The Physics of Racing

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Part 1
Weight Transfer

Most autocrossers and race drivers learn early in their careers the importance of balancing a car. Learning to do it consistently and automatically is one essential part of becoming a truly good driver. While the skills for balancing a car are commonly taught in drivers' schools, the rationale behind them is not usually adequately explained. That rationale comes from simple physics. Understanding the physics of driving not only helps one be a better driver, but increases one's enjoyment of driving as well. If you know the deep reasons why you ought to do certain things you will remember the things better and move faster toward complete internalization of the skills.

Balancing a car is controlling weight transfer using throttle, brakes, and steering. This article explains the physics of weight transfer. You will often hear instructors and drivers say that applying the brakes shifts weight to the front of a car and can induce oversteer. Likewise, accelerating shifts weight to the rear, inducing understeer, and cornering shifts weight to the opposite side, unloading the inside tires. But why does weight shift during these maneuvers? How can weight shift when everything is in the car bolted in and strapped down? Briefly, the reason is that inertia acts through the center of gravity (CG) of the car, which is above the ground, but adhesive forces act at ground level through the tire contact patches. The effects of weight transfer are proportional to the height of the CG off the ground. A flatter car, one with a lower CG, handles better and quicker because weight transfer is not so drastic as it is in a high car.

The rest of this article explains how inertia and adhesive forces give rise to weight transfer through Newton's laws. The article begins with the elements and works up to some simple equations that you can use to calculate weight transfer in any car knowing only the wheelbase, the height of the CG, the static weight distribution, and the track, or distance between the tires across the car. These numbers are reported in shop manuals and most journalistic reviews of cars.

Most people remember Newton's laws from school physics. These are fundamental laws that apply to all large things in the universe, such as cars. In the context of our racing application, they are:

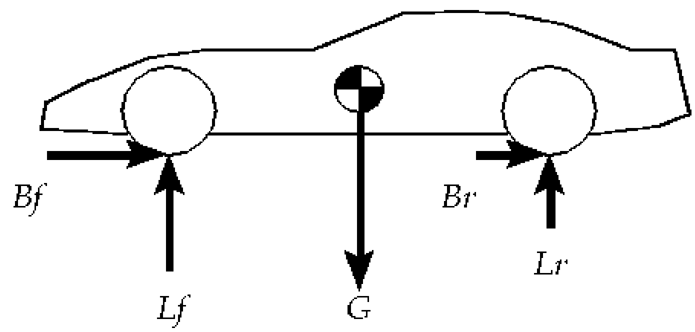
The first law: **a car in straight-line motion at a constant speed**

will keep such motion until acted on by an external force. The only reason a car in neutral will not coast forever is that friction, an external force, gradually slows the car down. Friction comes from the tires on the ground and the air flowing over the car. The tendency of a car to keep moving the way it is moving is the inertia of the car, and this tendency is concentrated at the CG point.

The second law: **When a force is applied to a car, the change in motion is proportional to the force divided by the mass of the car**. This law is expressed by the famous equation $F = ma$, where F is a force, m is the mass of the car, and a is the acceleration, or change in motion, of the car. A larger force causes quicker changes in motion, and a heavier car reacts more slowly to forces. Newton's second law explains why quick cars are powerful and lightweight. The more F and the less m you have, the more a you can get.

The third law: **Every force on a car by another object, such as the ground, is matched by an equal and opposite force on the object by the car**. When you apply the brakes, you cause the tires to push forward against the ground, and the ground pushes back. As long as the tires stay on the car, the ground pushing on them slows the car down.

Let us continue analyzing braking. Weight transfer during accelerating and cornering are mere variations on the theme. We won't consider subtleties such as suspension and tire deflection yet. These effects are very important, but secondary. The figure shows a car and the forces on it during a "one g" braking maneuver. One g means that the total braking force equals the weight of the car, say, in pounds.



A car braking at 1 "gee"

In this figure, the black and white "pie plate" in the center is the CG. G is the force of gravity that pulls the car toward the center of the Earth. This is the weight of the car; weight is just another word for the force of gravity. It is a fact of Nature, only fully explained by Albert Einstein, that gravitational forces act through the CG of an object, just like inertia. This fact can be explained at deeper levels, but such an explanation would take us too far off the subject of weight transfer.

L_f is the lift force exerted by the ground on the front tire, and L_r is the lift force on the rear tire. These lift forces are as real as the ones that keep an airplane in the air, and they keep the car from falling through the ground to the center of the Earth.

We don't often notice the forces that the ground exerts on objects because they are so ordinary, but they are at the essence of car dynamics. The reason is that the magnitude of these forces determine the ability of a tire to stick, and imbalances between the front and rear lift forces account for understeer and oversteer. The figure only shows forces on the car, not forces on the ground and the CG of the Earth. Newton's third law requires that these equal and opposite forces exist, but we are only concerned about how the ground and the Earth's gravity affect the car.

If the car were standing still or coasting, and its weight distribution were 50-50, then L_f would be the same as L_r. It is always the case that L_f plus L_r equals G, the weight of the car. Why? Because of Newton's first law. The car is not changing its motion in the vertical direction, at least as long as it doesn't get airborne, so the total sum of all forces in the vertical direction must be zero. G points down and counteracts the sum of L_f and L_r, which point up.

Braking causes L_f to be greater than L_r. Literally, the "rear end gets light," as one often hears racers say. Consider the front and rear braking forces, B_f and B_r, in the diagram. They push backwards on the tires, which push on the wheels, which push on the suspension parts, which push on the rest of the car, slowing it down. But these forces are acting at ground level, not at the level of the CG. The braking forces are indirectly slowing down the car by pushing at ground level, while the inertia of the car is 'trying' to keep it moving forward as a unit at the CG level.

The braking forces create a rotating tendency, or torque, about the CG. Imagine pulling a table cloth out from under some glasses and candelabra. These objects would have a tendency to tip or rotate over, and the tendency is greater for taller objects and is greater the harder you pull on the cloth. The rotational tendency of a car under braking is due to identical physics.

The braking torque acts in such a way as to put the car up on its nose. Since the car does not actually go up on its nose (we hope), some other forces must be counteracting that tendency, by Newton's first law. G cannot be doing it since it passes right through the cetner of gravity. The only forces that can counteract that tendency are the lift forces, and the only way they can do so is for L_f to become greater than L_r. Literally, the ground pushes up harder on the front tires during braking to try to keep the car from tipping forward.

By how much does L_f exceed L_r? The braking torque is proportional to the sum of the braking forces and to the height of the CG. Let's say that height is 20 inches. The counterbalancing torque resisting the braking torque is proportional to L_f and half the wheelbase (in a car with 50-50 weight distribution), minus L_r times half the wheelbase since L_r is helping the braking forces upend the car. L_f has a lot of work to do: it must resist the torques of both the braking forces and the lift on the rear tires. Let's say the wheelbase is 100 inches. Since we are braking at one g, the braking forces equal G, say, 3200 pounds. All this is summarized in the following equations:

$$3200\,\text{lbs times } 20\,\text{inches} = L_f \text{ times } 50\,\text{inches} - L_r \text{ times } 50\,\text{inches}$$

$$L_f + L_r = 3200\,\text{lbs (this is always true)}$$

With the help of a little algebra, we can find out that

$$L_f = 1600 + 3200/5 = 2240\,\text{lbs}$$

$$L_r = 1600 - 3200/5 = 960\,\text{lbs}$$

Thus, by braking at one g in our example car, we add 640 pounds of load to the front tires and take 640 pounds off the rears! This is very pronounced weight transfer.

By doing a similar analysis for a more general car with CG height of h, wheelbase w, weight G, static weight distribution d expressed as a fraction of weight in the front, and braking with force B, we can show that

$$L_f = dG + Bh/w$$

$$L_r = (1 - d)G - Bh/w$$

These equations can be used to calculate weight transfer during acceleration by treating acceleration force as negative braking force. If you have acceleration figures in gees, say from a *G-analyst* or other device, just multiply them by the weight of the car to get acceleration forces (Newton's second law!). Weight transfer during cornering can be analyzed in a similar way, where the track of the car replaces the wheelbase and d is always 50% (unless you account for the weight of the driver). Those of you with science or engineering backgrounds may enjoy deriving these equations for yourselves. The equations for a car doing a combination of braking and cornering, as in a trail braking maneuver, are much more complicated and require some mathematical tricks to derive.

Now you know why weight transfer happens. The next topic that comes to mind is the physics of tire adhesion, which explains how weight transfer can lead to understeer and oversteer conditions.

Part 2

Keeping Your Tires Stuck to the Ground

In last month's article, we explained the physics behind weight transfer. That is, we explained why braking shifts weight to the front of the car, accelerating shifts weight to the rear, and cornering shifts weight to the outside of a curve. Weight transfer is a side-effect of the tires keeping the car from flipping over during maneuvers. We found out that a one G braking maneuver in our 3200 pound example car causes 640 pounds to transfer from the rear tires to the front tires. The explanations were given directly in terms of Newton's fundamental laws of Nature.

This month, we investigate what causes tires to stay stuck and what causes them to break away and slide. We will find out that you can make a tire slide either by pushing too hard on it or by causing weight to transfer off the tire by your control inputs of throttle, brakes, and steering. Conversely, you can cause a sliding tire to stick again by pushing less hard on it or by tranferring weight to it. The rest of this article explains all this in term of (you guessed it) physics.

This knowledge, coupled with a good 'instinct' for weight transfer, can help a driver predict the consequences of all his or her actions and develop good instincts for staying out of trouble, getting out of trouble when it comes, and driving consistently at ten tenths. It is said of Tazio Nuvolari, one of the greatest racing drivers ever, that he knew at all times while driving the weight on each of the four tires to within a few pounds. He could think, while driving, how the loads would change if he lifted off the throttle or turned the wheel a little more, for example. His knowledge of the physics of racing enabled him to make tiny, accurate adjustments to suit every circumstance, and perhaps to make these adjustments better than his competitors. Of course, he had a very fast brain and phenomenal reflexes, too.

I am going to ask you to do a few physics "lab" experiments with me to investigate tire adhesion. You can actually do them, or you can just follow along in your imagination. First, get a tire and wheel off your car. If you are a serious autocrosser, you probably have a few loose sets in your garage. You can do the experiments with a heavy box or some object that is easier to handle than a tire, but the numbers you get won't apply directly to tires, although the principles we investigate will apply.

Weigh yourself both holding the wheel and not holding it on a bathroom scale. The difference is the weight of the tire and wheel assembly. In my case, it is 50 pounds (it would be a lot less if I had those $3000 Jongbloed wheels! Any sponsors reading?). Now put the wheel on the ground or on a table and push sideways with your hand against the tire until it slides. When you push it, push down low near the point where the tire touches the ground so it doesn't tip over.

The question is, how hard did you have to push to make the tire slide? You can find out by putting the bathroom scale between your hand and the tire when you push. This procedure doesn't give a very accurate reading of the force you need to make the tire slide, but it gives a rough estimate. In my case, on the concrete walkway in front of my house, I had to push with 85 pounds of force (my neighbors don't bother staring at me any more; they're used to my strange antics). On my linoleum kitchen floor, I only had to push with 60 pounds (but my wife does stare at me when I do this stuff in the house). What do these numbers mean?

They mean that, on concrete, my tire gave me $85/50 = 1.70$ gees of sideways resistance before sliding. On a linoleum race course (ahem!), I would only be able to get $60/50 = 1.20G$. We have directly experienced the physics of grip with our bare hands. The fact that the tire resists sliding, up to a point, is called the *grip phenomenon*. If you could view the interface between the ground and the tire with a microscope, you would see complex interactions between long-chain rubber molecules bending, stretching, and locking into concrete molecules creating the grip. Tire researchers look into the detailed workings of tires at these levels of detail.

Now, I'm not getting too excited about being able to achieve $1.70G$ cornering in an autocross. Before I performed this experiment, I frankly expected to see a number below $1G$. This rather unbelievable number of $1.70G$ would certainly not be attainable under driving conditions, but is still a testimony to the rather unbelievable state of tire technology nowadays. Thirty years ago, engineers believed that one G was theoretically impossible from a tire. This had all kinds of consequences. It implied, for example, that dragsters could not possibly go faster than 200 miles per hour in a quarter mile: you can go $\sqrt{2ax} = 198.48$ mph if you can keep $1G$ acceleration all the way down the track. Nowadays, drag racing safety watchdogs are working hard to keep the cars under 300 mph; top fuel dragsters launch at more than

3 gees.

For the second experiment, try weighing down your tire with some ballast. I used a couple of dumbells slung through the center of the wheel with rope to give me a total weight of 90 pounds. Now, I had to push with 150 pounds of force to move the tire sideways on concrete. Still about $1.70G$. We observe the fundamental law of adhesion: the force required to slide a tire is proportional to the weight supported by the tire. When your tire is on the car, weighed down with the car, you cannot push it sideways simply because you can't push hard enough.

The force required to slide a tire is called the *adhesive limit* of the tire, or sometimes the *stiction*, which is a slang combination of "stick" and "friction." This law, in mathematical form, is

$$F \leq \mu W$$

where F is the force with which the tire resists sliding; μ is the *coefficient of static friction* or *coefficient of adhesion*; and W is the weight or vertical load on the tire contact patch. Both F and W have the units of force (remember that weight is the force of gravity), so μ is just a number, a proportionality constant. This equation states that the sideways force a tire can withstand before sliding is less than or equal to μ times W. Thus, μW is the maximum sideways force the tire can withstand and is equal to the stiction. We often like to speak of the sideways acceleration the car can achieve, and we can convert the stiction force into acceleration in gees by dividing by W, the weight of the car. μ can thus be measured in gees.

