Constrained $\mathcal{H}_2/\mathcal{H}_\infty$ Control Design of Dynamic Virtual Power Plants via System Level Synthesis and Simple Pole Approximation

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Renewable Energy Transitions Require New Design Techniques

Converter-interfaced renewable generation is replacing conventional fossil fuel-fired synchronous machines

Sunlight Intensity

Wind Speed

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- traditional primary frequency control need to be provided by power converters
- provide fast frequency control on similar time scales to voltage control but without cross-coupling between them

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- traditional primary frequency control need to be provided by power converters
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- operational limitations of individual devices
  - state, input, and output constraints
  - steady state gain constraint
- nonconvexity

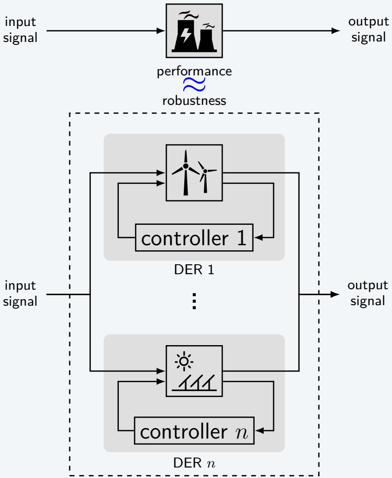
Dynamic Virtual Power Plant

DVPP aggregates a collection of DERs to provide

- specified as desired dynamic behavior/responses
- fast frequency and voltage control beyond mere set point tracking by (static) virtual power plant[1]

Reliability

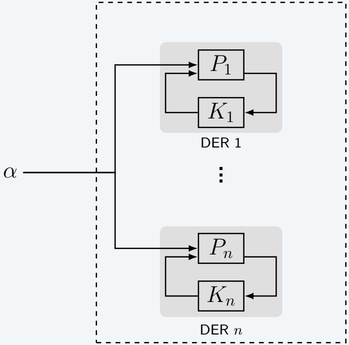
- DERs complement each other in terms of energy/power availability, response times, and weather dependency
- none of the DERs itself is able to do so



[1] B. Marinescu. "POSYTYF concept and objectives." 2020. [Online]. Available: https://posytyf-h2020.eu/project-overview/project-structure

Dynamic Virtual Power Plant Control Design

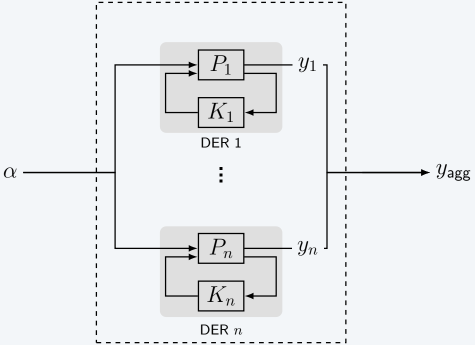
- broadcast input signal $\alpha = \begin{bmatrix} \Delta f \\ \Delta v \end{bmatrix}$



Dynamic Virtual Power Plant Control Design

- broadcast input signal $\alpha = \begin{bmatrix} \Delta f \\ \Delta v \end{bmatrix}$
- aggregate power output

$$\underbrace{\begin{bmatrix} \Delta p_{\text{agg}} \\ \Delta q_{\text{agg}} \end{bmatrix}}_{y_{\text{agg}}} = \sum_i \underbrace{\begin{bmatrix} \Delta p_i \\ \Delta q_i \end{bmatrix}}_{y_i}$$



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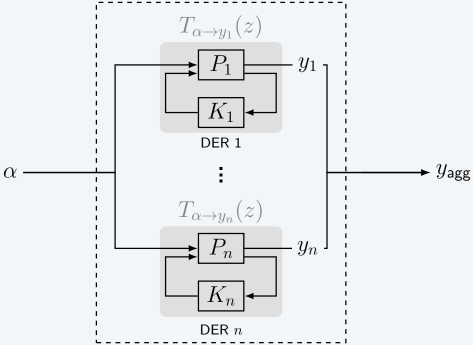
- DER i closed-loop behavior $T_{\alpha \to y_i}(z)$
- aggregate DVPP behavior

