Radius of Convergence by Top-line Analysis

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Problem statement

Given the initial-value problem (IVP)

$$x'(t) = f\,(t,(t)) \in \mathbb{R}^d \quad x(t_0) = x_0 \quad [t_0, t_{\text{end}}] \quad \text{tol} > 0$$

compute Taylor series (TS) approximate solution u of order p

$$u(t) = x_n + \sum_{i=1}^{p} (x_n)_i (t - t_n)^i \quad \text{on} \quad [t_n, t_{n+1}]$$

- INPUTS: Local IVP at (t_n, x_n), $n \geq 0$ and order p
- OUTPUTS: Taylor coefficients (TCs) $(x_n)_i$ at t_n

Radius of Convergence (ROC) is the stepsize that defines t_{n+1}

- INPUTS: real-valued, sufficiently differentiable functional, TCs
- OUTPUTS: ROC

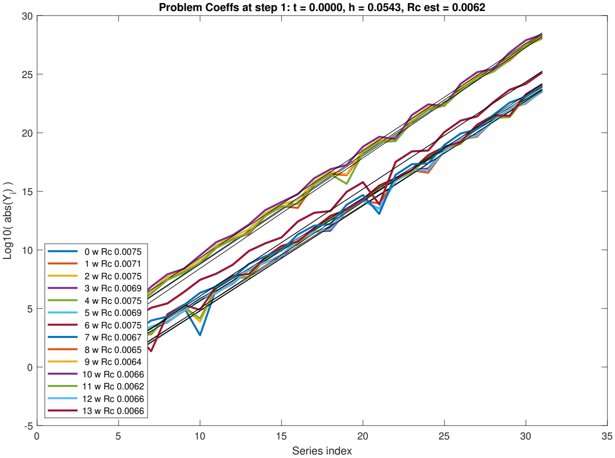

Figure 1: Top-line analysis shows divergent TS and rejected step

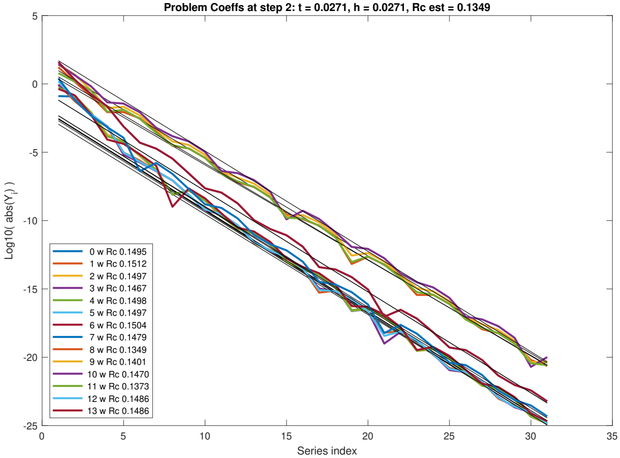

Figure 2: Top-line analysis indicates convergent TS and stepsize

The ROC and its applications

- Chang and Corliss defined ROC top-line analysis method
  Chang and Corliss (1982)
- Taylor series methods
  Jorba and Zou (2005)
  Bergsma and Mooij (2016)
  Chang and Corliss (1971)

Model refinement

Assumptions, Theoretical Models, Instance Models, General Definitions, Data Definitions

- A real valued, sufficiently differentiable function in one real variable
- A Realize TCs or TS
  - Fitting TS as data
  - Assigning meaning to the fitting parameters, $\beta_1 + \mathrm{idx}\beta_2$
  - Convergence $\beta_2 < 0$ or divergence $\beta_2 > 0$
- IM ODE IVP solving
  - The ROC $r_c = \mathrm{abs}(t_{n+1} - t_n)/10^{\beta_2}$
  - The ROC as a stepsize
  - This problem may be a candidate for Drasil after all