The coefficient of static friction is not exactly a constant. Under driving conditions, many effects come into play that reduce the stiction of a good autocross tire to somewhere around $1.10G$. These effects are deflection of the tire, suspension movement, temperature, inflation pressure, and so on. But the proportionality law still holds reasonably true under these conditions. Now you can see that if you are cornering, braking, or accelerating at the limit, which means at the adhesive limit of the tires, any weight transfer will cause the tires unloaded by the weight transfer to pass from sticking into sliding.

Actually, the transition from sticking 'mode' to sliding mode should not be very abrupt in a well-designed tire. When one speaks of a "forgiving" tire, one means a tire that breaks away slowly as it gets more and more force or less and less weight, giving the driver time to correct. Old, hard tires are, generally speaking, less forgiving than new, soft tires. Low-profile tires are less forgiving than high-profile tires. Slicks are less forgiving than DOT tires. But these are very broad generalities and tires must be judged individually, usually by getting some word-of-mouth recommendations or just by trying them out in an autocross. Some tires are so unforgiving that they break away virtually without warning, leading to driver dramatics usually resulting in a spin. Forgiving tires are much easier to control and much more fun to drive with.

"Driving by the seat of your pants" means sensing the slight changes in cornering, braking, and acceleration forces that signal that one or more tires are about to slide. You can sense these change literally in your seat, but you can also feel changes in steering resistance and in the sounds the tires make. Generally, tires 'squeak' when they are nearing the limit, 'squeal' at the limit, and 'squall' over the limit. I find tire sounds very informative and always listen to them while driving.

So, to keep your tires stuck to the ground, be aware that accelerating gives the front tires less stiction and the rear tires more, that braking gives the front tire more stiction and the rear tires less, and that cornering gives the inside tires less stiction and the outside tires more. These facts are due to the combination of weight transfer and the grip phenomenon. Finally, drive smoothly, that is, translate your awareness into gentle control inputs that always keep appropriate tires stuck at the right times. This is the essential knowledge required for car control, and, of course, is much easier said than done. Later articles will use the knowledge we have accumulated so far to explain understeer, oversteer, and chassis set-up.

Part 3
Basic Calculations

In the last two articles, we plunged right into some relatively complex issues, namely weight transfer and tire adhesion. This month, we regroup and review some of the basic units and dimensions needed to do dynamical calculations. Eventually, we can work up to equations sufficient for a full-blown computer simulation of car dynamics. The equations can then be 'doctored' so that the computer simulation will run fast enough to be the core of an autocross computer game. Eventually, we might direct this series of articles to show how to build such a game in a typical microcomputer programming language such as C or BASIC, or perhaps even my personal favorite, LISP. All of this is in keeping with the spirit of the series, the Physics of Racing, because so much of physics today involves computing. Software design and programming are essential skills of the modern physicist, so much so that many of us become involved in computing full time.

Physics is the science of measurement. Perhaps you have heard of highly abstract branches of physics such as quantum mechanics and relativity, in which exotic mathematics is in the forefront. But when theories are taken to the laboratory (or the race course) for testing, all the mathematics must boil down to quantities that can be measured. In racing, the fundamental quantities are distance, time, and mass. This month, we will review basic equations that will enable you to do quick calculations in your head while cooling off between runs. It is very valuable to develop a skill for estimating quantities quickly, and I will show you how.

Equations that don't involve mass are called *kinematic*. The first kinematic equation relates speed, time, and distance. If a car is moving at a constant speed or velocity, v, then the distance d it travels in time t is

$$d = vt$$

or velocity times time. This equation really expresses nothing more than the definition of velocity.

If we are to do mental calculations, the first hurdle we must jump comes from the fact that we usually measure speed in miles per hour (mph), but distance in feet and time in seconds. So, we must modify our equation with a conversion factor, like this

$$d\,(\text{feet}) = v\frac{\text{miles}}{\text{hour}}\,t\,(\text{seconds})\frac{5280\,\text{feet/mile}}{3600\,\text{seconds/hour}}$$

If you "cancel out" the units parts of this equation, you will see that you get feet on both the left and right hand sides, as is appropriate, since equality is required of any equation. The conversion factor is 5280/3600, which happens to equal 22/15. Let's do a few quick examples. How far does a car go in one second (remember, say, "one-one-thousand, two-one-thousand," *etc.* to yourself to count off seconds)? At fifteen mph, we can see that we go

$$d = 15\,\text{mph times 1 sec times } 22/15 = 22\text{ feet}$$

or about 1 and a half car lengths for a 14 and 2/3 foot car like a late-model Corvette. So, at 30 mph, a second is three car lengths and at 60 mph it is six. If you lose an autocross by 1 second (and you'll be pretty good if you can do that with all the good drivers in our region), you're losing by somewhere between 3 and 6 car lengths! This is because the average speed in an autocross is between 30 and 60 mph.

Everytime you plow a little or get a little sideways, just visualize your competition overtaking you by a car length or so. One of the reasons autocross is such a difficult sport, but also such a pure sport, from the driver's standpoint, is that you can't make up this time. If you blow a corner in a road race, you may have a few laps in which to make it up. But to win an autocross against good competition, you must drive nearly perfectly. The driver who makes the fewest mistakes usually wins!

The next kinematic equation involves acceleration. It so happens that the distance covered by a car at constant acceleration from a standing start is given by

$$d = \frac{1}{2}at^2$$

or 1/2 times the acceleration times the time, squared. What conversions will help us do mental calculations with this equation? Usually, we like to measure acceleration in Gs. One G happens to be 32.1 feet per second squared. Fortunately, we don't have to deal with miles and hours here, so our equation becomes,

$$d\,(\text{feet}) = 16a\,(G\text{s})\,t\,(\text{seconds})^2$$

roughly. So, a car accelerating from a standing start at $\frac{1}{2}G$, which is a typical number for a good, stock sports car, will go 8 feet in 1 second. Not very far! However, this picks up rapidly. In two seconds, the car will go 32 feet, or over two car lengths.

Just to prove to you that this isn't crazy, let's answer the question "How long will it take a car accelerating at $\frac{1}{2}G$ to do the quarter mile?" We invert the equation above (recall your high school algebra), to get

$$t = \sqrt{d\,(\text{feet})\;16a\,(G\text{s})}$$

and we plug in the numbers: the quarter mile equals 1320 feet, $a = \frac{1}{2}G$, and we get $t = \sqrt{1320/8} = \sqrt{165}$ which is about 13 seconds. Not too unreasonable! A real car will not be able to keep up full $\frac{1}{2}G$ acceleration for a quarter mile due to air resistance and reduced torque in the higher gears. This explains why real (stock) sports cars do the quarter mile in 14 or 15 seconds.

The more interesting result is the fact that it takes a full second to go the first 8 feet. So, we can see that the launch is critical in an autocross. With excessive wheelspin, which robs you of acceleration, you can lose a whole second right at the start. Just visualize your competition pulling 8 feet ahead instantly, and that margin grows because they are 'hooked up' better.

For doing these mental calculations, it is helpful to memorize a few squares. 8 squared is 64, 10 squared is 100, 11 squared is 121, 12 squared is 144, 13 squared is 169, and so on. You can then estimate square roots in your head with acceptable precision.

Finally, let's examine how engine torque becomes force at the drive wheels and finally acceleration. For this examination, we will need to know the mass of the car. Any equation in physics that involves mass is called *dynamic*, as opposed to kinematic. Let's say we have a Corvette that weighs 3200 pounds and produces 330 foot-pounds of torque at the crankshaft. The Corvette's automatic transmission has a first gear ratio of 3.06 (the auto is the trick set up for vettes—just ask Roger Johnson or Mark Thornton). A transmission is nothing but a set of circular, rotating levers, and the gear ratio is the leverage, multiplying the torque of the engine. So, at the output of the transmission, we have

$$3.06 \times 330 = 1010\,\text{foot-pounds}$$

of torque. The differential is a further lever-multiplier, in the case of the Corvette by a factor of 3.07, yielding 3100 foot pounds at the center of the rear wheels (this is a lot of torque!). The distance from the center of the wheel to the ground is about 13 inches, or 1.08 feet, so the maximum force that the engine can put to the ground in a rearward direction (causing the ground to push back forward—remember part 1 of this series!) in first gear is

$$3100\,\text{foot-pounds}/1.08\,\text{feet} = 2870\,\text{pounds}$$

Now, at rest, the car has about 50/50 weight distribution, so there is about 1600 pounds of load on the rear tires. You will remember from last month's article on tire adhesion that the tires cannot respond with a forward force much greater than the weight that is on them, so they simply will spin if you stomp on the throttle, asking them to give you 2870 pounds of force.

We can now see why it is important to squeeeeeeeze the throttle gently when launching. In the very first instant of a launch, your goal as a driver is to get the engine up to where it is pushing on the tire contact patch at about 1600 pounds. The tires will squeal or hiss just a little when you get this right. Not so coincidentally, this will give you a forward force of about 1600 pounds, for an $F = ma$ (part 1) acceleration of about $\frac{1}{2}G$, or half the weight of the car. The main reason a car will accelerate with only $\frac{1}{2}G$ to start with is that half of the weight is on the front wheels and is unavailable to increase the stiction of the rear, driving tires. Immediately, however, there will be some weight transfer to the rear. Remembering part 1 of this series again, you can estimate that about 320 pounds will be transferred to the rear immediately. You can now ask the tires to give you a little more, and you can gently push on the throttle. Within a second or so, you can be at full throttle, putting all that torque to work for a beautiful hole shot!

In a rear drive car, weight transfer acts to make the driving wheels capable of withstanding greater forward loads. In a front drive car, weight transfer works against acceleration, so you have to be even more gentle on the throttle if you have a lot of power. An all-wheel drive car puts all the wheels to work delivering force to the ground and is theoretically the best.

Technical people call this style of calculating "back of the envelope," which is a somewhat picturesque reference to the habit we have of writing equations and numbers on any piece of paper that happens to be handy. You do it without calculators or slide rules or abacuses. You do it in the garage or the pits. It is not exactly precise, but gives you a rough idea, say within

10 or 20 percent, of the forces and accelerations at work. And now you know how to do back-of-the-envelope calculations, too.

Part 4
There Is No Such Thing as Centrifugal Force

One often hears of "centrifugal force." This is the apparent force that throws you to the outside of a turn during cornering. If there is anything loose in the car, it will immediately slide to the right in a left hand turn, and *vice versa*. Perhaps you have experienced what happened to me once. I had omitted to remove an empty Pepsi can hidden under the passenger seat. During a particularly aggressive run (something for which I am not unknown), this can came loose, fluttered around the cockpit for a while, and eventually flew out the passenger window in the middle of a hard left hand corner.

I shall attempt to convince you, in this month's article, that centrifugal force is a fiction, and a consequence of the fact first noticed just over three hundred years ago by Newton that objects tend to continue moving in a straight line unless acted on by an external force.

When you turn the steering wheel, you are trying to get the front tires to push a little sideways on the ground, which then pushes back, by Newton's third law. When the ground pushes back, it causes a little sideways acceleration. This sideways acceleration is a change in the sideways velocity. The acceleration is proportional to the sideways force, and inversely proportional to the mass of the car, by Newton's second law. The sideways acceleration thus causes the car to veer a little sideways, which is what you wanted when you turned the wheel. If you keep the steering and throttle at constant positions, you will continue to go mostly forwards and a little sideways until you end up where you started. In other words, you will go in a circle. When driving through a sweeper, you are going part way around a circle. If you take skid pad lessons (highly recommended), you will go around in circles all day.

If you turn the steering wheel a little more, you will go in a tighter circle, and the sideways force needed to keep you going is greater. If you go around the same circle but faster, the necessary force is greater. If you try to go around too fast, the adhesive limit of the tires will be exceeded, they will slide, and you will not stick to the circular path—you will not "make it."

From the discussion above, we can see that in order to turn right, for example, a force, pointing to the right, must act on the car that veers it away from the straight line it naturally tries to follow. If the force stays constant, the car will go in a circle. From the point of view of the car, the force always points to the right. From a point of view outside the car, at rest with respect to the ground, however, the force points toward the center of the circle. From this point of view, although the force is constant in *magnitude*, it changes *direction*, going around and around as the car turns, always pointing at the geometrical center of the circle. This force is called *centripetal*, from the Greek for "center seeking." The point of view on the ground is privileged, since objects at rest from this point of view feel no net forces. Physicists call this special point of view an *inertial frame of reference*. The forces measured in an inertial frame are, in a sense, more correct than those measured by a physicist riding in the car. Forces measured inside the car are *biased* by the centripetal force.

Inside the car, all objects, such as the driver, feel the natural inertial tendency to continue moving in a straight line. The driver receives a centripetal force from the car through the seat and the belts. If you don't have good restraints, you may find yourself pushing with your knee against the door and tugging on the controls in order to get the centripetal force you need to go in a circle with the car. It took me a long time to overcome the habit of tugging on the car in order to stay put in it. I used to come home with bruises on my left knee from pushing hard against the door during an autocross. I found that a tight five- point harness helped me to overcome this unnecessary habit. With it, I no longer think about body position while driving—I can concentrate on trying to be smooth and fast. As a result, I use the wheel and the gearshift lever for steering and shifting rather than for helping me stay put in the car!

The 'forces' that the driver and other objects inside the car feel are actually centripetal. The term *centrifugal*, or "center fleeing," refers to the inertial tendency to resist the centripetal force and to continue going straight. If the centripetal force is constant in magnitude, the centrifugal tendency will be constant. There is no such thing as centrifugal force (although it is a convenient fiction for the purpose of some calculations).

