



UNIVERSITY OF
WATERLOO

FACULTY OF
ENGINEERING

Hybrid State Space and Frequency Domain System Level Synthesis for Sparsity-Promoting $\mathcal{H}_2/\mathcal{H}_\infty$ Control Design

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Optimal Linear State Feedback Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ Controller Synthesis Is Valuable and Challenging

Renewable resources require performance and robustness for uncertainties



Sunlight Intensity



Wind Speed

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Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ Synthesis

- has a long history
- is valuable on applications
- but challenging due to the nonconvexity

Our Objective: Improve Existing Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ Synthesis

Methods for Tackling Nonconvexity



Methods for Tackling Nonconvexity

Youla¹

Coprime decomposition



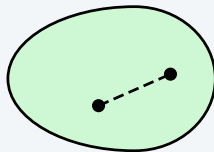
Nonconvex

System Level Synthesis²

State feedback control
Output feedback control

Input-Output Parameterization³

Output feedback control



Convex

Convex Reparameterization

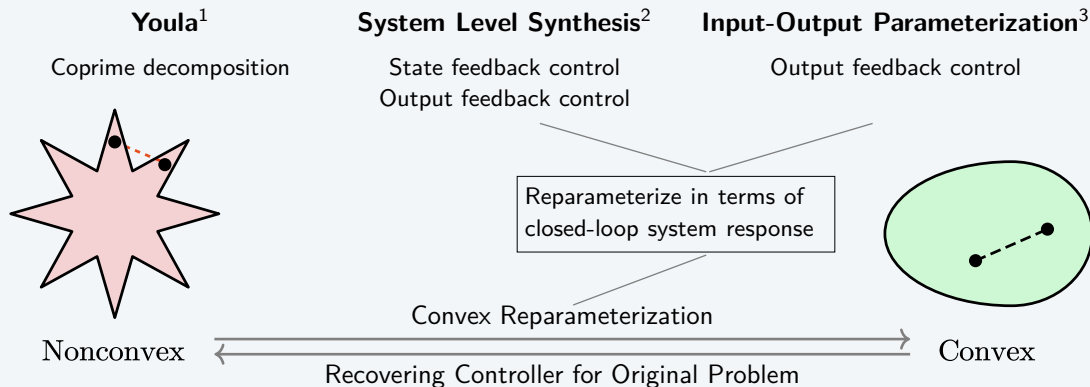
Recovering Controller for Original Problem

¹D. Youla, H. Jabr, and J. Bongiorno, "Modern wiener-hopf design of optimal controllers—part ii: The multivariable case," IEEE Transactions on Automatic Control, vol. 21, no. 3, pp. 319–338, 1976.

²J. Anderson, J. C. Doyle, S. H. Low, and N. Matni, "System level synthesis," Annual Reviews in Control, vol. 47, pp. 364–393, 2019.

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Methods for Tackling Nonconvexity

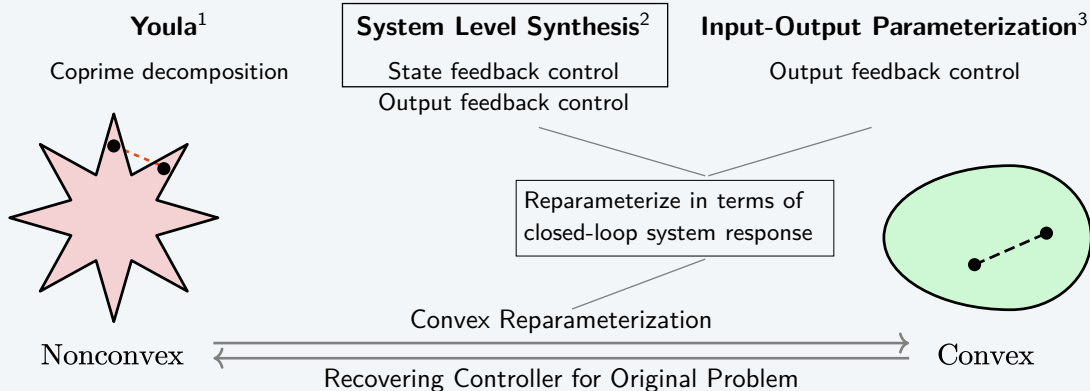


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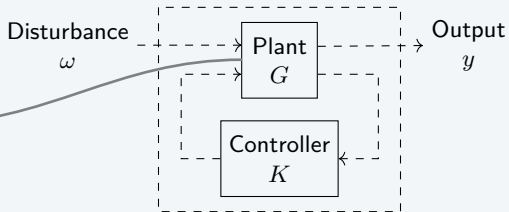
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Problem Formulation after System Level Synthesis

