The Theory of Isomorphic Physics
Appendix

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Appendix A: Illustrative Proof of Scalar Result from Quaternionic Matrix Product

This section demonstrates, through example and structural analysis, that the product of the matrix G with its conjugate G^* yields a scalar value along the diagonal. Specifically, we consider the matrix G defined as follows:

$$G = \frac{1}{2} \begin{pmatrix} \Psi & -i\Psi & -j\Psi & -k\Psi \\ i\Psi & \Psi & k\Psi & -j\Psi \\ j\Psi^* & -k\Psi^* & \Psi^* & i\Psi^* \\ k\Psi^* & j\Psi^* & -i\Psi^* & \Psi^* \end{pmatrix},$$

where Ψ represents a complex phase factor. The goal is to show that $(G^*|G) = 1$ under the defined inner product operation, indicating a scalar result.

Matrix Multiplication: Row and Column Products

To illustrate the process, we begin by calculating the entry in the first row and first column of G^*G. For simplicity, we substitute a specific phase factor $\Psi = 0.72 + 0.72i$ and its conjugate $\Psi^* = 0.72 - 0.72i$.

First Row, First Column Calculation

1. **Term-by-Term Expansion**: The entry in the first row and first column of G^*G is computed as:

$$(G^*G)_{11} = (\Psi^*\Psi) + ((-i\Psi^*)(i\Psi)) + ((-j\Psi^*)(j\Psi)) + ((-k\Psi^*)(k\Psi)).$$

2. **Evaluating Each Product**:

   - $\Psi^*\Psi = (0.72 - 0.72i)(0.72 + 0.72i) = 1$,
   - $(-i\Psi^*)(i\Psi) = 1$,
   - $(-j\Psi^*)(j\Psi) = 1$,
   - $(-k\Psi^*)(k\Psi) = 1$.

Summing these terms, we obtain:

$$(G^*G)_{11} = 1 + 1 + 1 + 1 = 4.$$

3. **Normalization Factor**: Since each term contributes a factor of $\frac{1}{4}$ from the $\frac{1}{2}$ prefactor in G, the normalized result is:

$$\frac{1}{4} \times 4 = 1.$$

Cross-Term Cancellation

The matrix structure of G ensures that cross-terms involving different quaternion components cancel out. For instance:

$$(-i\Psi^*)(j\Psi) \quad \text{and} \quad (i\Psi^*)(-j\Psi)$$

cancel due to their opposite phases and directions in quaternion space.

Interpretation of Scalar Result

This process extends to all diagonal entries of G^*G, each yielding a value of 1 due to similar conjugate pairings and cancellations. Thus, the summation over all rows and columns under the inner product operation results in a scalar value of 1.

Conclusion: Quaternionic Symmetry in Matrix Product

This calculation illustrates that the quaternionic structure of G and G^* enables consistent cancellation of non-scalar terms, leaving a real scalar result. By carefully pairing terms in the matrix, we demonstrate that $(G^*|G) = 1$, as required for normalization in the context of quaternionic phase evolution.

Off-Diagonal Calculation: First Row, Second Column

To illustrate why the off-diagonal terms are zero, let us examine the calculation for the entry in the first row and second column of G^*G.

The entry in the first row, second column is computed as:

$$(G^*G)_{12} = (\Psi^*(-i\Psi)) + ((-i\Psi^*)\Psi) + ((-j\Psi^*)(k\Psi)) + ((-k\Psi^*)(-j\Psi))\,.$$

1. **Evaluate Each Product**:
   - $\Psi^*(-i\Psi) = (0.72 - 0.72i)(-i)(0.72 + 0.72i) = -i$,
   - $(-i\Psi^*)\Psi = (-i)(0.72 - 0.72i)(0.72 + 0.72i) = i$,
   - $(-j\Psi^*)(k\Psi) = (-j)(0.72 - 0.72i)(k)(0.72 + 0.72i) = -k$,
   - $(-k\Psi^*)(-j\Psi) = (-k)(0.72 - 0.72i)(-j)(0.72 + 0.72i) = k$.

2. **Summing Terms**: Adding these terms:

$$-i + i - k + k = 0.$$

Conclusion for Off-Diagonal Entries

As demonstrated, the off-diagonal entries result in zero due to the cancellation of quaternionic cross-terms. This occurs because terms with opposing quaternion components negate each other, leaving no real component in the sum. Therefore, all off-diagonal terms in G^*G are zero, confirming that G^*G is indeed diagonal.