Let's figure out exactly how much sideways acceleration is needed to keep a car going at speed v in a circle of radius r. We can then convert this into force using Newton's second law, and then figure out how fast we can go in a circle before exceeding the adhesive limit—in other words, we can derive maximum cornering speed. For the following discussion, it will be helpful for you to draw little back-of-the-envelope pictures (I'm leaving them out, giving our editor a rest from transcribing my graphics into the newsletter).

Consider a very short interval of time, far less than a second. Call it dt (d stands for "delta," a Greek letter mathematicians use as shorthand for "tiny increment"). In time dt, let us say we go forward a distance dx and sideways a distance ds. The forward component of the velocity of the car is approximately $v = dx/dt$. At the beginning of the time interval dt, the car has no sideways velocity. At the end, it has sideways velocity ds/dt. In the time dt, the car has thus had a change in sideways velocity of ds/dt. Acceleration is, precisely, the change in velocity over a certain time, divided by the time; just as velocity is the change in position over a certain time, divided by the time. Thus, the sideways acceleration is

$$a = \frac{ds}{dt}\frac{1}{dt}$$

How is ds related to r, the radius of the circle? If we go forward by a fraction f of the radius of the circle, we must go sideways by exactly the same fraction of dx to stay on the circle. This means that $ds = f\ dx$. The fraction f is, however, nothing but dx/r. By this reasoning, we get the relation

$$ds = dx\frac{dx}{r}$$

We can substitute this expression for ds into the expression for a, and remembering that $v = dx/dt$, we get the final result

$$a = \frac{ds}{dt}\frac{1}{dt} = \frac{dx}{dt}\frac{dx}{dt}\frac{1}{r} = \frac{v^2}{r}$$

This equation simply says quantitatively what we wrote before: that the acceleration (and the force) needed to keep to a circular line increases with the velocity and increases as the radius gets smaller.

What was *not* appreciated before we went through this derivation is that the necessary acceleration increases as the *square* of the velocity. This means that the centripetal force your tires must give you for you to make it through a sweeper is very sensitive to your speed. If you go just a little bit too fast, you might as well go *much* too fast—your're not going to make it. The following table shows the maximum speed that can be achieved in turns of various radii for various sideways accelerations. This table shows the value of the expression

$$\frac{15}{22}\sqrt{32.1a\text{ (gees) } r(\text{feet})}$$

which is the solution of $a = v^2/r$ for v, the velocity. The conversion factor 15/22 converts v from feet per second to miles per hour, and 32.1 converts a from gees to feet per second squared. We covered these conversion factors in part 3 of this series.

TABLE 1: SPEED (MILES PER HOUR)

ACCELERATION	RADIUS (FEET)				
(GEES)	50.00	100.00	150.00	200.00	500.00
0.25	13.66	19.31	23.66	27.32	43.19
0.50	19.31	27.32	33.45	38.63	61.08
0.75	23.66	33.45	40.97	47.31	74.81
1.00	27.32	38.63	47.31	54.63	86.38
1.25	30.54	43.19	52.90	61.08	96.57
1.50	33.45	47.31	57.94	66.91	105.79
1.75	36.13	51.10	62.59	72.27	114.27
2.00	38.63	54.63	66.91	77.26	122.16

For autocrossing, the columns for 50 and 100 feet and the row for 1.00 G are most germane. The table tells us that to achieve 1.00 G sideways acceleration in a corner of 50 foot radius (this kind of corner is all too common in autocross), a driver must not go faster than 27.32 miles per hour. To go 30 mph, 1.25 G is required, which is probably not within the capability of an autocross tire at this speed. There is not much subjective difference between 27 and 30 mph, but the objective difference is usually between making a controlled run and spinning badly.

The absolute fastest way to go through a corner is to be just over the limit near the exit, in a controlled slide. To do this, however, you must be pointed in just such a way that when the car breaks loose and slides to the exit of the corner it will be pointed straight down the optimal racing line at the exit when it "hooks up" again. You can smoothly add throttle during this maneuver and be really moving out of the corner. But you must do it smoothly. It takes a long time to learn this, and probably a lifetime to perfect it, but it feels absolutely triumphal when done right. I have not figured out how to drive through a sweeper, except for the exit, at anything greater than the limiting velocity because sweepers are just too long to slide around. If anyone (Ayrton Senna, perhaps?) knows how, please tell me!

The chain of reasoning we have just gone through was first discovered by Newton and Leibniz, working independently. It is, in fact, a derivation in differential calculus, the mathematics of very small quantities. Newton keeps popping up. He was perhaps the greatest of all physicists, having discovered the laws of motion, the law of gravity, and calculus, among other things such as the fact that white light is made up of multiple colors mixed together.

It is an excellent diagnostic exercise to drive a car around a circle marked with cones or chalk and gently to increase the speed until the car slides. If the front breaks away first, your car has natural understeer, and if the rear slides first, it has natural oversteer. You can use this information for chassis tuning. Of course, this is only to be done in safe circumstances, on a rented skid pad or your own private parking lot. The police will gleefully give you a ticket if they catch you doing this in the wrong places.

Part 5

Introduction to the Racing Line

This month, we analyze the best way to go through a corner. "Best" means in the least time, at the greatest average speed. We ask "what is the shape of the driving line through the corner that gives the best time?" and "what are the times for some other lines, say hugging the outside or the inside of the corner?" Given the answers to these questions, we go on to ask "what shape does a corner have to be before the driving line I choose doesn't make any time difference?" The answer is a little surprising.

The analysis presented here is the simplest I could come up with, and yet is still quite complicated. My calculations went through about thirty steps before I got the answer. Don't worry, I won't drag you through the mathematics; I just sketch out the analysis, trying to focus on the basic principles. Anyone who would read through thirty formulas would probably just as soon derive them for him or herself.

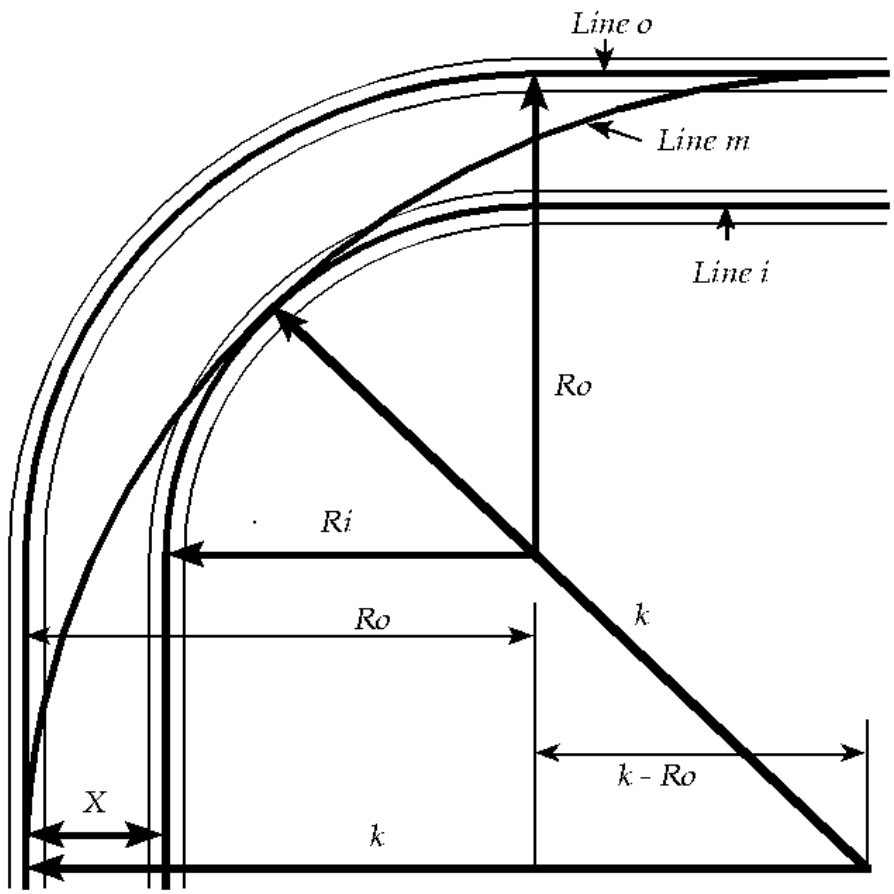
There are several simplifying assumptions I make to get through the analysis. First of all, I consider the corner in isolation; as an abstract entity lifted out of the rest of a course. The actual best driving line through a corner depends on what comes before it and after it. You usually want to optimize exit speed if the corner leads onto a straight. You might not apex if another corner is coming up. You may be forced into an unfavorable entrance by a prior curve or slalom.

Speaking of road courses, you will hear drivers say things like "you have to do such-and-such in turn six to be on line for turn ten and the front straight." In other words, actions in any one spot carry consequences pretty much all the way around. The ultimate drivers figure out the line for the entire course and drive it as a unit, taking a Zen-like approach. When learning, it is probably best to start out optimizing each kind of corner in isolation, then work up to combinations of two corners, three corners, and so on. In my own driving, there are certain kinds of three corner combinations I know, but mostly I work in twos. I have a long way to go.

It is not feasible to analyze an actual course in an exact, mathematical way. In other words, although science can provide general principles and hints, finding the line is, in practice, an art. For me, it is one of the most fun parts of racing.

Other simplifying assumptions I make are that the car can either accelerate, brake, or corner at constant speed, with abrupt transitions between behaviors. Thus, the lines I analyze are splices of accelerating, braking, and cornering phases. A real car can, must, and should do these things in combination and with smooth transitions between phases. It is, in fact, possible to do an exact, mathematical analysis with a more realistic car that transitions smoothly, but it is much more difficult than the splice-type analysis and does not provide enough more quantitative insight to justify its extra complexity for this article.

Our corner is the following ninety-degree right-hander:



This figure actually represents a family of corners with any constant width, any radius, and short straights before and after. First, we go through the entire analysis with a particular corner of 75 foot radius and 30 foot width, then we end up with times for corners of various radii and widths.

Let us define the following parameters:

$$
\begin{aligned}
r &= \text{radius of corner center line} = 75 \text{ feet} \\
W &= \text{width of course} = 30 \text{ feet} \\
r_o &= \text{radius of outer edge} = r + \tfrac{1}{2}W = 90 \text{ feet} \\
r_i &= \text{radius of inner edge} = r - \tfrac{1}{2}W = 60 \text{ feet}
\end{aligned}
$$

Now, when we drive this corner, we must keep the tires on the course, otherwise we get a lot of cone penalties (or go into the weeds). It is easiest (though not so realistic) to do the analysis considering the path of the center of gravity of the car rather than the paths of each wheel. So, we define an *effective* course, narrower than the real course, down which we may drive the center of the car.

$$
\begin{aligned}
w &= \text{width of car} = 6 \text{ feet} \\
R_o &= \text{effective outer radius} = r_o - \tfrac{1}{2}w = 87 \text{ feet} \\
R_i &= \text{effective inner radius} = r_i + \tfrac{1}{2}w = 63 \text{ feet} \\
X &= \text{effective width of course} = W - w = 24 \text{ feet}
\end{aligned}
$$

This course is indicated by the labels and the thick radius lines in the figure.

From last month's article, we know that for a fixed centripetal acceleration, the maximum driving speed increases as the square root of the radius. So, if we drive the largest possible circle through the effective corner, starting at the outside of the entrance straight, going all the way to the inside in the middle of the corner (the *apex*), and ending up at the outside of the exit straight, we can corner at the maximum speed. Such a line is shown in the figure as the thick circle labeled "line m." This is a simplified version of the classic racing line through the corner. Line m reaches the apex at the geometrical center of the circle, whereas the classic racing line reaches an apex after the geometrical center—a *late* apex—because it assumes we are accelerating out of the corner and must therefore have a continuously increasing radius in the second half and a slightly tighter radius in the first half to prepare for the acceleration. But, we continue analyzing the geometrically perfect line because it is relatively easy. The figure shows also Line i, the *inside* line, which come up the inside of the entrance straight, corners on the inside, and goes down the inside of the exit straight; and Line o, the *outside* line, which comes up the outside, corners on the outside, and exits on the outside.

One might argue that there are certain advantages of line i over line m. Line i is considerably shorter than Line m, and although we have to go slower through the corner part, we have less total distance to cover and might get through faster. Also, we can accelerate on part of the entrance chute and all the way on the exit chute, while we have to drive line m at constant speed. Let's find out how much time it takes to get through lines i and m. We include line o for completeness, even though it looks bad because it is both slower and longer than m.

