

# The Theory of Isomorphic Physics

## Part 1: Realigning Relativity and Classical Physics with Quantum Mechanics

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This theory will define an infinite set of states, and posit that a subset is capable of mapping to the universe one to one and onto. For this reason it is the most thoroughly and rigorously testable theory ever devised, limited only by our ability to calibrate it and compute it. This theory will be able to hone down the set via wave equations (or state equations, more generally), but will ultimately leave this as a prediction needing not only physical but also computational testing.

This theory proposes a groundbreaking framework with the potential to predict physical phenomena with unprecedented accuracy, limited only by computational resources. The theory introduces a robust set of equations that, once properly calibrated, could yield predictions for everything from dark matter and dark energy to resolving the vacuum energy catastrophe. The challenge lies in identifying the correctly calibrated subset of these equations—a task demanding significant computational innovation and theoretical insight.

This series of papers will embark on a transformative journey in theoretical physics. The first paper will critically examine and strip away what we perceive as unnecessary assumptions of classical physics. By removing these constraints, we aim to reveal new relationships and principles that align more closely with relativity and quantum mechanics, without the confines of classical interpretations.

The subsequent two papers will focus on defining and detailing a novel equation for a new state that supersedes the traditional roles of wavefunctions and scalar functions in quantum physics. We will explore how these new state equations integrate and interact within a structured matrix group, setting the stage for dynamic models reminiscent of atomic systems.

In the fifth paper, we will introduce the wave equations that directly map to foundational physical concepts such as Minkowski spacetime, the energy-momentum relationship, and the Hamiltonian. This paper will lay out the mathematical derivations and the theoretical underpinnings that connect these new equations to established physical laws.

The concluding papers will delve into the physical ramifications and empirical predictions of the theory. Here, we will discuss how this theory not only fits within the current experimental data but also makes novel predictions that

could be tested with future experiments. These discussions will include potential technological impacts, implications for cosmology and particle physics, and a thorough consideration of criticisms and alternative perspectives from the theoretical physics community.

By systematically developing and substantiating this theory through a series of methodically structured papers, we aim to provide a robust theoretical framework capable of redefining our understanding of the universe. Each step in this series is designed to build upon the last, culminating in a comprehensive and empirically testable theory presented for rigorous peer review and potential publication in top-tier journals.

## 1 Metaphysics

One of the central philosophical ideas of this theory is that the universe came from nothing. In the context of a theory where mathematics are isomorphic to physical reality, this means that these mathematics equate to zero, represented by the "zero equations." These zero equations are Minkowski spacetime (Minkowski spacetime) and the energy-momentum relationship, both of which can be equated to zero by subtracting time or energy. This foundational concept underpins the entire framework of the theory. It also leads into the idea that the universe can be expressed or be isomorphic to a set of scalars.

This paper introduces a novel approach to the fundamental structure of quantum mechanics and relativity. At its core, the theory redefines the state by inherently incorporating spacetime curvature, providing a more natural framework for describing quantum spin and the masses of particles in the standard model. This new state is represented within uniquely defined 4x4 quaternion-valued matrices, forming a group that gives rise to an algebra capable of describing both individual particles and composite systems, such as atoms.

Grounded in fundamental wave equations derived from Minkowski spacetime, the energy-momentum relationship, and aspects of the Schrödinger equation, the framework simplifies many of the classical constructs of physics, offering a more unified understanding of the universe. By grounding this theory in multiple foundational equations—Minkowski spacetime, the energy-momentum relationship, and the Schrödinger equation—we propose a framework that increases descriptive power at the foundational level. Consequently, many classical constructs and aspects of physics are rendered unnecessary, allowing for a more streamlined and unified understanding of the universe's fundamental workings.

In this theory, classical concepts like mass, length, and time, traditionally expressed in dimensional units, are replaced by pure scalar magnitudes that connect directly to the state. The use of quaternion-valued 4x4 matrices, which represent states and eigenfunctions with curvature and intrinsic quantum spins, enables novel dynamic descriptions that are well-suited for quantum physics.