$$y_{\text{agg}} = \sum_i T_{\alpha \to y_i}(z)\alpha$$



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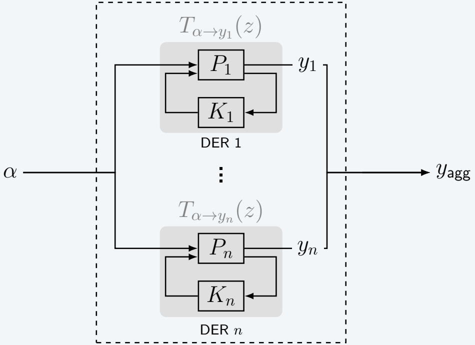
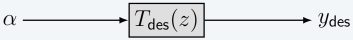
- desired DVPP behavior

$$\underbrace{\begin{bmatrix} \Delta p_{\text{des}} \\ \Delta q_{\text{des}} \end{bmatrix}}_{y_{\text{des}}} = \underbrace{\begin{bmatrix} \frac{-d_1 h}{\tau_p z - \tau_p + h} & 0 \\ 0 & \frac{-d_2 h}{\tau_q z - \tau_q + h} \end{bmatrix}}_{T_{\text{des}}(z)} \alpha$$





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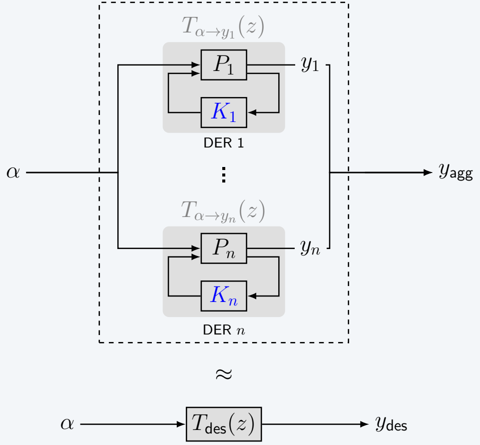
aggregation condition: $\sum_i T_{\alpha \to y_i}(z) \approx T_{\text{des}}(z)$



Goal: design controllers $K_i(z)$ such that aggregation condition and local DER limits are satisfied

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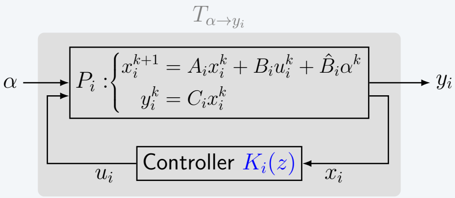

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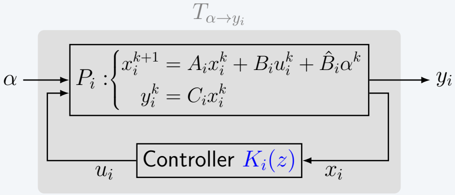


$$\underset{\{K_i(z)\}}{\text{minimize}} \left\| \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} \sum_i T_{\alpha \to y_i}(z) - T_{\text{des}}(z) \\ \sum_i T_{\alpha \to u_i}(z) \end{bmatrix} \right\|_{\mathcal{H}_2/\mathcal{H}_\infty}$$

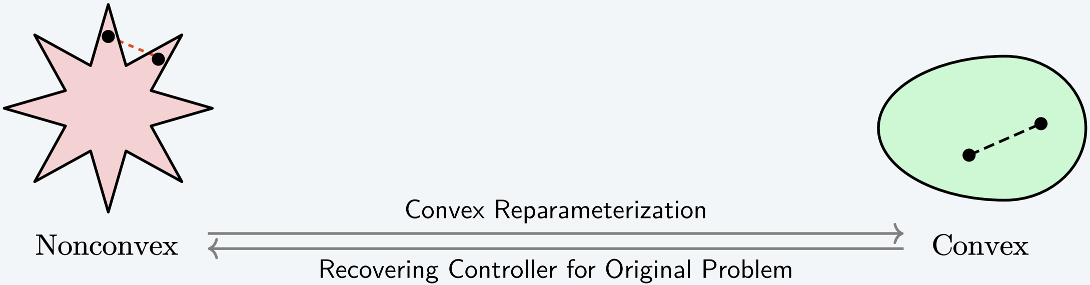
subject to $\hat{B}_i T_{\alpha \to x_i}(z), \hat{B}_i T_{\alpha \to u_i}(z)$ are real, rational, strictly proper and stable

state x_i, control input u_i, output y_i are limited

steady state of $y_i^\infty = \lim_{z \to 1} T_{\alpha \to y_i}(z)$

Goal: design controllers $K_i(z)$ such that aggregation condition and local DER limits are satisfied