If we assume a maximum centripetal acceleration of 1.10g, which is just within the capability of autocross tires, we get the following speeds for the cornering phases of Lines i, o, and m:

Cornering Speed (mph)		
Line i	Line o	Line m
32.16	37.79	48.78
v_i	v_o	v_m

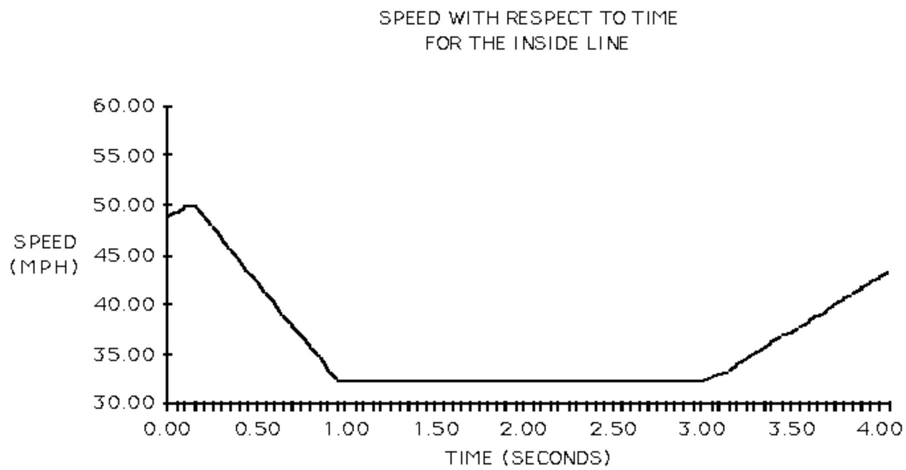
Line m is all cornering, so we can easily calculate the time to drive it once we know the radius, labeled k in the figure. A geometrical analysis results in

$$k = 3.414(R_o - 0.707R_i) = 145\,\text{feet}$$

and the time is

$$t_m = \left(\frac{\pi}{2}k\right) / \left(\frac{22}{15}v_m\right) = 3.18\,\text{seconds.}$$

For line i, we accelerate for a bit, brake until we reach 32.16 mph, corner at that speed, and then accelerate on the exit. Let's assume, to keep the comparison fair, that we have timing lights at the beginning and end of line m and that we can begin driving line i at 48.78 mph, the same speed that we can drive line m. Let us also assume that the car can accelerate at $\frac{1}{2}$g and brake at 1g. Our driving plan for line i results in the following velocity profile:



Because we can begin by accelerating, we start beating line m a little. We have to brake hard to make the corner. Finally, although we accelerate on the exit, we don't quite come up to 48.78 mph, the exit speed for line m. But, we don't care about exit speed, only time through the corner. Using the velocity profile above, we can calculate the time for line i, call it t_i, to be 4.08 seconds. Line i loses by 9/10ths of a second. It is a fair margin to lose an autocross by this much over a whole course, but this analysis shows we can lose it in just one typical corner! In this case, line i is a catastrophic mistake. Incidentally, line o takes 4.24 seconds = t_o.

What if the corner were tighter or of greater radius? The following table shows some times for 30 foot wide corners of various radii:

radius	30.00	45.00	60.00	75.00	90.00	95.00
t_o	3.99	4.06	4.15	4.24	4.35	4.38
t_i	3.94	3.94	4.00	4.08	4.17	4.21
t_m	2.64	2.83	3.01	3.18	3.34	3.39
margin	1.30	1.11	1.01	0.90	0.83	0.82

Line i *never* beats line m even though that as the radius increases, the margin of loss decreases. The trend is intuitive because corners of greater radius are also longer and the extra speed in line m over line i is less. The margin is greatest for tight corners because the width is a greater fraction of the length and the speed differential is greater.

How about for various widths? The following table shows times for a 75 foot radius corner of several widths:

width	10.00	30.00	50.00	70.00	90.00
t_o	2.68	4.24	5.47	6.50	7.41
t_i	2.62	4.08	5.32	6.45	7.51
t_m	2.46	3.18	3.77	4.27	4.73
margin	0.16	0.90	1.55	2.18	2.79

The wider the course, the greater the margin of loss. This is, again, intuitive since on a wide course, line m is a really large circle through even a very tight corner. Note that line o becomes better than line i for wide courses. This is because the speed differential between lines o and i is very great for wide courses. The most notable fact is that line m beats line i by 0.16 seconds even on a course that is only four feet wider than the car! You really must "use up the whole course."

So, the answer is, under the assumptions made, that the inside line is *never* better than the classic racing line. For the splice-type car behavior assumed, I conjecture that *no* line is faster than line m.

We have gone through a simplified kind of *variational analysis*. Variational analysis is used in all branches of physics, especially mechanics and optics. It is possible, in fact, to express all theories of physics, even the most arcane, in variational form, and many physicists find this form very appealing. It is also possible to use variational analysis to write a computer program that finds an approximately perfect line through a complete, realistic course.

Part 6
Speed and Horsepower

The title of this month's article consists of two words dear to every racer's heart. This month, we do some "back of the envelope" calculations to investigate the basic physics of speed and horsepower (the "back of the envelope" style of calculating was covered in part 3 of this series).

How much horsepower does it take to go a certain speed? At first blush, a physicist might be tempted to say "none," because he or she remembers Newton's first law, by which an object moving at a constant speed in a straight line continues so moving forever, even to the end of the Universe, unless acted on by an external force. Everyone knows, however, that it is necessary to keep your foot on the gas to keep a car moving at a constant speed. Keeping your foot on the gas means that you are making the engine apply a backward force to the ground, which applies a reaction force forward on the car, to keep the car moving. In fact, we know a few numbers from our car's shop manual. A late model Corvette, for example, has a top speed of about 150 miles per hour and about 240 hp. This means that if you keep your foot *all* the way down, using up all 240 hp, you can eventually go 150 mph. It takes a while to get there. In this car, you can get to 60 mph in about 6 seconds (if you don't spin the drive wheels), to 100 mph in about 15 seconds, and 150 in about a minute.

All this seems to contradict Newton's first law. What is going on? An automobile moving at constant speed in a straight line on level ground is, in fact, acted on by a number of external forces that tend to slow it down. Without these forces, the car *would* coast forever as guaranteed by Newton's first law. You must counteract these forces with the engine, which indirectly creates a reaction force that keeps the car going. When the car is going at a constant speed, the *net* force on the car, that is, the speeding-up forces minus the slowing-down forces, is zero.

The most important external, slowing-down force is *air resistance* or *drag*. The second most important force is friction between the tires and the ground, the so-called *rolling resistance*. Both these forces are called *resistance* because they always act to oppose the forward motion of the car in whatever direction it is going. Another physical effect that slows a car down is internal friction in the drive train and wheel bearings. Acting internally, these forces cannot slow the car. However, they push backwards on the tires, which push forward on the ground, which pushes back by Newton's third law, slowing the car down. The internal friction forces are opposed by external reaction forces, which act as slight braking forces, slowing the car. So, Newton and the Universe are safe; everything is working as it should.

How big are the resistance forces, and what role does horsepower play? The physics of air resistance is very complex and an area of vigorous research today. Most of this research is done by the aerospace industry, which is technologically very closely related to the automobile industry, especially when it comes to racing. We'll slog through some arithmetic here to come up with a table that shows how much horsepower it takes to sustain speed. Those who don't have the stomach to go through the math can skim the next few paragraphs.

We cannot derive equations for air resistance here. We'll just look them up. My source is *Fluid Mechanics*, by L. D. Landau and E. M. Lifshitz, two eminent Russian physicists. They give the following approximate formula:

$$F = \frac{1}{2} C_d A \rho v^2$$

The factors in this equation are the following:

$C_d =$ coefficient of friction, a factor depending on the shape of a car and determined by experiment; for a late model Corvette it is about 0.30;

$A =$ frontal area of the car; for a Corvette, it is about 20 square feet;

$\rho =$ Greek letter *rho*, density of air, which we calculate below;

$v =$ speed of the car.

Let us calculate the density of air using "back of the envelope" methods. We know that air is about 79% Nitrogen and 21% Oxygen. We can look up the fact that Nitrogen has a molecular weight of about 28 and Oxygen has a molecular weight of about 32. What is molecular weight? It is the mass (not the weight, despite the name) of 22.4 liters of gas. It is a number of historical convention, just like feet and inches, and doesn't have any real science behind it. So, we figure that air has an average molecular weight of

$$\frac{79\% \text{ of } 28 + 21\% \text{ of } 32 = 28.84 \,\text{grams}}{22.4 \,\text{liters}} = 1.29 \,\text{gm/l}$$

I admit to using a calculator to do this calculation, against the spirit of the "back of the envelope" style. So sue me.

We need to convert 1.29 gm/l to pounds of mass per cubic foot so that we can do the force calculations in familiar, if not convenient, units. It is worthwhile to note, as an aside, that a great deal of the difficulty of doing calculations in the physics of racing has to do with the traditional units of feet, miles, and pounds we use. The metric system makes all such calculations vastly simpler. Napoleon Bonaparte wanted to convert the world the metric system (mostly so his own soldiers could do artillery calculations quickly in their heads) but it is still not in common use in America nearly 200 years later!

Again, we look up the conversion factors. My source is *Engineering Formulas* by Kurt Gieck, but they can be looked up in almost any encyclopedia or dictionary. There are 1000 liters in a cubic meter, which in turn contains 35.51 cubic feet. Also, a pound-mass contains 453.6 grams. These figures give us, for the density of air

$$1.29\,\frac{\text{gm}}{\text{liter}}\,\frac{\text{lb-mass}}{453.6\text{ gm}}\,\frac{1000\text{ liters}}{1\text{ meter}^3}\,\frac{1\text{ meter}^3}{35.51\text{ ft}^3} = 0.0801\,\frac{\text{lb-mass}}{\text{ft}^3}$$

This says that a cubic foot of air weighs 8 hundredths of a pound, and so it does! Air is much more massive than it seems, until you are moving quickly through it, that is.

Let's finish off our equation for air resistance. We want to fill in all the numbers except for speed, v, using the Corvette as an example car so that we can calculate the force of air resistance for a variety of speeds. We get

$$F = \frac{1}{2}\,(0.30 = C_d)\left(20\text{ ft}^2 = A\right)\left(0.080\,\frac{\text{lb-mass}}{\text{ft}^3} = \rho\right)v^2 = 0.24v^2\,\frac{\text{lb-mass}}{\text{ft}}$$

We want, at the end, to have v in miles per hour, but we need v in feet per seconds for the calculations to come out right. We recall that there are 22 feet per second for every 15 miles per hour, giving us

$$\begin{aligned} F &= 0.24\left(\frac{22\text{ ft/sec}}{15\text{ mph}}\,v\,(\text{mph})\right)^2\frac{\text{lb-mass}}{\text{ft}} \\ &= 0.517\,(v\ \ (\text{mph}))^2\,\frac{\text{lb-mass ft}}{\text{sec}^2} \end{aligned}$$

Now (this gets confusing, and it wouldn't be if we were using the metric system), a pound mass is a phony unit. A lb-mass is concocted to have a weight of 1 pound under the action of the Earth's gravity. Pounds are a unit of force or weight, not of mass. We want our force of air resistance in pounds of force, so we have to divide lb-mass ft/sec^2 by 32.1, numerically equal to the acceleration of Earth's gravity in ft/sec^2, to get pounds of force. You just have to know these things. This was a lot of work, but it's over now. We finally get

$$F = \frac{0.517}{32.1}\left(v \ \text{(mph)}\right)^2 = 0.0161\left(v \ \text{(mph)}\right)^2 \text{ pounds}$$

Let's calculate a few numbers. The following table gives the force of air resistance for a number of interesting speeds:

v(mph)	15	30	60	90	120	150
F(pounds)	3.60	14.5	58.0	130	232	362

We can see that the force of air resistance goes up rapidly with speed, until we need over 350 pounds of constant force just to overcome drag at 150 miles per hour. We can now show where horsepower comes in.

Horsepower is a measure of *power*, which is a technical term in physics. It measures the amount of work that a force does as it acts over time. *Work* is another technical term in physics. It measures the actual effect of a force in moving an object over a distance. If we move an object one foot by applying a force of one pound, we are said to be doing one foot-pound of work. If it takes us one second to move the object, we have exerted one foot-pound per second of power. A horsepower is 550 foot-pounds per second. It is another one of those historical units that Napoleon hated and that has no reasonable origin in science.

We can expend one horsepower by exerting 550 pounds of force to move an object 1 foot in 1 second, or by exerting 1 pound of force to move an object 550 feet in 1 second, or by exerting 1 pound of force to move an object 1 foot in 0.001818 seconds, and so on. All these actions take the same amount of power. Incidentally, a horsepower happens to be equal also to 745 watts. So, if you burn about 8 light bulbs in your house, someone somewhere is expending at least one horsepower (and probably more like four or five) in electrical forces to keep all that going for you, and you pay for the service at the end of the month!.

All this means that to find out how much horsepower it takes to overcome air resistance at any speed, we need to multiply the force of air resistance by speed (in feet per second, converted from miles per hour), and divide by 550, to convert foot-lb/sec to horsepower. The formula is

$$P = Fv = \frac{0.0161}{550}\,\frac{22}{15}\,v^3 = \frac{0.354}{8{,}250}\,(v\ \text{(mph)})^3 \text{ horsepower}$$

and we get the following numbers from the formula for a few interesting speeds.

v(mph)	30	55	65	90	120	150	200
F(pounds)	14.5	48.7	68.0	130	232	362	644
horsepower	1.16	7.14	11.8	31.3	74.2	145	344

I put 55 mph and 65 mph in this table to show why some people think that the 55 mph national speed limit saves gasoline. It only requires about 7 hp to overcome drag at 55 mph, while it requires almost 12 hp to overcome drag at 65. Fuel consumption is approximately proportional to horsepower expended.

More interesting to the racer is the fact that it takes 145 hp to overcome drag at 150 mph. We know that our Corvette example car has about 240 hp, so about 95 hp must be going into overcoming rolling resistance and the slight braking forces arising from internal friction in the drive train and wheel bearings. Race cars capable of going 200 mph usually have at least 650 hp, about 350 of which goes into overcoming air resistance. It is probably possible to go 200 mph with a car in the 450–500 hp range, but such a car would have very good aerodynamics; expensive, low-friction internal parts; and low rolling resistance tires, which are designed to have the smallest possible contact patch like high performance bicycle tires, and are therefore not good for handling.

