Homework Ch.3 Boolean Algebra & Digital Circuits

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1 Construct a truth table for ...

A) $x\bar{y}z + \overline{xy}z + x\overline{yz}$

x	y	z	$x\bar{y}z$	$\overline{xy}z$	$x\overline{yz}$	$F(x,y,z)$
0	0	0	0	0	0	0
0	0	1	0	1	0	1
0	1	0	0	0	0	0
0	1	1	0	1	0	1
1	0	0	0	0	1	1
1	0	1	1	1	1	1
1	1	0	0	0	1	1
1	1	1	0	0	0	0

* DeMorgan's : $\overline{xy} = \bar{x} + \bar{y}$

$\Rightarrow x\bar{y}z + (\bar{x}+\bar{y})z + x(\bar{y}+\bar{z})$

B) $(x+\bar{z})(y+z)(\bar{x}+\bar{y})$

x	y	z	$(x+\bar{z})$	$(y+z)$	$(\bar{x}+\bar{y})$	$F(x,y,z)$
0	0	0	1	0	1	0
0	0	1	0	1	1	0
0	1	0	1	1	1	1
0	1	1	0	1	1	0
1	0	0	1	0	1	0
1	0	1	1	1	1	1
1	1	0	1	1	0	0
1	1	1	1	1	0	0

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4) Using DeMorgan's Law, write an expression for the complement of F

if $F(x,y,z) = xy + \bar{x}z + y\bar{z}$ ...

* Replace each variable by its complement and interchange ANDs and ORs :

$\rightarrow \bar{x}\bar{y} + x\bar{z} + \bar{y}z$

$\rightarrow (\bar{x}+\bar{y})(x+\bar{z})(\bar{y}+z)$

* Check w/ truth table :

x	y	z	xy	$\bar{x}z$	$y\bar{z}$	$(\bar{x}+\bar{y})$	$(x+\bar{z})$	$(\bar{y}+z)$	$F()$		$\bar{F}()$
0	0	0	0	0	0	1	1	1	0	≠	1
0	0	1	0	1	0	1	0	1	1	≠	0
0	1	0	0	0	1	1	1	0	1	≠	0
0	1	1	0	1	0	1	0	1	1	≠	0
1	0	0	0	0	0	1	1	1	0	≠	1
1	0	1	0	0	0	1	1	1	0	≠	1
1	1	0	1	0	1	0	1	0	1	≠	0
1	1	1	1	0	0	0	1	1	1	≠	0

✓

9) Show that $x = xy + x\bar{y}$

Ⓐ Using truth table

x	y	xy	$x\bar{y}$	$F_{(x,y)}$
0	0	0	0	0
0	1	0	0	0
1	0	0	1	1
1	1	1	0	1

=

Ⓑ Using Boolean Identities

→ Start by simplify $xy + x\bar{y}$

$= xy + x\bar{y}$

$= x(y+\bar{y})$ [Distributive]

$= x\,(1)$ [Inverse]

$= x$ [Identity]

11) Simplify the Boolean expression using algebraic identities. List the identity used at each step...

Ⓒ $F(x,y,z) = \overline{(x+y)}\,\overline{(\bar{x}+\bar{y})}$

$= (\bar{x}\bar{y})(\overline{\bar{x}+\bar{y}})$ [DeMorgan's : OR]

$= (\bar{x}\bar{y})(\bar{\bar{x}}\bar{\bar{y}})$ [DeMorgan's : OR]

$= (\bar{x}\bar{y})(x\,y)$ [Double Complement]

$= (\bar{y}\bar{x})(xy)$ [communtative]

$= \bar{y}(\bar{x}x)y$ [Associative]

$= (\bar{y}y)(\bar{x}x)$ [communtative]

$= 0 \cdot 0$ [Inverse]

$= 0$

15) Simplify the Boolean expression. List the identity used at each step...

© $\bar{x}(\bar{y}z + yz) + xyz$

$= \bar{x}z(\bar{y}+y) + xyz$ [distributive]

$= \bar{x}z(1) + xyz$ [Inverse]

$= \bar{x}z + xyz$ [Identity]

$= z(\bar{x} + xy)$ [distributive]

$= z(\bar{x} + xy)$

OR

$= \bar{x}\bar{y}z + \bar{x}yz + xyz$ [distributive]

$= \bar{x}\bar{y}z + yz(\bar{x}+x)$ [distributive]

$= \bar{x}\bar{y}z + yz(1)$ [Inverse]

$= \bar{x}\bar{y}z + yz$ [Identity]

$= z(\bar{x}\bar{y} + y)$ [distributive]

