The Theory of IsomorPhic Physics
Part 3: The Composite state

John Henke

November 24, 2024

[12pt]article amsmath, amssymb geometry hyperref graphicx authblk booktabs

margin=1in

Mapping Unitless Scalars to Physical Units in [Your Theory's Name] [Your Name]

1 Bridging Scalar Formalism and Classical Physics via Dimensionless Quantities

In our theoretical framework, we posit that the universe fundamentally operates on scalars without inherent units. Units such as length, time, and mass are constructs introduced through measurement tools like rulers, clocks, and scales. This perspective suggests that physical quantities possess magnitudes, and it is through our measurement systems that we assign units to these magnitudes.

Despite this philosophical stance, there exists common ground between our scalar formalism and classical physics, particularly through dimensionless quantities. Both theories acknowledge that products like xk_x and ωt are dimensionless scalars. Here, x represents position, k_x is the wavenumber, ω is the angular frequency, and t denotes time. The units of x and k_x (meters and inverse meters, respectively) cancel out in the product xk_x, rendering it dimensionless. Similarly, ωt is dimensionless due to the inverse seconds of ω and the seconds of t.

1.1 Arbitrary Scaling and the Significance of Dimensionless Products

In our theory, we focus on the value of the dimensionless products xk_x and ωt rather than the individual values of x, k_x, ω, or t. This focus allows us to scale k_x arbitrarily as long as we inversely scale x to maintain the same value of xk_x. The same principle applies to ω and t in the product ωt.

Mathematically, if we scale k_x by a factor of λ, we scale x by λ^{-1}:

$$k_x' = \lambda k_x, \quad x' = \frac{x}{\lambda}, \quad \text{such that} \quad x' k_x' = x k_x.$$

This proportional scaling demonstrates that the individual magnitudes of x and k_x are not uniquely determined in our theory; only their product xk_x holds physical significance. By treating x and k_x as magnitudes without fixed units, we maintain the scalar nature of our formalism while ensuring consistency with observable phenomena.

1.2 Mapping Scalars to Physical Reality

The dimensionless products xk_x and ωt provide a bridge between our unitless formalism and classical physics. In classical wave mechanics, these products appear in the phase of wave functions:

$$\Psi(x,t) = Ae^{i_c(k_x x - \omega t)},$$

where A is the amplitude and i_c is the imaginary unit. The exponent $i_c(k_x x - \omega t)$ is dimensionless, ensuring that $\Psi(x,t)$ is a well-defined mathematical function. By focusing on these dimensionless quantities, we can align our theoretical constructs with measurable physical phenomena without assigning specific units to x, k_x, ω, or t.

This approach allows us to map our theory's scalar values to physical reality for quantities such as position, time, wavenumber, and angular frequency. While we avoid directly mapping to classical terms like momentum p and energy E through relations like $p = \hbar k_x$ and $E = \hbar\omega$ (since these involve Planck's constant $\hbar$ and introduce units), we acknowledge that changes in k_x and ω influence the behavior of physical systems in ways that are consistent with observations.

1.3 Consistency with Classical Physics

By agreeing on the dimensionless nature of xk_x and ωt, our theory and classical physics share a common foundation. This agreement allows for:

- **Theoretical Coherence**: Our theory remains internally consistent, adhering to the principle that the universe operates on scalars without units.
- **Experimental Alignment**: Dimensionless products correspond to observable quantities, enabling comparisons between theoretical predictions and experimental data.
- **Flexibility in Scaling**: The arbitrary scaling of x and k_x (or ω and t) in our theory does not affect the physical predictions, as long as their products remain constant.

1.4 Implications for Other Physical Quantities

While we focus on dimensionless products, it is essential to consider how other physical quantities relate within our framework:

- **Momentum and Energy**: In classical physics, momentum and energy are linked to k_x and ω through Planck's constant. In our theory, we avoid this direct mapping to prevent conflicts with our definitions of mass and spacetime rates. Instead, we can explore how variations in k_x and ω affect system dynamics without invoking units.
- **Mass and Spacetime Rates**: By defining mass as a rate through spacetime $-S$, we maintain a scalar characterization consistent with our philosophical stance.

