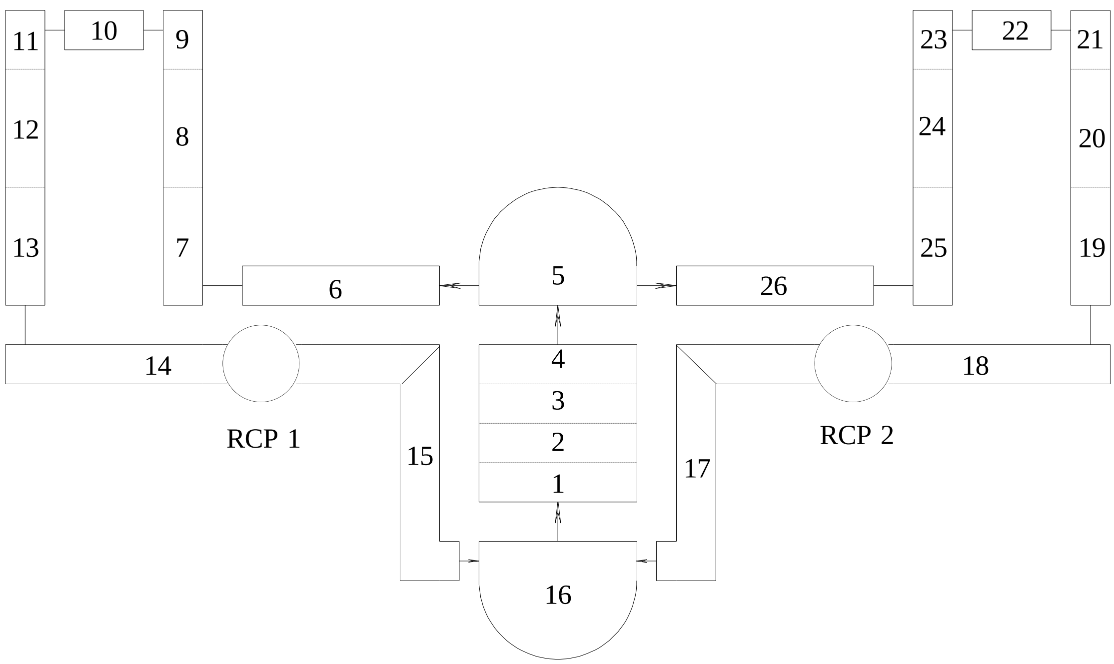
Spring 2024

NE 724 Reactor Heat Transfer

Course Project

Loop Momentum Balance Model of a Nuclear Power System

A simplified flow diagram of a Pressurized Water Reactor is given below.



where the physical components represented by the nodes are given in the table below.

Nodes	Component
1-4	Reactor Core
5	Upper Plenum
6,26	Hot Legs
7-13,19-25	Steam Generators
14,18	Cold legs
15,17	Downcomer
16	Lower Plenum

One form of the discretized fluid conservation equations for a flow system of this type is

Momentum

Loop 1

$$\frac{1}{g_c}\sum_i \frac{L_i}{A_{xi}}\left\{\frac{\dot{m}_1^{t+\Delta t}-\dot{m}_1^t}{\Delta t}\right\} = -\Delta P_{core}^{t+\Delta t} - \left\{\sum_i \frac{f_{1i}^{t+\Delta t}L_i}{D_{ei}}\left(\frac{\dot{m}_1^{t+\Delta t}}{A_{xi}}\right)^2 \frac{1}{2\rho g_c} + \sum_j \frac{k_{1j}}{2\rho g_c}\left(\frac{\dot{m}_1^{t+\Delta t}}{A_{xj}}\right)^2\right\} - \sum_k \rho_{1k}^t \frac{g}{g_c}\Delta H_k + \Delta P_{p1}$$