By emphasizing state dynamics, the theory eliminates the need for classical constructs, advocating for a novel approach centered on states, probability distributions, and the mathematical principles of relativity. By adopting this new

mathematical foundation, the theory aims to transcend the limitations inherent in classical approaches, which often relied on the assumptions of dimensional analysis, which I argue to be ultimately incompatible with the more streamlined version of relativity implemented in these papers. Instead, we emphasize the essential nature of the scalar magnitudes of physical quantities, stripping away unnecessary classical constructs to distill physics into its purest mathematical form. This approach unifies relativity, quantum mechanics, and states under a single, streamlined framework.

Our perceptions of length, time and space are abstractions, and so are units. What ultimately matters in a physical theory is that the mappings to experimental data be meaningful and well-described.

## 2 Reconsidering Units and Dimensional Analysis

Classical physics relies heavily on units and dimensional analysis, where quantities like energy, momentum, mass, and time are constrained by physical dimensions (e.g., mass ( $m$ ), length ( $L$ ), or time ( $T$ )). This theory challenges that reliance, instead proposing that the universe's true building blocks are states, along with their resulting probability distributions, eigenfunctions, and associated eigenvalues.

The limitations of dimensional analysis became evident when Minkowski introduced the idea of spacetime as a unified four-dimensional continuum, blending space and time into a single framework. This theory extends that concept further by centering everything on states, where space, time, and mass are fundamentally interconnected components governed by the same dynamics.

By rejecting the constraints of classical units and focusing on the scalar properties inherent to states, this theory provides a clearer, more unified view of the universe. The magnitudes that arise from states—such as energy, momentum, and mass—are the measurable quantities that ultimately lead to our physical measurements using scales, rulers, and clocks, all with the same magnitudes but without the need for units.

This scalar-based framework offers a more coherent way to describe physical reality, where the behavior of states dictates the evolution of the universe, free from the complications that arise from dimensional analysis. Notably, within the state, terms like  $\omega t$  and  $kx$  are already scalar-valued, suggesting that each component can be understood without reference to physical units, supporting the overall argument for a unitless framework.

The main motivation for this is that normally spacetime is treated as an analytical too, but in this formalism, we treat it as a physically manifest, as manifest as time or space. This theory is agnostic as to whether or not  $S$  is a dimension and maintains that with the rejection of dimensional analysis, this agnosticism may not only be permissible but also necessary as dimensions are normally defined in relation to dimensional analysis.

## Equivalence of Magnitudes: Energy, Momentum, and Their Rates of Change

In this framework, we establish the equivalence between the magnitudes of energy ( $E$ ) and angular frequency ( $\omega$ ), and the magnitudes of momentum ( $p$ ) and wavenumber ( $k$ ). This equivalence provides justification for substituting  $E$  for  $\omega$  and  $p$  for  $k$  in the context of a theory that rejects traditional dimensional analysis.

### Energy and Angular Frequency Magnitudes

Energy is traditionally related to angular frequency by Planck's constant:

$$E = \hbar\omega.$$

By taking the absolute value of both sides, we find:

$$|E| = \hbar|\omega|.$$

In this framework, we normalize Planck's constant ( $\hbar = 1$ ) to eliminate the need for a proportionality constant, yielding:

$$|E| = |\omega|.$$

Thus, the absolute magnitudes of  $E$  and  $\omega$  are equivalent, allowing us to use  $E$  as a direct representation of angular frequency within this theory.

### Momentum and Wavenumber Magnitudes

Similarly, momentum is related to wavenumber by Planck's constant:

$$p = \hbar k.$$

Taking the absolute value and normalizing  $\hbar$ , we have:

$$|p| = |k|.$$

This equivalence justifies the substitution of  $p$  for  $k$  when analyzing wavenumber-related dynamics in this framework.

### Implications for the Rejection of Dimensional Analysis

In classical physics, dimensional analysis strictly distinguishes between quantities like  $E$  and  $\omega$  or  $p$  and  $k$  due to their differing units. However, in this theory, we replace the concept of units with a focus on state magnitudes, treating energy, momentum, angular frequency, and wavenumber as scalar quantities that describe the rates of change in their respective components. By establishing that:

$$|E| = |\omega|, \quad |p| = |k|,$$

we remove the need for unit-specific distinctions and instead describe physical phenomena in terms of intrinsic rates.

