Modulbeschreibung (MATH)

Jonathan Bauer
2046213
MaCit
AhPiM

1) Einführung

Diese PDF enthält:

> detaillierte Auflistung der Moduleinheiten

> Einheiten und Grundlagen zur Umrechnung der Variablen

> Umsetzung der physikal. Gesetzmäßigkeiten in Funktionen + Subroutinen

MODULE_ PARAMETERS

$R = 8{,}314 \left[\frac{kg \cdot m^2}{K \cdot mol \cdot s^2}\right]$

$g = \begin{bmatrix} 0 \\ 0 \\ -9{,}81 \end{bmatrix} \left[\frac{m}{s^2}\right]$

$M_{H_2O} = 18{,}01588 \cdot 10^{-3} \left[\frac{kg}{mol}\right]$

$p_{atm} = 101325 \left[\frac{kg}{m\,s^2}\right]$

$M_{Luft} = 28{,}9644 \cdot 10^{-3} \left[\frac{kg}{mol}\right]$

$\rho_{H_2O} = 1000 \left[\frac{fg}{\mu m^3}\right]$

$\rho_{Cov2} = 1000 \left[\frac{fg}{\mu m^3}\right]$

$D_0 = 2{,}3 \cdot 10^{-5} \left[\frac{m^2}{s}\right]$

$T_0 = 273{,}15 \; [K]$

$\boxed{\begin{aligned} A &= 8{,}07 \; [1] & C &= 233{,}426 \; [1] \\ B &= 1730{,}63 \; [1] & mmHg\_to\_Pa &= 133{,}32 \; [1] \end{aligned}}$

Antoine - Gleichungs - Parameter

$\rho_{Luft} = \frac{p_{atm} \cdot M_{Luft}}{R \cdot T_{\infty}} \qquad \frac{\left[\frac{kg}{m\cancel{s^2}}\right] \cdot \left[\frac{\cancel{kg}}{\cancel{mol}}\right]}{\left[\frac{\cancel{kg} \cdot m^2}{\cancel{K} \cdot \cancel{mol} \cdot \cancel{s^2}}\right] \cdot [\cancel{K}]} = \left[\frac{kg}{m^3}\right] = \left[\frac{fg}{\mu m^3}\right]$

$\eta_L = m \cdot T + c$

$c = 1{,}338 \cdot 10^{-5} \left[\frac{kg}{m \cdot s}\right]$

$m = 0{,}968 \cdot 10^{-7} \left[\frac{kg}{m \cdot s \cdot K}\right]$

$$\nu_L = \frac{\eta_L}{\rho_L}$$

$$\frac{\left[\frac{\cancel{kg}}{\cancel{m}\cdot s}\right]}{\left[\frac{\cancel{kg}}{m^{\cancel{3}\,2}}\right]} = \left[\frac{m^2}{s}\right]$$

MODULE_PARTICLE

$r = \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix}$ [m] $v = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$ $\left[\frac{m}{s}\right]$ $f = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}$ [μN]

d_{core} [μm] d_{shell} [μm] T [K]

t_{emerge} [s] $core_{on\_g}$ [1] active [1]

$V_{wind} = \begin{bmatrix} v_{w,x} \\ v_{w,y} \\ v_{w,z} \end{bmatrix}$ $\left[\frac{m}{s}\right]$ T_{air} [K] humidity [%]

$$V_{euclid} = \sqrt{|v_x|^2 + |v_y|^2 + |v_z|^2} \qquad \sqrt{\left[\frac{m^2}{s^2}\right]} = \left[\frac{m}{s}\right]$$

$$V_{core} = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \quad [\mu m^3] \qquad (= V_{shell})$$

$$M_{core} = V_{core} \cdot \rho_{core} \quad (\approx M_{shell}) \qquad [\mu m^3]\left[\frac{fg}{\mu m^3}\right] = [fg]$$

$$dt_1 = c \cdot \frac{m}{f} \qquad \left[\frac{m}{s}\right]\frac{[fg]}{[\mu N]} = \frac{[fg][s]}{[\mu]} \approx 1 \cdot 10^{-9}\,[s]$$

$$dt_2 = \frac{r}{100 \cdot v} \qquad \frac{[\mu m]}{[m]}[s] = 1 \cdot 10^{-6}\,[s]$$

MODULE_INIT

1) initialize position

$$\vec{r} = (\text{random} - 0{,}5) \quad [m] \ ✓$$

2) initialize velocity

$$\vec{v} = 2(\text{random} - 0{,}5) \cdot v_{max} \cdot I \quad [\frac{m}{s}] \ ✓$$

3) initialize structure

10 nm ... 1000 nm

$$d_{core} = (10 + 890 \cdot \text{random}) \cdot 10^{-3} \quad [\mu m]$$

$$d_{shell} = d_{core} + 4000 \cdot \text{random} \cdot 10^{-3} \quad [\mu m]$$

< 5000 nm

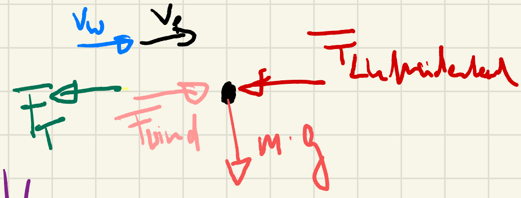
$$T = T_0 + 20 + 15 \cdot \text{random} \quad [K]$$

