Homework 1: due January 23rd 11:59PM.

1. [10 points] Create a Matlab function to give a rational approximation of sin(x) with predefined accuracy err (suggest to define function as:

   ```
   function [y,n] = hw1_sine(x,err) //argument err is truncation error
   ```

). Submit your m file to Canvas. In a separate file, show your test result and describe what your algorithm computes.

   Hint: by using the Taylor series, we know that

   $$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!} \qquad (|x| < \infty)$$

   To approximate sin(x) to an accuracy of err we need to find the value n where we should stop the summation. Since the series for sin(x) is an alternating one, we can find a n such that the k^{th} term a_k of the series $a_k <$ err.

2. [10 points] Do Exercises 1.3.1 on page 52 of the textbook.

3. [10 points] Do Exercises 1.3.3 on page 52 of the textbook.

4. [10 points] Implement the following algorithm in Matlab/Octave (the code is from the textbook on page 12).

```
program First
integer i, imax, n ← 30
real error, y, x ← 0.5, h ← 1, emin ← 1
for i = 1 to n
    h ← 0.25h
    y ← [sin(x + h) − sin(x)]/h
    error ← |cos(x) − y|
    output i, h, y, error
    if error < emin then
        emin ← error;   imin ← i
output imin, emin
end program First
```

   Submit your code to Canvas. In a separate file, explain what went wrong with its output (using the theory in Section 1.4). It is not required to correct the code above.