$= z((\overline{x+y}) + y)$ [DeMorgan's]

→ let $F(x,y,z) = \bar{x}(\bar{y}z + yz) + xyz$, $F_1(x,y,z) = z(\bar{x}+xy)$, $F_2(x,y,z) = z(\overline{x+y} + y)$

x	y	z	xy	yz	$\bar{y}z$	xyz	$(\bar{y}z+yz)$	$(\bar{x}+xy)$	$((\overline{x+y})+y)$	$F(x,y,z)$		F_1		F_2
0	0	0	0	0	0	0	0	1	1	0	=	0	=	0
0	0	1	0	0	1	0	1	1	1	1	=	1	=	1
0	1	0	0	0	0	0	0	1	1	0	=	0	=	0
0	1	1	0	1	0	0	1	1	1	1	=	1	=	1
1	0	0	0	0	0	0	0	0	0	0	=	0	=	0
1	0	1	0	0	1	0	1	0	0	0	=	0	=	0
1	1	0	1	0	0	0	0	1	1	0	=	0	=	0
1	1	1	1	1	0	1	1	1	1	1	=	1	=	1

→ Check with Karnaugh Map:

- 8 minterms

x	y	z	
0	0	0	$\bar{x}\bar{y}\bar{z}$
0	0	1	$\bar{x}\bar{y}z$
0	1	0	$\bar{x}y\bar{z}$
0	1	1	$\bar{x}yz$
1	0	0	$x\bar{y}\bar{z}$
1	0	1	$x\bar{y}z$
1	1	0	$xy\bar{z}$
1	1	1	xyz

x \ yz	00	01	11	10
0	0	1	1	0
1	0	0	1	0

$(\bar{x}z + yz)$
0
1
0
1
0
0
0
1

✓

Group A: 001 + 011 → $\bar{x}\bar{y}z + \bar{x}yz \Rightarrow \bar{x}z$

Group B: 011 + 111 → $\bar{x}yz + xyz \Rightarrow yz$

⇒ Group A + Group B : $\bar{x}z + yz$

21) Write the Boolean expression in sum-of-products form ...

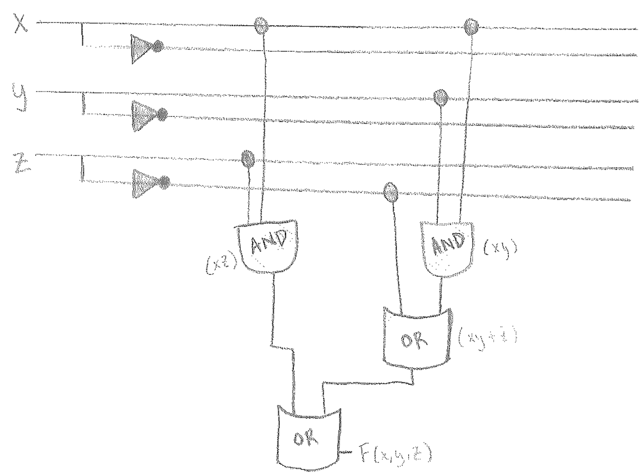
x	y	z	F
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

$\Rightarrow F(x,y,z) = \bar{x}\bar{y}\bar{z} + \bar{x}yz + xy\bar{z}$

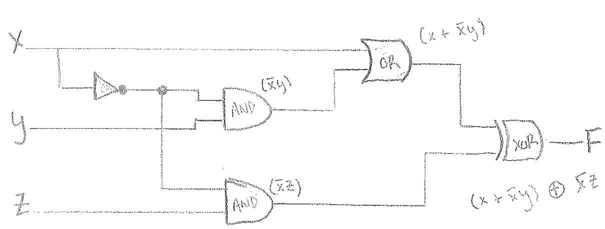
29) Draw the combinational circuit that directly implements the Boolean expression:

$$F(x,y,z) = xz + (xy + \bar{z})$$

→ For n inputs, 2^n possible combinations $\Rightarrow 2^3 = 8$ combinations



32) Find the truth table that describes the circuit:

