The Theory of Isomorphic Physics

Part 2: The Natural Wavefunction

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The Natural Wavefunction and Set Notation

We can multiply or divide primed wavefunctions ψ' to form a more general, unprimed version, which we will call the natural wavefunction:

$$\psi = \prod_{i=1}^{n} (\psi_i')^{a_i} \cdot \prod_{j=1}^{m} \left(\underline{\psi}_j'\right)^{b_j}$$

where a_i and b_j take values of 1, 0, or -1 for multiplication, exclusion, or division respectively.

We define the set of all wavefunctions with the properties of interest as:

$$\mathcal{S}(\text{properties}) = \{\psi \mid \text{specified properties of } \psi\}$$

For example, a specific wavefunction with a left-handed rate of rotation of $1/2$, a curvature type of $-a$, and a magnitude scale of e can be written as:

$$\underset{\sim}{\overset{\leftarrow}{\psi}}{}'_{(-a)\odot} \in \mathcal{S}\left(\frac{1}{2}, -a, e\right)$$

This signifies that the wavefunction $\underset{\sim}{\overset{\leftarrow}{\psi}}{}'_{(-a)\odot}$ belongs to the set of wavefunctions with those specific properties. Note that this does not refer to a unique wavefunction but rather to any member of the infinite set of wavefunctions that share these characteristics.

Arbitrary Natural Wavefunctions

When using the unprimed notation, ψ, we refer to an arbitrary element from the set $\mathcal{S}(\text{properties})$. For instance:

$$\underset{\sim}{\overset{\leftarrow}{\psi}}{}_{(-a)\odot} \in \mathcal{S}\left(\frac{1}{2}, -a, e\right)$$

This refers to any wavefunction with those specified properties but without any further restriction on its construction. This allows for flexibility in discussing the natural wavefunction while keeping the relevant properties explicit.

Transformation of Flat to Curved Space-Time with $\overline{\underline{\psi}}$

In this theory, we introduce $\overline{\underline{\psi}}$ as a wavefunction that transforms inputs from uncurved (flat) space-time into curved space-time. The function $\overline{\underline{\psi}}$ represents the behavior of physical systems under curvature, such that it provides the correct output even when curvature is present.

The general form of this transformation is given by:

$$\overline{\underline{\psi}} : \mathbb{R}^4 \to \mathcal{M}$$

where:

- $\mathbb{R}^4$ represents flat space-time with coordinates $(t^\flat, x^\flat, y^\flat, z^\flat)$.
- $\mathcal{M}$ represents curved space-time, where the wavefunction $\overline{\underline{\psi}}$ takes into account the effects of curvature due to forces, energy, and mass.

Mapping of Components

To further clarify, each of the flat space-time components is mapped to the corresponding curved-space component through $\overline{\underline{\psi}}$. This mapping relates the absolute values of the space-time coordinates and the curvature-adjusted outputs of $\overline{\underline{\psi}}$ as follows:

$$|t| = \overline{\underline{\psi}}_0$$
$$|x| = \overline{\underline{\psi}}_1$$
$$|y| = \overline{\underline{\psi}}_2$$
$$|z| = \overline{\underline{\psi}}_3$$
$$|S| = \overline{\underline{\psi}}_r$$

These equations show how the flat space-time coordinates $(t^\flat, x^\flat, y^\flat, z^\flat)$ and the scalar S (representing space-time curvature) are transformed through $\overline{\underline{\psi}}$, which outputs their curved counterparts. This transformation accounts for potential curvature in both space and time dimensions.

Behavior of Eigenvalues

When the wavefunction is uncurved but underlined (i.e., it has an eigenvalue), the output will match the input. In other words, the eigenvalues of the uncurved ψ correspond exactly to the values of the flat space-time coordinates $_\mu$. However, when curvature is introduced through $\overline{\underline{\psi}}$, the eigenvalues will still be correct, but they will reflect the curvature of space-time. This ensures that the outputs of the wavefunction remain consistent with the inputs, even when curvature is present.

