

# The Theory of Isomorphic Physics

## Part 2: Primitive states

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### Primitive Simple States

This theory proposes that states are the most fundamental constituents of the universe. Unlike standard definitions, it introduces **primitive states** (denoted by a prime,  $\psi'$ ) and **primitive position states** (denoted by an underline and a prime,  $\underline{\psi}'$ ). These primitive states serve as foundational building blocks, more fundamental than particles, as they can be combined through multiplication and division to form **composite states**. A subset of these composite states, theoretically, maps to known particles.

### Primitive Simple States

The **primitive state**  $\psi'$  serves as the fundamental building block for constructing the more general state  $\psi$  (without a prime). Instead of a single scalar expression, the primitive state  $\psi'(|\xi|)$  is composed of components corresponding to each index  $\mu = 0, 1, 2, 3, 4$ , defined as:

$$\psi'(|\xi|) = (\psi'_{\mu=0}, \psi'_{\mu=1}, \psi'_{\mu=2}, \psi'_{\mu=3}, \psi'_{\mu=4})$$

where each component  $\psi'_{\mu}$  is given by:

$$\psi'_{\mu} = \left( \zeta + \frac{\eta}{\lambda + |\xi_{\mu}|} \right)^{\theta(\lambda + |\xi_{\mu}|)}.$$

This notation highlights that the primitive state consists of five distinct components, each indexed by  $\mu$  and governed by the parameters  $\zeta$ ,  $\eta$ , and  $\theta$ .

### Parameters Governing the Primitive State

- \*\* $\zeta$ ,  $\eta$ , and  $\theta$ \*\*  
These parameters determine the properties of each component and take specific values (generally 1, -1,  $i$ , or  $-i$ ) as described in Table 1. Unique combinations of these parameters give rise to distinct physical characteristics

for each component. - **\*\*λ\*\***: This parameter controls the curvature of the state across spacetime components and is defined by:

$$\lambda = \sigma \xi^\nu$$

where: -  $\sigma$  is the **curvature magnitude** parameter. Larger values of  $\sigma$  correspond to lower curvature, while smaller values yield increased curvature. -  $\nu$  dictates how the curvature scales with respect to the scalar input  $\xi$ , shaping the behavior of the state as a function of position.

## Physical Interpretation of Primitive States

Primitive states, defined by  $\psi'$ , form the basic building blocks of **composite states**. They are foundational entities within the theory, conceptually similar to elementary particles in the Standard Model, though a direct correspondence with known particles remains speculative. By varying  $\zeta$ ,  $\eta$ , and  $\theta$ , primitive states can exhibit a broad range of behaviors that mimic characteristics of fundamental particles, including rotation rates and distinct magnitudes.

The behavior of primitive states is closely related to the mathematical structure of  $e$ , as evidenced by their resemblance to the limit:

$$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$$

## Parameter Combinations and Physical Properties

There are 40 unique configurations of primitive states, each defined by distinct combinations of  $\zeta$ ,  $\eta$ , and  $\theta$ . These combinations yield varied properties, such as rate of rotation, curvature type, magnitude scale, zero-point orientation, and peak offset, as outlined in Table 1. The influence of each parameter on these properties is crucial for understanding the diversity of physical phenomena these states can represent.

## The Properties of Primitive States

Below is an abridged version that gives the notation for the given primitive state, its zeta, eta, and theta values, and the resulting properties.

### rate of rotation

The rate of rotation, denoted by *rateofrotation*, is defined as:

$$\text{rate of rotation} = \frac{\text{Cycles}}{\Delta |X_\mu^b|} \Big|_{|P_\mu^b|=1}$$

The standard wavefunction,  $e^{ikx}$ , also exhibits this property. For instance,  $e^{ikx}$  completes one cycle as  $kx$  increases by  $2\pi$ . Therefore, the rate of rotation

Notation	$\zeta$	$\eta$	$\theta$	rate of rotation	Curvature Type	Magnitude Scale	Zero-Point Orientation	Peak Offset
$\overleftarrow{\psi}(-A)\odot$	$i$	$i$	1	-4	$-A$	$e$	1	0
$\overrightarrow{\psi}(A)\odot$	$i$	$-i$	-1	4	$A$	$e$	1	0
$\overrightarrow{\psi}(A)\wedge$	-1	$-i$	1	2	$A$	1	1	$1/e$
$\overleftarrow{\psi}(-B)\vee$	$i$	1	1	-4	$-B$	1	1	$-1/e$
$\overrightarrow{\psi}(F)\wedge$	1	1	$i$	0	$F$	1	1	$1/e$
$\overleftarrow{\psi}(-C)\odot$	1	-1	1	0	$-C$	$1/e$	1	0

Table 1: Abridged table of primitive states, showing some representative properties. For full details, refer to Appendix ??.

for the standard wavefunction is  $2\pi$ , and this rotation is right-handed by default. To reverse the handedness, the sign of  $k$  must be flipped.