1.5 Conclusion

Our focus on dimensionless, scalar quantities like xk_x and ωt offers a viable path to integrate our unitless formalism with classical physics. This approach allows us to:

- Maintain the theoretical integrity of our scalar-based framework.
- Provide a method to relate our theoretical constructs to physical measurements.
- Bridge the gap between our philosophy of a unitless universe and the practical necessity of units in experimental physics.

By leveraging the shared recognition of dimensionless quantities, we ensure that our theory remains both philosophically consistent and empirically relevant.

Abstract

In this paper, we establish mappings between the unitless quantities in our theoretical framework and physical units using fundamental constants. We focus on quantities such as energy, momentum, position, wave number, angular frequency, and force, providing explicit conversion factors that allow us to interpret our theoretical predictions in terms of measurable physical quantities. This mapping enhances the applicability of our theory and facilitates comparisons with experimental observations.

2 Introduction

Our theoretical framework operates with unitless scalars, yet physical measurements involve quantities with units. By establishing relationships between our unitless quantities and fundamental physical constants like Planck's constant ($\hbar$) and the speed of light (c), we can define mappings that assign physical units to our scalars. This approach enables us to interpret our theoretical predictions in terms of measurable physical quantities and aligns our framework with established physical laws.

3 Mapping Unitless Quantities to Physical Units

3.1 Overview of the Mapping

In our theoretical framework, we've established mappings between unitless quantities and physical units using fundamental constants like Planck's constant $\hbar$ and the speed of light c. The key relationships and mappings are:

3.1.1 Momentum Mapping

$$p_{\mathrm{phys}} = M_p \cdot p = \frac{\hbar}{\pi} \cdot p. \tag{1}$$

3.1.2 Wave Number Mapping

$$k_{\mathrm{phys}} = M_k \cdot k = \frac{1}{\pi} \cdot k. \tag{2}$$

3.1.3 Energy Mapping

$$E_{\mathrm{phys}} = M_E \cdot E = \frac{\hbar c}{\pi} \cdot E. \tag{3}$$

3.1.4 Angular Frequency Mapping

$$\omega_{\mathrm{phys}} = M_\omega \cdot \omega = \frac{c}{\pi} \cdot \omega. \tag{4}$$

These mappings are derived from the relationships in physics:

$$E_{\mathrm{phys}} = \hbar \omega_{\mathrm{phys}}, \tag{5}$$
$$p_{\mathrm{phys}} = \hbar k_{\mathrm{phys}}, \tag{6}$$
$$E_{\mathrm{phys}} = p_{\mathrm{phys}} c, \tag{7}$$
$$\omega_{\mathrm{phys}} = c k_{\mathrm{phys}}. \tag{8}$$

3.2 Mapping Constants

For clarity, we summarize the mapping constants:

- **Momentum Mapping Constant** M_p:

$$M_p = \frac{\hbar}{\pi}. \tag{9}$$

- **Wave Number Mapping Constant** M_k:

$$M_k = \frac{1}{\pi}. \tag{10}$$

- **Energy Mapping Constant** M_E:

$$M_E = \frac{\hbar c}{\pi}. \tag{11}$$

- **Angular Frequency Mapping Constant** M_ω:

$$M_\omega = \frac{c}{\pi}. \tag{12}$$

- **Position Mapping Constant** L_p:

$$L_p = \pi. \tag{13}$$

- **Time Mapping Constant** T_p:

$$T_p = \frac{2\pi^3}{\hbar c}. \tag{14}$$

3.3 Calculating Physical Units for Unitless Scalar Value 1

We now calculate the physical units corresponding to the unitless scalar value of 1 for various quantities in our theory.

3.3.1 Momentum $p = 1$

Using the momentum mapping:

$$p_{\text{phys}} = \frac{\hbar}{\pi} \cdot 1 = \frac{\hbar}{\pi}. \tag{15}$$

Numerical value:

$$\hbar = 1.0545718 \times 10^{-34}\,\mathrm{J \cdot s}, \tag{16}$$

$$p_{\text{phys}} = \frac{1.0545718 \times 10^{-34}}{3.1416} \approx 3.3585 \times 10^{-35}\,\mathrm{kg \cdot m/s}. \tag{17}$$

3.3.2 Wave Number $k = 1$

Using the wave number mapping:

$$k_{\text{phys}} = \frac{1}{\pi} \cdot 1 \approx 0.31831\,\mathrm{m}^{-1}. \tag{18}$$

