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2-11-20
Automata

Homework 3

1. Assume two regular languages X, Y.
   a. X⊖Y = (X-Y) U (Y-X), and we already know that Regular languages are closed under Union and difference. Hence it will be closed under symmetric difference.
2. If L is a language and x is a symbol, then let L/x be the set of strings w such that wx ∈ L. For example, if L = { 10, 110, 000, 11, 0 }, then L/0 = { 1, 11, 00, ε }. Prove that if L is a regular language, then L/x must be a regular language.
   a. If we recognize L/x as { w | wx∈L }, This fits out definition of a regular language, assuming L is a regular language.
3. Prove that the language L_2 <u>is</u> or <u>is not</u> regular. Let L_2 be the set of all strings { ww | w ∈ $\{0,1\}^*$, |w| > 0 }.
   a. We already know that {w | w ∈ $\{0,1\}^*$ } is a regular language. Since regular languages are closed under concatenation changing w with ww simply concatenates thus we are still dealing with a regular language. And in a similar vein, |w| > 0, simply prevents us from choosing no instances of {0, 1}*. So this means we still have a regular language.
4. Prove that the language L_3 <u>is</u> or <u>is not</u> regular. Let L_3 be the set of all strings { xwwy | x ∈ $\{0,1\}^*$, y ∈ $\{0,1\}^*$, w ∈ $\{0,1\}^*$, |w| > 0 }.
   a. We already know that {w | w ∈ $\{0,1\}^*$ } is a regular language. Since regular languages are closed under concatenation changing w with ww simply concatenates thus we are still dealing with a regular language. And in a similar vein, |w| > 0, simply prevents us from choosing no instances of {0, 1}*. So this means we still have a regular language. And since x and y are both symbols from regular languages, concatenating them does the same thing as adding more w's, but they simple can be null strings, which still allows the language to be regular.