To express this more formally:

$\overline{\underline{\psi}}(_\mu) =_\mu$ (if uncurved, where $_\mu$ is the input and output eigenvalue)

This relationship holds as long as no curvature is introduced. When curvature is present, the eigenvalues change according to the transformation $\overline{\underline{\psi}}$.

General Formulation of Ψ

To capture both flat and curved scenarios, we define Ψ as follows:

$$\Psi = \psi^\flat \Big|_{\mu \to \overline{\underline{\psi}}}$$

In this expression:

- $\psi^\flat$ is the wavefunction defined in flat space-time.
- $\overline{\underline{\psi}}$ is the transformation that takes flat space-time inputs and outputs curved space-time, modifying the space-time structure as needed.
- μ represents the flat space-time coordinates $(t^\flat, x^\flat, y^\flat, z^\flat, S^\flat)$.

This formulation ensures that the wavefunction retains the correct eigenvalues even in the presence of curvature, with the transformation handled by $\overline{\underline{\psi}}$.

Role of Φ

In cases where curvature is introduced, the wavefunction Ψ can be modified by the introduction of potential energy through Φ. As a simplified model, we treat Φ as the wavefunction ψ where the scaling parameter $P_\mu^\flat$ has been set to 1:

$$\Psi = \psi^\flat \Big|_{\mu \to |P_\mu^\flat| \overline{\underline{\psi}}}$$

This expression shows that the presence of potential energy alters the wavefunction in a manner proportional to $P_\mu^\flat$, but the curvature remains properly scaled within the system.

In particular:

$$\Phi = \psi^\flat \Big|_{P_\mu^\flat \to 1}$$

Here, the curvature term Φ becomes a simple wavefunction with normalized parameters, simplifying the treatment of potential energy.

Interpretation of $\overline{\underline{\psi}}$

The wavefunction $\overline{\underline{\psi}}$ not only transforms flat space-time into curved space-time but also ensures that the correct eigenvalues for position, energy, momentum, and mass are maintained in both cases. This allows us to use flat space-time as a reference, while accommodating the complexities of curved space-time when forces, energy, and mass are introduced.

By ensuring that eigenvalues match the expected outputs for both flat and curved scenarios, $\overline{\underline{\psi}}$ bridges the gap between classical and quantum systems, providing a coherent framework for understanding how curvature affects the wavefunction and its resulting eigenvalues.

The Properties of a Result of Multiplication

The Eigen-property under Multiplication

We define the set $\mathcal{P}$ as the set of all wavefunctions with a **non-multiplicative position eigenvalue**. That is, if $\underline{\psi} \in \mathcal{P}$, then $\underline{\psi}$ has a position eigenvalue without the need for multiplication by x or any additional terms.

Now, if we take two wavefunctions, $\underline{\tilde{\psi}}$ and $\underline{\overline{\psi}}$, both belonging to $\mathcal{P}$, and divide one by the other, the result will no longer have a position eigenvalue:

$$\underline{\tilde{\psi}}/\underline{\overline{\psi}} = \tilde{\psi}$$

Here, $\tilde{\psi} \notin \mathcal{P}$, as dividing two wavefunctions with position eigenvalues results in the loss of the position eigenvalue property.

If we multiply two elements from the set $\mathcal{P}$, which contains wavefunctions with first-order position eigenvalues, the resulting wavefunction no longer belongs to $\mathcal{P}$ because it now has a higher-order eigenvalue, for example:

$$\underline{\psi} \cdot \underline{\psi} \notin \mathcal{P}$$

where $\mathcal{P}$ is defined as:

$$\mathcal{P} = \{\psi \mid \text{has a linear (first-order) position eigenvalue}\}$$

By definition, elements of $\mathcal{P}$ have only first-order eigenvalues (e.g., corresponding to x). When we multiply an element of $\mathcal{P}$ by itself, the result has a second-order eigenvalue (e.g., x^2), and therefore, it no longer belongs to $\mathcal{P}$.