LTI system G :

$$x(k+1) = Ax(k) + Bu(k) + \underbrace{\hat{B}w(k)}_{v(k)}$$

$$y(k) = Cx(k)$$



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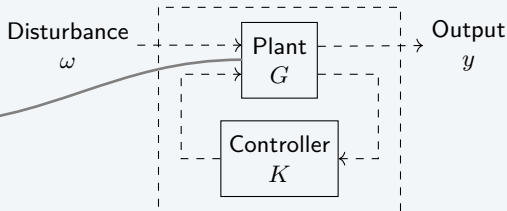


Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ Problem

$$\min_{K(z)} \left\| \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} T_{w \rightarrow y}(z) - T_{\text{des}}(z) \\ T_{w \rightarrow u}(z) \end{bmatrix} \right\|_{\mathcal{H}_2/\mathcal{H}_\infty}$$

$$\text{s.t. } T_{v \rightarrow x}(z), T_{v \rightarrow u}(z) \in \frac{1}{z} \mathcal{RH}_\infty$$

- Q, R can be chosen by the designer



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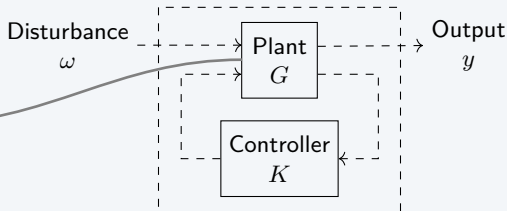


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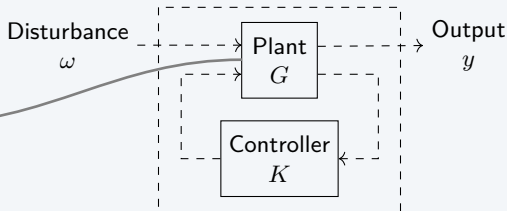


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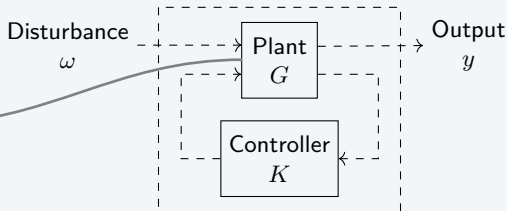


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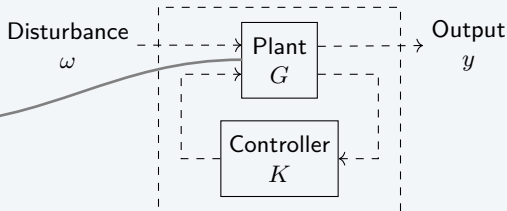


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SLS →

Convex SLS Problem

$$\begin{aligned} \min_{\Phi_x(z), \Phi_u(z)} & \left\| \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} C\Phi_x(z)\hat{B} - T_{\text{des}}(z) \\ \Phi_u(z)\hat{B} \end{bmatrix} \right\|_{\mathcal{H}_2/\mathcal{H}_\infty} \\ \text{s.t. } & (zI - A)\Phi_x(z) - B\Phi_u(z) = I \\ & \Phi_x(z), \Phi_u(z) \in \frac{1}{z} \mathcal{RH}_\infty \end{aligned}$$

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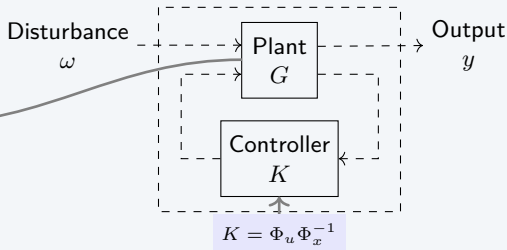


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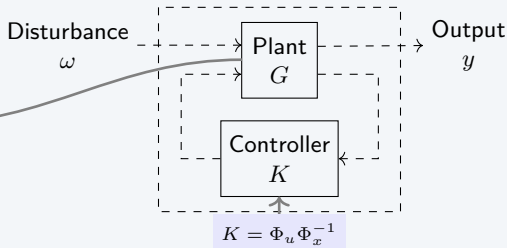
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Infinite Dimensionality

Simple Pole Approximation⁴ Addresses the Limitations of Finite Impulse Response

Finite Impulse Response (FIR)

closed-loop poles all lie at the origin



- infeasibility for stabilizable but uncontrollable systems
- high computational cost in systems with large separation of time scales
- unknown to incorporate prior knowledge about optimal closed-loop poles

⁴M. W. Fisher, G. Hug, and F. Dörfler, "Approximation by simple poles—part i: Density and geometric convergence rate in hardy space," IEEE Transactions on Automatic Control, vol. 69, no. 8, pp. 4894–4909, 2024.

