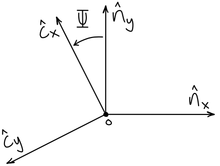
4 Wheel Steering Vehicle Model

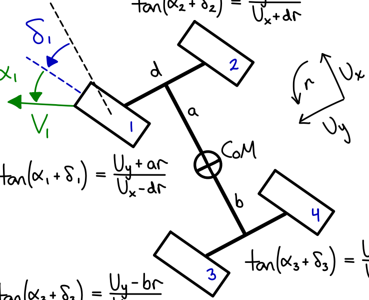
Linear $F_y = C_\alpha \alpha(\delta, U_x, U_y, r)$

Commanded F_x

Static Load



$$\tan(\alpha_2 + \delta_2) = \frac{U_y + ar}{U_x + dr}$$



$$\tan(\alpha_1 + \delta_1) = \frac{U_y + ar}{U_x - dr}$$

$$\tan(\alpha_3 + \delta_3) = \frac{U_y - br}{U_x - dr}$$

$$\tan(\alpha_3 + \delta_3) = \frac{U_y - br}{U_x - dr}$$

$$\sum_i {}^N\vec{F}_i^{\,c} = \frac{{}^N d\, {}^N\vec{p}^{\,c}}{dt} = m\, {}^N\vec{a}^{\,c}$$

$$\sum_{i}^{4} \left(F_{x_i} ¢\delta_i - F_{y_i} \$\delta_i\right)\hat{c}_x + \left(F_{x_i} \$\delta_i + F_{y_i} ¢\delta_i\right)\hat{c}_y = m \frac{{}^N d^2}{dt^2}\vec{r}^{\,c/o}$$

$$= m \frac{{}^N d^2}{dt^2} E\hat{n}_x + N\hat{n}_y = m \frac{{}^N d^2}{dt^2} x\hat{c}_x + y\hat{c}_y$$

$$= m \frac{{}^N d}{dt}\left[\left(\frac{{}^C d}{dt} x\hat{c}_x + y\hat{c}_y\right) + r\hat{c}_z \times \left(x\hat{c}_x + y\hat{c}_y\right)\right]$$

$$= m \frac{{}^N d}{dt}\left[\dot{x}\hat{c}_x + \dot{y}\hat{c}_y + r\left(x\hat{c}_y - y\hat{c}_x\right)\right] = m \frac{{}^N d}{dt}\left(U_x\hat{c}_x + U_y\hat{c}_y\right)$$

OxTS Measurements

$$= m\left[\frac{{}^C d}{dt}\left(U_x\hat{c}_x + U_y\hat{c}_y\right) + r\hat{c}_z \times \left(U_x\hat{c}_x + U_y\hat{c}_y\right)\right]$$

$$= m\left(\left(\dot{U}_x - rU_y\right)\hat{c}_x + \left(\dot{U}_y + rU_x\right)\hat{c}_y\right) = m\left(a_x\hat{c}_x + a_y\hat{c}_y\right)$$

$$\vec{F}_i \cdot \hat{c}_x = F_{x_i} ¢\delta_i - F_{y_i} \$\delta_i \qquad \vec{\tau} = \frac{{}^N d}{dt} \underline{\underline{I}}^{\,c/CoM} \cdot {}^N\vec{\omega}^{\,c}$$

$$\begin{pmatrix} d(-\vec{F}_1 + \vec{F}_2 - \vec{F}_3 + \vec{F}_4)\cdot\hat{c}_x + \\ a(\vec{F}_1 + \vec{F}_2)\cdot\hat{c}_y - b(\vec{F}_3 + \vec{F}_4)\cdot\hat{c}_y \end{pmatrix}\hat{c}_z = \frac{{}^N d}{dt} I_z r\hat{c}_z = I_z \dot{r}\hat{c}_z$$

$$\dot{x} = f(x, a) \qquad \dot{\Psi} = r$$

$$\dot{E} = V¢(\Psi + \beta) = -U_y ¢\Psi - U_x \$\Psi$$

$$\dot{N} = V\$(\Psi + \beta) = U_x ¢\Psi - U_y \$\Psi$$

$$\dot{r} = \frac{1}{I_z}\begin{pmatrix} d(-\vec{F}_1 + \vec{F}_2 - \vec{F}_3 + \vec{F}_4)\cdot\hat{c}_x + \\ a(\vec{F}_1 + \vec{F}_2)\cdot\hat{c}_y - b(\vec{F}_3 + \vec{F}_4)\cdot\hat{c}_y \end{pmatrix} \qquad \begin{matrix} \vec{F}_i \cdot \hat{c}_x = F_{x_i} ¢\delta_i - F_{y_i} \$\delta_i \\ \vec{F}_i \cdot \hat{c}_y = F_{x_i} \$\delta_i + F_{y_i} ¢\delta_i \end{matrix}$$

$$\dot{U}_x = rU_y + \frac{1}{m}\sum_{i}^{4} F_{x_i} ¢\delta_i - F_{y_i} \$\delta_i$$