Loop 2

$$\frac{1}{g_c}\sum_i \frac{L_i}{A_{xi}}\left\{\frac{\dot{m}_2^{t+\Delta t}-\dot{m}_2^t}{\Delta t}\right\} = -\Delta P_{core}^{t+\Delta t} - \left\{\sum_i \frac{f_{2i}^{t+\Delta t}L_i}{D_{ei}}\left(\frac{\dot{m}_2^{t+\Delta t}}{A_{xi}}\right)^2 \frac{1}{2\rho g_c} + \sum_j \frac{k_{2j}}{2\rho g_c}\left(\frac{\dot{m}_2^{t+\Delta t}}{A_{xj}}\right)^2\right\} - \sum_k \rho_{2k}^t \frac{g}{g_c}\Delta H_k + \Delta P_{p2}$$

Core

$$\frac{1}{g_c}\frac{L_c}{A_{xc}}\left\{\frac{\dot{m}_c^{t+\Delta t}-\dot{m}_c^t}{\Delta t}\right\} = \Delta P_{core}^{t+\Delta t} - \left\{\frac{f_c^{t+\Delta t}L_c}{D_{ec}}\left(\frac{\dot{m}_c^{t+\Delta t}}{A_{xc}}\right)^2 \frac{1}{2\rho g_c} + \sum_j \frac{k_{cj}}{2\rho g_c}\left(\frac{\dot{m}_c^{t+\Delta t}}{A_{xj}}\right)^2\right\} - \sum_k \rho_{ck}^t \frac{g}{g_c}\Delta H_{ck}$$

Mass

$$\dot{m}_c^{t+\Delta t} = \dot{m}_1^{t+\Delta t} + (n-1)\dot{m}_2^{t+\Delta t}$$

where *n*-1 is the number of symmetric loops.

Energy

The Internal Energy equation for any one dimensional segment can be written

$$V_k\rho_k^t\left\{\frac{u_k^{t+\Delta t}-u_k^t}{\Delta t}\right\} + \frac{\dot{m}_k^{t+\Delta t}}{2}\left\{1-\frac{\left|\dot{m}_k^{t+\Delta t}\right|}{\dot{m}_k^{t+\Delta t}}\right\}u_{k+1}^{t+\Delta t} + \left|\dot{m}_k^{t+\Delta t}\right|u_k^{t+\Delta t} - \frac{\dot{m}_k^{t+\Delta t}}{2}\left\{1+\frac{\left|\dot{m}_k^{t+\Delta t}\right|}{\dot{m}_k^{t+\Delta t}}\right\}u_{k-1}^{t+\Delta t} = \dot{q}_k^t$$

where the mass flow rate is taken to be positive from node *k* to *k*+1.

The heat transfer rate across the steam generators can be taken to be

$$\dot{q}_k = U_k(t) A_{s_k} (T_{sat} - T_k)$$

The heat transfer across the reactor core can be obtained from solution of the time dependent heat conduction equation in the fuel.

<u>State</u>

$$T_k = T_k(u_k, P)$$

$$\rho_k = \rho(T_k, P)$$

To insure the friction and form losses always act to ***<u>oppose</u>*** the flow, rewrite the momentum equations in the form

<u>Momentum</u>

<u>Loop 1</u>

$$\frac{1}{g_c}\sum_i \frac{L_i}{A_{xi}}\left\{\frac{\dot{m}_1^{t+\Delta t} - \dot{m}_1^t}{\Delta t}\right\} = -\Delta P_{core}^{t+\Delta t} - \left\{\sum_i \frac{f_{1i}^{t+\Delta t} L_i}{D_{ei}}\left(\frac{1}{A_{xi}}\right)^2 \frac{1}{2\rho g_c} + \sum_j \frac{k_{1j}}{2\rho g_c}\left(\frac{1}{A_{xj}}\right)^2\right\}\dot{m}_1^{t+\Delta t}\left|\dot{m}_1^{t+\Delta t}\right| - \sum_k \rho_{1k}^t \frac{g}{g_c}\Delta H_k + \Delta P_{p1}$$