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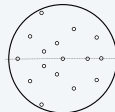
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Simple Pole Approximation (SPA)

any finite selection of stable poles that is closed under complex conjugation



- can apply for stabilizable but uncontrollable systems
- low computational cost in practice
- can easily include prior knowledge

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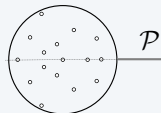
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The closed-loop system responses are

$$\Phi_x(z) = \sum_{p \in \mathcal{P}} \frac{G_p}{z - p}, \quad \Phi_u(z) = \sum_{p \in \mathcal{P}} \frac{H_p}{z - p},$$

G_p and H_p are complex coefficient matrices

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Increased Suboptimality in Prior Work⁵ Due to Finite Time Horizon Approximation

Finite Time Horizon Approximation

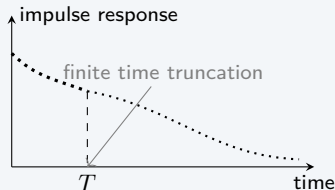
T := time horizon

\mathcal{I}_T := impulse response of size T

\mathcal{C}_T := convolution matrix of size T

$$\|\Phi(z)\|_{\mathcal{H}_2} = \lim_{T \rightarrow \infty} \|\mathcal{I}_T\|_F$$

$$\|\Phi(z)\|_{\mathcal{H}_\infty} = \lim_{T \rightarrow \infty} \|\mathcal{C}_T\|_2$$



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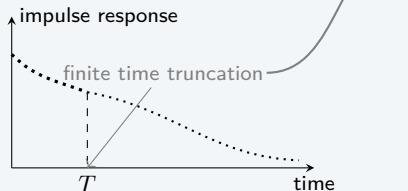
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- suboptimality bound is derived under the assumption of solving problem exactly
- suboptimality may not tend to zero as the number of poles diverges

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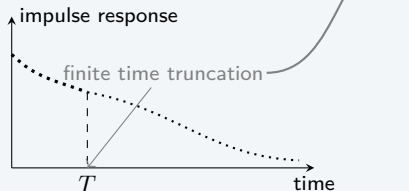
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- degraded performance
- higher memory and storage requirements and longer runtime

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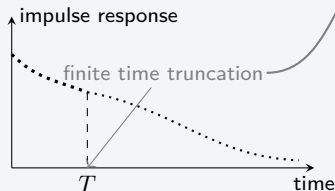
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Goal: eliminate the error of finite time horizon approximation

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KYP Lemma⁶ Expresses $\mathcal{H}_2/\mathcal{H}_\infty$ Norms as LMIs

For given transfer function $\Phi(z) = \tilde{C}(zI - \tilde{A})^{-1}\tilde{B}$, if \tilde{A} is stable in the discrete time then the following statements hold.

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1) $\|\Phi(z)\|_{\mathcal{H}_2} < \gamma_1$ if and only if there exist $K_1 \in \mathbb{S}^{n \times |\mathcal{P}|}$, $Z \in \mathbb{S}^m$, such that

$$\text{Trace}(Z) < \gamma_1, \begin{bmatrix} K_1 & K_1 \tilde{A} & K_1 \tilde{B} \\ \tilde{A}^\top K_1 & K_1 & 0 \\ \tilde{B}^\top K_1 & 0 & \gamma_1 I \end{bmatrix} \succ 0, \begin{bmatrix} K_1 & 0 & \tilde{C}^\top \\ 0 & I & 0 \\ \tilde{C} & 0 & Z \end{bmatrix} \succ 0.$$

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2) $\|\Phi(z)\|_{\mathcal{H}_\infty} < \gamma_2$ if and only if there exists $K_2 \in \mathbb{S}^{n \times |\mathcal{P}|}$,

$$\begin{bmatrix} K_2 & 0 & \tilde{A}^\top K_2 & \tilde{C}^\top \\ 0 & \gamma_2 I & \tilde{B}^\top K_2 & 0 \\ K_2 \tilde{A} & K_2 \tilde{B} & K_2 & 0 \\ \tilde{C} & 0 & 0 & \gamma_2 I \end{bmatrix} \succ 0$$

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Closed Loop Realizations Preserves Linearity

Diagram illustrating the structure of the closed-loop realization matrices \bar{A} , \bar{B} , \bar{C}_x , and \bar{C}_u .