$20°C \leq T \leq 35°C$

4) initialize circumstance

↳ Einheiten von v_w, T_∞, humidity werden beibehalten (einfache Zuweisung)

MODULE_FORCE



evaluate_force PUBLIC

→ Aufruf der einzelnen internen Unterroutinen (keine Berechnung)

reset PRIVATE

$\vec{f} = \vec{0}$ $[\mu N]$

gravitation PRIVATE

$$\vec{f} = \vec{f} + m_p \cdot \vec{g} \qquad [kg] \cdot \left[\frac{m}{s^2}\right] = \overset{[\mu N]}{\overset{\parallel}{\left[\frac{\cancel{\mu g} \cdot m}{s^2}\right]}} \cdot 10^{-9}$$

air resistance PRIVATE

$$\vec{f} = \vec{f} - 6\pi \cdot \eta_L \cdot \frac{d_p}{2} \cdot \vec{V} \qquad \left[\frac{kg}{\cancel{m} \cdot s}\right][\mu m]\left[\frac{\cancel{m}}{s}\right] = \overset{[\mu N]}{\overset{\parallel}{\left[\frac{\mu g \cdot m}{s^2}\right]}} \cdot 10^{3}$$

wind PRIVATE

$$\vec{f} = \vec{f} + \frac{\rho_L}{2} \cdot \vec{V}_w^2 \cdot \left(\frac{d_p}{2}\right)^2 \pi \qquad \left[\frac{kg}{\mu m^{\cancel{3}}}\right]\left[\frac{m^2}{s^2}\right]\left[\cancel{\mu m^2}\right] = \left[\frac{kg \cdot m^2}{\mu m \cdot s^2}\right] = [\mu N] \cdot 10^{-3}$$

MODULE_MOVEMENT

$\boxed{\text{dydt}}$

$$\dot{y} = \begin{bmatrix} \dot{r}_x \\ \dot{r}_y \\ \dot{r}_z \\ \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \\ a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} r_x \\ r_y \\ r_z \\ v_x \\ v_y \\ v_z \end{bmatrix}}_{y} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ f_x/m \\ f_y/m \\ f_z/m \end{bmatrix}$$

$$F = m \cdot a$$

$$a = \frac{F}{m} \quad \frac{[\mu N]}{[kg]} = \frac{\left[\frac{kg \cdot m}{s^2}\right]}{[kg]} = \left[\frac{m}{s^2}\right] \cdot 10^{3}$$

MODULE_EVAPORATION

<u>evaluate evaporation</u>

↳ rein operative SUBROUTINE
nicht berechnet

<u>D_Coeff</u> PRIVATE

$$T_m = \frac{T_p + T_\infty}{2} \qquad [K]$$

$$D_{T_m} = D_0 \cdot \left(\frac{T_m}{T_0}\right)^{1.8} \qquad \left[\frac{m^2}{s}\right]$$

<u>p0_H2O</u> PRIVATE

$$p^0_{H_2O} = 10^{\left(A - \frac{B}{C+T}\right)} \qquad [mmHg] \rightsquigarrow [Pa] \quad 133{,}32$$

<u>pw_H2O</u> PRIVATE

$$p_w = 1 \cdot p^0_{H_2O}(T_p) \qquad [Pa]$$

$p_\infty\_H_2O$ PRIVATE

$$p_\infty H_2O = humidity \cdot p^0_{H_2O}(T_\infty) \quad [Pa]$$

Sh PRIVATE

$$Sh = 2 + 0{,}552 \cdot Re^{1/2} \cdot Sc^{1/3} \quad [1]$$

Re PRIVATE

$$Re = \frac{V \cdot d_p}{\nu_L} \qquad \frac{\left[\frac{m}{\cancel{s}}\right] \cdot \left[\mu m\right]}{\left[\frac{\cancel{m^2}}{\cancel{s}}\right]} = [1] \cdot 10^{-6}$$

Sc PRIVATE

$$Sc = \frac{\nu_L}{D} \qquad \frac{\left[\frac{m^2}{s}\right]}{\left[\frac{m^2}{s}\right]} = [1]$$

h_m PRIVATE

$$h_m = \frac{D}{d_p} \cdot Sh \qquad \frac{\left[\frac{m^2}{s}\right]}{[\mu m]} \cdot [1] = \left[\frac{\mu m}{s}\right] \cdot 10^{12}$$

dddt PRIVATE

$$\boxed{y = d_p}$$

$$[\mu m] \rightarrow [dddt] \overset{!}{=} \left[\frac{\mu m}{s}\right]$$

$$\frac{dd}{dt} = \frac{2}{\rho_{H_2O}} \cdot k_m \cdot \frac{M_{H_2O}}{R} \cdot \left(\frac{p_{w,H_2O}}{T_p} - \frac{p_{\infty,H_2O}}{T_\infty} \right)$$

$$= \left[\frac{m^3}{kg}\right] \left[\frac{\mu m}{s}\right] \frac{\left[\frac{\cancel{kg}}{\cancel{mol}}\right]}{\left[\frac{\cancel{kg} \cdot m^2}{\cancel{K} \cdot \cancel{mol} \cdot s}\right]} \cdot \frac{\left[\frac{kg}{m \cdot \cancel{s}}\right]}{[\cancel{K}]}$$

$$= \left[\frac{\cancel{m^3}}{\cancel{kg}}\right] \left[\frac{\mu m}{s}\right] \left[\frac{\cancel{kg}}{\cancel{m^3}}\right] = \left[\frac{\mu m}{s}\right]$$