<u>Loop 2</u>

$$\frac{1}{g_c}\sum_i \frac{L_i}{A_{xi}}\left\{\frac{\dot{m}_2^{t+\Delta t} - \dot{m}_2^t}{\Delta t}\right\} = -\Delta P_{core}^{t+\Delta t} - \left\{\sum_i \frac{f_{2i}^{t+\Delta t} L_i}{D_{ei}}\left(\frac{1}{A_{xi}}\right)^2 \frac{1}{2\rho g_c} + \sum_j \frac{k_{2j}}{2\rho g_c}\left(\frac{1}{A_{xj}}\right)^2\right\}\dot{m}_2^{t+\Delta t}\left|\dot{m}_2^{t+\Delta t}\right| - \sum_k \rho_{2k}^t \frac{g}{g_c}\Delta H_k + \Delta P_{p2}$$

<u>Core</u>

$$\frac{1}{g_c}\frac{L_c}{A_{xc}}\left\{\frac{\dot{m}_c^{t+\Delta t} - \dot{m}_c^t}{\Delta t}\right\} = \Delta P_{core}^{t+\Delta t} - \left\{\frac{f_c^{t+\Delta t} L_c}{D_{ec}}\left(\frac{1}{A_{xc}}\right)^2 \frac{1}{2\rho g_c} + \sum_j \frac{k_{cj}}{2\rho g_c}\left(\frac{1}{A_{xj}}\right)^2\right\}\dot{m}_c^{t+\Delta t}\left|\dot{m}_c^{t+\Delta t}\right| - \sum_k \rho_{ck}^t \frac{g}{g_c}\Delta H_{ck}$$

The momentum equations are nonlinear in the new time mass flow rates.

Linearize these terms to give

Momentum

Loop 1

$$\frac{1}{g_c}\sum_i \frac{L_i}{A_{xi}}\left\{\frac{\dot{m}_1^{n+1}-\dot{m}_1^t}{\Delta t}\right\} = -\Delta P_{core}^{n+1} - \left\{\sum_i \frac{f_{1i}^n L_i}{D_{ei}}\left(\frac{1}{A_{xi}}\right)^2 \frac{1}{2\rho g_c} + \sum_j \frac{k_{1j}}{2\rho g_c}\left(\frac{1}{A_{xj}}\right)^2\right\}\left(2\dot{m}_1^{n+1}-\dot{m}_1^n\right)\left|\dot{m}_1^n\right| - \sum_k \rho_{1k}^t \frac{g}{g_c}\Delta H_k + \Delta P_{p1}$$

Loop 2

$$\frac{1}{g_c}\sum_i \frac{L_i}{A_{xi}}\left\{\frac{\dot{m}_2^{n+1}-\dot{m}_2^t}{\Delta t}\right\} = -\Delta P_{core}^{n+1} - \left\{\sum_i \frac{f_{2i}^n L_i}{D_{ei}}\left(\frac{1}{A_{xi}}\right)^2 \frac{1}{2\rho g_c} + \sum_j \frac{k_{2j}}{2\rho g_c}\left(\frac{1}{A_{xj}}\right)^2\right\}\left(2\dot{m}_2^{n+1}-\dot{m}_2^n\right)\left|\dot{m}_2^n\right| - \sum_k \rho_{2k}^t \frac{g}{g_c}\Delta H_k + \Delta P_{p2}$$

Core

$$\frac{1}{g_c}\frac{L_c}{A_{xc}}\left\{\frac{\dot{m}_c^{n+1}-\dot{m}_c^t}{\Delta t}\right\} = \Delta P_{core}^{n+1} - \left\{\frac{f_c^n L_c}{D_{ec}}\left(\frac{1}{A_{xc}}\right)^2 \frac{1}{2\rho g_c} + \sum_j \frac{k_{cj}}{2\rho g_c}\left(\frac{1}{A_{xj}}\right)^2\right\}\left(2\dot{m}_c^{n+1}-\dot{m}_c^n\right)\left|\dot{m}_c^n\right| - \sum_k \rho_{ck}^t \frac{g}{g_c}\Delta H_{ck}$$

Mass

$$\dot{m}_c^{n+1} = \dot{m}_1^{n+1} + (n-1)\dot{m}_2^{n+1}$$

The mass conservation equation and the three linearized momentum equations can be solved for the mass flow rates and core pressure drop at each interation.