The matrix \bar{A} is partitioned into two main sections:

- Real Poles:** The first section, corresponding to real poles p_1^r, \dots, p_m^r , has a block diagonal structure with blocks of the form $\begin{bmatrix} \text{Re}(p_i^c)I & \text{Im}(p_i^c)I \\ -\text{Im}(p_i^c)I & \text{Re}(p_i^c)I \end{bmatrix}$.
- Complex Poles:** The second section, corresponding to complex poles p_n^c , has a block diagonal structure with blocks of the form $\begin{bmatrix} \text{Re}(p_n^c)I & \text{Im}(p_n^c)I \\ -\text{Im}(p_n^c)I & \text{Re}(p_n^c)I \end{bmatrix}$.

The matrix \bar{B} is partitioned into two main sections:

- Real Poles:** The first section, corresponding to real poles, has a block diagonal structure with blocks of the form $\begin{bmatrix} I \\ \vdots \\ I \end{bmatrix}$.
- Complex Poles:** The second section, corresponding to complex poles, has a block diagonal structure with blocks of the form $\begin{bmatrix} 2I \\ 0 \\ \vdots \\ 2I \\ 0 \end{bmatrix}$.

The matrices \bar{C}_x and \bar{C}_u are partitioned into two main sections:

- Real Poles:** The first section, corresponding to real poles, has a block diagonal structure with blocks of the form $\begin{bmatrix} G_{p_1^r} & \dots & G_{p_m^r} \end{bmatrix}$.
- Complex Poles:** The second section, corresponding to complex poles, has a block diagonal structure with blocks of the form $\begin{bmatrix} \text{Re}(G_{p_1^c}) & \text{Im}(G_{p_1^c}) & \dots & \text{Re}(G_{p_n^c}) & \text{Im}(G_{p_n^c}) \end{bmatrix}$.

The matrices \bar{C}_x and \bar{C}_u are shown as:

$$\bar{C}_x = \begin{bmatrix} G_{p_1^r} & \dots & G_{p_m^r} & \text{Re}(G_{p_1^c}) & \text{Im}(G_{p_1^c}) & \dots & \text{Re}(G_{p_n^c}) & \text{Im}(G_{p_n^c}) \end{bmatrix}$$

$$\bar{C}_u = \begin{bmatrix} H_{p_1^r} & \dots & H_{p_m^r} & \text{Re}(H_{p_1^c}) & \text{Im}(H_{p_1^c}) & \dots & \text{Re}(H_{p_n^c}) & \text{Im}(H_{p_n^c}) \end{bmatrix}$$

Closed Loop Realizations Preserves Linearity

$$\begin{array}{c}
 \overbrace{\left[\begin{array}{c} p_1^r I \\ \vdots \\ p_m^r I \end{array} \right]}^{\bar{A}} \quad \overbrace{\left[\begin{array}{c} I \\ \vdots \\ I \end{array} \right]}^{\bar{B}} \\
 \left[\begin{array}{c|cc} \text{Re}(\bar{p}_1^c)I & \text{Im}(\bar{p}_1^c)I & \\ -\text{Im}(\bar{p}_1^c)I & \text{Re}(\bar{p}_1^c)I & \\ \vdots & \vdots & \ddots \\ \text{Re}(\bar{p}_n^c)I & \text{Im}(\bar{p}_n^c)I & \\ -\text{Im}(\bar{p}_n^c)I & \text{Re}(\bar{p}_n^c)I & \end{array} \right] \quad \left[\begin{array}{c} 2I \\ 0 \\ \vdots \\ 2I \\ 0 \end{array} \right] \\
 \left. \begin{array}{l} \bar{C}_x = \left[G_{p_1^r} \cdots G_{p_m^r} \right] \\ \bar{C}_u = \left[H_{p_1^r} \cdots H_{p_m^r} \right] \end{array} \right\} \left[\begin{array}{c|cc} \text{Re}(G_{p_1^c}) & \text{Im}(G_{p_1^c}) & \cdots & \text{Re}(G_{p_n^c}) & \text{Im}(G_{p_n^c}) \\ \text{Re}(H_{p_1^c}) & \text{Im}(H_{p_1^c}) & \cdots & \text{Re}(H_{p_n^c}) & \text{Im}(H_{p_n^c}) \end{array} \right]
 \end{array}$$

contribution of real poles
 contribution of complex poles
 \bar{C}_x is linear w.r.t G_p
 \bar{C}_u is linear w.r.t H_p

Closed Loop Realizations Preserves Linearity

Diagram illustrating the construction of a closed-loop realization matrix \bar{A} from real and complex poles.

