Let $X = \begin{pmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{pmatrix} = A^{-1}RA$ where A is the (column) basis of the parent (fine) lattice and R is one of the generators of the point-group for A. Thus X is a matrix of integers with determinant ± 1.

Assume that $H = \begin{pmatrix} a & 0 & 0 \\ p & b & 0 \\ q & r & c \end{pmatrix}$ is the intended HNF, with index n, for the new (courser) lattice with basis AH which must satisfy the same symmetry R. Thus $H^{-1}XH$ must be an integer matrix, leading to the following divisibility conditions (simplified as much as possible). Note they are recursive in that the integer quotients from each set show up in the later sets.

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a divides $pX_{12} + qX_{13}$.
Let $\alpha_1 = (pX_{12} + qX_{13})/a$.

a divides $bX_{12} + rX_{13}$.
Let $\alpha_2 = (bX_{12} + rX_{13})/a$.

a divides cX_{13}.
Let $\alpha_3 = cX_{13}/a$.

a divides n.
Let $\alpha_4 = n/a$.

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b divides $-pX_{11} + aX_{21} - p\alpha_1 + pX_{22} + qX_{23}$.
Let $\beta_1 = (-pX_{11} + aX_{21} - p\alpha_1 + pX_{22} + qX_{23})/b$.

b divides $-p\alpha_2 + rX_{23}$.
Let $\beta_2 = (-p\alpha_2 + rX_{23})/b$.

b divides $-p\alpha_3 + cX_{23}$
Let $\beta_3 = (-p\alpha_3 + cX_{23})/b$.

b divides α_4.
And $c = \alpha_4/b$.

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c divides $aX_{31} + pX_{32} + qX_{33} - \beta_1 r - \alpha_1 q - qX_{11}$

c divides $-rX_{22} + bX_{32} + rX_{33} - r\beta_2 - q\alpha_2$