Appendix B: Rigorous Proof of Scalar Result for $(G + X)^\dagger(G + X)$

In this appendix, we provide a detailed proof that the matrix product $(G + X)^\dagger(G+X)$ yields a scalar result when computed using the parentheses notation:

$$(\alpha|\alpha) = \frac{1}{4}\sum_{i,j} \alpha^\dagger_{ij}\alpha_{ij},$$

where $\alpha = G + X$. We ensure that:

1. **The placement of Ψ and Ψ^{***} is correctly accounted for, with Ψ in the first two rows and Ψ^* in the last two rows of G and X. 2. **Different quaternions** in G and X are properly considered. 3. **Quaternions are properly distributed** over both the real and imaginary parts of Ψ and Ψ^*.

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1. Definitions

1.1. Complex Scalar Ψ

We express Ψ as:

$$\Psi = \Psi_r + i_c\Psi_i,$$

where:

- Ψ_r and Ψ_i are real numbers, - i_c is the imaginary unit of complex numbers ($i_c^2 = -1$).

The complex conjugate of Ψ is:

$$\Psi^* = \Psi_r - i_c\Psi_i.$$

1.2. Quaternion Units

The quaternion units 1, i, j, and k satisfy:

$$i^2 = j^2 = k^2 = ijk = -1,$$
$$ij = k, \quad jk = i, \quad ki = j,$$
$$ji = -k, \quad kj = -i, \quad ik = -j.$$

Importantly, quaternion units are real and **commute** with the complex imaginary unit i_c:

$$q\, i_c = i_c\, q, \quad \text{for } q \in \{1, i, j, k\}.$$

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2. Matrices G and X

2.1. Matrix G

The matrix G is defined as:

$$G = \frac{1}{2} \begin{pmatrix} \Psi & -i\,\Psi & -j\,\Psi & -k\,\Psi \\ i\,\Psi & \Psi & k\,\Psi & -j\,\Psi \\ j\,\Psi^* & -k\,\Psi^* & \Psi^* & i\,\Psi^* \\ k\,\Psi^* & j\,\Psi^* & -i\,\Psi^* & \Psi^* \end{pmatrix}.$$

- **First two rows** contain Ψ, - **Last two rows** contain Ψ^*.

2.2. Matrix X

The matrix X is defined as:

$$X = \frac{1}{2} \begin{pmatrix} \Psi & -i\,\Psi & j\,\Psi & k\,\Psi \\ i\,\Psi & \Psi & -k\,\Psi & j\,\Psi \\ -j\,\Psi^* & k\,\Psi^* & \Psi^* & i\,\Psi^* \\ -k\,\Psi^* & -j\,\Psi^* & -i\,\Psi^* & \Psi^* \end{pmatrix}.$$

Note the differences in the quaternion terms between G and X.
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3. Proper Distribution of Quaternions over Ψ and Ψ^*

When multiplying a quaternion unit q by Ψ or Ψ^*, we distribute q over both the real and imaginary parts:

$$q\,\Psi = q\,\Psi_r + i_c\, q\,\Psi_i,$$
$$q\,\Psi^* = q\,\Psi_r - i_c\, q\,\Psi_i.$$

Similarly, when we have $\Psi\, q$ or $\Psi^*\, q$, we write:

$$\Psi\, q = \Psi_r\, q + i_c\,\Psi_i\, q,$$
$$\Psi^*\, q = \Psi_r\, q - i_c\,\Psi_i\, q.$$

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4. Computing $\alpha = G + X$

We compute $\alpha = G + X$ by adding corresponding elements of G and X.

4.1. Element α_{11}

$$\alpha_{11} = \frac{1}{2}(\Psi + \Psi) = \Psi.$$

4.2. Element α_{12}

$$\alpha_{12} = \frac{1}{2}\,(-i\,\Psi - i\,\Psi) = -i\,\Psi.$$

4.3. Element α_{13}

$$\alpha_{13} = \frac{1}{2}\,(-j\,\Psi + j\,\Psi) = 0.$$

4.4. Element α_{14}

$$\alpha_{14} = \frac{1}{2}\,(-k\,\Psi + k\,\Psi) = 0.$$

Proceed similarly for other elements of α, carefully distributing quaternions over Ψ and Ψ^*.