The matrix \bar{A} is partitioned into blocks corresponding to real poles and complex poles. The first part shows the contribution of real poles, where the matrix is block diagonal with blocks $p_1^r I, \dots, p_m^r I$. The second part shows the contribution of complex poles, where the matrix is block diagonal with blocks $\begin{bmatrix} \text{Re}(p_1^c)I & \text{Im}(p_1^c)I \\ -\text{Im}(p_1^c)I & \text{Re}(p_1^c)I \end{bmatrix}, \dots, \begin{bmatrix} \text{Re}(p_n^c)I & \text{Im}(p_n^c)I \\ -\text{Im}(p_n^c)I & \text{Re}(p_n^c)I \end{bmatrix}$.

The output matrix \bar{C}_x is linear w.r.t G_p , and the input matrix \bar{C}_u is linear w.r.t H_p .

$$\bar{A} = \begin{bmatrix} p_1^r I & & & & \\ & \ddots & & & \\ & & p_m^r I & & \\ & & & \begin{bmatrix} \text{Re}(p_1^c)I & \text{Im}(p_1^c)I \\ -\text{Im}(p_1^c)I & \text{Re}(p_1^c)I \end{bmatrix} & \\ & & & \ddots & \\ & & & & \begin{bmatrix} \text{Re}(p_n^c)I & \text{Im}(p_n^c)I \\ -\text{Im}(p_n^c)I & \text{Re}(p_n^c)I \end{bmatrix} \end{bmatrix}$$

$$\bar{B} = \begin{bmatrix} I \\ \vdots \\ I \\ 2I \\ 0 \\ \vdots \\ 2I \\ 0 \end{bmatrix}$$

contribution of real poles

contribution of complex poles

$$\bar{C}_x = \begin{bmatrix} G_{p_1^r} & \dots & G_{p_m^r} & \text{Re}(G_{p_1^c}) & \text{Im}(G_{p_1^c}) & \dots & \text{Re}(G_{p_n^c}) & \text{Im}(G_{p_n^c}) \end{bmatrix}$$

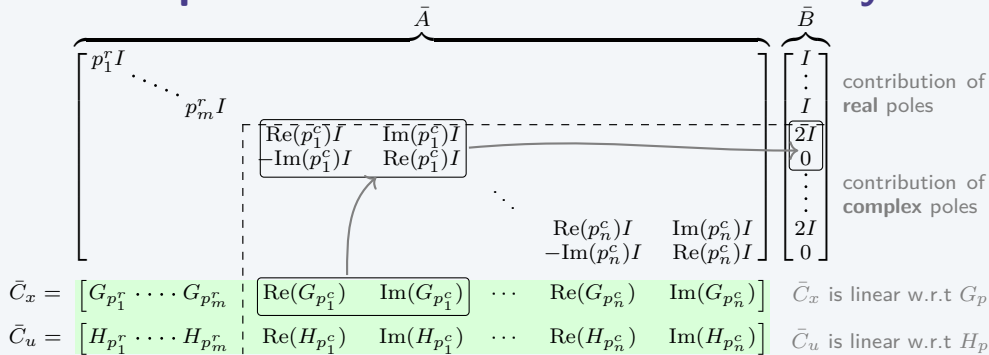
$$\bar{C}_u = \begin{bmatrix} H_{p_1^r} & \dots & H_{p_m^r} & \text{Re}(H_{p_1^c}) & \text{Im}(H_{p_1^c}) & \dots & \text{Re}(H_{p_n^c}) & \text{Im}(H_{p_n^c}) \end{bmatrix}$$

\bar{C}_x is linear w.r.t G_p

\bar{C}_u is linear w.r.t H_p

$(\bar{A}, \bar{B}, \bar{C}_x, 0)$ is a real state space realization of $\Phi_x(z)$
 $(\bar{A}, \bar{B}, \bar{C}_u, 0)$ is a real state space realization of $\Phi_u(z)$

Closed Loop Realizations Preserves Linearity



$(\bar{A}, \bar{B}, \bar{C}_x, 0)$ is a real state space realization of $\Phi_x(z)$
 $(\bar{A}, \bar{B}, \bar{C}_u, 0)$ is a real state space realization of $\Phi_u(z)$

$$\tilde{A} = \begin{bmatrix} \bar{A} & 0 \\ 0 & A_{\text{des}} \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} \bar{B} \hat{B} \\ B_{\text{des}} \end{bmatrix}, \quad \tilde{C} = \begin{bmatrix} QC\bar{C}_x & -QC_{\text{des}} \\ R\bar{C}_u & 0 \end{bmatrix}, \quad \Phi(z) = \left[\begin{array}{c|c} \tilde{A} & \tilde{B} \\ \hline \tilde{C} & 0 \end{array} \right]$$