These equations may be written in matrix form as

$$\begin{bmatrix} a_1 & 0 & 0 & 1 \\ 0 & a_2 & 0 & 1 \\ 0 & 0 & a_c & -1 \\ 1 & (n-1) & -1 & 0 \end{bmatrix}\begin{bmatrix} \dot{m}_1^{n+1} \\ \dot{m}_2^{n+1} \\ \dot{m}_c^{n+1} \\ \Delta P_{core}^{n+1} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_c \\ 0 \end{bmatrix}$$

where

$$a_1 = \frac{1}{g_c}\sum_i \frac{L_i}{A_{xi}}\frac{1}{\Delta t} + 2\left\{\sum_i \frac{f_{1i}^n L_i}{D_{ei}}\left(\frac{1}{A_{xi}}\right)^2 \frac{1}{2\rho g_c} + \sum_j \frac{k_{1j}}{2\rho g_c}\left(\frac{1}{A_{xj}}\right)^2\right\}\left|\dot{m}_1^n\right|$$

$$b_1 = \frac{1}{g_c}\sum_i \frac{L_i}{A_{xi}}\frac{1}{\Delta t}\dot{m}_1^t + \left\{\sum_i \frac{f_{1i}^n L_i}{D_{ei}}\left(\frac{1}{A_{xi}}\right)^2 \frac{1}{2\rho g_c} + \sum_j \frac{k_{1j}}{2\rho g_c}\left(\frac{1}{A_{xj}}\right)^2\right\}\left(\dot{m}_1^n\right)\left|\dot{m}_1^n\right| - \sum_k \rho_{1k}^t \frac{g}{g_c}\Delta H_k + \Delta P_{p1}$$

$$a_2 = \frac{1}{g_c}\sum_i \frac{L_i}{A_{xi}}\frac{1}{\Delta t} + 2\left\{\sum_i \frac{f_{2i}^n L_i}{D_{ei}}\left(\frac{1}{A_{xi}}\right)^2 \frac{1}{2\rho g_c} + \sum_j \frac{k_{2j}}{2\rho g_c}\left(\frac{1}{A_{xj}}\right)^2\right\}\left|\dot{m}_2^n\right|$$

$$b_2 = \frac{1}{g_c}\sum_i \frac{L_i}{A_{xi}}\frac{1}{\Delta t}\dot{m}_2^t + \left\{\sum_i \frac{f_{2i}^n L_i}{D_{ei}}\left(\frac{1}{A_{xi}}\right)^2 \frac{1}{2\rho g_c} + \sum_j \frac{k_{2j}}{2\rho g_c}\left(\frac{1}{A_{xj}}\right)^2\right\}\left(\dot{m}_2^n\right)\left|\dot{m}_2^n\right| - \sum_k \rho_{2k}^t \frac{g}{g_c}\Delta H_k + \Delta P_{p2}$$

$$a_c = \frac{1}{g_c}\frac{L_c}{A_{xc}}\frac{1}{\Delta t} + 2\left\{\sum_i \frac{f_c^n L_c}{D_{ec}}\left(\frac{1}{A_{xc}}\right)^2 \frac{1}{2\rho g_c} + \sum_j \frac{k_{cj}}{2\rho g_c}\left(\frac{1}{A_{xc}}\right)^2\right\}\left|\dot{m}_c^n\right|$$

$$b_c = \frac{1}{g_c}\frac{L_c}{A_{xc}}\frac{1}{\Delta t}\dot{m}_c^t + \left\{\sum_i \frac{f_c^n L_c}{D_{ec}}\left(\frac{1}{A_{xc}}\right)^2 \frac{1}{2\rho g_c} + \sum_j \frac{k_{cj}}{2\rho g_c}\left(\frac{1}{A_{xj}}\right)^2\right\}\left(\dot{m}_c^n\right)\left|\dot{m}_c^n\right| - \sum_k \rho_{ck}^t \frac{g}{g_c}\Delta H_k$$