Methods for Tackling Nonconvexity



Methods for Tackling Nonconvexity

Youla[2]

Coprime decomposition

System Level Synthesis[3]

State feedback control
Output feedback control

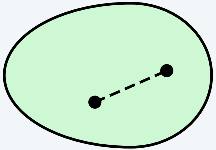
Input-Output Parameterization[4]

Output feedback control

Nonconvex

Convex Reparameterization

Recovering Controller for Original Problem



Convex

[2] D. Youla, H. Jabr, and J. Bongiorno, "Modern wiener-hopf design of optimal controllers–part ii: The multivariable case," IEEE Transactions on Automatic Control, vol. 21, no. 3, pp. 319–338, 1976.

[3] J. Anderson, J. C. Doyle, S. H. Low, and N. Matni, "**System level synthesis**," Annual Reviews in Control, vol. 47, pp. 364–393, 2019.

[4] L. Furieri, Y. Zheng, A. Papachristodoulou, and M. Kamgarpour, "An **input–output parametrization** of stabilizing controllers: Amidst youla and system level synthesis," IEEE Control Systems Letters, vol. 3, no. 4, pp. 1014–1019, 2019.

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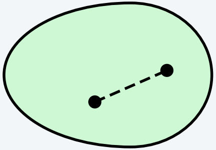
Reparameterize in terms of closed-loop system response



Convex Reparameterization

Nonconvex ⇄ Convex

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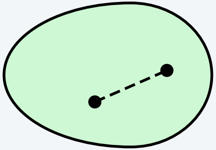

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Convex Formulation after System Level Synthesis

- reparameterized variables

$$\Phi_{x,i}(z) \overset{\text{SLS}}{\leftarrow\!\!\dashv} \hat{B}_i T_{\alpha\to x_i}(z)$$
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- compute objective transfer functions as

$$T_{\alpha\to y_i} = C_i \Phi_{x,i} \hat{B}_i$$
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- $\frac{1}{z}\mathcal{RH}_\infty$: the infinite dimensional space of real, rational, strictly proper, stable transfer functions

$$\begin{aligned} &\underset{\{\Phi_{x,i},\Phi_{u,i}\}}{\text{minimize}} && \left\| \underbrace{\begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} \sum_i C_i \Phi_{x,i} \hat{B}_i - T_{\text{des}} \\ \sum_i \Phi_{u,i} \hat{B}_i \end{bmatrix}}_{\Phi_{\text{obj}}} \right\|_{\mathcal{H}_2/\mathcal{H}_\infty} \\ &\text{subject to} && \Phi_{x,i}(z), \Phi_{u,i}(z) \in \frac{1}{z}\mathcal{RH}_\infty \end{aligned}$$

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- $m_{x,i}$, $m_{u,i}$, $m_{y,i}$: DER i state, input, output bounds

$$\begin{array}{rl}
\underset{\{\Phi_{x,i},\Phi_{u,i}\}}{\text{minimize}} & \left\| \underbrace{\begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} \sum_i C_i \Phi_{x,i} \hat{B}_i - T_{\text{des}} \\ \sum_i \Phi_{u,i} \hat{B}_i \end{bmatrix}}_{\Phi_{\text{obj}}} \right\|_{\mathcal{H}_2/\mathcal{H}_\infty} \\
\text{subject to} & \Phi_{x,i}(z), \Phi_{u,i}(z) \in \frac{1}{z}\mathcal{RH}_\infty \\
\text{[SLS]} & (zI - A_i)\Phi_{x,i}(z) - B_i \Phi_{u,i}(z) = I \\
\text{[state]} & x_i^k = \sum_{l=0}^{k} \mathcal{J}^{k-l}\left[\Phi_{x,i}\right] \hat{B}_i \alpha^l \le m_{x,i} \\
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$$\text{[steady gain]} \quad y_i^d = C_i \Phi_{x,i}(1) \hat{B}_i$$

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- recover $K_i(z) = \Phi_{u,i}(z)\Phi_{x,i}^{-1}(z)$ after solving the problem

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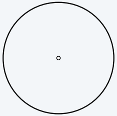
Infinite Dimensionality

Simple Pole Approximation[5] Addresses the Limitations of Finite Impulse Response

Finite Impulse Response (FIR)

closed-loop poles all lie at the origin



- infeasibility for stabilizable but uncontrollable systems
- high computational cost in systems with large separation of time scales
- unknown to incorporate prior knowledge about optimal closed-loop poles

[5] M. W. Fisher, G. Hug, and F. Dörfler, “Approximation by simple poles—part i: Density and geometric convergence rate in hardy space,” IEEE Transactions on Automatic Control, vol. 69, no. 8, pp. 4894–4909, 2024.