In contrast, this version of the state allows for multiple rates of rotation, defined by  $rateofrotation = 0, \pm 2, \pm 4$ . These correspond to: -  $rateofrotation = 0$  for scalar-valued states that do not rotate. -  $rateofrotation = 2$  for a right-handed state completing one cycle as  $|X_\mu^b|$  grows by 2. -  $rateofrotation = -2$  for the left-handed version of the same rotation. -  $rateofrotation = \pm 4$  for a right- or left-handed state completing one cycle per increase of 4 in  $|X_\mu^b|$ .

This modified state is naturally "two-handed," meaning that you can flip the sign of the momentum (or the direction the wave will travel) without needing to flip the sign of  $\xi$ . In essence, this two-handedness allows for a richer representation of physical phenomena, and though it is not itself quantum spin, it will later be connected closely to it.

The exact combination of  $\zeta$ ,  $\eta$ , and  $\theta$  that results in these varying rotation rates is influenced primarily by  $\zeta$  and  $\theta$  (while  $\eta$  does not affect the rate). Specifically: - If we have  $1^X$ , this will not rotate as a function of  $X$ . -  $i^X$  will rotate in a left-handed way as  $X$  grows, while  $-i^X$  will rotate in a right-handed way. -  $\theta$  flips the sign of the exponent, which flips the handedness. For instance,  $(-1)^X$  will rotate in a left-handed way as  $X$  grows, and flipping the sign of  $\theta$  will flip the handedness to right-handed.

Notably, many elements in the table differ from others only in the sign of  $rateofrotation$ . This suggests that these states naturally describe conjugated versions of  $\psi$ , as the negative- $rateofrotation$  version can be interpreted as the conjugate of the positive- $rateofrotation$  counterpart.

## Flat Notation for Limiting States

The **\*\*flat notation\*\*** is used to denote a state that has been "flattened" by taking a parameter, such as  $\lambda$ , to an exceedingly high value. We define the flattened version of a state, denoted as  $\psi_\alpha^b$ , as follows:

$$\psi_\alpha^b = \lim_{\lambda \rightarrow \infty} \psi_\alpha$$

The subscript  $\alpha$  is optional here; it serves to emphasize that  $\psi_\alpha^b$  is related to  $\psi_\alpha$ , with the only difference being the value of  $\lambda$ . In contexts where it is clear which state is being flattened, the subscript may be omitted for simplicity.

In this theory, the flat symbol (<sup>b</sup>) represents a conceptual approach where the parameter  $\lambda$  is taken to an exceedingly high value, approaching what might be thought of as an "infinite" value in mathematical terms. However, it is important to note that this language is somewhat figurative. Since this theory attempts to describe mathematics that are isomorphic to physical reality,  $\lambda$  is treated as having a definite, albeit extremely high, physical value.

Thus, the notation  $\lim_{\lambda \rightarrow \infty}$  is intended to communicate that the parameter  $\lambda$  is exceedingly large—sufficiently large to yield a negligible curvature in the context of the state. In physical terms,  $\lambda$  is not truly infinite but is treated as if it were for practical purposes of analysis, allowing  $\psi_\alpha^b$  to approximate a "flat" state with minimal curvature or deviation.

The **flat notation** is therefore an effective shorthand to indicate the concept of taking  $\lambda$  to a value where the state's behavior becomes effectively constant or "flat." This simplifies discussions about limiting behaviors and highlights the relationship between the unmodified state and its flattened version.

## Category-Theoretic Representation of Related States

This theory frequently involves states that differ only by a specific transformation or modifier, such as a limit, curvature, or position magnitude. To represent these relationships efficiently, we use a category-theoretic formalism.