Control Design Derivation with Two Parts

Objective

$$\min_{\Phi(z) \in \frac{1}{z} \mathcal{RH}_\infty} \|\Phi(z)\|_{\mathcal{H}_2} + \lambda \|\Phi(z)\|_{\mathcal{H}_\infty}$$

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$$\begin{aligned} \min_{\gamma_1, \gamma_2, \Phi(z) \in \frac{1}{z} \mathcal{RH}_\infty} \quad & \gamma_1 + \lambda \gamma_2 \\ \text{s.t.} \quad & \|\Phi(z)\|_{\mathcal{H}_2} < \gamma_1 \\ & \|\Phi(z)\|_{\mathcal{H}_\infty} < \gamma_2 \end{aligned}$$

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KYP



Control Design Derivation with Two Parts

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KYP



SLS Constraint

$$(zI - A)\Phi_x(z) - B\Phi_u(z) = I$$

Control Design Derivation with Two Parts

Objective

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SLS Constraint

$$(zI - A)\Phi_x(z) - B\Phi_u(z) = I$$

SPA

$$\sum_{p \in \mathcal{P}} G_p = I$$

$$(pI - A)G_p - BH_p = 0$$

Control Design Derivation with Two Parts

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Control Design Framework

Hybrid Domain Control Design Yields a SDP

$$\begin{array}{ll}
 \min & \gamma_1 + \lambda \gamma_2 \\
 \text{subject to} & \text{Tr}(Z) < \gamma_1 \\
 & \begin{bmatrix} K_1 & K_1 \tilde{A} & K_1 \tilde{B} \\ \tilde{A}^\top K_1 & K_1 & 0 \\ \tilde{B}^\top K_1 & 0 & \gamma_1 I \end{bmatrix} \succ 0 \\
 & \begin{bmatrix} K_1 & 0 & \tilde{C}(G_p, H_p)^\top \\ 0 & I & 0 \\ \tilde{C}(G_p, H_p) & 0 & Z \end{bmatrix} \succ 0 \\
 & \begin{bmatrix} K_2 & 0 & \tilde{A}^\top K_2 & \tilde{C}(G_p, H_p)^\top \\ 0 & \gamma_2 I & \tilde{B}^\top K_2 & 0 \\ K_2 \tilde{A} & K_2 \tilde{B} & K_2 & 0 \\ \tilde{C}(G_p, H_p) & 0 & 0 & \gamma_2 I \end{bmatrix} \succ 0 \\
 \hline
 & \sum_{p \in \mathcal{P}} G_p = I \\
 & (pI - A) G_p - B H_p = 0
 \end{array}$$

Hybrid Domain Control Design Yields a SDP

$$\begin{aligned}
 & \min_{K_1, K_2, Z, G_p, H_p, \gamma_1, \gamma_2} \quad \gamma_1 + \lambda \gamma_2 \text{ --- linear} \\
 & \text{subject to} \quad \text{Tr}(Z) < \gamma_1 \\
 & \quad \begin{bmatrix} K_1 & K_1 \tilde{A} & K_1 \tilde{B} \\ \tilde{A}^\top K_1 & K_1 & 0 \\ \tilde{B}^\top K_1 & 0 & \gamma_1 I \end{bmatrix} \succ 0 \\
 & \quad \begin{bmatrix} K_1 & 0 & \tilde{C}(G_p, H_p)^\top \\ 0 & I & 0 \\ \tilde{C}(G_p, H_p) & 0 & Z \end{bmatrix} \succ 0 \\
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LMIs

Optimization Structure

- objective is a linear combination of new variables
- $\mathcal{H}_2/\mathcal{H}_\infty$ norms are transferred to LMIs in the **state space**

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 & \quad \sum_{p \in \mathcal{P}} G_p = I \text{ --- affine} \\
 & \quad (pI - A) G_p - B H_p = 0 \text{ --- linear}
 \end{aligned}$$

LMIs

Optimization Structure

- objective is a linear combination of new variables
- $\mathcal{H}_2/\mathcal{H}_\infty$ norms are transferred to LMIs in the **state space**
- SLS constraints remain linear/affine in the **frequency domain**
- hybrid domain control design becomes a **semidefinite program (SDP)**