Part 7

The Traction Budget

This month, we introduce the traction budget. This is a way of thinking about the traction available for car control under various conditions. It can help you make decisions about driving style, the right line around a course, and diagnosing handling problems. We introduce a diagramming technique for visualizing the traction budget and combine this with a well-known visualization tool, the "circle of traction," also known as the circle of friction. So this month's article is about tools, conceptual and visual, for thinking about some aspects of the physics of racing.

To introduce the traction budget, we first need to visualize a tire in contact with the ground. Figure 1 shows how the bottom surface of a tire might look if we could see that surface by looking down from above. In other words, this figure shows an imaginary "X-ray" view of the bottom surface of a tire. For the rest of the discussion, we will always imagine that we view the tire this way. From this point of view, "up" on the diagram corresponds to forward forces and motion of the tire and the car, "down" corresponds to backward forces and motion, "left" corresponds to leftward forces and motion, and "right" on the diagram corresponds to rightward forces and motion.

The figure shows a shaded, elliptical region, where the tire presses against the ground. All the interaction between the tire and the ground takes place in this *contact patch*: that part of the tire that touches the ground. As the tire rolls, one bunch of tire molecules after another move into the contact patch. But the patch itself more-or-less keeps the same shape, size, and position relative to the axis of rotation of the tire and the car as a whole. We can use this fact to develop a simplified view of the interaction between tire and ground. This simplified view lets us quickly and easily do approximate calculations good within a few percent. (A full-blown, mathematical analysis requires tire coordinates that roll with the tire, ground coordinates fixed on the ground, car coordinates fixed to the car, and many complicated equations relating these coordinate systems; the last few percent of accuracy in a mathematical model of tire-ground interaction involves a great deal more complexity.)

You will recall that forces on the tire from the ground are required to make

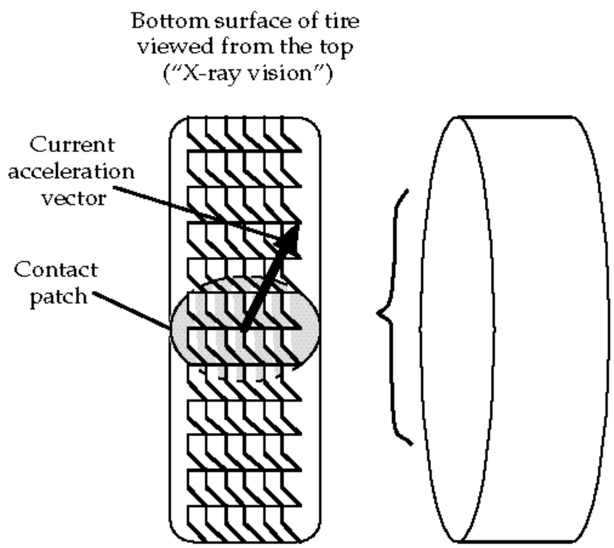


Figure 1: The bottom surface of a tire viewed from the top as though with "X ray vision."

a car change either its speed of motion or its direction of motion. Thinking of the X-ray vision picture, forces pointing up are required to make the car accelerate, forces pointing down are required to make it brake, and forces pointing right and left are required to make the car turn. Consider forward acceleration, for a moment. The engine applies a torque to the axle. This torque becomes a force, pointing backwards (down, on the diagram), that the tire applies to the ground. By Newton's third law, the ground applies an equal and opposite force, therefore pointing forward (up), on the contact patch. This force is transmitted back to the car, accelerating it forward. It is easy to get confused with all this backward and forward action and reaction. Remember to think only about the forces on the tire and to ignore the forces on the ground, which point the opposite way.

You will also recall that a tire has a limited ability to stick to the ground. Apply a force that is too large, and the tire slides. The maximum force that a tire can take depends on the weight applied to the tire:

$$F \leq \mu W$$

where F is the force on the tire, μ is the coefficient of adhesion (and depends on tire compound, ground characteristics, temperature, humidity, phase of the moon, *etc.*), and W is the weight or load on the tire.

By Newton's second law, the weight on the tire depends on the fraction of the car's mass that the tire must support and the acceleration of gravity, $g = 32.1\ \text{ft/sec}^2$. The fraction of the car's mass that the tire must support depends on geometrical factors such as the wheelbase and the height of the center of gravity. It also depends on the acceleration of the car, which completely accounts for weight transfer.

It is critical to separate the geometrical, or *kinematic*, aspects of weight transfer from the mass of the car. Imagine two cars with the same geometry but different masses (weights). In a one g braking maneuver, the same *fraction* of each car's total weight will be transferred to the front. In the example of Part 1 of this series, we calculated a 20% weight transfer during one g braking because the height of the CG was 20% of the wheelbase. This weight transfer will be the same 20% in a 3500 pound, stock Corvette as in a 2200 pound, tube-frame, Trans-Am Corvette so long as the geometry (wheelbase, CG height, *etc.*) of the two cars is the same. Although the actual weight, in pounds, will be different in the two cases, the fractions of the cars' total weight will be equal.

Separating kinematics from mass, then, we have for the weight

$$W = f(a)mg$$

where $f(a)$ is the fraction of the car's mass the tire must support and also accounts for weight transfer, m is the car's mass, and g is the acceleration of gravity.

Finally, by Newton's second law again, the acceleration of the tire due to the force F applied to it is

$$a = F/f(a)m$$

We can now combine the expressions above to discover a fascinating fact:

$$a = F/f(a)m \leq a_{max}$$

$$a_{max} = \frac{\mu W}{f(a)m} = \frac{\mu f(a)mg}{f(a)m} = \mu g$$

The maximum acceleration a tire can take is μg, a constant, independent of the mass of the car! While the maximum *force* a tire can take depends very much on the current vertical load or weight on the tire, the acceleration of that tire does not depend on the current weight. If a tire can take one g before sliding, it can take it on a lightweight car as well as on a heavy car, and it can take it under load as well as when lightly loaded. We hinted at this fact in Part 2, but the analysis above hopefully gives some deeper insight into it. We note that a_{max} being constant is only approximately true, because μ changes slightly as tire load varies, but this is a second-order effect (covered in a later article).

So, in an approximate way, we can consider the available acceleration from a tire independently of details of weight transfer. The tire will give you so many gees and that's that. This is the essential idea of the traction budget. What you do with your budget is your affair. If you have a tire that will give you one g, you can use it for accelerating, braking, cornering, or some combination, but you cannot use more than your budget or you will slide. The front-back component of the budget measures accelerating and braking, and the right-left component measures cornering acceleration. The front-back component, call it a_y, combines with the left-right component, a_x, not by adding, but by the Pythagorean formula:

$$a = \sqrt{a_x^2 + a_y^2}$$

Rather than trying to deal with this formula, there is a convenient, visual representation of the traction budget in the *circle of traction.* Figure 2 shows the circle. It is oriented in the same way as the X-ray view of the contact patch, Figure 1 , so that up is forward and right is rightward. The circular boundary represents the limits of the traction budget, and every point inside the circle represents a particular choice of how you spend your budget. A point near the top of the circle represents pure, forward acceleration, a point near the bottom represents pure braking. A point near the right boundary, with no up or down component, represents pure rightward cornering acceleration. Other points represent Pythagorean combinations of cornering and forward or backward acceleration.

The beauty of this representation is that the effects of weight transfer are factored out. So the circle remains approximately the same no matter what the load on a tire.

In racing, of course, we try to spend our budget so as to stay as close to the limit, *i.e.* , the circular boundary, as possible. In street driving, we try to stay well inside the limit so that we have lots of traction available to react to unforeseen circumstances.

I have emphasized that the circle is only an approximate representation of the truth. It is probably close enough to make a computer driving simulation

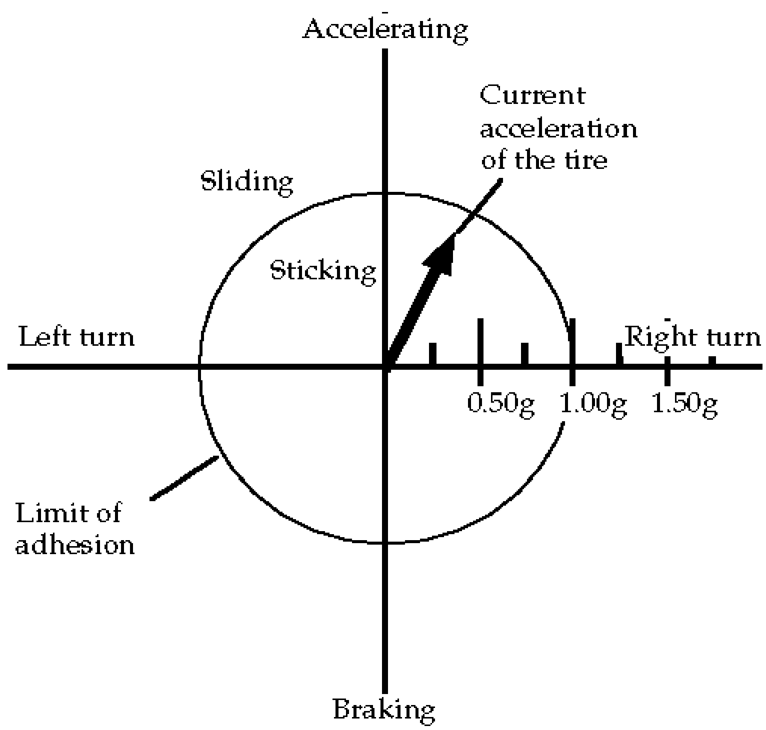


Figure 2: The Circle of Traction

that feels right (I'm pretty sure that "Hard Drivin' " and other such games use it). As mentioned, tire loads do cause slight, dynamic variations. Car characteristics also give rise to variations. Imagine a car with slippery tires in the back and sticky tires in the front. Such a car will tend to oversteer by sliding. Its traction budget will not look like a circle. Figure 3 gives an indication of what the traction budget for the whole car might look like (we have been discussing the budget of a single tire up to this point, but the same notions apply to the whole car). In Figure 3 , there is a large traction circle for the sticky front tires and a small circle for the slippery rear tires. Under acceleration, the slippery rears dominate the combined traction budget because of weight transfer. Under braking, the sticky fronts dominate. The combined traction budget looks something like an egg, flattened at top and wide in the middle. Under braking, the traction available for cornering is considerably greater than the traction available during acceleration because the sticky fronts are working. So, although this poorly handling car tends to oversteer by sliding the rear, it also tends to understeer during acceleration because the slippery rears will not follow the steering front tires very effectively.

The traction budget is a versatile and simple technique for analyzing and visualizing car handling. The same technique can be applied to developing driver's skills, planning the line around a course, and diagnosing handling problems.

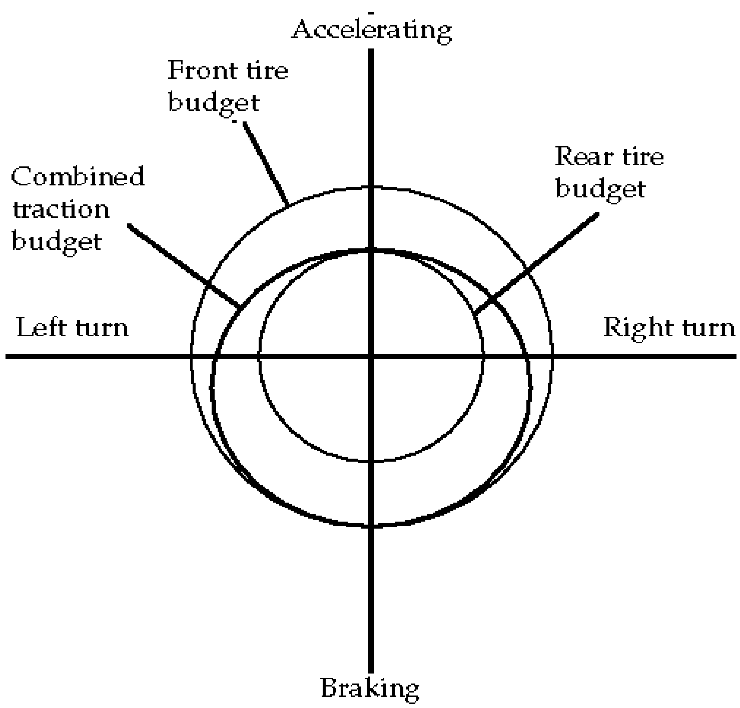


Figure 3: A traction budget diagram for a poorly handling car.

Part 8
Simulating Car Dynamics with a Computer Program

This month, we begin writing a computer program to simulate the physics of racing. Such a program is quite an ambitious one. A simple racing video game, such as "Pole Position," probably took an expert programmer several months to write. A big, realistic game like "Hard Drivin'" probably took three to five people more than a year to create. The point is that the topic of writing a racing simulation is one that we will have to revisit many times in these articles, assuming your patience holds out. There are many 'just physics' topics still to cover too, such as springs and dampers, transients, and thermodynamics. Your author hopes you will find the computer programming topic an enjoyable sideline and is interested, as always, in your feedback.

We will use a computer programming language called Scheme. You have probably encountered BASIC, a language that is very common on personal computers. Scheme is like BASIC in that it is *interactive*. An interactive computer language is the right kind to use when inventing a program as you go along. Scheme is better than BASIC, however, because it is a good deal simpler and also more powerful and modern. Scheme is available for most PCs at very modest cost (MIT Press has published a book and diskette with Scheme for IBM compatibles for about $40; I have a free version for Macintoshes). I will explain everything we need to know about Scheme as we go along. Although I assume little or no knowledge about computer programming on your part, we will ultimately learn some very advanced things.