**Objects and Morphisms**

We define a **category**  $\mathcal{S}$ , where:

- **Objects** are **states**, denoted by symbols like  $\psi_\alpha$ .
- **Morphisms** are **transformations** applied to states, such as:
  - **Flattening morphism** (<sup>b</sup>), which takes the limit  $\lambda \rightarrow \infty$ .
  - **Position magnitude morphism** (<sub>p</sub>), introducing position-related properties.

The subscript  $\alpha$  denotes that states are related, sharing the same underlying properties (e.g., energy, momentum, mass), but differing by the applied **morphism**.

**Morphisms and State Transformations**

Consider related states  $\psi_\alpha$  and  $\psi_\alpha^b$ :

- $\psi_\alpha$ : the original state.
- $\psi_\alpha^b$ : the **flattened** version of  $\psi_\alpha$ , where the **flattening morphism** <sup>b</sup> has been applied.

The expression:

$$\Phi = \psi_\alpha^b - \psi_\alpha$$

represents the **difference** between the two related states, corresponding to their **curvature** or **deviation**.

**Category-Theoretic Representation**

In the category  $\mathcal{S}$ :

- **Objects**: labeled by  $\psi_\alpha$ ,  $\psi_\alpha^b$ , etc.
- **Morphisms**: denote **transformations** applied to states, such as  $b : \psi_\alpha \rightarrow \psi_\alpha^b$  and (<sub>p</sub>) (position magnitude).

The **subscript  $\alpha$**  indicates the states belong to the same **family** but differ by specific morphisms.

Example and General Case

For a general transformation  $f : \psi_\alpha \rightarrow f(\psi_\alpha)$ :

-  $\psi_\alpha$  is the \*\*original state\*\*. -  $f(\psi_\alpha)$  is the \*\*transformed state\*\*.

## Magnitude Scale

The magnitude scale of a state refers to its amplitude. The subscript notation—such as  $\odot$  and  $\oslash$ —indicates the scale associated with a given state:

-  $\odot$ : Magnitude scale of  $e$ . -  $\oslash$ : Magnitude scale of  $e^{-1}$ . - No subscript: Magnitude scale of 1.

These scales can be described mathematically as follows:

$$|\psi'_{\odot}| = e$$

$$|\psi'| = 1$$

$$|\psi'_{\oslash}| = e^{-1}$$

The magnitude scale depends on the parameters  $\zeta$ ,  $\eta$ , and  $\theta$  of the state. Below are examples illustrating the different magnitude scales:

Scale  $e$  For a magnitude scale of  $e$ , consider  $\zeta = i$ ,  $\eta = i$ , and  $\theta = 1$ . The state can be expressed as:

$$\psi'(|X|) \approx \left[ i \left( 1 + \frac{1}{\lambda} \right) \right]^{\lambda+|X|} = i^{\lambda+|X|} \left( 1 + \frac{1}{\lambda} \right)^{\lambda+|X|}$$

As  $\lambda + |X|$  approaches infinity, the expression resembles the definition of  $e$ , resulting in a magnitude proportional to  $e$ .

Scale 1 For a magnitude scale of 1, let  $\zeta = i$ ,  $\eta = 1$ , and  $\theta = 1$ . The state becomes:

$$\psi'(|X|) = \left( i + \frac{1}{\lambda + |X|} \right)^{\lambda+|X|}$$

In the limit as  $\lambda + |X|$  approaches infinity, the term involving the fraction vanishes, resulting in:

$$\psi'(|X|) \approx i^{|X|}$$

Since  $i^{|X|}$  has a constant magnitude of 1, the state exhibits a magnitude scale of 1.

Scale  $e^{-1}$  For a magnitude scale of  $e^{-1}$ , consider  $\zeta = \eta = i$  and  $\theta = -1$ . The negative exponent effectively inverts the scale, leading to:

$$\psi'(|X|) = \left( i + \frac{i}{\lambda + |X|} \right)^{-(\lambda+|X|)}$$

This results in an overall magnitude scale of  $e^{-1}$ .

Summary of Magnitude Scales - **Scale  $e$** : Occurs when  $\zeta = \eta = i, \theta = 1$ .  
- **Scale 1**: Occurs when  $\zeta = i, \eta = 1, \theta = 1$ . - **Scale  $e^{-1}$** : Occurs when  $\zeta = \eta = i, \theta = -1$ .

