EE263 Homework 1

Fall 2025

2.61. Matrix representation of polynomial differentiation. We can represent a polynomial of degree less than n,

$$p(x) = a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0,$$

as the vector $(a_0, a_1, \ldots, a_{n-1}) \in \mathbb{R}^n$. Consider the linear transformation $\mathcal{D}$ that differentiates polynomials, *i.e.*, $\mathcal{D}p = dp/dx$. Find the matrix D that represents $\mathcal{D}$ (*i.e.*, if the coefficients of p are given by a, then the coefficients of dp/dx are given by Da).

2.100. A mass subject to applied forces. Consider a unit mass subject to a time-varying force $f(t)$ for $0 \leq t \leq n$. Let the initial position and velocity of the mass both be zero. Suppose that the force has the form $f(t) = x_j$ for $j-1 \leq t < j$ and $j = 1, \ldots, n$. Let y_1 and y_2 denote, respectively, the position and velocity of the mass at time $t = n$.

a) Find the matrix $A \in \mathbb{R}^{2\times n}$ such that $y = Ax$.

b) For $n = 4$, find a sequence of input forces $x_1, \ldots, x_n$ that moves the mass to position 1 with velocity 0 at time n.

2.110. Counting paths in an undirected graph. Consider an undirected graph with n nodes, and no self loops (*i.e.*, all branches connect two different nodes). Let $A \in \mathbf{R}^{n\times n}$ be the *node adjacency matrix*, defined as

$$A_{ij} = \begin{cases} 1 & \text{if there is a branch from node } i \text{ to node } j \\ 0 & \text{if there is no branch from node } i \text{ to node } j \end{cases}$$

Note that $A = A^\mathsf{T}$, and $A_{ii} = 0$ since there are no self loops. We can intrepret A_{ij} (which is either zero or one) as the number of branches that connect node i to node j. Let $B = A^k$, where $k \in \mathbb{Z}$, $k \geq 1$. Give a simple interpretation of B_{ij} in terms of the original graph. (You might need to use the concept of a *path* of length m from node p to node q.)

2.150. Gradient of some common functions. Recall that the gradient of a differentiable function $f : \mathbb{R}^n \to \mathbb{R}$, at a point $x \in \mathbb{R}^n$, is defined as the vector

$$\nabla f(x) = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{bmatrix},$$

where the partial derivatives are evaluated at the point x. The first order Taylor approximation of f, near x, is given by

$$\hat{f}_{\text{tay}}(z) = f(x) + \nabla f(x)^\mathsf{T}(z - x).$$

This function is affine, *i.e.*, a linear function plus a constant. For z near x, the Taylor approximation $\hat{f}_{\text{tay}}$ is very near f. Find the gradient of the following functions. Express the gradients using matrix notation.

a) $f(x) = a^\mathsf{T}x + b$, where $a \in \mathbb{R}^n$, $b \in \mathbb{R}$.

b) $f(x) = x^\mathsf{T} A x$, for $A \in \mathbb{R}^{n\times n}$.

c) $f(x) = x^\mathsf{T} A x$, where $A = A^\mathsf{T} \in \mathbb{R}^{n\times n}$. (Yes, this is a special case of the previous one.)

2.210. Express the following statements in matrix language. You can assume that all matrices mentioned have appropriate dimensions. Here is an example: "Every column of C is a linear combination of the columns of B" can be expressed as "$C = BF$ for some matrix F".

There can be several answers; one is good enough for us.

a) Suppose Z has n columns. For each i, row i of Z is a linear combination of rows $i, \ldots, n$ of Y.

b) W is obtained from V by permuting adjacent odd and even columns (*i.e.*, 1 and 2, 3 and 4, ...).

c) Each column of P makes an acute angle with each column of Q.

d) Each column of P makes an acute angle with the corresponding column of Q.

e) The first k columns of A are orthogonal to the remaining columns of A.