## Unified Representation of State Dynamics

This equivalence enables a streamlined representation of state dynamics where energy and momentum are treated as intrinsic rates of temporal and spatial change, respectively. For instance:

$$E \leftrightarrow \omega, \quad p \leftrightarrow k,$$

underscoring the unified nature of these quantities within this theoretical framework. By rejecting dimensional analysis and emphasizing scalar magnitudes, we simplify the formalism while preserving its alignment with observable physics.

This reinterpretation aligns with the broader goals of this theory to replace traditional constructs with a category-theoretic and state-centric approach, ensuring compatibility with experimental predictions while discarding the restrictions imposed by classical dimensional analysis.

## 3 Energy, Momentum, and Mass as Rates

In this section, we explore the rates of change through different components of the state, focusing on spatial dimensions, time, and an additional scalar dimension denoted as  $S$ . This scalar component  $S$  appears in the equation for Minkowski Spacetime (Minkowski spacetime) and is treated, in this theory, as an eigenvalue of a scalar function that affects the position in Minkowski spacetime without directly altering wave propagation in the usual spatial dimensions. By incorporating  $S$ , the theory reconciles discrepancies between traditional wave mechanics and the behavior of massive particles.

In this framework, the rate of change of the state with respect to time ( $t$ ) is proportional to energy ( $E$ ), while the rates of change with respect to spatial dimensions ( $x, y, z$ ) correspond to momentum components. Extending this concept, the rate of change of the scalar component  $S$  is proportional to mass ( $m$ ). The inclusion of  $S$  allows for an extended interpretation of mass, incorporating it as an active parameter that affects the system's configuration in Minkowski spacetime.

Unlike energy and momentum, which directly influence state dynamics, the scalar nature of  $m$  and  $S$  ensures that they affect the overall configuration of the system without altering its oscillatory dynamics. This scalar property enables mass to shift the balance between time and space without disrupting the fundamental oscillations of the state.

The framework aligns with the familiar energy-momentum relation:

$$-E^2 + p^2 + m^2 = 0$$

Correspondingly, the Minkowski spacetime relation is expressed as:

$$-|t|^2 + |x|^2 + |y|^2 + z^2 + S^2 = 0$$

where  $\omega$ ,  $k_x$ ,  $k_y$ ,  $k_z$ , and  $m$  ensure that the eigenvalues sum to zero.

The association of mass with the rate of change through the scalar component  $S$  is rooted in the concept of balancing "rise" (through time) and "run" (through space). For instance, a particle moving at the speed of light has zero mass and does not move through  $S$ . In such a scenario, the particle maximizes its rise versus run ratio, emphasizing pure spatial motion. Conversely, a particle with no momentum but with mass has all rise (motion through time) and no run (motion through space). The scalar  $S$  allows for combinations between these two extremes, with mass emphasizing rise while momentum emphasizes run. By definition,  $S$  influences the ratio of rise to run, with mass  $m$  acting as the rate through  $S$ .

The presence of  $S$  as an independent scalar dimension enhances the ability of this theory to align seamlessly with the energy-momentum relationship and Minkowski spacetime. In traditional quantum mechanics, mass is treated primarily as a parameter affecting the curvature of the state, without explicit consideration of a scalar dimension like  $S$ . In contrast, this theory posits that  $S$  actively affects the system's configuration within Minkowski spacetime without directly modifying wave propagation in the usual spatial dimensions.

This additional scalar degree of freedom offers a richer structure for understanding how mass influences the evolution of wave packets. Rather than being a passive parameter, mass, through  $S$ , actively contributes to the geometric configuration of Minkowski spacetime, shaping how wave packets evolve without altering their inherent oscillatory characteristics.

Mapping this theory to the standard quantum mechanical framework involves understanding the role of  $S$ . In standard quantum mechanics, the state is a wavefunction that does not account for  $S$  directly. So by describing a state that maps to the wavefunction but also has an additional scalar component whose position eigenvalue, loosely speaking, yields  $S$ , we can incorporate both  $S$  and its rate, mass, into the state.

In conclusion, the inclusion of the scalar dimension  $S$  provides an expanded understanding of mass as an active participant in the dynamics of Minkowski spacetime, bridging gaps between traditional descriptions of states and their observed behavior in the presence of mass. This framework supports the natural compatibility of the energy-momentum relationship and Minkowski spacetime while offering new insights into the interplay between mass, energy, and momentum.

