Software Requirements Specification for ROC: software estimating the radius of convergence of a power series

John M Ernsthausen

October 12, 2020

Contents

Revision History

Date	Version	Notes
12 October 2020	1.0	First submission

1 Reference Material

This section records information for easy reference.

1.1 Table of Units

A Table of Units is not applicable to ROC.

1.2 Table of Symbols

The table that follows summarizes the mathematical notation used in this document.

symbol	description
$z \in \mathbb{C}$	A complex number z is a member of the complex numbers $\mathbb{C}$
$x \in \mathbb{R}$	A real number x is a member of the real numbers $\mathbb{R}$
$\{c_n\} \subset \mathbb{C}$	An ordered set (sequence) of complex numbers whose n^{th} element is $c_n \in \mathbb{C}$
$\{c_n\}_{n=0}^{N-1} \subset \mathbb{C}$	An finite sequence of N complex numbers whose n^{th} element is $c_n \in \mathbb{C}$
$\{x_n\} \subset \mathbb{C}$	An ordered set (sequence) of real numbers whose n^{th} element is $x_n \in \mathbb{R}$
$\sum_{n=0}^{\infty} c_n z^n$	A power series. $c_n \in \mathbb{C}$ is the n^{th} coefficient. z^n is the n^{th} power of $z \in \mathbb{C}$
$\liminf_{n\to\infty}$	Lower subsequential limit
$\limsup_{n\to\infty}$	Upper subsequential limit
R_c	Radius of the circle of convergence
$\mathbb{R}^d$	A d-dimension real vector space
$\mathcal{D}$	An open subset of $\mathbb{R}^d$
$[t_0, t_{\text{end}}]$	Interval of real numbers, $t \in [t_0, t_{\text{end}}]$ means $t_0 \leq t \leq t_{\text{end}}$
$[t_n, t_{n+1}]$	Interval of real numbers, $t \in [t_n, t_{n+1}]$ means $t_n \leq t \leq t_{n+1}$
$(x_n)_n$	The n^{th} Taylor coefficient

1.3 Abbreviations and Acronyms

symbol	description
A	Assumption
BM	Bergsma and Mooij Bergsma and Mooij (2016)
CC	Chang and Corliss Chang and Corliss (1982)
C++	The programming language
DD	Data Definition
FORTRAN 77	The programming language
GD	General Definition
GS	Goal Statement
IM	Instance Model
IVP	Initial Value Problem
JZ	Jorba and Zou Jorba and Zou (2005)
LC	Likely Change
ODE	Ordinary Differential Equation
R	Requirement
ROC	Radius of Convergence software developed for this project
SRS	Software Requirements Specification
T	Theoretical Model
TC	Taylor coefficient
TS	Taylor series

2 Introduction

Given a sequence $\{c_n\} \subset \mathbb{C}$ of complex numbers, the series

$$\sum_{n=0}^{\infty} c_n z^n \tag{1}$$

is called a *power series*. The number c_n is the n^{th} coefficient of z^n. The term z^n denotes the n^{th} power of the complex number z.

In general, a power series will converge or diverge, depending on the magnitude of z. With every power series, there is associated a circle of convergence such that (1) converges if z is in the interior of the circle of convergence or diverges if z is in the exterior of the circle of convergence. The convergence/divergence behavior of (1) on the circle of convergence can not be described so simply. By convention, the entire complex plane is the interior of a circle of infinite radius, and a point is the interior of a circle of zero radius.

This project is concerned with estimating the radius of the circle of convergence, R_c.

2.1 Purpose of Document

The purpose of this SRS document is to facilitate the communication, planning, design, development/testing maintenance, and documentation of ROC by communicating and reflecting on its software requirements. Success of this project depends on ROC meeting these requirement. The scientific and business problem ROC solves is described in the "Problem Description" section (Section 4.1).

2.2 Scope of Requirements

The problem ROC solves will be restricted to the real case.

The software ROC will estimate the radius of convergence. It will not compute R_c exactly.

The software ROC will produce its estimated R_c from a finite number of terms in the power series.

2.3 Characteristics of Intended Reader

This document assumes the intended reader has familiarity with real analysis and complex analysis.

2.4 Organization of Document

This document is built from the template Smith (2006); Smith et al. (2017) that seeks to standardize software documentation. The suggested order for reading this SRS document is: Goal Statement (Subsection 4.1.3), Instance Models (Subsection 4.2.5), Requirements (Section 5), Introduction (Section 2), and Specific System Description (Section 4).

3 General System Description

This section provides general information about the system. It identifies the interfaces between the system and its environment, describes the user characteristics and lists the system constraints.

3.1 System Context

The following equation depicts ROC abstractly:

$$\{c_n\}_{n=0}^{N-1} \to R_c \tag{2}$$

3.2 User Characteristics

One intended user of ROC is a user of MAPLE or MATLAB. While neither MATLAB nor MAPLE currently implements an estimate for R_c, presumably the companies that develop MATLAB and MAPLE will want to provide such a facility when robust, reliable, and available.