The first thing we need to do is create a *data structure* that contains the mathematical state of the car at any time. This data structure is a block of computer memory. As simulated time progresses, mathematical operations performed on the data structure simulate the physics. We create a new instance of this data structure by typing the following on the computer keyboard at the Scheme prompt:

```
(new-race-car)
```

This is an example of an *expression*. The expression includes the parentheses. When it is typed in, it is `evaluated` immediately. When we say that Scheme is an interactive programming language, we mean that it evaluates expressions immediately. Later on, I show how we *define* this expression. It is by defining such expressions that we write our simulation program.

Everything in Scheme is an expression (that's why Scheme is simple). Every expression has a value. The value of the expression above is the new data structure itself. We need to give the new data structure a name so we can refer to it in later expressions:

```
(define car-161 (new-race-car))
```

This expression illustrates two Scheme features. The first is that expressions can contain sub-expressions inside them. The inside expressions are called *nested*. Scheme figures out which expressions are nested by counting parentheses. It is partly by nesting expressions that we build up the complexity needed to simulate racing. The second feature is the use of the special Scheme word `define`. This causes the immediately following word to become a stand-in synonym for the value just after. The technical name for such a stand-in synonym is *variable*. Thus, the expression `car-161`, wherever it appears after the `define` expression, is a synonym for the data structure created by the nested expression `(new-race-car)`.

We will have another data structure (with the same format) for `car-240`, another for `car-70`, and so on. We get to choose these names to be almost anything we like [1]. So, we would create all the data structures for the cars in our simulation with expressions like the following:

```
(define car-161 (new-race-car))
(define car-240 (new-race-car))
(define car-70  (new-race-car))
```

The state of a race car consists of several numbers describing the physics of the car. First, there is the car's position. Imagine a map of the course. Every position on the map is denoted by a pair of coordinates, x and y. For elevation changes, we add a height coordinate, z. The position of the center of gravity of a car at any time is denoted with expressions such as the following:

[1] It so happens, annoyingly, that we can't use the word `car`. This is a Scheme reserved word, like `define`. Its use is explained later

```
(race-car-x car-161)
(race-car-y car-161)
(race-car-z car-161)
```

Each of these expressions performs *data retrieval* on the data structure `car-161`. The value of the first expression is the x coordinate of the car, *etc.*Normally, when running the Scheme interpreter, typing an expression simply causes its value to be printed, so we would see the car position coordinates printed out as we typed. We could also store these positions in another block of computer memory for further manipulations, or we could specify various mathematical operations to be performed on them.

The next pieces of state information are the three components of the car's velocity. When the car is going in any direction on the course, we can ask "how fast is it going in the x direction, ignoring its motion in the y and z directions?" Similarly, we want to know how fast it is going in the y direction, ignoring the x and z directions, and so on. Decomposing an object's velocity into separate components along the principal coordinate directions is necessary for computation. The technique was originated by the French mathematician Descartes, and Newton found that the motion in each direction can be analyzed independently of the motions in the other directions at right angles to the first direction.

The velocity of our race car is retrieved via the following expressions:

```
(race-car-vx car-161)
(race-car-vy car-161)
(race-car-vz car-161)
```

To end this month's article, we show how velocity is computed. Suppose we retrieve the position of the car at simulated time t_1 and save it in some variables, as follows:

```
(define x1 (race-car-x car-161))
(define y1 (race-car-y car-161))
(define z1 (race-car-z car-161))
```

and again, at a slightly later instant of simulated time, t_2:

```
(define x2 (race-car-x car-161))
(define y2 (race-car-y car-161))
(define z2 (race-car-z car-161))
```

We have used `define` to create some new variables that now have the values of the car's positions at two times. To calculate the average velocity of the car between the two times and store it in some more variables, we evaluate the following expressions:

```
(define vx (/ (- x2 x1) (- t2 t1)))
(define vy (/ (- y2 y1) (- t2 t1)))
(define vz (/ (- z2 z1) (- t2 t1)))
```

The nesting of expressions is one level deeper than we have seen heretofore, but these expressions can be easily analyzed. Since they all have the same form, it suffices to explain just one of them. First of all, the `define` operation works as before, just creating the variable `vx` and assigning it the value of the following expression. This expression is

```
(/ (- x2 x1) (- t2 t1))
```

In normal mathematical notation, this expression would read

$$\frac{x_2 - x_1}{t_2 - t_1}$$

and in most computer languages, it would look like this:

```
(x2 - x1) / (t2 - t1)
```

We can immediately see this is the velocity in the x direction: a change in position divided by the corresponding change in time. The Scheme version of this expression looks a little strange, but there is a good reason for it: consistency. Scheme requires that all operations, including everyday mathematical ones, appear in the first position in a parenthesized expression, immediately after the left parenthesis. Although consistency makes mathematical expressions look strange, the payback is simplicity: all expressions have the same form. If Scheme had one notation for mathematical expressions and another notation for non-mathematical expressions, like most computer languages, it would be more complicated. Incidentally, Scheme's notation is called Polish notation. Perhaps you have been exposed to Hewlett-Packard calculators, which use reverse Polish, in in which the operator always appears in the *last* position. Same idea, and advantages, as Scheme, only reversed.

So, to analyze the expression completely, it is a division expression

```
(/ ...)
```

whose two arguments are nested subtraction expressions

```
(- ...) (- ...)
```

The whole expression has the form

```
(/ (- ...) (- ...))
```

which, when the variables are filled in, is

```
(/ (- x2 x1) (- t2 t1))
```

After a little practice, Scheme's style for mathematics becomes second nature and the advantages of consistent notation pay off in the long run.

Finally, we should like to store the velocity values in our data structure. We do so as follows:

```
(set-race-car-vx! car-161 vx)
(set-race-car-vy! car-161 vy)
(set-race-car-vz! car-161 vz)
```

The `set` operations change the values in the data structure named `car-161`. The exclamation point at the end of the names of these operations doesn't do anything special. It's just a Scheme idiom for operations that change data structures.

Part 9

Straights

We found in part 5 of this series, "Introduction to the Racing Line," that a driver can lose a shocking amount of time by taking a bad line in a corner. With a six-foot-wide car on a ten-foot-wide course, one can lose sixteen hundredths by 'blowing' a single right-angle turn. This month, we extend the analysis of the racing line by following our example car down a straight. It is often said that the most critical corner in a course is the one before the longest straight. Let's find out how critical it is. We calculate how much time it takes to go down a straight as a function of the speed entering the straight. The results, which are given at the end, are not terribly dramatic, but we make several, key improvements in the mathematical model that is under continuing development in this series of articles. These improvements will be used as we proceed designing the computer program begun in Part 8.

The mathematical model for traveling down a straight follows from Newton's second law:

$$F = ma, \tag{1}$$

where F is the force on the car, m is the mass of the car, and a is the acceleration of the car. We want to solve this equation to get time as a function of distance down the straight. Basically, we want a table of numbers so that we can look up the time it takes to go any distance. We can build this table using accountants' columnar paper, or we can use the modern version of the columnar pad: the electronic spreadsheet program.

To solve equation 1, we first invert it:

$$a = F/m. \tag{2}$$

Now a, the acceleration, is the rate of change of velocity with time. *Rate of change* is simply the ratio of a small change in velocity to a small change in time. Let us assume that we have filled in a column of times on our table. The times start with 0 and go up by the same, small amount, say 0.05 sec. Physicists call this small time the *integration step*. It is standard practice to begin solving an equation with a fixed integration step. There are sometimes good reasons to vary the integration step, but those reasons do not arise in this problem. Let us call the integration step Δt. If we call the time in the

i-th row t_i, then for every row except the first,

$$\Delta t = t_i - t_{i-1} = \text{constant}. \quad (3)$$

We label another column *velocity*, and we'll call the velocity in the i-th row v_i. For every row except the first, equation 2 becomes:

$$\frac{v_i - v_{i-1}}{\Delta t} = F/m. \quad (4)$$

We want to fill in velocities as we go down the columns, so we need to solve equation 4 for v_i. This will give us a formula for computing v_i given v_{i-1} for every row except the first. In the first row, we put the speed with which we enter the straight, which is an input to the problem. We get:

$$v_i = v_{i-1} + \Delta t F/m. \quad (5)$$

We label another column *distance*, and we call the distance value in the i-th row x_i. Just as acceleration is the rate of change of velocity, so velocity is the rate of change of distance over time. Just as before, then, we may write:

$$v_i = \frac{x_i - x_{i-1}}{\Delta t}. \quad (6)$$

Solved for x_i, this is:

$$x_i = x_{i-1} + \Delta t v_i. \quad (7)$$

Equation 7 gives us a formula for calculating the distance for any time given the previous distance and the velocity calculated by equation 5. Physicists would say that we have a scheme for *integrating the equations of motion.*

A small detail is missing: what is the force, F? Everything to this point is *kinematic.* The real modeling starts now with formulas for calculating the force. For this, we will draw on all the previous articles in this series. Let's label another column *force*, and a few more with *drag*, *rolling resistance*, *engine torque*, *engine rpm*, *wheel rpm*, *trans gear ratio*, *drive ratio*, *wheel torque*, and *drive force.* As you can see, we are going to derive a fairly complete, if not accurate, model of accelerating down the straight. We need a few constants:

CONSTANT	SYMBOL	EXAMPLE VALUE
rear end ratio	R	3.07
density of air	ρ	0.0025 slugs/ft^3
coeff. of drag	C_d	0.30
frontal area	A	20 ft^2
wheel diameter	d	26 in = 2.167 ft
roll resist factor	r_r	0.696 lb/(ft/sec)
car mass	m	100 slug
first gear ratio	g_1	2.88
second gear ratio	g_2	1.91
third gear ratio	g_3	1.33
fourth gear ratio	g_4	1.00

and a few variables:

VARIABLE	SYMBOL	EXAMPLE VALUE
engine torque	T_E	330 ft-lbs
drag	F_d	45 lbs
rolling resistance	F_r	54 lbs
engine rpm	E	4000
wheel rpm	W	680
wheel torque	T_W	1930 ft-lbs
wheel force	F_W	1780 lbs
net force	F	1681 lbs

All the example values are for a late model Corvette. *Slugs* are the English unit of mass, and 1 slug weighs about 32.1 lbs at sea level (another manifestation of $F = ma$, with F in lbs, m in slugs, and a being the acceleration of gravity, 32.1 ft/sec$_2$).

The most basic modeling equation is that the force we can use for forward acceleration is the propelling force transmitted through the wheels minus drag and rolling resistance:

$$F = F_W - F_d - F_r. \tag{8}$$

The force of drag we get from Part 6:

$$F_d = \frac{1}{2} C_d A \rho v_i^2. \tag{9}$$

Note that to calculate the force at step i, we can use the velocity at step i. This force goes into calculating the acceleration at step i, which is used to calculate the velocity and distance at step $i+1$ by equations 5 and 7. Those two equations represent the only 'backward references' we need. Thus, the only inputs to the integration are the initial distance, 0, and the entrance velocity, v_0.

The rolling resistance is approximately proportional to the velocity:

$$F_r = r_r v_i = 0.696 v_1. \tag{10}$$

This approximation is probably the weakest one in the model. I derived it by noting from a Corvette book that 8.2 hp were needed to overcome rolling resistance at 55 mph. I have nothing else but intuition to go on for this equation, so take it with a grain of salt.

Finally, we must calculate the forward force delivered by the ground to the car by reaction to the rearward force delivered to the ground *via* the engine and drive train:

$$F_W = \frac{T_E R g_k}{d/2}. \tag{11}$$

This equation simply states that we take the engine torque multiplied by the rear axle ratio and the transmission drive ratio in the k-th gear, which is the torque at the drive wheels, T_W, and divide it by the radius of the wheel, which is half the diameter of the wheel, d.

To calculate the forward force, we must decide what gear to be in. The logic we use to do this is the following: from the velocity, we can calculate the wheel rpm:

$$W = 60 \, \frac{\text{sec}}{\text{min}} \, \frac{v_i}{\pi d}. \tag{12}$$

From this, we know the engine rpm:

$$E = W R g_k. \tag{13}$$

At each step of integration, we look at the current engine rpm and ask "is it past the torque peak of the engine?" If so, we shift to the next highest gear, if possible. Somewhat arbitrarily, we assume that the torque peak is at 4200 rpm. To keep things simple, we also make the optimistic assumption that the engine puts out a constant torque of 330 ft-lbs. To make the model more realistic, we need merely look up a torque curve for our engine, usually expressed as a function of rpm, and read the torque off the curve at each step of the integration. The current approximation is not terrible however; it merely gives us artificially good times and speeds. Another important

Table 1: Exit speeds and times for several entrance speeds

	200 ft straight		500 ft straight	
Entrance speed (mph)	Exit speed (mph)	Time (sec)	Exit speed (mph)	Time (sec)
25	61.51	2.972	81.12	5.811
27	61.77	2.916	81.51	5.748
29	62.15	2.845	82.02	5.676
31	62.34	2.793	82.19	5.599
35	63.18	2.691	82.78	5.472
40	64.65	2.548	83.49	5.282
45	66.85	2.392	84.68	5.065
50	69.27	2.261	85.83	4.875

improvement on the logic would be to check whether the wheels are spinning, *i.e.* , that acceleration is less than about $\frac{1}{2}G$, and to 'lift off the gas' in that case.