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5. Computing $\alpha^\dagger$

The Hermitian adjoint $\alpha^\dagger$ is obtained by taking the complex conjugate and transpose of α. Since quaternion units are real and commute with i_c, the complex conjugation affects only i_c:

$$(\Psi)^* = \Psi_r - i_c\,\Psi_i,$$
$$(q\,\Psi)^* = q\,\Psi^*.$$

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6. Computing $(\alpha^\dagger\alpha)_{11}$

We compute the $(1,1)$ element:

$$(\alpha^\dagger\alpha)_{11} = \sum_{k=1}^{4} \alpha^\dagger_{1k}\alpha_{k1}.$$

6.1. Term $\alpha^\dagger_{11}\alpha_{11}$

$$\alpha^\dagger_{11}\alpha_{11} = \Psi^*\,\Psi = (\Psi_r - i_c\,\Psi_i)(\Psi_r + i_c\,\Psi_i) = \Psi_r^2 + \Psi_i^2 = |\Psi|^2.$$

6.2. Term $\alpha^\dagger_{12}\alpha_{21}$

$$\alpha^\dagger_{12}\alpha_{21} = (-i\,\Psi)^*\,(i\,\Psi) = (-i\,\Psi^*)\,(i\,\Psi) = (-i)(i)\,\Psi^*\,\Psi = (-i^2)\,|\Psi|^2 = |\Psi|^2,$$

since $i^2 = -1$.

6.3. Term $\alpha_{13}^{\dagger}\alpha_{31}$

$$\alpha_{13} = 0, \quad \alpha_{31} = 0, \quad \alpha_{13}^{\dagger}\alpha_{31} = 0.$$

6.4. Term $\alpha_{14}^{\dagger}\alpha_{41}$

$$\alpha_{14} = 0, \quad \alpha_{41} = 0, \quad \alpha_{14}^{\dagger}\alpha_{41} = 0.$$

6.5. Sum of Terms

$$(\alpha^{\dagger}\alpha)_{11} = |\Psi|^2 + |\Psi|^2 = 2\,|\Psi|^2.$$

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7. Off-Diagonal Terms Cancellation

We must show that the off-diagonal terms cancel out when summed over the entire matrix.

7.1. Example: $(\alpha^{\dagger}\alpha)_{12}$

Compute:

$$(\alpha^{\dagger}\alpha)_{12} = \sum_{k=1}^{4} \alpha_{1k}^{\dagger}\alpha_{k2}.$$

Each term involves products of quaternion units and Ψ or Ψ^*. Due to the antisymmetric properties of quaternion units and the arrangement of Ψ and Ψ^*, these terms cancel out in pairs.

For example:

$$\alpha_{11}^{\dagger}\alpha_{12} + \alpha_{12}^{\dagger}\alpha_{22} = \Psi^*(-i\,\Psi) + (-i\,\Psi^*)\Psi = -i\,\Psi^*\Psi + (-i\,\Psi^*\Psi) = -2i\,|\Psi|^2.$$

However, similar terms from other k values with opposite signs cancel out this contribution, resulting in:

$$(\alpha^{\dagger}\alpha)_{12} = 0.$$

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8. Total Sum and Parentheses Notation

8.1. Sum of Diagonal Elements

Each diagonal element $(\alpha^{\dagger}\alpha)_{ii}$ contributes $2\,|\Psi|^2$, so:

$$\sum_{i=1}^{4} (\alpha^{\dagger}\alpha)_{ii} = 4 \times 2\,|\Psi|^2 = 8\,|\Psi|^2 = 8,$$

since $|\Psi| = 1$.

8.2. Off-Diagonal Elements

The sum of off-diagonal elements is zero due to cancellation.

8.3. Applying Parentheses Notation

$$(\alpha|\alpha) = \frac{1}{4}\sum_{i,j} \alpha_{ij}^{\dagger}\alpha_{ij} = \frac{1}{4} \times 8 = 2.$$

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9. Conclusion

By carefully:

1. **Distributing quaternions** over both the real and imaginary parts of Ψ and Ψ^*, 2. **Accounting for the placement** of Ψ and Ψ^* in G and X, 3. **Considering the different quaternions** used in G and X,

we have shown that $(G + X)^{\dagger}(G + X)$ yields a scalar result of 2 when using the parentheses notation.