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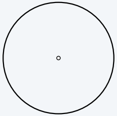
Finite Impulse Response (FIR)

closed-loop poles all lie at the origin



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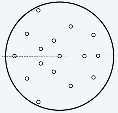
any finite selection of stable poles that is closed under complex conjugation



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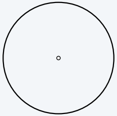

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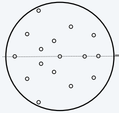

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The closed-loop system responses are

$$\Phi_{x,i}(z) = \sum_{p \in \mathcal{P}_i} \frac{G_{p,i}}{z-p}, \quad \Phi_{u,i}(z) = \sum_{p \in \mathcal{P}_i} \frac{H_{p,i}}{z-p},$$

$G_{p,i}$ and $H_{p,i}$ are complex coefficient matrices

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KYP Lemma[6] Expresses $\mathcal{H}_2/\mathcal{H}_\infty$ Norms as LMIs

For given transfer function $\tilde{\Phi}(z) = \tilde{C}(zI - \tilde{A})^{-1}\tilde{B}$, if $\tilde{A}$ is stable in the discrete time then the following statements hold.

linear matrix inequality (LMI)

- a convex constraint requiring a symmetric matrix to be positive semidefinite

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- avoid the potential bilinear matrix inequalities

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Control Design Derivation with Two Parts

Objective

$$\min_{\Phi_{\mathsf{obj}}(z)\in\frac{1}{z}\mathcal{RH}_\infty} \|\Phi_{\mathsf{obj}}(z)\|_{\mathcal{H}_2} + \lambda\|\Phi_{\mathsf{obj}}(z)\|_{\mathcal{H}_\infty}$$

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$$\begin{aligned} \min_{\gamma_1,\gamma_2,\Phi_{\mathsf{obj}}(z)\in\frac{1}{z}\mathcal{RH}_\infty} \quad & \gamma_1 + \lambda\gamma_2 \\ \text{s.t.} \quad & \|\Phi_{\mathsf{obj}}(z)\|_{\mathcal{H}_2} < \gamma_1 \\ & \|\Phi_{\mathsf{obj}}(z)\|_{\mathcal{H}_\infty} < \gamma_2 \end{aligned}$$

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KYP

Constraints

SLS constraint
state, input, output constraints
steady gain constraint

Control Design Derivation with Two Parts

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affine or linear constraints

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Control Design Framework

DVPP Control Design Yields an SDP

$$
\begin{aligned}
\underset{K_1, K_2, Z, \{G_{p,i}, H_{p,i}\}, \gamma_1, \gamma_2}{\text{minimize}} \quad & \gamma_1 + \lambda \gamma_2 \text{ — linear} \\
\text{subject to} \quad & \mathrm{Tr}(Z) < \gamma_1 \\
& \begin{bmatrix} K_1 & K_1 \tilde{A} & K_1 \tilde{B} \\ \tilde{A}^\top K_1 & K_1 & 0 \\ \tilde{B}^\top K_1 & 0 & \gamma_1 I \end{bmatrix} \succ 0 \\
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LMIs

Optimization Structure

- $\mathcal{H}_2/\mathcal{H}_\infty$ norms are transferred to LMIs

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LMIs

[SLS constraint]

$$\textstyle\sum_{p \in \mathcal{P}_i} G_{p,i} = I \text{ — affine}$$

$$(pI - A_i)\, G_{p,i} - B_i H_{p,i} = 0 \text{ — linear}$$

Optimization Structure

- $\mathcal{H}_2/\mathcal{H}_\infty$ norms are transferred to LMIs

DVPP Control Design Yields an SDP

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[state]

$$\sum_{l=0}^{k} \sum_{p \in \mathcal{P}_i} G_{p,i} p^{k-l-1} \hat{B}_i \alpha^l \leq m_{x,i} \quad \text{affine}$$

[input]

$$\sum_{l=0}^{k} \sum_{p \in \mathcal{P}_i} H_{p,i} p^{k-l-1} \hat{B}_i \alpha^l \leq m_{u,i} \quad \text{- affine}$$