These magnitude scales can manifest with different rates of rotation. However, the physical relevance of these scales remains unclear, and later sections will discuss how these properties might be canceled or normalized.

## Zero-Point Orientation

The zero-point orientation of a state describes the value of the state at its first peak nearest to zero. For the primitive states discussed in this paper, this value is consistently 1. However, for other forms of primitive states, the zero-point orientation may vary, resulting in different initial values at the first peak. This distinction is significant in describing the initial configuration and behavior of various states.

Again, physical experiments don't seem to have an analog for this, so later sections will explore how to keep this a constant at 1 in all cases.

The standard wavefunction has this property and its zero-point orientation is 1.

## Peak Offset

For primitive states with a non-zero rate of rotation, the term "peak offset" refers to the location of the state's first peak. In some cases, this peak occurs at  $\xi = 0$ , whereas in others, it may occur at  $\xi = \frac{1}{e}$  or  $\xi = -\frac{1}{e}$ . The presence of peak offset is directly influenced by the rate of rotation and other parameters that define the state.

In primitive states with a rate of rotation equal to zero, the concept of peak offset becomes more abstract. Here, the state is not scalar-valued but rather complex-valued, and it does not rotate through the complex plane. Despite this, a connection to peak offset remains, as evidenced by the behavior of composite states. Specifically, if we divide a state with a peak offset of  $\frac{1}{e}$  and a rate of rotation of 1 by a scalar-valued state with a peak offset of  $\frac{1}{e}$  and a rate of rotation of 0, the resulting state will not exhibit a peak offset, and its first peak will be centered at  $\xi = 0$ .

This abstract treatment of peak offset underscores the interplay between rotation rates and the initial configuration of states, providing deeper insight into how these fundamental building blocks behave under different circumstances.

Once again, physical experiments don't seem to have an analog for this, so later sections will explore how to keep this a constant at 1 in all cases.

The standard wavefunction has this property and its peak offset is 0.

## Primitive Position States

Primitive position states are analogous to position eigenfunctions. Unlike traditional eigenfunctions, however, they do not require a position operator to determine their magnitude (which serves as an analog to an eigenvalue). Instead, these states intrinsically possess a magnitude that is theoretically proportional to the position. This formalism presents an alternative approach to quantum mechanics, one that does not rely on operator formalism. The aim is to provide a more direct representation of physical properties, avoiding the abstraction of operators, which can be challenging to define physically.

### Primitive Position States with Rotation

The first type of primitive position state is defined for states that rotate through the complex plane. This is achieved by combining two primitive states, each having the same functional form but differing in the sign of their spatial component ( $|X_\mu^b|$ ). The mathematical expression for such a primitive position state  $\underline{\psi}'(|X_\mu^b|)$  is given by:

$$\underline{\psi}'(|X_\mu^b|) = \Lambda^2 \left( \left( Z + \frac{H}{\Lambda + |X_\mu^b|} \right)^{\Theta(\Lambda + |X_\mu^b|)} - \left( -Z - \frac{H}{\Lambda - |X_\mu^b|} \right)^{\Theta(\Lambda - |X_\mu^b|)} \right)$$

This construction subtracts two components of the primitive state, with one component having  $|X_\mu^b|$  added and the other having  $|X_\mu^b|$  subtracted. The subtraction yields a very small remainder, which is then scaled by  $\Lambda^2$  to restore the state to a normal scale (e.g.,  $e$ ,  $1$ , or  $e^{-1}$ ).

**\*\*Synchronizing the Phases:\*\*** - Since the two components have opposite signs for  $|X_\mu^b|$ , they naturally rotate in opposite directions within the complex plane. To meaningfully subtract one from the other, their phases need to be synchronized. - To achieve synchronization, both  $\zeta$  and  $\eta$  are negated in one of the components. This ensures that the two components rotate through the complex plane in a synchronized manner, despite their opposite directions of rotation. - Alternatively, one could negate  $\theta$ , but this approach involves more complexity, and the various options for phase synchronization will be addressed in greater detail in a later section.

By using  $\Lambda^2$  as a scaling factor, the resulting state is brought back to a magnitude on the scale of  $e$ ,  $1$ , or  $e^{-1}$ . As the two components diverge over time, the magnitude of their difference grows, resulting in an object that behaves analogously to a position eigenfunction.