3.3.3 Energy $E = 1$

Using the energy mapping:

$$E_{\text{phys}} = \frac{\hbar c}{\pi} \cdot 1. \tag{19}$$

Numerical value:

$$c = 2.99792458 \times 10^8 \,\text{m/s}, \tag{20}$$

$$E_{\text{phys}} = \frac{1.0545718 \times 10^{-34} \times 2.99792458 \times 10^8}{3.1416} \approx 1.0063 \times 10^{-26} \,\text{J}. \tag{21}$$

3.3.4 Angular Frequency $\omega = 1$

Using the angular frequency mapping:

$$\omega_{\text{phys}} = \frac{c}{\pi} \cdot 1 \approx 9.5493 \times 10^7 \,\text{rad/s}. \tag{22}$$

3.3.5 Position $x = 1$

Using the position mapping:

$$x_{\text{phys}} = \pi \cdot 1 \approx 3.1416 \,\text{meters}. \tag{23}$$

3.3.6 Wavelength λ Corresponding to $k = 1$

The physical wavelength is related to the wave number k_{phys}:

$$\lambda_{\text{phys}} = \frac{2\pi}{k_{\text{phys}}}. \tag{24}$$

Substituting $k_{\text{phys}} = \frac{1}{\pi}$:

$$\lambda_{\text{phys}} = \frac{2\pi}{\frac{1}{\pi}} = 2\pi^2 \approx 19.7392 \,\text{meters}. \tag{25}$$

3.3.7 Time t Corresponding to $\omega = 1$

The time period T_{phys} is related to the angular frequency ω_{phys}:

$$T_{\text{phys}} = \frac{2\pi}{\omega_{\text{phys}}}. \tag{26}$$

Using $\omega_{\text{phys}} \approx 9.5493 \times 10^7$ rad/s:

$$T_{\text{phys}} = \frac{2\pi}{9.5493 \times 10^7} \approx 6.585 \times 10^{-8} \,\text{seconds}. \tag{27}$$

3.3.8 Force F

Given the small values of force in our theory, we adjust the unitless value to a more appropriate scale. Let's consider $F = 1 \times 10^{-1}$:

Using the force mapping constant derived later:

$$F_{\text{phys}} = M_F \cdot F. \tag{28}$$

We will calculate M_F in Section 4.

3.4 Summary Table

Quantity	Unitless Value	Physical Value	Units
Momentum p	1	$p_{\text{phys}} \approx 3.3585 \times 10^{-35}$	kg·m/s
Wave Number k	1	$k_{\text{phys}} \approx 0.31831$	m^{-1}
Energy E	1	$E_{\text{phys}} \approx 1.0063 \times 10^{-26}$	Joules
Angular Frequency ω	1	$\omega_{\text{phys}} \approx 9.5493 \times 10^{7}$	rad/s
Position x	1	$x_{\text{phys}} \approx 3.1416$	Meters
Wavelength λ	$k = 1$	$\lambda_{\text{phys}} \approx 19.7392$	Meters
Time Period T	$\omega = 1$	$T_{\text{phys}} \approx 6.585 \times 10^{-8}$	Seconds

Table 1: Physical values corresponding to unitless scalar value 1 in our theory.

4 Derivation of the Mappings

4.1 Relationships Between Quantities

- **Energy and Angular Frequency:**

$$E_{\text{phys}} = \hbar\omega_{\text{phys}}. \tag{29}$$

- **Momentum and Wave Number:**

$$p_{\text{phys}} = \hbar k_{\text{phys}}. \tag{30}$$

- **Energy and Momentum:**

$$E_{\text{phys}} = p_{\text{phys}}c. \tag{31}$$

- **Angular Frequency and Wave Number:**

$$\omega_{\text{phys}} = ck_{\text{phys}}. \tag{32}$$

4.2 Consistency Checks

4.2.1 Energy Consistency

Starting with:

$$E_{\text{phys}} = \hbar\omega_{\text{phys}} = \hbar\left(\frac{c}{\pi}\cdot\omega\right) = \frac{\hbar c}{\pi}\cdot\omega. \tag{33}$$

This matches the energy mapping:

$$E_{\text{phys}} = \frac{\hbar c}{\pi}\cdot E. \tag{34}$$

Therefore, in our theory, $E = \omega$.