After generating a real-valued Taylor series (TS) approximate solution of order p to the ordinary differential equation (ODE) initial-value problem (IVP)

$$y'(t) = f\big(t, y(t)\big), \quad y(t_0) = y_0 \in \mathcal{D} \subset \mathbb{R}^d, \quad t \in [t_0, t_{\text{end}}] \subset \mathbb{R}, \tag{3}$$

the TS methods defined in Jorba and Zou Jorba and Zou (2005) (JZ), Bergsma and Mooij Bergsma and Mooij (2016) (BM), and Chang and Corliss Chang and Corliss (1982) (CC) explicitly require an estimate for the TS radius of convergence.

JZ permit the order p to change from step to step. At each (t_n, x_n), $n \geq 0$, the TS method for the numerical solution of (3) computes Taylor coefficients (TCs) $(x_n)_i$ at t_n to construct the TS approximate solution

$$T(t) = x_n + \sum_{i=1}^{p} (x_n)_i (t - t_n)^i \quad \text{on} \quad [t_n, t_{n+1}]. \tag{4}$$

Another type of intended user is any developers like BM, JZ, or CC of a TS method seeking the accuracy assurance from knowing the domain $[t_n, t_{n+1}]$ is in the circle of convergence.

3.3 System Constraints

The method developed in this project is expected to be independent of system constraints. However most TS methods are developed in C++ or FORTRAN 77, the goto languages of scientific computing. Certainly a scripting language would not be sufficient for large systems.

4 Specific System Description

This section first presents the problem description, which gives a high-level view of the problem to be solved. This is followed by the solution characteristics specification, which presents the assumptions, theories, definitions and finally the instance models.

4.1 Problem Description

ROC is intended to estimate the radius of the circle of convergence of a power series.

Indeed, ROC accepts a sufficiently large integer N and a finite sequence $\{c_0, c_1, \ldots, c_{N-1}\}$ with each $c_n \in \mathbb{C}$ as inputs. ROC returns as output an estimate for the radius of the circle of convergence, $R_c \in \mathbb{R}$.

4.1.1 Terminology and Definitions

This subsection doesn't apply to ROC.

4.1.2 Physical System Description

This subsection doesn't apply to ROC.

4.1.3 Goal Statements

Given the inputs, the goal statements are:

GS1: Implement CC top line analysis (Chang and Corliss, 1982, pp. 127–128) to estimate the R_c, one of several sub-algorithms of the CC algorithm.

GS2: Use top line analysis to estimate R_c for three problems from real analysis where R_c is known. Report on the accuracy of the result.

GS3: Use top line analysis to estimate the order of a singularity as described in CC (Chang and Corliss, 1982, pp. 127–128).

GS4: Use top line analysis to estimate R_c while solving ODEIVP by the TS method.

4.2 Solution Characteristics Specification

This section characterizes the attributes of an acceptable solution. Both analysts and stakeholders should agree on these attributes so that the solution can be accepted when the project is complete.

4.2.1 Assumptions

This section simplifies the original problem and helps in developing the theoretical model by filling in the missing information for the physical system. The numbers given in the square brackets refer to the theoretical model [T], general definition [GD], data definition [DD], instance model [IM], or likely change [LC], in which the respective assumption is used.

A1:

A2: Linear Fit to data.

A3:

4.2.2 Theoretical Models

This section focuses on the general equations and laws that ROC is based on.

4.2.3 General Definitions

This section collects the laws and equations that will be used in building the instance models.

4.2.4 Data Definitions

This section collects and defines all the data needed to build the instance models. The dimension of each quantity is also given.

4.2.5 Instance Models

This section transforms the problem defined in Section 4.1 into one which is expressed in mathematical terms. It uses concrete symbols defined in Section 4.2.4 to replace the abstract symbols in the models identified in Sections 4.2.2 and 4.2.3.

4.2.6 Input Data Constraints

5 Requirements

This section provides the functional requirements, the business tasks that the software is expected to complete, and the nonfunctional requirements, the qualities that the software is expected to exhibit.

5.1 Functional Requirements

R1:

R2:

R3:

R4:

R5:

5.2 Nonfunctional Requirements

6 Likely Changes

LC1:

7 Unlikely Changes

LC2:

8 Traceability Matrices and Graphs

9 Values of Auxiliary Constants

ROC does not have symbolic parameters at this time.

References

Michiel Bergsma and Erwin Mooij. Application of taylor series integration to reentry problems. In *AIAA Atmospheric Flight Mechanics Conference*, page 0024, 2016.

YF Chang and G Corliss. Solving ordinary differential equations using Taylor series. *ACM TOMS*, 8(2):114–144, 1982.

Àngel Jorba and Maorong Zou. A software package for the numerical integration of ODEs by means of high-order Taylor methods. *Experimental Mathematics*, 14(1):99–117, 2005.

W. Spencer Smith. Systematic development of requirements documentation for general purpose scientific computing software. In *Proceedings of the 14th IEEE International Requirements Engineering Conference, RE 2006*, pages 209–218, Minneapolis / St. Paul, Minnesota, 2006. URL http://www.ifi.unizh.ch/req/events/RE06/.

W. Spencer Smith, John McCutchan, and Jacques Carette. Commonality analysis for a family of material models. Technical Report CAS-17-01-SS, McMaster University, Department of Computing and Software, 2017.