which may be solved <u>directly</u> for the new iterate values as

$$\dot{m}_1^{n+1} = \frac{b_1 a_2 + (n-1) b_1 a_c - (n-1) a_c b_2 + a_2 b_c}{a_1 a_2 + (n-1) a_1 a_c + a_2 a_c}$$

$$\Delta P_c^{n+1} = b_1 - a_1 \dot{m}_1^{n+1}$$

$$\dot{m}_2^{n+1} = \frac{b_2 - \Delta P_c^{n+1}}{a_2}$$

$$\dot{m}_c^{n+1} = \dot{m}_1^{n+1} + (n-1)\dot{m}_2^{n+1}$$

During normal operation, the pressure increase across the reactor coolant pumps can be obtained from a pump performance curve of the form

$$\frac{\Delta P_p}{(\Delta P_p)_{\mathrm{RATED}}} = 1.094 + 0.089 \times \left(\frac{G}{G_{\mathrm{RATED}}}\right) - 0.183 \times \left(\frac{G}{G_{\mathrm{RATED}}}\right)^2$$

where $(\Delta P_p)_{\mathrm{RATED}}$ and G_{RATED} are the rated values. During a pump trip, the pressure increase across the pump can be approximated by

$$\Delta P_p(t) = \frac{\Delta P_p(0)}{1 + t/\beta}$$

where $\Delta P_p(0)$ is the nominal steady state pressure increase across the pump, and β is dictated by the fly wheel inertia (it essentially indicates the time required to half the initial pump pressure increase).

For example, for a fly wheel with high enertia, β would be large and the pump would take a long time to coast down.

To prevent reverse flow under normal conditions, the pumps are equipped with an anti reverse gear, which prohibits the pump impeller from turning in the reverse direction. This can be modelled by applying a loss coefficient to the pump node of the form

$$K = \frac{1}{2}(K^{+} + K^{-}) + \frac{1}{2}\frac{|\dot{m}|}{\dot{m}}(K^{+} - K^{-})$$

where K^{+} is some relatively small positive loss coefficient associated with positive flow through the pump, and $K^{-}$is a large positive loss coefficient to inhibit reverse flow in the loop.

a) For the PWR operating parameters given in the attached table, determine the value of $(\Delta P_p)_{\mathrm{RATED}}$ such that the given core mass flux is achieved. You should assume G_{RATED} is the nominal steady state mass flux in the cold leg.

b) Determine the minimum value of β such that the core exit temperature and the maximum clad surface temperature remain below the fluid saturation temperature following a trip of *all* reactor coolant pumps. Assume all pumps trip from steady state, full power conditions. You can assume the reactor trips 2 seconds after the pump trip and core heat generation is given by an appropriate decay heat correlation.

   The overall heat transfer coefficient across a steam generator node can be approximated as

$$U_k(t)A_{sk} = \frac{n\pi D_i \Delta z_k}{\frac{1}{h_i(t)} + \xi}$$

   where ξ is a constant and $h_i(t)$ is an appropriate single phase forced convection heat transfer coefficient. The value of $\Delta P_p(0)$ should be chosen to satisfy the given nominal steady state flow rate. The value of ξ is chosen to produce the correct nominal steady state core inlet temperature.

c) Once you have determined the appropriate value of β, rerun the transient assuming *one* reactor coolant pump trips and coast down naturally.

d) A transient that must be considered when performing a reactor safety analysis, assumes a reactor coolant pump locked rotor. Under these conditions the $\Delta P_p(0)$ of the affected pump goes to zero immediately and both K^+ and K^- are large and positive, effectively eliminating flow in that loop. Analyze this transient.

e) An additional transient which must be considered when performing a safety analysis is that of a broken pump shaft, where the pump impeller is free to rotate in either direction, allowing for reverse flow in the affected loop. Under these conditions the $\Delta P_p(0)$ of the affected pump goes to zero immediately and the loss coefficient associated with the pump node is *small and positive* in either the positive or negative directions. Analyze this transient.

f) One source of asymmetric loop behavior is plugging of leaky steam generator tubes, leading to a reduction in the flow and heat transfer areas of the affected loop. Determine the number of tubes required to reduce the flow rate in the affected loop by 10%. You can model this by simply reducing the number of tubes in the affected steam generator.