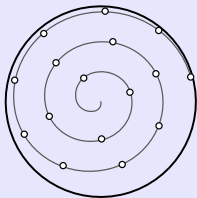
The Control Design Method

- can be solved efficiently
- eliminates the error of finite time horizon approximation

Suboptimality Bound⁷ Works for Our Method

Spiral Pole Selection

$\mathcal{P}_n :=$ selected n poles on the spiral in the complex conjugate

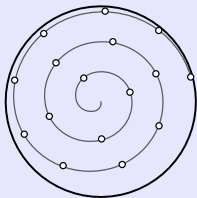


⁷M. W. Fisher, G. Hug, and F. Dörfler, "Approximation by simple poles – part ii: System level synthesis beyond finite impulse response," IEEE Transactions on Automatic Control, pp. 1–16, 2024.

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Suboptimality Bound

J^* := ground-truth optimal cost

$J(\mathcal{P}_n)$:= optimal cost with approximating \mathcal{P}_n

Then under mild assumptions there exists a constant $K > 0$ and $N > 0$ such that $n \geq N$ implies

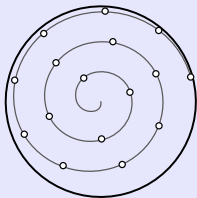
$$\frac{J(\mathcal{P}_n) - J^*}{J^*} \leq \frac{K}{\sqrt{n}}$$

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$$\frac{J(\mathcal{P}_n) - J^*}{J^*} \leq \frac{K}{\sqrt{n}}$$

- applies directly to our method
- ensures that the suboptimality converges to zero as the number of poles approaches infinity

⁷M. W. Fisher, G. Hug, and F. Dörfler, "Approximation by simple poles – part ii: System level synthesis beyond finite impulse response," IEEE Transactions on Automatic Control, pp. 1–16, 2024.

Sparsity-Promoting Method Enhances Robustness and Fixed-Order Performance

The Number of Poles \uparrow

- increases the computational cost of the design problem
- reduces the robustness of the resulting controller

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Sparsity constraints:

$$\sum_{p \in \mathcal{P}} \mathbb{1}(G_p) \leq l, \quad \sum_{p \in \mathcal{P}} \mathbb{1}(H_p) \leq l$$

⁸M. Yin, A. Iannelli, M. Khosravi, A. Parsi, and R. S. Smith, "Linear time-periodic system identification with grouped atomic norm regularization," IFAC-PapersOnLine, vol. 53, no. 2, pp. 1237–1242, 2020.

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nonconvex \Downarrow group lasso⁸

Sparsity penalty function:

$$\gamma_1 + \lambda \gamma_2 + \sigma_x \sum_{p \in \mathcal{P}} \|G_p\|_F^2 + \sigma_u \sum_{p \in \mathcal{P}} \|H_p\|_F^2$$

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Optimal Sparse Selection

$$\min_{K_1, K_2, Z, G_p, H_p, \gamma_1, \gamma_2} \gamma_1 + \lambda \gamma_2 + \sigma_x \sum_{p \in \mathcal{P}} \|G_p\|_F^2 + \sigma_u \sum_{p \in \mathcal{P}} \|H_p\|_F^2$$

s.t. Same Constraints

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conic program

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Numerical Example: Wind Turbine Interfaced to the Power Grid



$$A = \begin{bmatrix} 0.8046 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1177 & -0.0112 & -0.1332 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9889 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0.0111 & 0 \\ 0 & 1 \end{bmatrix},$$
$$C = \begin{bmatrix} 0 & 0.9066 & -0.0364 & 1.0218 & 0 \\ 0 & 0.364 & 0.9066 & -0.2406 & -1.0201 \end{bmatrix}, \hat{B} = \begin{bmatrix} -0.4885 & 0 \\ 0 & 1.0069 \\ 2.5 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$
$$A_{\text{des}} = \begin{bmatrix} 0.944 & \\ & 0.944 \end{bmatrix}, B_{\text{des}} = \begin{bmatrix} -1.1052 & \\ & -1.1389 \end{bmatrix}, C_{\text{des}} = I_2.$$

A wind turbine interfaced to the power grid via a power converter

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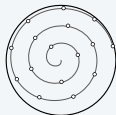
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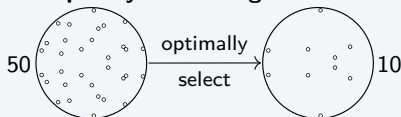
Pole Selection: first incorporate the plant poles and the poles of the desired transfer function