[output]

$$\sum_{l=0}^{k} \sum_{p \in \mathcal{P}_i} p^{k-l-1} C_i G_{p,i} \hat{B}_i \alpha^l \leq m_{y,i} \quad \text{- affine}$$

[steady gain]

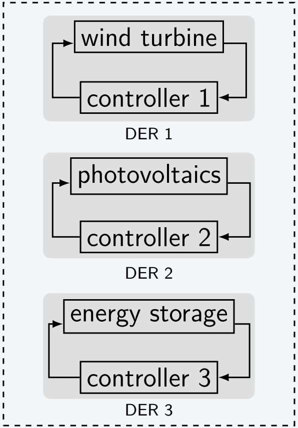
$$C_i \sum_{p \in \mathcal{P}_i} G_{p,i} \frac{1}{1-p} \hat{B}_i = y_i^d \quad \text{— affine}$$

Optimization Structure

- $\mathcal{H}_2/\mathcal{H}_\infty$ norms are transferred to LMIs
- SLS constraint, state, input, output and steady gain constraints remain linear/affine
- DVPP control design becomes a **semidefinite program (SDP)**

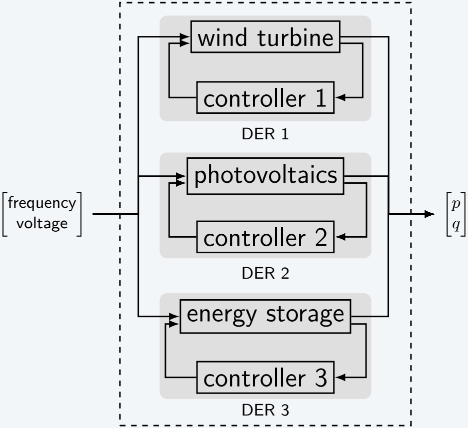
Test Case: IEEE 9-Bus System

- replace thermal-based power plant at bus 3 by a DVPP



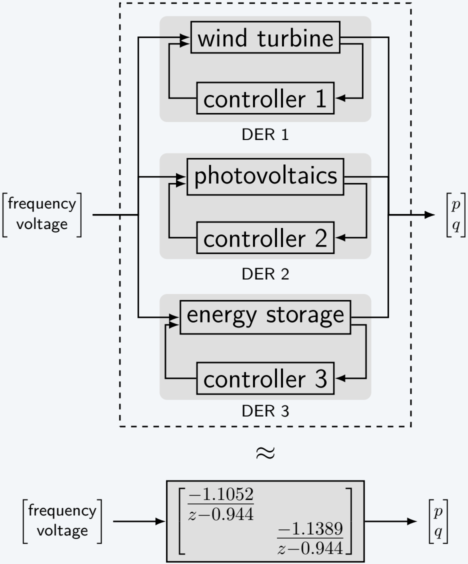
Test Case: IEEE 9-Bus System

- replace thermal-based power plant at bus 3 by a DVPP
- provide fast frequency and voltage regulation services



Test Case: IEEE 9-Bus System

- replace thermal-based power plant at bus 3 by a DVPP
- provide fast frequency and voltage regulation services
- employ aggregative control method to the desired behavior



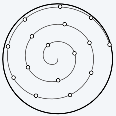
Test Case: IEEE 9-Bus System

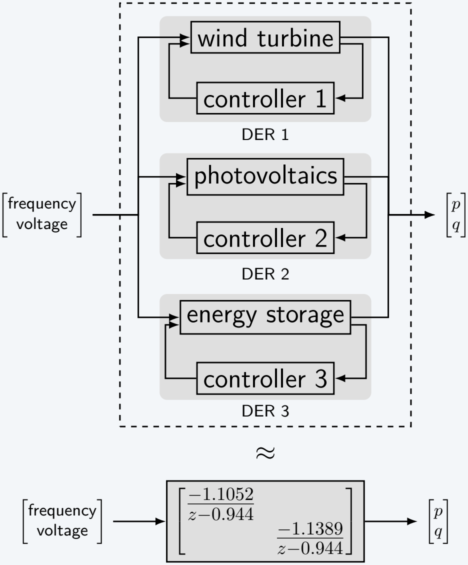
- replace thermal-based power plant at bus 3 by a DVPP
- provide fast frequency and voltage regulation services
- employ aggregative control method to the desired behavior

Pole Selection: first incorporate the plant poles and the poles of the desired transfer function

