1) x, u: $x = [E\ N\ \Psi\ U_x\ U_y\ r]$ $\quad u = [\delta, \Upsilon_r]$

$|\Psi| \le \Upsilon$

$|\delta| \le \delta_{MAX}$

$\Upsilon_{MIN} \le \Upsilon_r \le \Upsilon_{MAX}$

$\Upsilon_{MAX} = \min(5{,}400\ NM, \mu F_{zr} R_w \cos\alpha_r)$

$\Upsilon_{MIN} = -\mu F_{zr} R_w \cos\alpha_r$

Lifted Variable Eqns:

$$\alpha_f = \tan^{-1}\left(\frac{U_y + ar}{U_x}\right) - \delta \qquad \alpha_r = \tan^{-1}\left(\frac{U_y - br}{U_x}\right)$$

$$F_{zf} = \frac{1}{a+b}(mbg - hF_{xr}) \qquad F_{zr} = \frac{1}{a+b}(mag + hF_{xr}) \qquad F_{xf} = 0$$

$$F_{yf} = \begin{cases} -C_\alpha \tan\alpha_f + \dfrac{C_\alpha^2}{3\mu F_{zf}}|\tan\alpha_f|\tan\alpha_f - \dfrac{C_\alpha^3}{27\mu^2 F_{zf}^2}\tan^3\alpha_f \\ -\mu F_{zf}\,\mathrm{sgn}(\alpha_f) \quad \text{otherwise} \end{cases} \quad \text{if } |\alpha_f| < \tan^{-1}(3\mu F_{zf}/C_\alpha)$$

$$F_{xr} = \Upsilon_r / R_w \qquad \Sigma = \frac{\sqrt{(\mu F_{zr})^2 - F_{xr}^2}}{\mu F_{zr}}$$

$$F_{yr} = \begin{cases} -C_\alpha \tan\alpha_r + \dfrac{C_\alpha^2}{3\Sigma\mu F_{zr}}|\tan\alpha_r|\tan\alpha_r - \dfrac{C_\alpha^3}{27\Sigma^2\mu^2 F_{zr}^2}\tan^3\alpha_r \\ -\Sigma\mu F_{zf}\,\mathrm{sgn}(\alpha_f) \quad \text{otherwise} \end{cases} \quad \text{if } |\alpha_r| < \tan^{-1}(3\Sigma\mu F_{zr}/C_\alpha)$$

$\dot{x} = f(x,u)$:

$$F_{xf}\cos\delta - F_{yf}\sin\delta + F_{xr} = m(\dot{U}_x - rU_y) = m(\ddot{N}\cos\Psi - \ddot{E}\sin\Psi)$$

$$F_{xf}\sin\delta + F_{yf}\cos\delta + F_{yr} = m(\dot{U}_y + rU_x) = m(-\ddot{N}\sin\Psi - \ddot{E}\cos\Psi)$$

$$aF_{yf}\cos\delta + aF_{xf}\sin\delta - bF_{yr} = I_z\dot{r} = I_z\ddot{\Psi}$$