The limit of this construction can be expressed as:

$$\lim_{\Lambda \rightarrow \infty} \left( \left( 1 + \frac{1}{\Lambda + |X_\mu^b|} \right)^{(\Lambda + |X_\mu^b|)} - \left( 1 + \frac{1}{\Lambda - |X_\mu^b|} \right)^{(\Lambda - |X_\mu^b|)} \right) = \frac{e|X_\mu^b|}{\Lambda^2}$$

The magnitude of the difference is proportional to the scale of the primitive state, multiplied by  $|X_\mu^b|$ , and divided by  $\Lambda^2$ . Multiplying by  $\Lambda^2$  brings the state back to a scale of  $e$ , 1, or  $e^{-1}$ , resulting in a state that grows approximately at a rate equal to its magnitude scale multiplied by the input  $|X_\mu^b|$ .

## Primitive Position States without Rotation

The second type of primitive position state is defined for states that do not rotate through the complex plane, i.e., they have a rotation rate of zero. These states are represented as:

$$\underline{\bar{\psi}}'(|X_\mu^b|) = \bigotimes_{\mu=0}^4 \Lambda^2 \left( \left( Z' + \frac{H'}{\Lambda + |X_\mu^b|} \right)^{\Theta(\Lambda + |X_\mu^b|)} - \left( Z' + \frac{H'}{\Lambda - |X_\mu^b|} \right)^{\Theta(\Lambda - |X_\mu^b|)} \right)$$

For these non-rotating states, there is no need to synchronize the phases of the components, as they do not rotate. Consequently, there is no requirement to negate  $\zeta$ ,  $\eta$ , or  $\theta$ . The subtraction of the two components in this case is straightforward, as their phases are already aligned.

**\*\*Behavior and Scaling:\*\*** - In these states, the magnitude grows as a direct function of the input  $|X_\mu^b|$ , and there is no oscillatory behavior in the complex plane. - The output magnitude is directly proportional to the input, as shown in the case of zero curvature and scale 1:

$$\left| \overline{\psi}_\vee^b \right| = \Xi$$

In the case of primitive position states with a non-zero rate of rotation, the behavior is similar to that of position eigenfunctions, where the state grows outward in a manner akin to  $x\psi$ . Thus, primitive position states naturally describe dynamic behavior similar to that of position eigenfunctions, without the need for operators that impart eigenvalue multipliers.

## Complications with $\Lambda$

In order for the primitive position states to exhibit coherent behavior, the parameter  $\Lambda$  must take on specific values, especially for states with a non-zero rate of rotation. Specifically,  $\Lambda$  must be an integer multiple of 2. If  $\Lambda$  is allowed to take continuous values while  $|X_\mu^b|$  is also continuous, the state may become misaligned, resulting in phases that are not synchronized for each term in the position state. Below, we present several potential solutions to address this issue:

1. **\*\*Quantization of  $\Lambda$  and  $|X_\mu^b|$ \*\***: This approach requires both  $\Lambda$  and  $|X_\mu^b|$  to be integer-valued. By quantizing  $\xi$  into sufficiently small units, even for particles with high energy, the product  $P \cdot \xi$  remains very small. This ensures



that the steps in the state are sufficiently small to approximate continuous rotation through the complex plane, preserving coherence.

2. **\*\*Rounding  $\Lambda$  to the Nearest Integer\*\***: A ceiling function can be applied to round  $\Lambda$  to the nearest integer. While this introduces minor discontinuities, it ensures that  $\Lambda$  is appropriately quantized, thereby maintaining overall coherence in the state.

3. **\*\*Using Non-Rotating Primitive Position States\*\***: Only primitive position states with non-zero rates of rotation require  $\Lambda$  to be an integer multiple of 2. As an alternative, the non-rotating version of the primitive position states can be used, taking the limit as  $\lambda \rightarrow \infty$ . This results in states without curvature, and the curvature types of other primitive states can be utilized instead.

4. **\*\*Applying a Ceiling Function to  $\Lambda$ \*\***: To further ensure coherence, a ceiling function can be applied, rounding  $\Lambda$  up to the nearest integer multiple of 2. This avoids potential desynchronization by ensuring consistent alignment of the oscillatory components.

5. **\*\*Defining a Rotation Function to Align Peaks\*\***: For cases where  $\Lambda$  is not an integer multiple of 2, a rotation function can be defined to synchronize the phases of the state components, ensuring that  $\psi(|X_\mu^b|)$  always peaks at the correct values. This approach is explained in more detail below.