## 4 Unified Inputs into the State: Space, Time, and Mass

In this theory, we unify space, time, and the scalar component  $S$  as inputs into the state through a single scalar variable, denoted by  $\xi$ . The scalar  $\xi$  encapsulates the fundamental relationship between space, time, and mass, offering a comprehensive interpretation of states as fields that expand in adherence to the Minkowski spacetime interval as  $\xi$  increases.

A central challenge in field-based interpretations is achieving compatibility with discrete relationships like the Minkowski spacetime interval and the energy-momentum relationship in a continuous field framework. This theory addresses this challenge by defining a unified scalar input  $\xi$ , where:

$$\xi = |t^b| = |x^b| = |y^b| = |z^b| = |S^b|,$$

governing the state across all dimensions. Here, space, time, and mass components are expressed as scaled versions of the input  $\xi$ , represented as:

$$\begin{aligned} |E^b|\xi &= |t^\circ|, \\ |p_x^b|\xi &= |x^\circ|, \\ |p_y^b|\xi &= |y^\circ|, \quad |p_z^b|\xi = |z^\circ|, \\ |m^b|\xi &= |S^\circ|, \end{aligned}$$

where  $t'$ ,  $x'$ , and  $S'$  denote the scaled versions of time, space, and the scalar mass component, respectively, determined by the particle's energy, momentum, and mass. By defining inputs as  $-E\xi$ ,  $p_x\xi$ , and  $m\xi$ , we ensure that when  $E$ ,  $p$ , and  $m$  satisfy the energy-momentum relationship, it naturally follows that  $t'$ ,  $x'$ , and  $S'$  satisfy the Minkowski spacetime equation.

To demonstrate how this scaling unifies the Minkowski spacetime interval and the energy-momentum relationship, consider a higher-dimensional Pythagorean-like relationship where each dimension of time, space, and mass contributes as independent components:

$$-t^\circ = -E\xi, \quad x^\circ = p_x\xi, \quad y^\circ = p_y\xi, \quad z^\circ = p_z\xi, \quad S^\circ = m\xi.$$

These scaled components form the basis for the Minkowski spacetime relationship:

$$-(t^\circ)^2 + (x^\circ)^2 + (y^\circ)^2 + (z^\circ)^2 - (S^\circ)^2 = 0,$$

which generalizes the Pythagorean theorem, with each squared component representing an independent contribution. Likewise, energy, momentum, and mass satisfy a similar form:

$$-E^2 + p_x^2 + p_y^2 + p_z^2 + m^2 = 0,$$

indicating an intrinsic compatibility between the Minkowski spacetime interval and the energy-momentum relationship. Scaling  $\xi$  effectively resizes this multidimensional space but preserves the fundamental relationships between its components, allowing continuous fields to adhere to these key physical equations.

In this framework, the total energy ( $E$ ) can be viewed as the hypotenuse in a higher-dimensional space, while momentum components ( $p_x$ ,  $p_y$ ,  $p_z$ ) and mass ( $m$ ) correspond to the “legs” of this multidimensional figure. This generalization

of the Pythagorean theorem aligns space, time, and mass, allowing a cohesive relationship among these elements without compromising continuity.

This model introduces two perspectives on space, represented by  $x$  and  $x^\circ$ . The input  $x$  represents the unscaled spatial coordinate, while  $x^\circ = p \cdot x$  provides a momentum-scaled perspective, where spatial coordinates are modified by the particle's momentum. From this viewpoint,  $x'$  appears larger than  $x$  in the momentum frame, analogous to length contraction in relativity. Here, the underlying space remains unchanged, but measurements differ depending on perspective.

To further refine localization within this field-based approach, a shift parameter  $\tau_\mu$  localizes position eigenfunctions by shifting their starting points in space and time. This shift enables precise localization, with the probability of collapsing to a particular point defined by the state's amplitude at the transformed coordinates.

In summary, the unified scalar input  $\xi$  serves as the foundation for understanding the state as a field encompassing all dimensions, integrating space, time, and mass. By interpreting these elements as scaled versions of  $\xi$ , this theory aligns with the Minkowski spacetime and energy-momentum relationships, offering an elegant, multidimensional representation of how states evolve and interact across spacetime.