Spiral Method

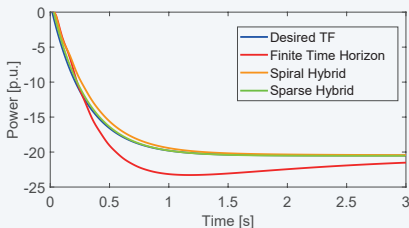
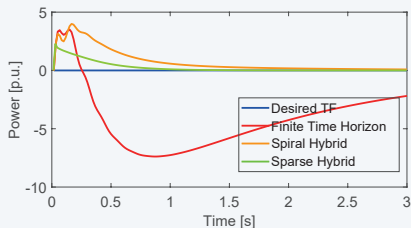
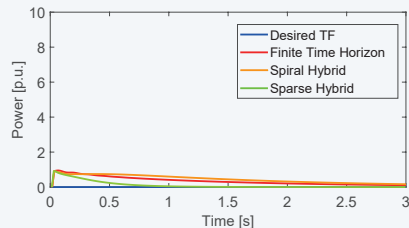
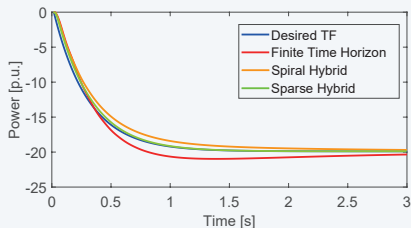
select the remaining 10 poles along an Archimedes spiral



Sparsity-Promoting Method



Sparse Hybrid Method Outperforms Finite Time Horizon Method



Conclusion

We developed a novel hybrid state space and frequency domain method for mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control design which

- reduced suboptimality
- improved performance
- less computational cost

Hybrid Domain Method

State Space

- mixed $\mathcal{H}_2/\mathcal{H}_\infty$ norm

Frequency Domain

- SLS constraint
- objective

LMIs

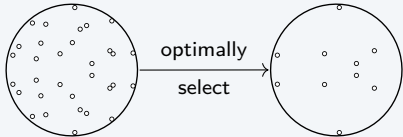
Linear/Affine

Semidefinite Program

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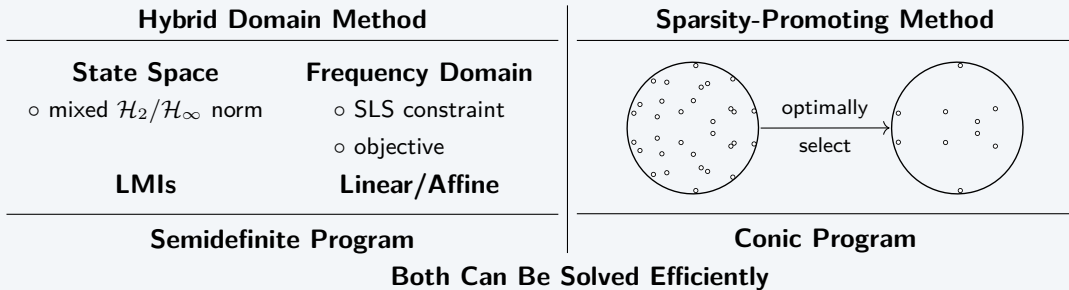
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| Hybrid Domain Method | | Sparsity-Promoting Method | |
|---|---------------------------------|---|--|
| State Space | Frequency Domain |  | |
| ○ mixed $\mathcal{H}_2/\mathcal{H}_\infty$ norm | ○ SLS constraint ○ objective | | |
| LMIs | Linear/Affine | | |
| Semidefinite Program | | Conic Program | |

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- improved performance
- less computational cost

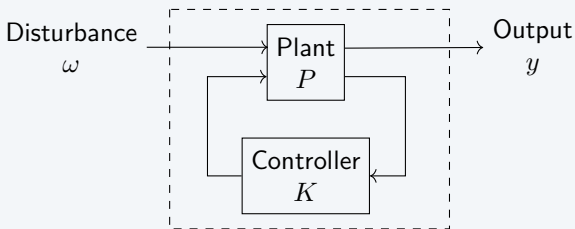


We demonstrated on the test case of control design for a wind turbine with power converter interface, and showed superior performance compared to the prior methods.

Ongoing Work

Ongoing Work

A recent work on \mathcal{H}_2 and \mathcal{H}_∞ synthesis combining SLS and SPA in continuous time⁹



while we focus on

- Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ synthesis in continuous time
- Suboptimality bound for SPA combined with SLS is nontrivial and has been certified

An upcoming manuscript is coming soon!

⁹Y. Du and J. S. Li, “State feedback system level synthesis in continuous time,” arXiv preprint arXiv:2410.08135, 2024.



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Thank you for your attention

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