We have all the ingredients necessary to calculate how much time it takes to cover a straight given an initial speed. You can imagine doing the calculations outlined above by hand on columnar paper, or you can check my results (below) by programming them up in a spreadsheet program like Lotus 1-2-3 or Microsoft Excel. Eventually, of course, if you follow this series, you will see these equations again as we write our Scheme program for simulating car dynamics. Integrating the equations of motion by hand will take you many hours. Using a spreadsheet will take several hours, too, but many less than integrating by hand.

To illustrate the process, we show below the times and exit speeds for a 200 foot straight, which is a fairly long one in autocrossing, and a 500 foot straight, which you should only see on race tracks. We show times and speeds for a variety of speeds entering the straight from 25 to 50 mph in Table 1. The results are also summarized in the two plots, Figures 1 and 2.

The notable facts arising in this analysis are the following. The time difference resulting from entering the 200' straight at 27 mph rather than 25 mph is about 6 hundredths. Frankly, not as much as I expected. The

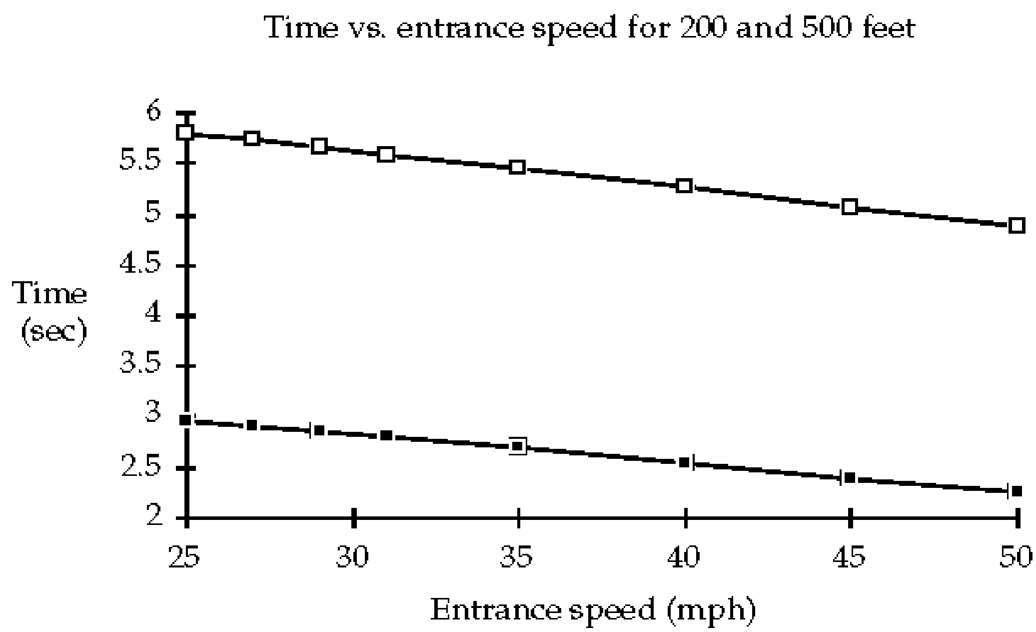

Time vs. entrance speed for 200 and 500 feet
Time (sec)
6
5.5
5
4.5
4
3.5
3
2.5
2
25
30
35
40
45
50
Entrance speed (mph)

Figure 1:

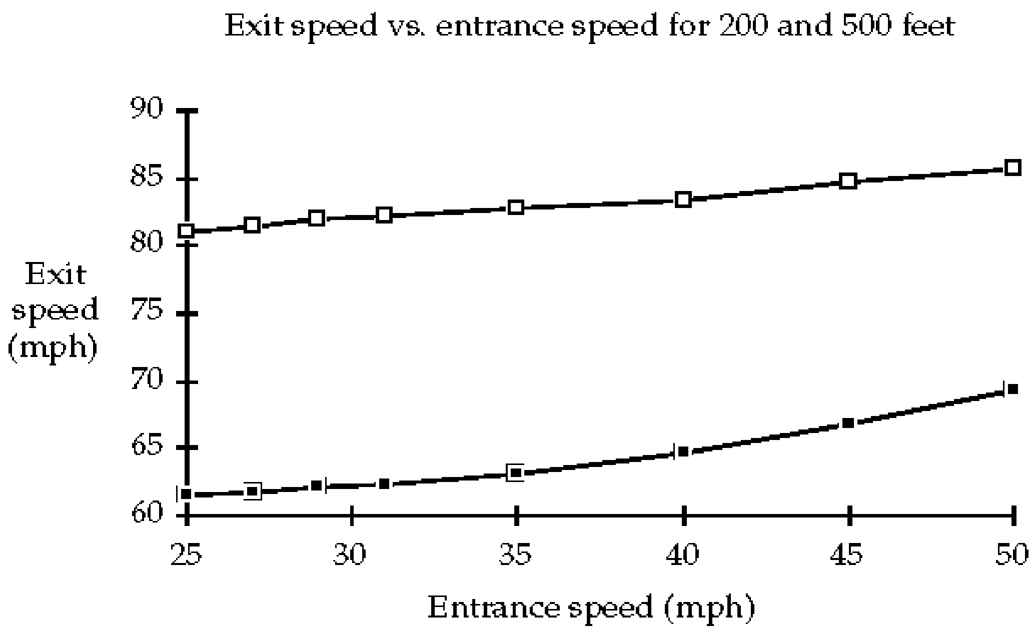

Exit speed vs. entrance speed for 200 and 500 feet
90
85
80
75
70
65
60
Exit speed (mph)
25
30
35
40
45
50
Entrance speed (mph)

Figure 2:

time difference between entering at 31 mph over 25 mph is about 2 tenths, again less than I would have guessed. The speed difference at the end of the straight between entering at 25 mph and 50 mph is only 8 mph, a result of the fact that the car labors against friction and higher gear ratios at high speeds. It is also a consequence of the fact that there is so much torque available at 25 mph in low gear that the car can almost make up the difference over the relatively short 200' straight. In fact, on the longer 500' straight, the exit speed difference between entering at 25 mph and 50 mph is not even 5 mph, though the time difference is nearly a full second.

This analysis would most likely be much more dramatic for a car with less torque than a Corvette. In a Corvette, with 330 ft-lbs of torque on tap, the penalty for entering a straight slower than necessary is not so great as it would be in a more typical car, where recovering speed lost through timidity or bad cornering is much more difficult.

Again, the analysis can be improved by using a real torque curve and by checking whether the wheels are spinning in lower gears.

Part 10
Grip Angle

In many ways, tire mechanics is an unpleasant topic. It is shrouded in uncertainty, controversy, and trade secrecy. Both theoretical and experimental studies are extremely difficult and expensive. It is probably the most uncontrollable variable in racing today. As such, it is the source of many highs and lows. An improvement in modeling or design, even if it is found by lucky accident, can lead to several years of domination by one tire company, as with BFGoodrich in autocrossing now. An unfortunate choice of tire by a competitor can lead to frustration and a disastrous hole in the budget.

This month, we investigate the physics of tire adhesion a little more deeply than in the past. In Parts 2, 4, and 7, we used the simple friction model given by $F \leq \mu W$, where F is the maximum traction force available from a tire; μ, assumed constant, is the coefficient of friction; and W is the instantaneous vertical load, or weight, on a tire. While this model is adequate for a rough, intuitive feel for tire behavior, it is grossly inadequate for quantitative use, say, for the computer program we began in Part 8 or for race car engineering and set up.

I am not a tire engineer. As always, I try to give a fresh look at any topic from a physicist's point of view. I may write things that are heretical or even wrong, especially on such a difficult topic as tire mechanics. I invite debate and corrections from those more knowledgeable than I. Such interaction is part of the fun of these articles for me.

I call this month's topic 'grip angle.' The grip angle is a quantity that captures, for many purposes, the complex and subtle mechanics of a tire. Most writers call this quantity 'slip angle.' I think this name is misleading because it suggests that a tire works by slipping and sliding. The truth is more complicated. Near maximum loads, the contact patch is partly gripping and partly slipping. The maximum net force a tire can yield occurs at the threshold where the tire is still gripping but is just about to give way to total slipping. Also, I have some difficulties with the analyses of slip angle in the literature. I will present these difficulties in these articles, unfortunately, probably without resolution. For these reasons, I give the quantity a new name.

A tire is an elastic or deformable body. It delivers forces to the car by stretching, compressing, and twisting. It is thus a very complex sort of spring with several different ways, or *modes*, of deformation. The hypothetical tire implied by $F \leq \mu W$ with constant μ would be a non-elastic tire. Anyone who has driven hard tires on ice knows that non-elastic tires are basically uncontrollable, not just because μ is small but because regular tires on ice do not twist appreciably.

The first and most obvious mode of derformation is radial. This deformation is along the radius of the tire, the line from the center to the tread. It is easily visible as a bulge in the sidewall near the contact patch, where the tire touches the ground. Thus, radial compression varies around the circumference.

Second is circumferential deformation. This is most easily visble as wrinkling of the sidewalls of drag tires. These tires are intentionally set up to deform dramatically in the circumferential direction.

Third is axial deformation. This is a deflection that tends to pull the tire off the (non-elastic) wheel or rim.

Last, and most important for cornering, is *torsional* deformation. This is a difference in axial deflection from the front to the back of the contact patch. Fundamentally, radial, circumferential, and axial deformation furnish a complete description of a tire. But it is very useful to consider the *differences* in these deflections around the circumference.

Let us examine exactly how a tire delivers cornering force to the car. We can get a good intuition into the physics with a pencil eraser. Get a block eraser, of the rectangular kind like 'Pink Pearl' or 'Magic Rub.' Stand it up on a table or desk and think of it as a little segment of the circumference of a tire. Think of the part touching the desk as the contact patch. Grab the top of the eraser and think of your hand as the wheel or rim, which is going to push, pull, and twist on the segment of tire circumference as we go along the following analysis.

Consider a car traveling at speed v in a straight line. Let us turn the steering wheel slightly to the right (twist the top of the eraser to the right). At the instant we begin turning, the rim (your hand on the eraser), at a circumferential position just behind the contact patch, pushes slightly leftward on the bead of the tire. Just ahead of the contact patch, likewise, the rim pulls the bead a little to the right. The push and pull together are called a *force couple*. This couple delivers a torsional, clockwise stress to the inner part of the tire carcass, near the bead. This stress is communicated to the contact patch by the elastic material in the sidewalls (or the main body of the eraser). As a result of turning the steering wheel, therefore, the rim twists the contact patch clockwise.

The car is still going straight, just for an instant. How are we going to explain a net rightward force from the road on the contact patch? This net force *must* be there, otherwise the tire and the car would continue in a straight line by Newton's First Law.

Consider the piece of road just under the contact patch at the instant the turn begins. The rubber particles on the left side of the patch are going a little bit faster with respect to the road than the rest of the car and the rubber particles on the right side of the patch are going a little bit slower than the rest of the car. As a result, the left side of the patch grips a little bit less than the right. The rubber particles on the left are more likely to slide and the ones on the right are more likely to grip. Thus, the left edge of the patch 'walks' a little bit upward, resulting in a net clockwise twisting motion of the patch. The torsional stress becomes a torsional motion. As this motion is repeated from one instant to the next, the tire (and the eraser—I hope you are still following along with the eraser) walks continuosly to the right.

The better grip on the right hand side of the contact patch adds up to a net rightward force on the tire, which is transmitted back through the sidewall to the car. The chassis of the car begins to yaw to the right, changing the direction of the rear wheels. A torsional stress on the rear contact patches results, and the rear tires commence a similar 'walking' motion.

The wheel (your hand) is twisted more away from the direction of the car than is the contact patch. The angular difference between the direction the wheel is pointed and the direction the tire walks is the grip angle. All quantities of interest in tire mechanics—forces, friction coefficients, *etc.*, are conventionally expressed as functions of grip angle.

In steady state cornering, as in sweepers, an understeering car has larger grip angles in front, and an oversteering car has larger grip angles in the rear. How to control grip angles statically with wheel alignment and dynamically with four-wheel steering are subjects for later treatment.

The greater the grip angle, the larger the cornering force becomes, up to a point. After this point, greater grip angle delivers less force. This point is analogous to the idealized adhesive limit mentioned earlier in this series. Thus, a real tire behaves *qualitatively* like an ideal tire, which grips until the adhesive limit is exceeded and then slides. A real tire, however, grips gradually better as cornering force increases, and then grips gradually worse as the limit is exceeded.

The walking motion of the contact patch is not entirely smooth, or in otherwords, somewhat *discrete*. Individual blocks of rubber alternately grip and slide at high frequency, thousands of times per second. Under hard cornering, the rubber blocks vibrating on the road make an audible squaling sound. Beyond the adhesive limit, squealing becomes a lower frequency sound, 'squalling,' as the point of optimum efficiency of the walking process is bypassed.

There is a lot more to say on this subject, and I admit that my first attempts at a mathematical analysis of grip angle and contact patch mechanics got bogged down. However, I think we now have an intuitive, conceptual basis for better modeling in the future.

Speaking of the future, summarize briefly the past of and plans for the *Physics of Racing* series. The following overlapping threads run through it:

Tire Physics concerns adhesion, grip angle, and elastic modeling. This has been covered in Parts 2, 4, 7, and 10, and will be covered in several later parts.

Car Dynamics concerns handling, suspension movement, and motion of a car around a course; has been covered in Parts 1, 4, 5, and 8 and will continue.

Drive Line Physics concerns modeling of engine performance and acceleration. Has been covered in Parts 3, 6, and 9 and will also continue.

Computer Simulation concerns the design of a working program that captures all the physics. This is the ultimate goal of the series. It was begun in Part 8 and will eventually dominate discussion.