## 5 Interpreting Position and Momentum as Complementary Perspectives

Building on the unified scalar input  $\xi$ , we further explore how space and momentum emerge as complementary perspectives within this framework. With  $\xi$  serving as a foundational input for time, space, and mass components, we obtain a cohesive model where spatial and momentum-scaled coordinates,  $x$  and  $x^\circ$ , represent two interconnected ways of observing the same underlying reality.

In conventional quantum mechanics, a state's spatial component is often observed directly as the position eigenvalue. However, by associating  $\xi$  with both the unscaled coordinate  $x$  and the scaled version  $x^\circ = p_x \xi$ , we identify  $x$  as an intrinsic spatial parameter and  $x'$  as its momentum-scaled counterpart. This introduces a duality where:

1. **\*\*Momentum as Contraction\*\***: In the wavefunction's exponential terms (e.g.,  $e^{ikx}$ ), higher momentum values contract spatial cycles, resulting in increased frequency. This contraction effect reflects the quantum behavior of higher momentum leading to tighter spatial cycles.
2. **\*\*Momentum as Dilation\*\***: On a spatial level, momentum scales  $x$  to  $x^\circ$ , yielding a spatial coordinate that grows with  $p_x$ . From this perspective, momentum "dilates" the spatial view, scaling the effective space through the particle's momentum.

Thus, we interpret  $x$  and  $x^\circ$  as distinct yet interdependent views of space, with  $x'$  incorporating the influence of momentum. When measuring the position



in an experiment, the measured value aligns with  $x^\circ$ , though it is perceived as  $x$  due to this underlying relationship.

In this way, mass acts as a sort of "filler" taking up some portion of the total energy without contributing meaningfully to the wave dynamics.

## 5.1 Mapping to Experimental Observations

This dual perspective allows the theory to remain compatible with experimental observations, where the measured position reflects the momentum's scaling effect. The momentum-scaling aligns with the notion that spatial cycles are modified by  $p_x$  and measured coordinates reflect a momentum-scaled view. Hence, while  $x$  represents the coordinate in its intrinsic form,  $x^\circ$  aligns with the observed position, incorporating momentum's influence as part of the spatial measurement.

## 5.2 Underlying Intuition: Contraction and Dilation of Space and Time

Metaphorically, if all particles moved at a constant rate, contracting or dilating space or time would make particles appear to move faster or slower, even though they inherently move at the same rate. More literally,  $\xi$  corresponds to this underlying constant rate of motion, while the scaling factors  $E$ ,  $p$ , and  $m$  represent contractions or dilations of space, time, or spacetime. These scaling factors create the observed effect of particles moving at different rates through time, space, or spacetime, despite their constant foundational rate as governed by  $\xi$ .

In relativity, spatial measurements—like the length of a ruler—can warp or contract due to the relative dilation or contraction of space and time. The analog to this "ruler" in our framework is one cycle of the wavefunction, or more generally, one cycle of the state. In a basic perspective where  $p_x = 1$ , one cycle occurs when  $x$  grows by  $2\pi$ , corresponding to the standard wavefunction behavior. However, in a perspective where  $p_x = 5$ , the wavefunction completes five cycles over the same interval of  $x = 2\pi$ , creating an apparent contraction of the cycle length by a factor of  $1/5$ . Thus, from the  $p_x = 5$  perspective, each cycle—our effective "ruler"—has a contracted length relative to the  $p_x = 1$  perspective.

Moreover, this framework naturally introduces a superposition of perspectives on the contraction of space, as the wavefunction embodies multiple such states. These overlapping perspectives interfere constructively or destructively, producing the observed oscillatory behavior of quantum states and contributing to the complex structure of quantum probability distributions. In this way, these varying perspectives have quant

An intuition for time is that it is radial and collapses information upon the given perspective like the light from many stars collapsing to the eye of an observer their information.

### 5.3 Unified Relationship with Minkowski Spacetime and Energy-Momentum Equations

Evaluating the unified scalar input at a given value of  $\xi$  allows both the Minkowski spacetime and energy-momentum equations to hold. For example, when  $\xi = 1$  and values such as  $E = 5$ ,  $p_x = 3$ , and  $m = 4$  are used, we find that  $t^\circ = 5$ ,  $x^\circ = 3$ , and  $|S|^\circ = 4$ , satisfying both the energy-momentum relationship:

$$-|E|^2 + |p_x|^2 + |p_y|^2 + |p_z|^2 + |m|^2 = 0,$$

and the Minkowski spacetime relationship:

$$-|t^\circ|^2 + |x^\circ|^2 + |y^\circ|^2 + |z^\circ|^2 - |S^\circ|^2 = 0.$$

By treating  $\xi$  as the continuous input governing the relationships between space, time, and mass, we allow both  $x$  and  $x'$  perspectives to coexist. This ensures that each relationship holds across different dimensions, unifying the framework with both relativity and quantum mechanics.