However, if we multiply an eigenfunction by a non-eigen-primitive that does not have a position eigenvalue, the result still belongs to $\mathcal{P}$. Therefore:

$$\underline{\psi} \in \{\psi \mid \text{has a non-multiplicative, first-order position eigenvalue}\}$$

Rate of Rotation under Multiplication

When we multiply two wavefunctions, their rates of rotation (denoted by κ) are additive. Below are three examples that illustrate this principle:

1. **Multiplying two wavefunctions with $\kappa = -1/2$:** Two elements from the set of wavefunctions with a rate of rotation $\kappa = -1/2$ multiply to form a wavefunction with $\kappa = -1$:

$$\overleftarrow{\psi} \cdot \overleftarrow{\psi} = \overleftarrow{\psi}$$

2. **Multiplying a wavefunction with $\kappa = 1$ by a wavefunction with no rotation:** An element from the set of wavefunctions with $\kappa = 1$ multiplies an element from the set with no rotation ($\kappa = 0$) to form a wavefunction that also belongs to the set with $\kappa = 1$:

$$\overrightarrow{\psi} \cdot \overline{\psi} = \overrightarrow{\psi}$$

3. **Multiplying a wavefunction with $\kappa = 1/2$ and the eigen-property by a wavefunction with no rotation:** An element from the set of wavefunctions with $\kappa = 1/2$ and the eigen-property multiplies an element with no rotation to form a wavefunction that belongs to the set with $\kappa = 0$ and still retains the eigen-property. This example illustrates that one property (e.g., rate of rotation) does not affect another (e.g., the eigen-property):

$$\underline{\overrightarrow{\psi}} \cdot \underline{\overleftarrow{\psi}} = \underline{\overline{\psi}}$$

Curvature Type under Multiplication

The potential, indicated with a subscript, approximately adds under multiplication, a behavior linked to concepts in perturbation theory and logarithmic approximations. The smaller the deviation from an ideal value like 1 or e, the closer the approximation becomes to exact. On the cosmic scale used in this theory, where quantum forces are considered on the scale of galaxies, this approximation is nearly exact:

$$\psi_a \cdot \psi_b \approx \psi_{a+b} \tag{1}$$

Curvature as Deviation

Curvature can be understood as a deviation from an ideal value, such as 1 or e. Small deviations compound multiplicatively in a manner resembling addition for sufficiently small values, a phenomenon well-known in perturbation theory.

For example, consider a wavefunction with magnitude 0.99. Its curvature Φ can be defined as:

$$\Phi_1 = 1 - 0.99 = 0.01$$

Multiplying two such wavefunctions results in:

$$\psi_1 \cdot \psi_2 = 0.99 \cdot 0.99 = 0.9801$$

The resulting curvature is:

$$\Phi_{\text{result}} = 1 - 0.9801 = 0.0199$$

Thus, the curvatures approximately add:

$$\Phi_1 + \Phi_2 \approx 0.02$$

Improved Approximation for Smaller Deviations

This approximation becomes more accurate as the curvature decreases. For instance, for a wavefunction with magnitude 0.999, we get:

$$\psi_1 \cdot \psi_2 = 0.999 \cdot 0.999 = 0.998001$$

The resulting curvature is:

$$\Phi_{\text{result}} = 1 - 0.998001 = 0.001999$$

which is much closer to exact addition.

Curvature Additivity on a Cosmic Scale

On a cosmic scale, where quantum forces are small, the approximation becomes nearly exact:

$$\lim_{\Phi \to 0} (1 - \Phi_1)(1 - \Phi_2) \approx 1 - (\Phi_1 + \Phi_2)$$

Thus, small curvatures add linearly under multiplication, resembling first-order perturbation theory.

Implications for Physical Systems

For systems on a cosmic scale, where deviations from 1 or e are minimal, additive curvature under multiplication becomes highly accurate. This simplifies the modeling of systems with many interacting wavefunctions.

$$\psi_a / \psi_a = \psi$$

Magnitude Scale under Multiplication

The magnitude scale is multiplicative under multiplication, meaning the resulting scale of two primitives is the product of their respective scales.