Rotation Function to Correct  $\Lambda$

When  $\Lambda$  is not an integer multiple of 2, the state components associated with  $\pm|X_\mu^b|$  may become desynchronized, resulting in a misalignment of peak values. To correct this, a phase rotation function is defined that shifts the zero-point of the state to align with a peak. The phase adjustment is given by:

$$e^{i\pi(\Lambda - \lfloor \Lambda/2 \rfloor \times 2)}$$

This rotation adjusts the state by an amount proportional to the deviation of  $\Lambda$  from an integer multiple of 2. For instance, if  $\Lambda = 101$ , the phase shift becomes:

$$e^{i\pi(101-100)} = e^{i\pi}$$

which corresponds to a half-cycle shift. Consequently, the general state is adjusted as follows:

$$\psi'(|X_\mu^b|) \rightarrow e^{i\pi(\Lambda - \lfloor \Lambda/2 \rfloor \times 2)} \psi'(|X_\mu^b|)$$

This phase correction ensures that the zero-point of the state is aligned with a peak, even when  $\Lambda$  is not an integer multiple of 2. By applying this adjustment, the coherence of the oscillatory behavior of the state is preserved, preventing desynchronization and maintaining the intended physical representation.

These corrections to  $\Lambda$  are crucial for ensuring that the primitive position states exhibit consistent and coherent oscillatory behavior, which is fundamental for accurately modeling physical phenomena within the framework of this theory.

## The Properties of Primitive Position States

Notation	$Z$	$H$	$\Theta$	rate of rotation ( <i>rateofrotation</i> )	Curvature Type	Magnitude Scale	Zero-Point Orientation	Peak Offset
$\frac{\psi}{\psi}(-a)\odot$	$i$	$i$	1	-4	$-a$	$e$	1	0
$\frac{\psi}{\psi}(a)-1\odot$	$i$	$-i$	-1	4	$a$	$e$	-1	0
$\frac{\psi}{\psi}(-a)1\odot$	1	1	1	0	$-a$	$e$	1	0
$\frac{\psi}{\psi}(-g)i\vee$	1	1	$-i$	0	$-g$	1	$i$	$-1/e$
$\frac{\psi}{\psi}(b)\wedge$	1	$i$	1	0	$b$	1	-1	$1/e$

Table 2: Abridged table showing representative properties of a given set of  $Z$ ,  $H$ , and  $\Theta$ . For the complete table, refer to Appendix ??.

## The Position Magnitude Property

Every primitive position state possesses a "position magnitude property" by definition. This means that a primitive position state intrinsically yields position magnitudes without the need for an operator or additional multiplication by position variables. This behavior is key to the formalism of primitive position states, which allows us to avoid an operator-based approach by treating position as an inherent property that can be transferred, acquired, or canceled through interaction.

## Space-Time Curvature

Since primitive position states have position magnitudes, we define space-time curvature in terms of these position magnitudes:

$$\Phi = \frac{\psi}{\psi}_{\alpha}^b - \frac{\psi}{\psi}_{\alpha}$$

This definition suggests that space-time curvature could map to the curvature described in general relativity, potentially offering a pathway towards unifying quantum mechanics with general relativity.

## Magnitude Scale

The magnitude scale of primitive position states refers to the scale of their position magnitudes. For consistency, we assume that all magnitude scales used will have a value of 1 to ensure that the position magnitude is on the correct scale. Primitive states with non-unit magnitude scales will interact with other primitive states such that the scales effectively cancel. For example, a primitive state with magnitude scale  $e$  might interact with another state of magnitude  $e^{-1}$ , or be divided by a state with magnitude  $e$ , resulting in a magnitude scale of 1.

## Zero-Point Orientation

The zero-point orientation for primitive position states varies and can take values of 1,  $i$ ,  $-1$ , or  $-i$ , which means that the first peak (the first location with a purely  $i$ ,  $-i$ , 1, or  $-1$  value) can have different initial values, as indicated in the table.

## Peak Offset

The peak offset for primitive position states is defined similarly to primitive states. If we divide a primitive position state by the input  $\xi$ , then the resulting state's peak offset property remains the same.

We often find it necessary to work with scalars that have a position magnitude. Peak offset can cause these to be non-scalars, so like magnitude scale, we will assume