Problem Data

REACTOR

Power	3411	Mw
Active Core Height	144	inches
Number of Rod Locations	55,777	
Number of Fuel Rods	50,952	
Rod Diameter	0.374	inches
Pellet Diameter	0.3225	inches
Clad Thickness	0.0225	inches
Gap Conductance	1000	Btu/hr-ft^2-F
Clad Thermal Conductivity	9.6	Btu/hr-ft-F
Rod Pitch	0.496	inches
Number of Spacer Grids	8	
Grid Loss Coefficient	0.5	
Mass Flux	2.48×10^6	lbm/hr-ft^2
Inlet Temperature	552	F
Outlet Temperature	616	F
Pressure	2250	psia
Core Inlet Loss Coefficient	4.25	
Core Exit Loss Coefficient	4.25	

UPPER PLENUM (including RPV Head)

Length	1.5	ft
Effective Diameter	158	inches
Volume	1373.7	ft^3

HOT LEGS (each)

Number	4	
Length	20	ft
Diameter	29	inches
Hot Leg Equivalent L/D	20	
Inlet Loss Coefficient	0.5	
Exit Loss Coefficient	1	

STEAM GENERATORS (all parameters per steam generator)

Number	4	
Secondary Side Pressure	1000	psia
Number of Steam Generator Tubes	6633	
Steam Generator Tube ID	0.6075	inches
Length of Steam Generator Tubes	66.8	ft
Bend Equivalent L/D	55	
Bundle Inlet Loss Coefficient	0.5	
Bundle Exit Loss Coefficient	1	

COLD LEGS (each)

Number	4	
Length	40	ft
Diameter	27.5	inches
Inlet Loss Coefficient	0.5	
Cold Leg Equivalent L/D	18	
Exit Loss Coefficient	4.6	

DOWNCOMER (treat as *n* unconnected 1-D flow paths and part of cold legs)

Vessel ID	173	inches
Core Barrel OD	158	inches
Length	18.4	ft

LOWER PLENUM

Length	7.2	ft
Diameter	173	inches
Volume	784.4	ft^3

The node powers are give by $\dot{Q} = \sum_{k=1}^{4} \dot{q}_k$ where $\dot{q}_k = \bar{\dot{q}} F_k$. $\dot{Q}$ is the total core thermal output, $\bar{\dot{q}}$ is the average power per node and F_k is the power distribution factor give in the table below.

Geometrical Data (per loop)

Component	Node	Length (ft)	D_e (in)	A_x (ft^2)	P_h (ft)	ΔH (ft)
Core	1	3.0	0.4635	52.74	4988.86	3.0
Core	2	3.0	0.4635	52.74	4988.86	3.0
Core	3	3.0	0.4635	52.74	4988.86	3.0
Core	4	3.0	0.4635	52.74	4988.86	3.0
Upper Plenum	5	1.5	158	136.2	0	1.5
Hot Leg	6	20	29	4.59	0	0
Steam Generator	7	9.54	0.6075	13.35	1054.9	9.54
Steam Generator	8	9.54	0.6075	13.35	1054.9	9.54
Steam Generator	9	9.54	0.6075	13.35	1054.9	9.54
Steam Generator	10	9.54	0.6075	13.35	1054.9	0
Steam Generator	11	9.54	0.6075	13.35	1054.9	-9.54
Steam Generator	12	9.54	0.6075	13.35	1054.9	-9.54
Steam Generator	13	9.54	0.6075	13.35	1054.9	-9.54
Cold Leg	14	40	27.5	4.12	0	0
Downcomer	15	18.4	15	6.77	0	-13.5
Lower Plenum	16	7.2	173	163.2	0	0

Power Distribution Factor

Node	F_k
1	.586
2	1.414
3	1.414
4	.586