4.2.2 Momentum Consistency

Starting with:

$$p_{\text{phys}} = \hbar k_{\text{phys}} = \hbar\left(\frac{1}{\pi}\cdot k\right) = \frac{\hbar}{\pi}\cdot k. \tag{35}$$

This matches the momentum mapping:

$$p_{\text{phys}} = \frac{\hbar}{\pi}\cdot p. \tag{36}$$

Therefore, in our theory, $p = k$.

5 Force Mapping

5.1 Deriving the Force Mapping Constant

5.1.1 Force from Change in Energy over Time

We consider force as the rate of change of energy over time:

$$F_E = \frac{dE}{dt}. \tag{37}$$

Mapping to physical units:

$$F^{(E)}_{\text{phys}} = \left(\frac{M_E}{T_p}\right)F_E. \tag{38}$$

Calculating the mapping constant:

$$\frac{M_E}{T_p} = \frac{\frac{\hbar c}{\pi}}{\frac{2\pi^3}{\hbar c}} = \frac{(\hbar c)^2}{2\pi^4}. \tag{39}$$

Thus,

$$F_{\text{phys}}^{(E)} = \left(\frac{(\hbar c)^2}{2\pi^4}\right) F_E. \tag{40}$$

5.1.2 Force from Change in Momentum over Space

Alternatively, force can be expressed as the rate of change of momentum over space:

$$F_p = \frac{dp}{dx}. \tag{41}$$

Mapping to physical units:

$$F_{\text{phys}}^{(p)} = \left(\frac{M_p}{L_p}\right) F_p = \left(\frac{\hbar}{\pi^2}\right) F_p. \tag{42}$$

5.2 Calculating Physical Force for a Given Unitless Value

Given that force values are extremely small in our theory, we choose an appropriate unitless value for force, say $F = 0.1$.

5.2.1 Calculating Physical Force from Change in Energy over Time

Using $F_E = 0.1$:

$$F_{\text{phys}}^{(E)} = \left(\frac{(\hbar c)^2}{2\pi^4}\right) \cdot 0.1. \tag{43}$$

Numerical value:

$$F_{\text{phys}}^{(E)} = \left(\frac{(1.0545718 \times 10^{-34} \times 2.99792458 \times 10^{8})^2}{2 \times (3.1416)^4}\right) \cdot 0.1 \approx 5.131 \times 10^{-55}\,\text{N}. \tag{44}$$

5.2.2 Calculating Physical Force from Change in Momentum over Space

Using $F_p = 0.1$:

$$F_{\text{phys}}^{(p)} = \left(\frac{\hbar}{\pi^2}\right) \cdot 0.1. \tag{45}$$

Numerical value:

$$F_{\text{phys}}^{(p)} = \frac{1.0545718 \times 10^{-34}}{9.8696} \cdot 0.1 \approx 1.068 \times 10^{-36}\,\text{N}. \tag{46}$$

Quantity	Unitless Value	Physical Value	Units
Force $F^{(E)}$	0.1	$F^{(E)}_{\text{phys}} \approx 5.131 \times 10^{-55}$	Newtons
Force $F^{(p)}$	0.1	$F^{(p)}_{\text{phys}} \approx 1.068 \times 10^{-36}$	Newtons

Table 2: Physical force values corresponding to unitless force value 0.1 in our theory.

5.3 Including Force in the Summary Table

6 Conclusion

By deriving and demonstrating the correct mappings from standard physics to our theory, we've established a clear correspondence between unitless quantities and physical units. The mappings for momentum, energy, position, wave number, angular frequency, and force allow us to interpret our theoretical predictions in terms of measurable physical quantities. The comprehensive tables provide a summary of these mappings, culminating our analysis and reinforcing the consistency and applicability of our theoretical framework.

Acknowledgments

[Your acknowledgments, if any.]

References

[Include any references here.]

Appendix A

Mapping Unitless Scalars to Physical Units Using Planck's Constant

Introduction

Our theoretical framework operates with unitless scalars, yet physical measurements involve quantities with units. By establishing a relationship between our unitless quantities and fundamental physical constants like Planck's constant ($\hbar$), we can define a mapping that assigns physical units to our scalars. This mapping enables us to interpret our theoretical predictions in terms of measurable physical quantities.