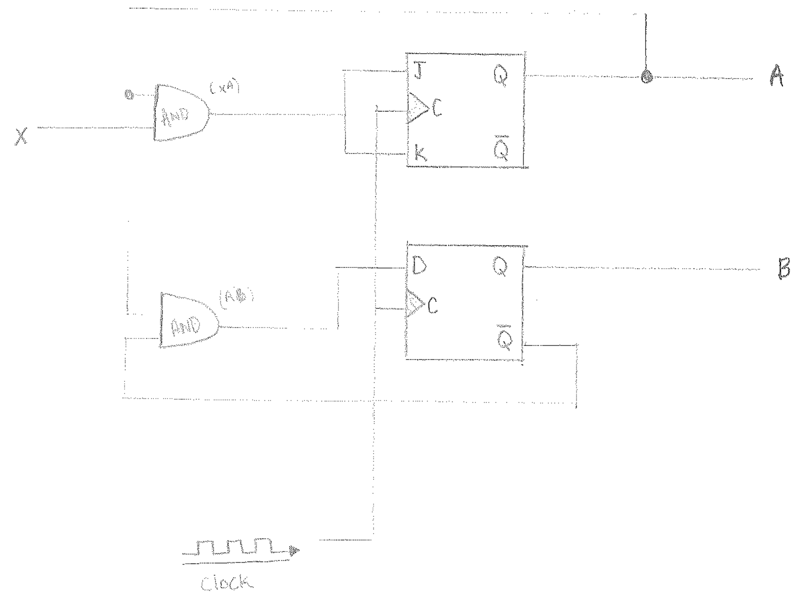


x	y	z	$\bar{x}y$	$\bar{x}z$	$(x+\bar{x}y)$	F
0	0	0	0	0	0	0
0	0	1	0	1	0	1
0	1	0	1	0	1	1
0	1	1	1	1	1	0
1	0	0	0	0	1	1
1	0	1	0	0	1	1
1	1	0	0	0	1	1
1	1	1	0	0	1	1

XOR

x	y	F
0	0	0
0	1	1
1	0	1
1	1	0

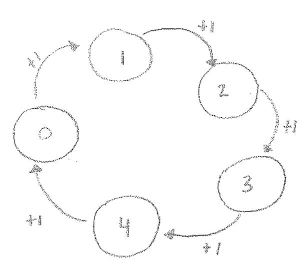
50) Complete the truth table for the sequential circuit:



A	B	X	NEXT STATE A	NEXT STATE B
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	0	0
1	0	0	0	1
1	0	1	1	1
1	1	0	0	0
1	1	1	1	0

59) Construct a Moore machine that counts modulo 5:

→ 5 possible outcomes: 0, 1, 2, 3, 4



0 mod 5 = 0
1 mod 5 = 1
2 mod 5 = 2
3 mod 5 = 3
4 mod 5 = 4

note
5 mod 5 = 0
6 mod 5 = 1
...

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2a) Create the K-maps and then simplify the function:

$F(x,y,z) = \bar{x}\bar{y}\bar{z} + \bar{x}yz + \bar{x}y\bar{z}$

x \ yz	00	01	11	10
0	1	0	1	1
1	0	0	0	0

Group A: 011 + 010 ⇒ $\bar{x}yz + \bar{x}y\bar{z}$
⇒ $\bar{x}y$

Group B: 000 + 010 ⇒ $\bar{x}\bar{y}\bar{z} + \bar{x}y\bar{z}$
⇒ $\bar{x}\bar{z}$

⇒ $A + B = \bar{x}y + \bar{x}\bar{z}$ [distributive]

$= \bar{x}(y + \bar{z})$

3) Write a simplified expression for the Boolean function defined by the following Karnaugh maps...

(A)

wx \ yz	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	0	0	1	0
10	1	0	1	0

Group A : 0000 + 0100 + 0010 + 0110 $\Rightarrow \bar{w}\bar{x}\bar{y}\bar{z} + \bar{w}x\bar{y}\bar{z} + \bar{w}\bar{x}y\bar{z} + \bar{w}xy\bar{z} \Rightarrow \bar{w}\bar{z}$

Group B : 0000 + 1000 $\Rightarrow \bar{w}\bar{x}\bar{y}\bar{z} + w\bar{x}\bar{y}\bar{z} \Rightarrow \bar{x}\bar{y}\bar{z}$

Group C : 1111 + 1011 $\Rightarrow wxyz + w\bar{x}yz \Rightarrow wyz$

$F(w,x,y,z) = \bar{w}\bar{z} + \bar{x}\bar{y}\bar{z} + wyz$

(C)

wx \ yz	00	01	11	10
00		1		1
01		1	1	1
11	1	1		
10	1	1		1

Group A : 1100 + 1000 + 1101 + 1001 $\Rightarrow w\bar{y}$

Group B : 0001 + 0101 + 1101 + 1001 $\Rightarrow \bar{y}z$

Group C : 0111 + 0110 $\Rightarrow \bar{w}xy$

Group D : 0010 + 1010 $\Rightarrow \bar{x}y\bar{z}$

$F(w,y,x,z) = w\bar{y} + \bar{y}z + \bar{w}xy + \bar{x}y\bar{z}$