## 6 Three-Layer System for Space and Time Dilation/Contraction

In this section, we describe the three-layer system of contraction and dilation of space and time, focusing on the scaling by energy, momentum, and mass, the curvature inherent in the states themselves, and its connection to relativistic dynamics.

The first layer explains why different dimensions evolve at varying rates. The evolution of time and space is driven by the scaling effects of energy ( $E$ ), momentum ( $p$ ), and mass ( $m$ ). This scaling determines how the inputs—time ( $t$ ), space ( $x$ ), and the scalar eigenvalue ( $S$ )—evolve according to these quantities. Specifically:

- Time Scaling (by Energy): The time coordinate is scaled by energy, leading to the transformation  $t^\circ = E\xi$ , ensuring that when energy satisfies the energy-momentum relationship, the scaled time  $t^\circ$  also satisfies the Minkowski spacetime interval (Minkowski spacetime).
- Space Scaling (by Momentum): The spatial dimension is scaled by momentum, with the transformation  $x^\circ = p\xi$ . This ensures that when momentum is related through the energy-momentum relationship, the scaled spatial coordinate  $x'$  also satisfies the Minkowski spacetime equation.
- Scalar Component Scaling (by Mass): The scalar eigenvalue  $S$  is scaled by mass, with the transformation  $S' = m\xi$ . When mass  $m = 0$ , there is no motion through the scalar component  $S$ , resulting in a light-like interval.

The second layer introduces additional curvature inherent in the state itself, in particular, in a state that is an analog to the position eigenfunction. In this layer, the output is not simply a scaled version of the input; rather, it undergoes a further transformation that results in a slightly adjusted output due to the intrinsic curvature properties of the state. This will be detailed later but X'

approximately equates to  $X'$ , meaning this second layer of curvature, is small as it has to do with a macro perspective on quantum forces. This layer accounts for deviations from the expected magnitudes after the first scaling. We denote this transformation by adding a second prime to the output, such that:

- Curvature Due to State Properties: The transformation at this layer takes the form  $x' = \bar{\psi}(x')$ , where  $\bar{\psi}$  represents the curvature inherent in the state. This curvature affects the magnitudes and ensures that the state correctly reflects the physical curvature present in space and time.

The third layer represents the familiar relativistic effects described by the Lorentz transformation. This layer describes how time and space are contracted or dilated as seen by a moving observer, based on the velocities or rates from the first two layers. For example, when energy equals momentum ( $E = p$ ), the mass term becomes zero, resulting in a light-like interval—mirroring the behavior of massless particles, such as photons, in relativity.

The Lorentz transformation introduces contraction or dilation as seen by a moving observer:

$$t' = \gamma \left( t^\circ - \frac{vx^\circ}{c^2} \right), \quad x' = \gamma (x^\circ - vt^\circ),$$

where  $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$  is the Lorentz factor, and  $v$  is the relative velocity between observers.

The three-layer system integrates quantum and relativistic perspectives:

- The first layer provides the initial scaling of dimensions by energy, momentum, and mass.
- The second layer introduces the intrinsic curvature of the state, resulting in slight deviations from the initial scaling.
- The third layer applies the relativistic contraction and dilation to these dimensions as seen by a moving observer.

-This theory complicates the fundamentals such that the higher level concepts are simplified. This theory describes the universe entirely in terms of states and the probability distributions and position, energy, momentum or mass magnitudes (eigenvalues) they yield. Therefore, in order to map to the universe, it is unavoidable that the description of these states are going to be complicated and technical. However, the equations we will ultimately arrive at that map to the energy momentum relationship, the equation for Minkowski spacetime and the Hamiltonian will all be exceedingly succinct and eloquent. So I ask the reader to bear through the highly technical and challenging first second, third, and fourth papers as they will be rewarded with the eloquent equations in the fifth paper.