Establishing the Mapping Through Planck's Constant

In our formalism, we've related the unitless spin angular momentum S to the unitless momentum p_χ via the proportionality constant K:

$$S = K \cdot p_\chi. \tag{47}$$

From the mapping in previous sections, we determined that:

$$K = \frac{\hbar}{\pi}. \tag{48}$$

This relationship implies that when p_χ takes on specific unitless values (e.g., $\frac{\pi}{2}$ or π), the corresponding physical spin angular momentum S matches the quantized values $\frac{\hbar}{2}$ and $\hbar$.

Defining the Unitless Momentum Scale

Using the proportionality constant K, we can define a mapping between our unitless momentum p and the physical momentum p_{phys}:

$$p_{\text{phys}} = K \cdot p = \frac{\hbar}{\pi} \cdot p. \tag{49}$$

This equation allows us to assign physical units to our unitless momentum values. For example:
- When $p = \dfrac{\pi}{2}$:

$$p_{\text{phys}} = \frac{\hbar}{\pi} \cdot \frac{\pi}{2} = \frac{\hbar}{2}. \tag{50}$$

- When $p = \pi$:

$$p_{\text{phys}} = \frac{\hbar}{\pi} \cdot \pi = \hbar. \tag{51}$$

This mapping shows that our unitless momentum values correspond to fundamental units of momentum in quantum mechanics.

Mapping Length and Time Scales

To map unitless lengths and times to physical units, we consider the relationships between momentum, length, and time in quantum mechanics. The de Broglie wavelength λ is related to momentum by:

$$\lambda = \frac{h}{p_{\text{phys}}}. \tag{52}$$

Similarly, the angular frequency ω is related to energy E (and thus momentum) by:

$$E = \hbar\omega = p_{\text{phys}}c. \tag{53}$$

Defining a Unitless Length Scale

We can define a unitless length ℓ corresponding to our unitless momentum p:

$$\ell = \frac{2\pi}{p}. \tag{54}$$

Using the mapping for p_{phys}, the physical wavelength λ_{phys} becomes:

$$\lambda_{\text{phys}} = \frac{h}{p_{\text{phys}}} = \frac{h}{\frac{\hbar}{\pi}p} = \frac{2\pi\hbar}{\hbar p} = \frac{2\pi}{p} = \ell. \tag{55}$$

Thus, our unitless length ℓ directly corresponds to the physical wavelength λ_{phys}.

Defining a Unitless Time Scale

Similarly, we can define a unitless time τ related to the angular frequency ω:

$$\omega = \frac{2\pi}{\tau}. \tag{56}$$

Since $E = \hbar\omega$ and $E = p_{\text{phys}}c$, we have:

$$\omega = \frac{p_{\text{phys}}c}{\hbar}. \tag{57}$$

Substituting $p_{\text{phys}} = \frac{\hbar}{\pi}p$:

$$\omega = \frac{\frac{\hbar}{\pi}p \cdot c}{\hbar} = \frac{pc}{\pi}. \tag{58}$$

Thus, the unitless time τ is:

$$\tau = \frac{2\pi}{\omega} = \frac{2\pi^2}{pc}. \tag{59}$$

Establishing the Mapping Constants

By comparing our unitless quantities to their physical counterparts, we can establish mapping constants for length, time, and momentum:

1. **Momentum Mapping Constant (M_p):**

$$M_p = \frac{\hbar}{\pi}. \tag{60}$$

2. **Length Mapping Constant (L_p)**:

   From $\lambda_{\text{phys}} = \ell$:

$$L_p = 1 \quad \text{(since unitless length directly maps to physical wavelength).} \tag{61}$$

3. **Time Mapping Constant (T_p)**:

   Using $\tau = \frac{2\pi^2}{pc}$ and $p_{\text{phys}} = M_p p$:

$$T_p = \frac{2\pi^2}{M_p c}. \tag{62}$$

Example Calculation

Suppose we have a unitless momentum $p = \pi$:
- **Physical Momentum**:

$$p_{\text{phys}} = M_p \cdot p = \frac{\hbar}{\pi} \cdot \pi = \hbar. \tag{63}$$

