric) *order relations* on $m, n : \mathsf{NAT}$

$n < m, \quad n \leq m, \quad n \geq m, \quad n > m$

and (arithmetic) operators, as *total* or *partial functions*

$\mathsf{succ}\ n$	successor of n	$\widehat{=}\ n + 1$
$\mathsf{pred}\ n$	predecessor of n, provided $n > 0$	$\widehat{=}\ n - 1$
$n + m$	addition	
$n - m$	subtraction, provided $n \geq m$	
$n * m$	multiplication	
$n\ \hat{}\ m$	n to the power m	
$n\ \mathsf{div}\ m$	natural division, provided $m > 0$	
$n\ \mathsf{mod}\ m$	natural modulus, provided $m > 0$	

together with the pre-defined *subset* constructor for $m_1, m_2 : \mathsf{NAT}$

$m_1 \mathinner{..} m_2$	the interval from m_1 through m_2	$\widehat{=}\ \{n : \mathsf{NAT} \bullet\ m_1 \leq n \leq m_2\}$

and two *selector functions* on non-empty subsets $N : \wp\, \mathsf{NAT} \setminus \emptyset$

$\mathsf{min}\ N$	minimum element of N	$\widehat{=}\ m : N \bullet (\forall\, n : N \bullet m \leq n)$
$\mathsf{max}\ N$	maximum element of N	$\widehat{=}\ m : N \bullet (\forall\, n : N \bullet m \geq n)$

$\mathsf{PRODUCTS}$, assuming S_1, S_2, $\ldots$ S_n are sets (of possibly-different 'types')
(and tuples), assuming values $x_1 : S_1$, $x_2 : S_2$, $\ldots$ $x_n : S_n$

$S_1 \times S_2$	cartesian product: the set of all tuple values (v_1, v_2), such that $v_1 \in S_1$ and $v_2 \in S_2$
$x_1 \mapsto x_2$	the ordered tuple $(x_1, x_2) \in S_1 \times S_2$
	assuming a tuple $t \mathrel{\widehat{=}} (x_1, x_2)$
$t[1],\ t[2]$	first, second components of t: $t[1] = x_1$, $t[2] = x_2$
$S_1 \times \ldots\ S_n$	extended product: the set of all n-tuples $(v_1, \ldots\ v_n)$, such that the i^{th} component-value $v_i \in S_i$
$x_1 \mapsto \ldots\ x_n$	the ordered n-tuple $(x_1, \ldots\ x_n) \in S_1 \times \ldots\ S_n$
	assuming an n-tuple $t \mathrel{\widehat{=}} (x_1, \ldots\ x_n)$ and $i : 1 \mathinner{..} n$
$t[i]$	i^{th} component-value of t: $t[i] = x_i$

BINARY RELATIONS for (defined) 'source/target types' S, T

$S \leftrightarrow T$	the set of all *relations* from S to T	$\mathrel{\widehat=}\ \wp\,(S \times T)$
	assuming $R : S \leftrightarrow T$	
R^{-1}	inverse of relation R, $R^{-1} \mathrel{\widehat=} \{y \mapsto x : T \times S \bullet x \mapsto y \in R\}$	$R^{-1} : T \leftrightarrow S$
$\mathsf{im}\ R$	image of a subset through relation R, $y \in (\mathsf{im}\ R)X \mathrel{\widehat\equiv}\ \exists x : X \bullet x \mapsto y \in R$	$\mathsf{im}\, R : \wp\, S \to \wp\, T$
$\mathsf{cf}\ R$	characteristic function of relation R, $y \in (\mathsf{cf}\ R)\, x\ \mathrel{\widehat\equiv}\ x \mapsto y \in R$	$\mathsf{cf}\ R : S \to \wp\, T$
$\mathsf{dom}\ R$	domain of the relation R, $\mathsf{dom}\, R\ \mathrel{\widehat=} (\mathsf{im}\, R^{-1})\, T$	$\mathsf{dom}\, R : \wp\, S$
$\mathsf{cod}\ R$	codomain of the relation R, $\mathsf{cod}\, R\ \mathrel{\widehat=} (\mathsf{im}\, R)\, S$	$\mathsf{cod}\, R : \wp\, T$
	assuming $X : \wp\, S$ and $Y : \wp\, T$	
$X \lhd R$	domain restriction to X on R	$\mathrel{\widehat=} \{x \mapsto y : R \bullet x \in X\}$
$X \ntriangleleft R$	domain exclusion of X from R	$\mathrel{\widehat=} \{x \mapsto y : R \bullet x \notin X\}$
$R \rhd Y$	codomain restriction to Y on R	$\mathrel{\widehat=} \{x \mapsto y : R \bullet y \in Y\}$
$R \ntriangleright Y$	codomain exclusion of Y from R	$\mathrel{\widehat=} \{x \mapsto y : R \bullet y \notin Y\}$
	assuming $R_1 : X \leftrightarrow Y$ and $R_2 : Y \leftrightarrow Z$ for sets X, Y, Z	
$R_2 \circ R_1$	relation R_2 composed with R_1, $x \mapsto z \in (R_2 \circ R_1) \mathrel{\widehat\equiv}\ \exists y : Y \bullet x \mapsto y \in R_1 \wedge y \mapsto z \in R_2$	$R_2 \circ R_1 : X \leftrightarrow Z$
	assuming $R_1 : X \leftrightarrow Y$ and $R_2 : X \leftrightarrow Z$ for sets X, Y, Z	
$R_1\ \&\ R_2$	relational join of R_1 and R_2, $x \mapsto (y, z) \in (R_1 \,\&\, R_2) \mathrel{\widehat\equiv}\ x \mapsto y \in R_1 \wedge x \mapsto z \in R_2$	$R_1 \,\&\, R_2 : X \leftrightarrow Y \times Z$
	assuming $R_1 : U \leftrightarrow V$ and $R_2 : X \leftrightarrow Y$ for sets U, V, X, Y	
$R_1 \otimes R_2$	relational product of R_1 and R_2, $(u, x) \mapsto (v, y) \in (R_1 \otimes R_2) \mathrel{\widehat\equiv}\ u \mapsto v \in R_1 \wedge x \mapsto y \in R_2$	$R_1 \otimes R_2 : U \times X \leftrightarrow V \times Y$
	assuming $R_1, R_2 : X \leftrightarrow Y$ for sets X, Y	
$R_1 \lessdot R_2$	relational overriding of R_1 by R_2, $x \mapsto y \in (R_1 \lessdot R_2) \mathrel{\widehat\equiv}\ x \mapsto y \in (\mathsf{dom}\, R_2 \ntriangleleft R_1) \vee x \mapsto y \in R_2$	$R_1 \lessdot R_2 : X \leftrightarrow Y$
	assuming 'homogeneous' $R_0 : S \leftrightarrow S$ on set S and $n : \mathsf{POS}$	
${R_0}^n$	relation R_0 composed with itself n times,	${R_0}^n : S \leftrightarrow S$
${R_0}^+$	*transitive* closure of relation R_0, ${R_0}^+ \mathrel{\widehat=} \bigcup n : \mathsf{POS} \bullet {R_0}^n$	${R_0}^+ : S \leftrightarrow S$
${R_0}^*$	*reflexive* transitive closure of R_0, ${R_0}^* \mathrel{\widehat=} {R_0}^+ \cup \mathsf{id}\,(\mathsf{dom}\, R_0)$	${R_0}^* : S \leftrightarrow S$
$\mathsf{id}\ S$	*identity function* on the set S, $\mathsf{id}\ S \mathrel{\widehat=} \bigcup x : S \bullet x \mapsto x$	$\mathsf{id}\ S : S \to S$