The following is a list of articles that have appeared so far:

1. Weight Transfer

2. Keeping Your Tires Stuck to the Ground

3. Basic Calculations

4. There is No Such Thing as Centrifugal Force

5. Introduction to the Racing Line

6. Speed and Horsepower

7. The Circle of Traction

8. Simulating Car Dynamics with a Computer Program

9. Straights

10. Grip Angle

and the following is a *tentative* list of articles I have planned for the near future (naturally, this list is 'subject to change without notice'):

Springs and Dampers, presenting a detailed model of suspension movement (suggested by Bob Mosso)

Transients, presenting the dynamics of entering and leaving corners, chicanes, and slaloms (this one suggested by Karen Babb)

Stability, explaining why spins and other losses of control occur

Smoothness, exploring what, exactly, is meant by smoothness

Modeling Car Data in a computer program; in several articles

Modeling Course Data in a computer program; also in several articles

In practice, I try to keep the lengths of articles about the same, so if a topic is getting too long (and grip angle definitely did), I break it up in to several articles.

Part 11
Braking

I was recently helping to crew Mark Thornton's effort at the Silver State Grand Prix in Nevada. Mark had built a beautiful car with a theoretical top speed of over 200 miles per hour for the 92 mile time trial from Lund to Hiko. Mark had no experience driving at these speeds and asked me as a physicist if I could predict what braking at 200 mph would be like. This month I report on the back-of-the-envelope calculations on braking I did there in the field.

There are a couple of ways of looking at this problem. Brakes work by converting the energy of motion, *kinetic* energy, into the energy of heat in the brakes. Converting energy from useful forms (motion, electrical, chemical, *etc.*) to heat is generally called *dissipating* the energy, because there is no easy way to get it back from heat. If we assume that brakes dissipate energy at a constant rate, then we can immediately conclude that it takes four times as much time to stop from 200 mph as from 100 mph. The reason is that kinetic energy goes up as the square of the speed. Going at twice the speed means you have four times the kinetic energy because $4 = 2^2$. The exact formula for kinetic energy is $\frac{1}{2}mv^2$, where m is the mass of an object and v is its speed. This was useful to Mark because braking from 100 mph was within the range of familiar driving experience.

That's pretty simple, but is it right? Do brakes dissipate energy at a constant rate? My guess as a physicist is 'probably not.' The efficiency of the braking process, dissipation, will depend on details of the friction interaction between the brake pads and disks. That interaction is likely to vary with temperature. Most brake pads are formulated to grip harder when hot, but only up to a point. Brake fade occurs when the pads and rotors are overheated. If you continue braking, heating the system even more, the brake fluid will eventually boil and there will be no braking at all. Brake fluid has the function of transmitting the pressure of your foot on the pedal to the brake pads by hydrostatics. If the fluid boils, then the pressure of your foot on the pedal goes into crushing little bubbles of gaseous brake fluid in the brake lines rather than into crushing the pads against the disks. Hence, no brakes.

We now arrive at the second way of looking at this problem. Let us assume

Starting Speed (mph)	Starting Speed (fps)	Time to brake (sec)	Distance to brake (feet)	Distance to brake (yards)
30	44	1.37	30.16	10.05
60	88	2.74	120.62	40.21
90	132	4.11	271.40	90.47
120	176	5.48	482.49	160.83
150	220	6.85	753.89	251.30
180	264	8.22	1085.61	361.87
210	308	9.60	1477.63	492.54

Table 2: Times and distances for braking to zero from various speeds.

that we have good brakes, so that the braking process is limited *not* by the interaction between the pads and disks but by the interaction between the tires and the ground. In other words, let us assume that our brakes are better than our tires. To keep things simple and back-of-the-envelope, assume that our tires will give us a constant deceleration of

$$1G \equiv a = 32.1 \frac{\text{feet}}{\text{sec}^2}$$

The time t required for braking from speed v can be calculated from:

$$t = v/a$$

which simply follows from the definition of constant acceleration. Given the time for braking, we can calculate the distance x, again from the definitions of acceleration and velocity:

$$x = vt - \frac{1}{2}at^2$$

Remembering to be careful about converting miles per hour to feet per second, we arrive at the numbers in Table 1.

We can immediately see from this table (and, indeed, from the formulas) that it is the *distance*, not the time, that varies as the square of the starting speed v. The braking time only goes up linearly with speed, that is, in simple proportion.

The numbers in the table are in the ballpark of the braking figures one reads in published tests of high performance cars, so I am inclined to believe that the second way of looking at the problem is the right way. In other words, the assumption that the brakes are better than the tires, so long as they are not overheated, is probably right, and the assumption that brakes dissipate energy at a constant rate is probably wrong because it leads to the conclusion that braking takes more time than it actually does.

My final advice to Mark was to leave *lots of room*. You can see from the table that stopping from 210 mph takes well over a quarter mile of very hard, precise, threshold braking at $1G$!

Part 12
CyberCar, Every Racer's DWIM Car?

The cybernetic DWIM car is coming. DWIM stands for "Do What I Mean."[2] It is a commonplace term in the field of Human-machine Interfaces, and refers to systems that automatically interpret the user's intent from his or her inputs.

Cybernetics (or at least one aspect of it) is the science of unifying humans and machines. The objective of cybernetics is usually to amplify human capability with 'intelligent' machines, but sometimes the objective is the reverse. Most of the work in cybernetics has been under the aegis of defense, for building advanced tanks and aircraft. There is a modest amount of cybernetics in the automotive industry, as well. Anti-lock Braking (ABS), Acceleration Slip Reduction (ASR), Electronic Engine Management, and Automatic Traction Control (ATC) are cybernetic DWIM systems—of a kind—already in production. They all make 'corrections' on the driver's input based on an assumed intention. Steer-by-wire, Continuously Variable Transmissions (CVT), and active suspensions are on the immediate horizon. All these features are part of a distinct trend to automate the driving experience. This month, we take a break from hard physics to look at the better and the worse of increased automation, and we look at one concept of the ultimate result, CyberCar.

Among the research directions in cybernetics are advanced sensors for human inputs. One of the more incredible is a system that reads brain waves and figures out what a fighter pilot wants to do directly from patterns in the waves.

A major challenge in the fighter cockpit is information overload. Pilots have far too many instruments, displays, horns, buzzers, radio channels, and idiot lights competing for their attention. In stressful situations, such as high speed dogfights, the pilot's brain simply ignores inputs beyond its capacity, so the pilot may not hear a critical buzzer or see a critical warning light. In the 'intelligent cockpit,' however, the pilot *consciously* suppresses certain displays and auditory channels, thus reducing sensory clutter. By the same token, the intelligent cockpit must be able to override the pilot's choices and

[2]and the word play on 'dream' was too much to resist.

to put up critical displays and to sound alarms in emergencies. In the reduced clutter of the cockpit, then, it is much less likely that a pilot will miss critical information.

How does the pilot select the displays that he[3] wants to see? The pilot cannot afford the time to scroll through menus like those on a personal computer screen or hunt-and-peck on a button panel like that on an automatic bank teller machine.

There are already sensors that can read a pilot's brain waves and anticipate what he wants to look at next. Before the pilot even consciously knows that he wants to look at a weapon status display, for example, the cybernetic system can infer the intention from his brain waves and pop up the display. If he thinks it is time to look at the radar, before he could speak the command, the system reads his brain waves, pops up the radar display, and puts away the weapon status display.

How does it work? During a training phase, the system reads brain waves and gets explicit commands through a button panel. The system analyzes the brain waves, looking for certain unique features that it can associate with the intention (inferred from the command from the button panel) to see the radar display, and other unique features to associate with the intention to look at weapon status, and so on. The system must be trained individually for each pilot. Later, during operation, whenever the system sees the unique brain wave patterns, it 'knows' what the pilot wants to do.

The implications of technology like this for automobiles is amazing. Already, things like ABS are a kind of rudimentary cybernetics. When a driver stands all over the brake pedal, it is assumed that his intention is to stop, not to skid. The ABS system 'knows,' in a manner of speaking, the driver's intention and manages the physical system of the car to accomplish that goal. So, instead of being a mere mechanical linkage between your foot and the brakes, the brake pedal becomes a kind of intentional, DWIM control. Same goes for traction control and ASR. When the driver is on the gas, the system 'knows' that he wants to go forward, not to spin out or do doughnuts. In the case of TC, the system regulates the torque split to the drive wheels, whether there be two or four. In the case of ASR, the system backs off the throttle when there is wheel spin. Cybernetics again.

ABS, TC, and ASR exist now. What about the future? Consider steer-by-wire. CyberCar, the total cybernetic car, infers the driver's intended

[3]Everywhere, 'he' means 'he or she,' 'his' means 'his or her,' etc.

direction from the steering wheel position. It makes corrections to the actual direction of the steered wheels and to the throttle and brakes much more quickly and smoothly than any driver can do. Coupled with slip angle[4] sensors[1] and inertial guidance systems, perhaps based on miniaturized laser/fiber optic gyros (no moving parts), cybernetic steering, throttle, and brake controls will make up a formidable racing car that could drive a course in practically optimal fashion given only the driver's *desired* racing line.

In an understeering situation, when a car is not turning as much as desired, a common driver mistake is to turn the steering wheel more. That is a mistake, however, only because the driver is treating the steering wheel as an *intentional* control rather than the physical control it actually is. In CyberCar, however, the steering wheel *is* an intentional control. When the driver adds more lock in a corner, CyberCar 'knows' that the driver just wants more steering. Near the limits of adhesion, CyberCar knows that the appropriate *physical* reaction is, in fact, some weight transfer to the front, either by trailing throttle or a little braking, and a little less steering wheel lock. When the fronts hook up again, CyberCar can immediately get back into the throttle and add a little more steering lock, all the while tracking the driver's desires through the intentional steering wheel in the cockpit. Similarly, in an oversteer situation, when the driver gives opposite steering lock, CyberCar knows what to do. First, CyberCar determines whether the condition is trailing throttle oversteer (TTO) or power oversteer (PO). It can do this by monitoring tire loads through suspension deflection and engine torque output over time. In TTO, CyberCar adds a little throttle and countersteers. When the drive wheels hook up again, it modulates the throttle and dials in a little forward lock. In PO, CyberCar gently trails off the throttle and countersteers. All the while, CyberCar monitors driver's intentional inputs and the physical status of the car at the rate of several kilohertz (thousands of times per second).

The very terms 'understeer' and 'oversteer' carry cybernetic implication, for these are terms of intent. Understeer means the car is not steering as much as wanted, and oversteer means it is steering too much.

The above description is within current technology. What if we get *really* fantastic? How about doing away with the steering wheel altogether? CyberCar, version II, knows where the driver wants to go by watching his eyes, and it knows whether to accelerate or brake by watching brain waves. With

[4] Also known as grip angle; see Part 10 of this series.

Virtual Reality and teleoperation, the driver does not even have to be inside the car. The driver, wearing binocular video displays that control incar cameras (or even synthetic computer graphics) *via* head position, sits in a virtual cockpit in the pits.

Now we must ask how much cybernetics is desirable? Autocrossing is, largely, a pure driver skill contest. Wheel-to-wheel racing adds racecraft—drafting, passing, deception, *etc.*—to car control skills. Does it not seem that cybernetics eliminates driver skill as a factor by automating it? Is it not just another way for the 'haves' to beat the 'have-nots' by out-spending them? Drivers who do not have ABS have already complained that it gives their competition an unfair advantage. On the other hand, drivers who *do* have it have complained that it reduces their feel of control and their options while braking. I think they doth protest too much.

In the highest forms of racing, where money is literally no object, cybernetics is already playing a critical role. The clutchless seven speed transmissions of the Williams/Renault team dominated the latter half of the 1991 Formula 1 season. But for some unattributable bad luck, they would have won the driver's championship and the constructor's cup. Carrol Smith, noted racing engineer, has been predicting for years that ABS will show up in Formula 1 as soon as systems can be made small and light enough[2]. It seems inevitable to me that cybernetic systems will give the unfair advantage to those teams most awash in money. However, autocrossers, club racers, and other grass roots competitors will be spared the expense, and the experience of being relieved of the enjoyment of car control, for at least another decade or two.

Acknowledgements

Thanks to Phil Ethier for giving me a few tips on car control that I might be able to teach to CyberCar and to Ginger Clark for bringing slip angle sensors to my attention.

References

[1] Patrick Borthelow. "Sensing Tire Slip Angles At the Racetrack." *Sensors*, September 1991.

[2] Carrol Smith. *Engineer to Win, Prepare to Win, Build to Win.* Classic Motorbooks, P.O. Box 1/RT021, Osceola, WI 54020.

Colophon

Brian Beckman began publishing the Physics of Racing series in June 1990 as a series of articles in his sports car club's newsletter. The articles were originally formatted in LaTeX[5] and typeset using TeXtures on the Apple Macintosh computer.

The articles have been widely distributed as Adobe PostScript files for many years, and were converted to HTML by Robert Keller *circa* 1995 for display on the World Wide Web. They can be seen on Keller's home page at http://members.home.net/rck/phor/.

In August 1999, I undertook a conversion of the Physics of Racing series to Adobe Acrobat PDF format. The original LaTeX sources were converted to the newer LaTeX 2_ε format, and a PDF file was generated using the pdftex package developed by Han The Thanh at Masaryk University, Czech Republic. Graphics were sourced from the GIF files in Keller's Web pages, and converted to PDF and PNG formats using the ImageMagick converter by John Cristy and gif2png by The PNG Development Group.

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[5] LaTeX is a collection of macros for the TeX typesetting system developed by Professor Donald Knuth of Stanford University. TeX is a trademark of the American Mathematical Society.