- **Physical Wavelength**:

$$\lambda_{\text{phys}} = \ell = \frac{2\pi}{p} = \frac{2\pi}{\pi} = 2. \tag{64}$$

- **Physical Time**:

$$\tau = \frac{2\pi^2}{pc} = \frac{2\pi^2}{\pi c} = \frac{2\pi}{c}. \tag{65}$$

The mapping constant for time is:

$$T_p = \tau = \frac{2\pi}{c}. \tag{66}$$

Generalizing the Mapping

By establishing these mapping constants, we can relate any unitless quantity in our theory to its physical counterpart:
- **Unitless Momentum p to Physical Momentum p_{phys}**:

$$p_{\text{phys}} = M_p \cdot p = \frac{\hbar}{\pi} \cdot p. \tag{67}$$

- **Unitless Length ℓ to Physical Length ℓ_{phys}**:

$$\ell_{\text{phys}} = L_p \cdot \ell = \ell. \tag{68}$$

- **Unitless Time τ to Physical Time τ_{phys}**:

$$\tau_{\text{phys}} = T_p \cdot \tau = \left(\frac{2\pi^2}{M_p c}\right) \cdot \tau. \tag{69}$$

Implications for Our Theory

- **Establishing Units**: We now have a method to assign physical units to the scalars in our theory, based on fundamental constants.

- **Consistency with Quantum Mechanics**: This mapping ensures that our unitless quantities correspond to the correct physical quantities in quantum mechanics.

- **Scalability**: By adjusting the unitless values, we can model physical systems at different scales while maintaining consistency with observed physical laws.

Conclusion

By leveraging the mapping to Planck's constant, we've established a bridge between our unitless theoretical framework and physical quantities with units. This mapping allows us to assign physical units to our scalars, enabling us to interpret our theoretical predictions in terms of measurable physical quantities.

—

Example: Electron in a Hydrogen Atom (Bohr Model)

Introduction

To test whether our mapping is working, we apply it to a well-understood physical system: the electron in a hydrogen atom, as described by the Bohr model. The Bohr model provides established relationships between energy levels, angular momentum, and wavelengths, which we can use to validate our mapping.

Bohr Model Fundamentals

The key equations in the Bohr model are:

1. **Quantization of Angular Momentum**:

$$L = n\hbar, \tag{70}$$

where n is the principal quantum number.

2. **Radius of Orbit**:

$$r_n = n^2 a_0, \tag{71}$$

where a_0 is the Bohr radius ($a_0 = 5.29177 \times 10^{-11}\,\mathrm{m}$).

3. **Electron Speed**:

$$v_n = \frac{e^2}{2\epsilon_0 \hbar} \cdot \frac{1}{n} = \frac{\alpha c}{n}, \tag{72}$$

where α is the fine-structure constant ($\alpha \approx \frac{1}{137}$).

4. **Electron Momentum**:

$$p_{\text{phys}} = m_e v_n. \tag{73}$$

5. **Electron Energy**:

$$E_n = -\frac{13.6\,\text{eV}}{n^2}. \tag{74}$$

6. **De Broglie Wavelength**:

$$\lambda_{\text{phys}} = \frac{h}{p_{\text{phys}}}. \tag{75}$$

7. **Number of Wavelengths in Orbit**:

$$n = \frac{2\pi r_n}{\lambda_{\text{phys}}}. \tag{76}$$

Calculations for $n = 1$

1. Angular Momentum:

$$L = 1 \times \hbar = \hbar. \tag{77}$$

2. Radius of Orbit:

$$r_1 = 1^2 \times a_0 = a_0 = 5.29177 \times 10^{-11}\,\text{m}. \tag{78}$$

3. Electron Speed:

$$v_1 = \alpha c \approx \frac{c}{137} \approx 2.1877 \times 10^6\,\text{m/s}. \tag{79}$$

4. Electron Momentum:

$$p_{\text{phys}} = m_e v_1 = (9.109 \times 10^{-31}\,\text{kg}) \times (2.1877 \times 10^6\,\text{m/s}) \approx 1.992 \times 10^{-24}\,\text{kg} \cdot \text{m/s}. \tag{80}$$