If we divide an element from the subset of natural wavefunctions with magnitude e by an element of the subset of wavefunctions with magnitude of $\mathrm{e}^{-}1, the result belongs to the subset with magnitude 1. \psi_{\oslash}\psi_{\odot} = \psi$If we divide a element from the subset of elements with magnitude $\mathrm{e}^{-}1 by a element from the same subset, the result belongs to the ψIf we divide a element from the subset of elements with magnitude e by an element from the same subset, the result belongs to the subset with magnitude 1.

$$\psi_{\odot}\psi_{\odot} = \psi$$

Since magnitude scale, in this case, applies applies to position eigenvalues, normalization cannot simply eliminate these variations in magnitude. It becomes essential to maintain the correct magnitude scale for wavefunctions. Specifically, we require the wavefunction to have a scale factor of 1, via the cancelation of other scale factors due to such multiplication or division.

Zero-Point Orientation under Multiplication

Zero-point orientation is also multiplicative. For example, if an element of the subset with zero-point orientation i multiplies an element from the subset with

zero-point orientation of -i, then the result belongs to the subset with zero point orientation of 1:

$$\psi_i \psi_- i = \psi$$

Peak Offset under Multiplication

For instance, dividing a primitive with a non-zero rate of rotation and a peak offset of $\frac{1}{e}$ by a primitive with no rate of rotation and the same peak offset will yield a function with no peak offset. This suggests that while peak offset is abstract for primitives with zero rotation, the property remains mathematically consistent.

In general, peak offset cancels under division and adds under multiplication.

Uppercase Ψ

With the language and notation now established, we can describe the wavefunction Ψ, which serves to describe particles and, theoretically, map to them. The wavefunction Ψ is given by:

$$\Psi = \psi_\mu \cdot \phi_\mu$$

where ψ_μ and ϕ_μ are plane waves. Ψ is thus a plane wave that can be normalized, and superpositions can be formed using normalization constants. ψ_μ and ϕ_μ are defined as follows:

$$\psi_\mu = \begin{cases} \overleftrightarrow{\psi}_\mu & \text{for } \mu = 0, 1, 2, 3 \\ \overline{\psi}_4 & \text{for } \mu = 4 \end{cases}$$

This definition implies that for $\mu = 0, 1, 2, 3$, ψ_μ belongs to the subset of wavefunctions with $\kappa = \pm 1$, representing the most standard form of the wavefunction. The right-left arrow notation, $\overleftrightarrow{\psi}$, indicates that the wavefunction can have either right- or left-handed rates of rotation.

In constructing Ψ, there are two approaches for handling momentum direction: 1. We can allow ρ (momentum direction) to be positive or negative and restrict ψ to only right-handed rotation. 2. Alternatively, we can allow ψ to have both right- and left-handed rotation while restricting ρ to positive values only.

This choice depends on how we wish to interpret the sign of momentum. Since ϕ_μ will later be defined to control the handedness that determines the sign of momentum, the above equation allows ψ_μ to have either handedness while constraining ρ to positive values.

For $\mu = 4$, ψ_4 is a plane wave, where ψ_4 corresponds to an S eigenvalue (related to spacetime curvature) in its eigenform and to a mass eigenvalue in its rate-eigenvalue form. Since S and mass (m) do not affect the wavefunction's rate of rotation or dynamics, we assume these are scalar-valued eigenfunctions, represented by overlined ψ functions, behaving similarly to scalar wavefunctions.

$$\phi_\mu = \begin{cases} \tilde{\phi}_\mu & \text{for } \mu = 0, 1, 2, 3 \\ \overline{\phi}_4 & \text{for } \mu = 4 \end{cases}$$

The ϕ_μ functions belong to the subset with $\rho = 1$, meaning they have quantized rates of rotation on the complex plane, defined by the κ of the given ϕ_μ. While ϕ_4 is included for symmetry with ψ_4, its necessity is debatable. Since ϕ_μ can have both eigenfunctions and eigenvalues, they are associated with position eigenvalues where $\rho = 1$, meaning the position grows at a default rate of 1.

Abstract Interpretation of S and Mass

If we define S (spacetime curvature) and mass more abstractly as eigenvalues without associated rotation rates, we can make some interesting observations. The wavefunction $\overset{\leftrightarrow}{\phi}$, representing momentum, may be interpreted as massless. Meanwhile, $\overset{\rightarrow}{\phi}$, having partial rotation, could represent an object that possesses both mass and momentum, as half of its eigenvalue is associated with rotation and the other half remains unrepresented.