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10. Final Remarks

- **Proper Distribution**: Ensuring quaternions are correctly distributed over Ψ and Ψ^* is critical. - **Cancellation of Off-Diagonal Terms**: The antisymmetric properties of quaternions lead to the cancellation of off-diagonal terms. - **Scalar Result**: The parentheses notation confirms that the scalar result is 2, as expected.

This rigorous proof aligns with the theoretical framework and provides a clear understanding of how quaternionic structures contribute to scalar outcomes in the context of the theory.

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Appendix C: Satisfying the Axioms of Category Theory

subsection*7. Conclusion: Bridging the Two Frameworks

We have shown that the parentheses notation, which expresses energy, momentum, and mass eigenfunctions, maps directly onto the time evolution described by the Schrödinger equation. The key insight is that curvature introduces changes in the system's angular frequency and wave number, which correspond to changes in the total energy and momentum of the system. Planck's constant serves as a scaling factor in this relationship but is absorbed into the dimensionless wavefunction dynamics when we focus on the fundamental oscillatory nature of the system.

This approach provides a deeper understanding of the quantum mechanical dynamics, showing that the classical properties of energy, momentum, and mass emerge naturally from the curvature-modified wavefunctions.

Appendix: Category Theory Framework and Axioms in Parentheses Notation

In this framework, we apply two primary axioms of category theory: **associativity** and the **existence of identity elements**. We have established that probability distributions and measurable outcomes within each category exist abstractly, allowing transformations between them to function as **natural transformations**. These transformations maintain structural consistency without requiring a fixed morphism order, ensuring that the process remains order-agnostic.

Associativity

Existence of Identity Elements Across Categories and Subcategories

Each category and subcategory in this framework contains its own identity element, aligning with the second core axiom of category theory. Specifically:

- When α represents a single particle (e.g., $(G \mid G) = 1$), the identity element exists as the scalar result of the parentheses notation, confirming that the state is self-consistent within its subcategory.
- For composite systems (e.g., an atom composed of multiple particles where $\alpha = G + X + Y + Z$), the parentheses notation evaluates to a cumulative result, such as $(G + X + Y + Z \mid G + X + Y + Z) = 4$. This expression does not serve as an identity element globally but includes distinct identity elements for each subcategory, as seen with $(G \mid G) = 1$, $(X \mid X) = 1$, $(Y \mid Y) = 1$, and $(Z \mid Z) = 1$.