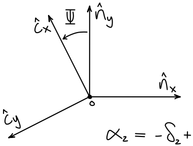
$$\dot{U}_y = -rU_x + \frac{1}{m}\sum_{i}^{4} F_{x_i} \$\delta_i + F_{y_i} ¢\delta_i$$

4 Wheel Steering Linear Vehicle Model

Linear $F_y = C_\alpha \alpha(\delta, U_x, U_y, r)$

$F_x = 0$

Static Load



$$\dot{x} = f(x, a)$$

$$\dot{E} = -U_y ¢\Psi - U_x \$\Psi$$

$$\dot{N} = U_x ¢\Psi - U_y \$\Psi$$

$$\dot{r} = \frac{1}{I_z}\left(d(-\vec{F}_1+\vec{F}_2-\vec{F}_3+\vec{F}_4)\cdot\hat{c}_x + a(\vec{F}_1+\vec{F}_2)\cdot\hat{c}_y - b(\vec{F}_3+\vec{F}_4)\cdot\hat{c}_y \right)$$

$$\dot{\Psi} = r$$

$$\vec{F}_i\cdot\hat{c}_x = F_{x_i} ¢\delta_i - F_{y_i} \$\delta_i$$

$$\vec{F}_i\cdot\hat{c}_y = F_{x_i} \$\delta_i + F_{y_i} ¢\delta_i$$

$$\dot{U}_x = rU_y + \frac{1}{M}\sum_{i}^{4} F_{x_i} ¢\delta_i - F_{y_i} \$\delta_i$$

$$\dot{U}_y = -rU_x + \frac{1}{M}\sum_{i}^{4} F_{x_i} \$\delta_i + F_{y_i} ¢\delta_i$$

local path
small $\Delta\psi$
const U_x
$F_x = 0$
δ small
$\alpha + \delta$ small
$F_y \delta$ small
$dr \ll U_x$
$F_y = -C_\alpha \alpha$

$$\dot{x} = Ax + Bu$$

$$\dot{s} = U_x - (U_y \Delta\psi_0 + U_{y_0}\Delta\psi)$$

Maybe ϕ

$$\dot{e} = U_y + U_x \Delta\psi$$

$$\dot{r} = \frac{1}{I_z}\left(a(F_{y_1}+F_{y_2}) - b(F_{y_3}+F_{y_4}) \right)$$

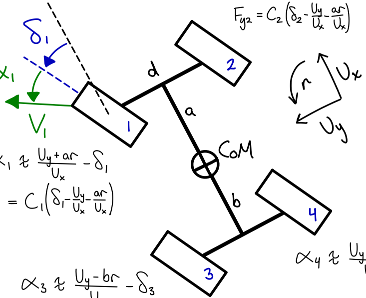
$$\Delta\dot{\psi} = r + KU_x$$

$$\dot{U}_x = 0$$

$$\dot{U}_y = -rU_x + \frac{1}{M}(F_{y_1}+F_{y_2}+F_{y_3}+F_{y_4})$$

$$\alpha_2 = -\delta_2 + \tan^{-1}\left(\frac{U_y+ar}{U_x+dr}\right) \approx \frac{U_y+ar}{U_x} - \delta_2$$

$$F_{y_2} = C_2\left(\delta_2 - \frac{U_y}{U_x} - \frac{ar}{U_x}\right)$$



$$\alpha_1 \approx \frac{U_y+ar}{U_x} - \delta_1$$

$$F_{y_1} = C_1\left(\delta_1 - \frac{U_y}{U_x} - \frac{ar}{U_x}\right)$$

$$\alpha_4 \approx \frac{U_y-br}{U_x} - \delta_4$$

$$\alpha_3 \approx \frac{U_y-br}{U_x} - \delta_3$$

$$F_{y_3} = C_3\left(\delta_3 - \frac{U_y}{U_x} + \frac{br}{U_x}\right)$$

$$\dot{x} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & U_x & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{\Sigma C}{U_x} & Q-U_x \\ 0 & 0 & 0 & Q/I_z & R \end{bmatrix} \begin{bmatrix} s \\ e \\ \Delta\psi \\ U_y \\ r \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ C_1/M & C_2/M & C_3/M & C_4/M \\ aC_1/I_z & aC_2/I_z & -bC_3/I_z & -bC_4/I_z \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{bmatrix} + \begin{bmatrix} U_x \\ 0 \\ KU_x \\ 0 \\ 0 \end{bmatrix}$$

$$R = -(a^2 C_f + b^2 C_r)/U_x I_z$$

$$Q = (C_r b - C_f a)/U_x$$