Finally, $\overline{\phi}$ would represent pure mass, while $\overset{\rightarrow}{\phi}$ would exist between the pure momentum (represented by $\overset{\leftrightarrow}{\phi}$) and the pure mass extremes (represented by $\overline{\phi}$). Importantly, while these three wavefunctions differ in their influence on the probability distribution, they equally affect the eigenvalues associated with their respective properties.

Spin Angular Momentum

In this formalism, spin angular momentum arises naturally from the wavefunction ϕ_μ, where the intrinsic quantized rates of rotation $\kappa = 0, \pm 2, \pm 4$ correspond to the spin states. Unlike in standard quantum mechanics, where operators act on wavefunctions to yield eigenvalues, here the wavefunctions themselves possess intrinsic eigenvalues.

The wavefunction ϕ_μ can be interpreted as having quantized angular momentum directly proportional to its κ. For example, when measuring the spin angular momentum along μ_1, the wavefunction collapses into specific momentum eigenstates:

$$L_1 \implies \phi_{\mu=2} \to \overset{\rightarrow}{\phi}_2, \quad \phi_{\mu=3} \to \overset{\rightarrow}{\phi}_3$$

The quantized spin states are reflected in the rotation rates of ϕ_μ, which have default values of $\kappa = 0, \pm 2, \pm 4$. These values determine the intrinsic spin of the system.

Since the magnitude of Planck's constant $\hbar$ maps to a wavefunction with $\rho = 1$, this formalism suggests that ϕ_μ is analogous to the quantum mechanical spin wavefunction, but with eigenvalues intrinsic to the wavefunction itself. Thus, the spin angular momentum is directly related to the rotation rate of ϕ_μ.

Orbital Angular Momentum

Orbital angular momentum is encoded in the wavefunction ψ_μ. In this formalism, ψ_μ also possesses intrinsic angular momentum, but unlike ϕ_μ, which has fixed quantized rates of rotation (κ), ψ_μ allows for more flexible values of ρ, leading to larger angular momentum values.

For example, ψ_μ may have an angular momentum eigenvalue that corresponds to longer orbits and larger magnitudes of momentum. This flexibility arises because ρ is not restricted to 1, allowing the wavefunction to describe a broader range of angular momentum values. Orbital angular momentum in this context is similar to that in traditional quantum mechanics, but it arises directly from the wavefunction without the need for an operator to extract the eigenvalue.

Total Angular Momentum

The total angular momentum is the sum of the spin and orbital angular momentum. In this theory, the total wavefunction $\Psi = \psi_\mu \cdot \phi_\mu$ combines the contributions from both ψ_μ and ϕ_μ. Since the wavefunctions are multiplied, their intrinsic angular momenta (encoded in their respective rotation rates κ) add to give the total angular momentum.

The total angular momentum is thus given by the sum of the spin contribution (from ϕ_μ) and the orbital contribution (from ψ_μ):

$$\kappa_{\text{total}} = \kappa_\psi + \kappa_\phi$$

Since both ψ_μ and ϕ_μ can have different signs for their rotation rates, the total angular momentum can either increase or decrease depending on the relative handedness of the two wavefunctions. This naturally leads to a broad range of possible total angular momentum values, all derived from the intrinsic properties of the wavefunctions.

Thus, the total angular momentum of a system is determined by the interplay between the spin and orbital components of the wavefunction, and these contributions are additive in this formalism. The resulting Ψ represents a complete picture of the angular momentum, with no need for external operators.

Pauli Exclusion Principle (PEP)

In this theory, the Pauli Exclusion Principle (PEP) is naturally enforced by the structure of the wavefunction Ψ. Specifically, if fermions are in a bound state, their wavefunctions must form a standing wave. This occurs because the ψ components of Ψ, representing the angular momentum, must have equal and opposite values. This results in the momenta of ψ_1 and ψ_2 summing to zero:

$$\overrightarrow{\psi}_1 + \overleftarrow{\psi}_2 = \psi_{\text{standing wave}}.$$

The standing wave condition imposes that the two wavefunctions, ψ_1 and ψ_2, must be anti-aligned in terms of their angular momentum. Only in this case can their sum form a standing wave with zero total angular momentum.