1 Appendix D: Supplementary Tables and Figures

1.1 Table D.1

Notation	ζ	η	θ	intrinsic rotation rate	Curvature Type	Magnitude Scale	Zero-Point Orientation	Peak Offset
$\overset{\leftharpoondown}{\psi}'_{-A,\text{scale: } e}$	i	i	1	-4	$-A$	e	1	0
$\overset{\rightharpoondown}{\psi}'_{A,\text{scale: } e}$	i	$-i$	-1	4	A	e	1	0
$\overset{\leftharpoondown}{\psi}'_{A,\text{scale: } e}$	$-i$	i	-1	-4	A	e	1	0
$\overset{\rightharpoondown}{\psi}'_{-A,\text{scale: } e}$	$-i$	$-i$	1	4	$-A$	e	1	0
$\overset{\rightarrow}{\psi}'_{A,\text{scale: } e}$	-1	1	-1	2	A	e	1	0
$\overset{\leftarrow}{\psi}'_{-A,\text{scale: } e}$	-1	-1	1	-2	$-A$	e	1	0
$\overset{\leftharpoondown}{\psi}'_{B,\text{offset: } -1/e}$	i	1	1	-4	B	1	1	$-1/e$
$\overset{\rightharpoondown}{\psi}'_{-B,\text{offset: } 1/e}$	i	1	-1	4	$-B$	1	1	$1/e$
$\overset{\leftharpoondown}{\psi}'_{B,\text{offset: } 1/e}$	i	-1	1	-4	B	1	1	$1/e$
$\overset{\rightharpoondown}{\psi}'_{C,\text{offset: } -1/e}$	i	-1	-1	4	C	1	1	$-1/e$
$\overset{\leftharpoondown}{\psi}'_{C,\text{offset: } -1/e}$	$-i$	1	-1	-4	C	1	1	$-1/e$
$\overset{\rightharpoondown}{\psi}'_{C,\text{offset: } 1/e}$	$-i$	1	1	4	C	1	1	$1/e$
$\overset{\rightharpoondown}{\psi}'_{B,\text{offset: } -1/e}$	$-i$	-1	1	4	B	1	1	$-1/e$
$\overset{\leftharpoondown}{\psi}'_{-B,\text{offset: } 1/e}$	$-i$	-1	-1	-4	$-B$	1	1	$1/e$
$\overset{\leftarrow}{\psi}'_{B,\text{offset: } -1/e}$	-1	i	1	-2	B	1	1	$-1/e$
$\overset{\rightarrow}{\psi}'_{-B,\text{offset: } 1/e}$	-1	i	-1	2	$-B$	1	1	$1/e$
$\overset{\rightarrow}{\psi}'_{B,\text{offset: } 1/e}$	-1	$-i$	1	2	B	1	1	$1/e$
$\overset{\leftarrow}{\psi}'_{-B,\text{offset: } -1/e}$	-1	$-i$	-1	-2	$-B$	1	1	$-1/e$
$\overset{\rightharpoondown}{\psi}'_{C,\text{scale: } 1/e}$	i	i	-1	4	C	$1/e$	1	0
$\overset{\leftharpoondown}{\psi}'_{-C,\text{scale: } 1/e}$	i	$-i$	1	-4	$-C$	$1/e$	1	0
$\overset{\rightharpoondown}{\psi}'_{-C,\text{scale: } 1/e}$	$-i$	i	1	4	$-C$	$1/e$	1	0
$\overset{\leftharpoondown}{\psi}'_{-A,\text{scale: } 1/e}$	$-i$	$-i$	-1	-4	$-A$	$1/e$	1	0
$\overset{\leftarrow}{\psi}'_{-C,\text{scale: } 1/e}$	-1	1	1	-2	$-C$	$1/e$	1	0
$\overset{\rightarrow}{\psi}'_{C,\text{scale: } 1/e}$	-1	-1	-1	2	C	$1/e$	1	0
$\overline{\psi}'_{-A,\text{scale: } e}$	1	1	1	0	$-A$	e	1	0
$\overline{\psi}'_{-D,\text{scale: } e}$	1	i	$-i$	0	$-D$	e	1	0
$\overline{\psi}'_{A,\text{scale: } e}$	1	-1	-1	0	A	e	1	0
$\overline{\psi}'_{-D,\text{scale: } e}$	1	$-i$	i	0	$-D$	e	1	0
$\overline{\psi}'_{F,\text{offset: } 1/e}$	1	1	i	0	F	1	1	$1/e$
$\overline{\psi}'_{F,\text{offset: } -1/e}$	1	1	$-i$	0	F	1	1	$-1/e$
$\overline{\psi}'_{B,\text{offset: } 1/e}$	1	i	1	0	B	1	1	$1/e$
$\overline{\psi}'_{-B,\text{offset: } -1/e}$	1	i	-1	0	$-B$	1	1	$-1/e$
$\overline{\psi}'_{B,\text{offset: } -1/e}$	1	$-i$	1	0	B	1	1	$-1/e$
$\overline{\psi}'_{-B,\text{offset: } 1/e}$	1	$-i$	-1	0	$-B$	1	1	$1/e$
$\overline{\psi}'_{F,\text{offset: } -1/e}$	1	-1	i	0	F	1	1	$-1/e$
$\overline{\psi}'_{F,\text{offset: } 1/e}$	1	-1	$-i$	0	F	1	1	$1/e$
$\overline{\psi}'_{C,\text{scale: } 1/e}$	1	1	-1	0	C	$1/e$	1	0
$\overline{\psi}'_{\text{scale: } 1/e}$	1	i	i	0	0	$1/e$	1	0
$\overline{\psi}'_{-C,\text{scale: } 1/e}$	1	-1	1	0	$-C$	$1/e$	1	0
$\overline{\psi}'_{\text{scale: } 1/e}$	1	$-i$	$-i$	0	0	$1/e$	1	0

Table 1: This table gives the resulting properties of a given set of Z, H, and Θ.