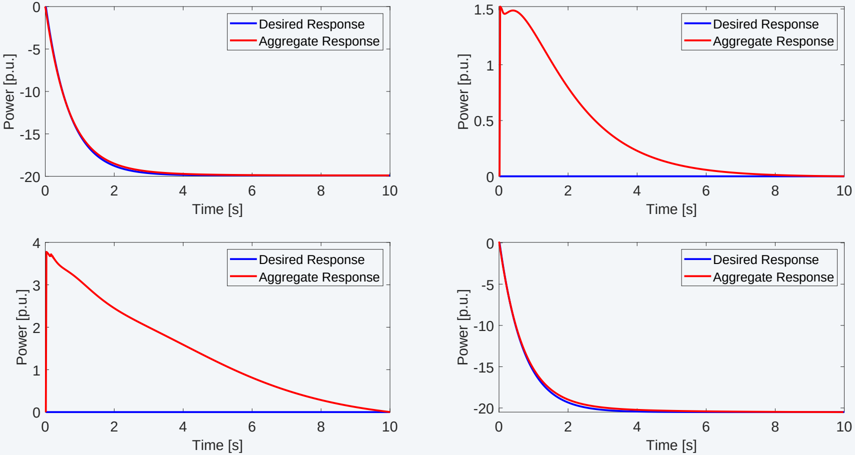
Spiral Method

select the remaining 10 poles along an Archimedes spiral





Control Design Method Shows Excellent Performance



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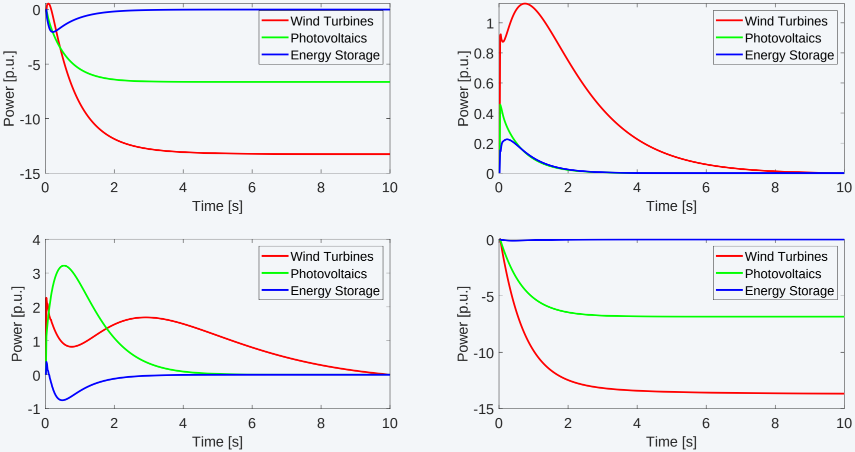

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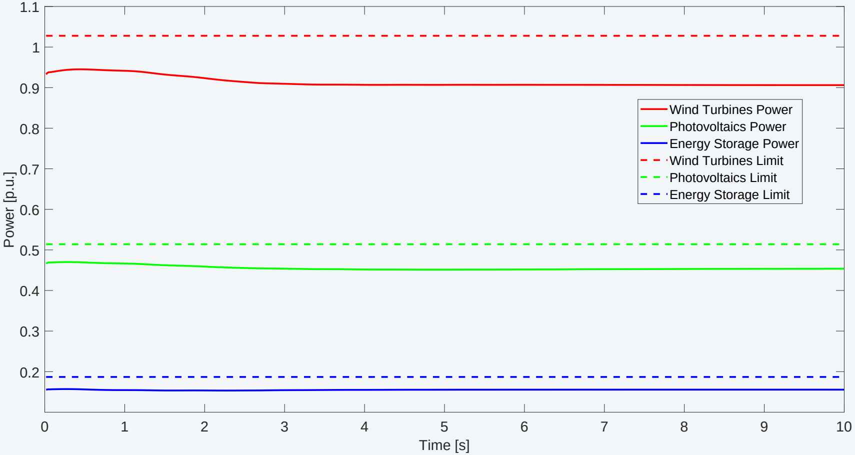

Conclusion

We developed a new DVPP control design method that addresses the joint challenges of

- renewable variability
- device-level limitations

We demonstrated on the case study of IEEE 9-bus system that heterogeneous DERs can deliver reliable fast frequency and voltage regulation in aggregate to the power grid while respecting individual device limitations

Thank you for your attention

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October 17, 2025