5. De Broglie Wavelength:

$$\lambda_{\text{phys}} = \frac{h}{p_{\text{phys}}} = \frac{6.626 \times 10^{-34}\,\text{J} \cdot \text{s}}{1.992 \times 10^{-24}\,\text{kg} \cdot \text{m/s}} \approx 3.328 \times 10^{-10}\,\text{m}. \tag{81}$$

6. Number of Wavelengths in Orbit:

$$n = \frac{2\pi r_1}{\lambda_{\text{phys}}} = \frac{2\pi \times 5.29177 \times 10^{-11}\,\text{m}}{3.328 \times 10^{-10}\,\text{m}} \approx 1. \tag{82}$$

Mapping to Our Unitless Formalism

Mapping the Physical Momentum to Unitless Momentum p

From our mapping:

$$p_{\text{phys}} = \frac{\hbar}{\pi} \cdot p \implies p = \frac{\pi p_{\text{phys}}}{\hbar}. \tag{83}$$

Using $\hbar = 1.055 \times 10^{-34}\,\text{J} \cdot \text{s}$:

$$p = \frac{\pi \times 1.992 \times 10^{-24}\,\text{kg} \cdot \text{m/s}}{1.055 \times 10^{-34}\,\text{J} \cdot \text{s}} \approx 5.936 \times 10^{10}. \tag{84}$$

Calculating the Unitless Wavelength ℓ

From our formalism:

$$\ell = \frac{2\pi}{p} = \frac{2\pi}{5.936 \times 10^{10}} \approx 1.058 \times 10^{-10}. \tag{85}$$

Mapping Unitless Length to Physical Length

From our mapping, considering the relationship:

$$\lambda_{\text{phys}} = \pi \ell, \tag{86}$$

we get:

$$\lambda_{\text{phys}} = \pi \times 1.058 \times 10^{-10} \approx 3.328 \times 10^{-10}\,\text{m}, \tag{87}$$

which matches the calculated de Broglie wavelength.

Calculating Angular Momentum in Our Formalism

From our mapping:

$$S = K \cdot p = \frac{\hbar}{\pi} \cdot p = \frac{\hbar}{\pi} \cdot \frac{\pi p_{\text{phys}}}{\hbar} = p_{\text{phys}}. \tag{88}$$

Thus, the spin angular momentum S in our formalism equals the physical momentum p_{phys}.

But in the Bohr model, the angular momentum is:

$$L = n\hbar = \hbar. \tag{89}$$

Using $L = p_{\text{phys}} r_1$:

$$L = p_{\text{phys}} r_1 = (1.992 \times 10^{-24}\,\text{kg} \cdot \text{m/s}) \times (5.29177 \times 10^{-11}\,\text{m}) \approx 1.055 \times 10^{-34}\,\text{kg} \cdot \text{m}^2/\text{s} = \hbar. \tag{90}$$

This confirms consistency.

Verifying Consistency

- The unitless momentum p mapped to the physical momentum p_{phys} correctly. - The unitless wavelength ℓ mapped to the physical wavelength λ_{phys} correctly. - The calculated angular momentum in our formalism corresponds to the physical angular momentum $L = \hbar$.

Conclusion

In this example, the mappings in our theoretical framework produce results consistent with standard quantum mechanics. The relationships between momentum, wavelength, angular momentum, and the number of cycles are maintained.

General Observations

- **Unitless Quantities Mapping**: The unitless momentum p and length ℓ map to their physical counterparts through scaling factors involving fundamental constants.

- **Consistency with Physical Laws**: The calculations confirm that the mappings yield values consistent with the Bohr model, supporting the viability of our theoretical framework.

- **Applicability to Other Systems**: This approach can be extended to other quantum systems, such as particles in a box or harmonic oscillators, to further test the mapping.

Next Steps

- **Test with Higher Energy Levels**: Apply the mapping to electrons in higher energy levels ($n > 1$) to verify consistency across different states.

- **Explore Other Quantum Systems**: Use the particle in a box or harmonic oscillator models to test the mapping in systems with different boundary conditions.

- **Refine the Mapping**: If discrepancies arise in more complex systems, consider adjusting the scaling factors or revisiting the assumptions in the mapping.

Final Remarks

By using the Bohr model as a test case, we've demonstrated that our mapping between unitless scalars and physical quantities can yield results consistent with established quantum physics. This provides a solid starting point for further exploration and validation of our theoretical framework.