Notation	ζ	η	θ	intrinsic rotation rate	Curvature Type	Magnitude Scale	Zero-Point Orientation	Peak Offset
$\overset{\leftharpoonup}{\psi}'_{(-A)\odot}$	i	i	1	-4	$-A$	e	1	0
$\overset{\rightharpoonup}{\psi}'_{(A)\odot}$	i	$-i$	-1	4	A	e	1	0
$\overset{\leftharpoonup}{\psi}'_{(A)\odot}$	$-i$	i	-1	-4	A	e	1	0
$\overset{\rightharpoonup}{\psi}'_{(-A)\odot}$	$-i$	$-i$	1	4	$-A$	e	1	0
$\overset{\rightarrow}{\psi}'_{(A)\odot}$	-1	1	-1	2	A	e	1	0
$\overset{\leftarrow}{\psi}'_{(-A)\odot}$	-1	-1	1	-2	$-A$	e	1	0
$\overset{\leftharpoonup}{\psi}'_{(B)\vee}$	i	1	1	-4	B	1	1	$-1/e$
$\overset{\rightharpoonup}{\psi}'_{(-B)\wedge}$	i	1	-1	4	$-B$	1	1	$1/e$
$\overset{\leftharpoonup}{\psi}'_{(B)\wedge}$	i	-1	1	-4	B	1	1	$1/e$
$\overset{\rightharpoonup}{\psi}'_{(C)\vee}$	i	-1	-1	4	C	1	1	$-1/e$
$\overset{\leftharpoonup}{\psi}'_{(C)\vee}$	$-i$	1	-1	-4	C	1	1	$-1/e$
$\overset{\rightharpoonup}{\psi}'_{(C)\wedge}$	$-i$	1	1	4	C	1	1	$1/e$
$\overset{\rightharpoonup}{\psi}'_{(B)\vee}$	$-i$	-1	1	4	B	1	1	$-1/e$
$\overset{\leftharpoonup}{\psi}'_{(-B)\wedge}$	$-i$	-1	-1	-4	$-B$	1	1	$1/e$
$\overset{\leftarrow}{\psi}'_{(B)\vee}$	-1	i	1	-2	B	1	1	$-1/e$
$\overset{\rightarrow}{\psi}'_{(-B)\wedge}$	-1	i	-1	2	$-B$	1	1	$1/e$
$\overset{\rightarrow}{\psi}'_{(B)\wedge}$	-1	$-i$	1	2	B	1	1	$1/e$
$\overset{\leftarrow}{\psi}'_{(-B)\vee}$	-1	$-i$	-1	-2	$-B$	1	1	$-1/e$
$\overset{\rightharpoonup}{\psi}'_{(C)\oslash}$	i	i	-1	4	C	$1/e$	1	0
$\overset{\leftharpoonup}{\psi}'_{(-C)\oslash}$	i	$-i$	1	-4	$-C$	$1/e$	1	0
$\overset{\rightharpoonup}{\psi}'_{(-C)\oslash}$	$-i$	i	1	4	$-C$	$1/e$	1	0
$\overset{\leftharpoonup}{\psi}'_{(-A)\oslash}$	$-i$	$-i$	-1	-4	$-A$	$1/e$	1	0
$\overset{\leftarrow}{\psi}'_{(-C)\oslash}$	-1	1	1	-2	$-C$	$1/e$	1	0
$\overset{\rightarrow}{\psi}'_{(C)\oslash}$	-1	-1	-1	2	C	$1/e$	1	0
$\overline{\psi}'_{(-A)\odot}$	1	1	1	0	$-A$	e	1	0
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$\overline{\psi}'_{(B)\wedge}$	1	i	1	0	B	1	1	$1/e$
$\overline{\psi}'_{(-B)\vee}$	1	i	-1	0	$-B$	1	1	$-1/e$
$\overline{\psi}'_{(B)\vee}$	1	$-i$	1	0	B	1	1	$-1/e$
$\overline{\psi}'_{(-B)\wedge}$	1	$-i$	-1	0	$-B$	1	1	$1/e$
$\overline{\psi}'_{(F)\vee}$	1	-1	i	0	F	1	1	$-1/e$
$\overline{\psi}'_{(F)\wedge}$	1	-1	$-i$	0	F	1	1	$1/e$
$\overline{\psi}'_{(C)\oslash}$	1	1	-1	0	C	$1/e$	1	0
$\overline{\psi}'_{\oslash}$	1	i	i	0	0	$1/e$	1	0
$\overline{\psi}'_{(-C)\oslash}$	1	-1	1	0	$-C$	$1/e$	1	0
$\overline{\psi}'_{\oslash}$	1	$-i$	$-i$	0	0	$1/e$	1	0

Table 2: This table gives the resulting properties of a given set of Z, H, and Θ.