Now, since $\Psi = \psi_\mu \cdot \phi_\mu$, this standing wave condition also applies to the ϕ components, which are responsible for encoding spin. The ϕ components must also have equal and opposite values in order to enforce the PEP. Specifically,

$$\overrightarrow{\phi}_1 + \overleftarrow{\phi}_2 = \phi_{\text{standing wave}}.$$

In this context, the ϕ components represent the intrinsic spin of the fermions. Since ϕ is responsible for the angular momentum quantization in the form of spin, this condition ensures that the two particles must have opposite spins in a bound state. Thus, in order for two fermions to form a standing wave, they must have opposite spin (i.e., one must be spin-up and the other spin-down), which directly enforces the PEP.

In contrast, bosons, which can but do not have to form standing waves, are not subject to the Pauli Exclusion Principle. The condition $\overrightarrow{\psi}_1 + \overleftarrow{\psi}_2 = \psi_{\text{standing wave}}$ does not restrict them, allowing them to occupy the same quantum state.

Therefore, this formulation not only leads to a natural description of the PEP for fermions but also explains why bosons are exempt from the exclusion principle.

Symbol	Set-Theoretic Definition
ψ	Formed by multiplying/dividing Ψ' and $\underline{\Psi}'$, and potentially adding the results.
$\tilde{\psi}$	$\tilde{\psi} \in \{\psi \mid \text{rate of rotation (Cycles per } \Delta\xi_\mu = 2) \in \{0, \pm\frac{1}{2}, \pm 1\}\}$
$\overline{\psi}$	$\overline{\psi} \in \{\psi \mid \text{rate of rotation (Cycles per } \Delta\xi_\mu = 2) = 0\}$
$\vec{\psi}$	$\vec{\psi} \in \{\psi \mid \text{rate of rotation (Cycles per } \Delta\xi_\mu = 2) = +\frac{1}{2}\}$
$\overleftarrow{\psi}$	$\overleftarrow{\psi} \in \{\psi \mid \text{rate of rotation (Cycles per } \Delta\xi_\mu = 2) = -\frac{1}{2}\}$
$\overrightarrow{\psi}$	$\overrightarrow{\psi} \in \{\psi \mid \text{rate of rotation (Cycles per } \Delta\xi_\mu = 2) = +1\}$
$\overleftarrow{\psi}$	$\overleftarrow{\psi} \in \{\psi \mid \text{rate of rotation (Cycles per } \Delta\xi_\mu = 2) = -1\}$
$\overleftrightarrow{\psi}$	$\overleftrightarrow{\psi} \in \{\psi \mid \text{rate of rotation (Cycles per } \Delta\xi_\mu = 2) = \pm 1\}$
$\overleftrightarrow{\psi}$	$\overleftrightarrow{\psi} \in \{\psi \mid \text{rate of rotation (Cycles per } \Delta\xi_\mu = 2) = \pm\frac{1}{2}\}$
$\psi^\flat$	$\psi^\flat \in \{\psi \mid \text{has an exceedingly high value of } \lambda \text{ or } \Lambda \text{ such that curvature approaches } 0\}$
Ψ_p	$\Psi_p \in \{\psi \mid \text{represents the wavefunction of particle } p \text{ with unknown parameters}\}$
ϕ	$\phi \in \{\psi \mid \rho_\mu = 1 \text{ for all } \mu\}$
$\underline{\psi}$	$\underline{\psi} \in \{\psi \mid \text{has a non-multiplicative position eigenvalue}\}$
$\underline{\underline{\psi}}$	$\underline{\underline{\psi}} \in \{\psi \mid \underline{\psi}/\overline{\phi}\}$
$\psi_{(U_1,U_2,\dots)}$	$\psi_{(U_1,U_2,\dots)} \in \{\psi \mid \text{has curvature properties denoted1} U_1, U_2, \dots\}$

Table 1: Definitions of various subsets of ψ using set theory.