Classifying RELATIONS and FUNCTIONS, with 'source/target types' S, T

$R : S \leftrightarrow T$	general case: R is a *partial* relation from S to T
	a relation $R : S \leftrightarrow T$ is a set of pairs ($R \subseteq S \times T$), so any operation on such sets may be applied to a relation as well
$R : S \leftrightarrow T$	special case: R is a *total* relation from S to T $\mathrel{\widehat{\equiv}} R : S \leftrightarrow T \wedge \mathsf{dom}\, R = S$
$F : S \nrightarrow T$	special case: F is a *partial* function from S to T $\mathrel{\widehat{\equiv}} F : S \leftrightarrow T \wedge \forall\, x_1 \mapsto y_1, x_2 \mapsto y_2 : F \bullet x_1 = x_2 \Rightarrow y_1 = y_2$
	a function $F : S \nrightarrow T$ is a relation with the added property that *at most* one 'target' value is associated to all possible 'source' values, so operations on relations or sets may be applied to functions as well – but note, for example, that the *union* of two functions is not necessarily a function ...
$F : S \rightarrow T$	special case: F is a *total* function from S to T $\mathrel{\widehat{\equiv}} F : S \nrightarrow T \wedge \mathsf{dom}\, F = S$
$F\,x$	*application* of a function F to value x, often written $F(x)$ $F : X \nrightarrow Y \wedge x \mapsto y \in F \mathrel{\widehat{\equiv}} F\,x = y$
	further special cases, for a relation (or function) R
inj R	R is an *injection*, $\mathrel{\widehat{\equiv}} R^{-1} \in T \nrightarrow S$ // *inverse is* functional
surj R	R is a *surjection*, $\mathrel{\widehat{\equiv}} R^{-1} \in T \leftrightarrow S$ // *inverse is* total *(all T)*
bij R	R is a *bijection*, $\mathrel{\widehat{\equiv}}$ inj $R \wedge$ surj R // *inverse is* total function

Finite SEQUENCES, with elements from some defined set T (their 'type')

seq T	the set of all *finite* sequences that have elements of type T $\mathrel{\widehat{=}} \bigcup n : \mathsf{NAT} \bullet (1 \mathinner{..} n) \rightarrow T$
	a sequence S : seq T is a finite *total* function that is either *empty* or has some *contiguous* interval $1 \mathinner{..} n$ as its domain, such that its length is then n – so operations on relations, functions and sets may be applied to a sequence as well; in particular, $S(i)$ selects the ith element (provided $i : 1 \mathinner{..} n$)
$\# S$	the *length* of sequence S (i.e. cardinality of that function)
	assuming S_1, S_2 : seq T, $x_1, \ldots\ x_n : T$ and $i : 1 \mathinner{..} n$
$\langle\,\rangle\,[T]$	an empty sequence (where its type $[T]$ is usually omitted)
$\langle x_1 \rangle$	the sequence with one element x_1
$\langle x_1, \ldots\ x_n \rangle$	the sequence with n elements $x_1, \ldots\ x_n$
$\langle x_1 \rangle S_2$	the sequence S_2 *preceded* by element x_1
$S_1 \langle x_n \rangle$	the sequence S_1 *extended* by element x_n
$S_1 \langle x_i \rangle S_2$	sequences S_1, S_2 *separated* by element x_i
$S_1 \langle\,\rangle S_2$	*concatenation* of the sequences S_1, S_2