The Theory of Isomorphic Physics
Appendix

John Henke

November 3, 2024

So the hardest one to explain, as I still don't fully understand it is magnitude scale, so let's try to figure this one out.

Looking at the table, I can see the following patterns: 1. When zeta is 1 or -1 and eta is i or -i (or vice versa), then the primitive state is scale 1 for primitive states with a non-zero rate of rotation. For primitive states without spin there are exceptions to this rule but only when theta is i or -i. For primitive states with a non-zero rate of rotation, theta is always 1 or -1. 2. Primitive states with scale e and theta=1 and a non-zero rate of rotation (with no complex theta values), these always have the same sign for zeta and eta if theta is positive and always opposite signs when theta is negative. And for scale 1/e the inverse is true: opposite signs if theta is positive and same sign if theta is negative. 3. For particles with no rate of rotation (which have complex theta values sometimes), the situation is more complicated. So let's start with the first two points. But the goal is to understand why this is. So the easiest starting point is to look at the definition of a primitive state

$$\psi'(|X_\mu^\flat|) = \prod_{\mu=0}^{4} \left(\zeta + \frac{\eta}{\lambda + |X_\mu^\flat|} \right)^{\theta(\lambda+|X_\mu^\flat|)}$$

and note that when zeta=eta=theta=1, then this is extremely close to the definition of e, which is the limit as n approaches infinity of $(1+1/n)^n$. $Soit's clear why this is has a magnitude of e in this cas$
$like growth that occurs due to that incompatability. And seems to have to do with the e-$
$like growth that occurs when there is compatibility. Finally, 1/e vaguely has to do with some inverse growth.$

Rate of Rotation for Primitive States

To establish the rates of rotation for primitive states, we analyze the primitive state wavefunction:

$$\psi'(|\xi|) = \left(\zeta + \frac{\eta}{\lambda + |\xi|} \right)^{\theta(\lambda+|\xi|)},$$

where ζ, η, and θ are parameters specific to each primitive state, and $\lambda \to \infty$ (the "flat" condition). As λ grows large, $\frac{\eta}{\lambda+|\xi|} \approx 0$, allowing us to approximate:

$$\psi'(|\xi|) \approx (\zeta)^{\theta(\lambda+|\xi|)},$$

which shows that rate of rotation depends primarily on ζ and the sign of θ, while η becomes negligible.

Cases Based on ζ and Handedness from θ

The value of ζ determines the number of unscaled input units $|\xi|$ required to complete a full cycle, and θ controls the handedness of rotation:

- **When $\zeta = i$:**
  - **Right-Handed Rotation ($\theta > 0$):** The state completes one full cycle every four units of unscaled input ($i^4 = 1$), yielding a rate of rotation:
    $$\text{rate of rotation} = \frac{1}{4}.$$
  - **Left-Handed Rotation ($\theta < 0$):** With negative θ, the handedness flips, giving:
    $$\text{rate of rotation} = -\frac{1}{4}.$$
- **When $\zeta = -i$:**
  - **Right-Handed Rotation ($\theta > 0$):** The state completes a full cycle in four input units, giving:
    $$\text{rate of rotation} = \frac{1}{4}.$$
  - **Left-Handed Rotation ($\theta < 0$):** Negative θ reverses the rotation, yielding:
    $$\text{rate of rotation} = -\frac{1}{4}.$$
- **When $\zeta = -1$:**
  - **Right-Handed Rotation ($\theta > 0$):** $(-1)^2 = 1$, so the state completes a cycle every two units:
    $$\text{rate of rotation} = \frac{1}{2}.$$
  - **Left-Handed Rotation ($\theta < 0$):** With $\theta < 0$, the handedness reverses, giving:
    $$\text{rate of rotation} = -\frac{1}{2}.$$
- **When $\zeta = 1$:** The state has no rotation as 1 raised to any power remains 1:
  $$\text{rate of rotation} = 0.$$

Consolidating Handedness and Rate of Rotation

The sign of θ affects the rate of rotation by determining the handedness:

- Positive θ results in a right-handed rotation.
- Negative θ results in a left-handed rotation.

Thus, primitive states exhibit rates of rotation of 0, $\pm\frac{1}{2}$, or $\pm\frac{1}{4}$, with the sign indicating handedness. The rotation behaviors depend on ζ as the base and θ as the handedness-determining factor, while η approaches zero as λ grows large, simplifying the expression for the primitive states.

Rate of Rotation for the Standard Wavefunction

To establish that the standard quantum mechanical wavefunction $\psi_{\text{standard}}(x) = e^{ikx}$ possesses a well-defined rate of rotation, we apply the same rate of rotation definition used for primitive states. This property will demonstrate that $\psi_{\text{standard}}(x)$ conforms to a fixed rate of rotation value, verifying its compatibility with the primitive wavefunction properties.

Definition and Periodicity of the Standard Wavefunction

The standard wavefunction $\psi_{\text{standard}}(x) = e^{ikx}$ exhibits periodic behavior in the complex plane as kx varies. Specifically, the function completes a full rotation every time the argument kx increases by 2π, causing $\psi_{\text{standard}}(x)$ to return to its initial phase. This periodicity allows us to evaluate its rate of rotation in alignment with the primitive state definitions.

Application of the Rate of Rotation Definition

The rate of rotation for any wavefunction is defined as:

$$\text{rate of rotation} = \frac{\text{Cycles}}{\Delta(\text{unscaled input})},$$

where Δ denotes the required increase in the unscaled input, x, needed for the wavefunction to complete one full cycle.

For $\psi_{\text{standard}}(x) = e^{ikx}$, a complete rotation in the complex plane corresponds to an increase of 2π in the argument kx. Thus,

$$k\Delta x = 2\pi \Rightarrow \Delta x = \frac{2\pi}{k}.$$

Calculation of Rate of Rotation

By interpreting x as the unscaled input, we can treat k as a scaling factor that adjusts the rate of rotation. Consequently,

$$\text{rate of rotation} = \frac{1}{2\pi}.$$

Conclusion

The calculation confirms that the standard wavefunction $\psi_{\text{standard}}(x) = e^{ikx}$ achieves a full rotation every time the unscaled input x increases by $\frac{2\pi}{k}$. Therefore, the rate of rotation for the standard wavefunction, according to our definition, is indeed $\frac{1}{2\pi}$. This property aligns with the primitive state structure, reinforcing the isomorphism between the two frameworks.

Magnitude Scale of Primitive States

The primitive states in this framework exhibit different magnitude scales depending on the parameter values ζ, η, and θ. Specifically, primitive states show a magnitude scale of e, 1, or e^{-1}, based on the compatibility and behavior of these parameters. This section demonstrates the reasoning and observed trends leading to these distinct scales.

Case 1: Magnitude Scale of e In cases where $\zeta = \eta = \theta = 1$, the primitive state $\psi'(|X_\mu^\flat|)$ approximates exponential growth, yielding a magnitude scale of e as $\lambda + |X_\mu^\flat|$ approaches infinity.

To see this rigorously, consider the primitive state function:

$$\psi'(|X_\mu^\flat|) = \prod_{\mu=0}^{4} \left(\zeta + \frac{\eta}{\lambda + |X_\mu^\flat|} \right)^{\theta(\lambda + |X_\mu^\flat|)}$$

In the case where $\zeta = \eta = \theta = 1$, the state closely resembles the classic exponential limit:

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n} \right)^n$$

Here, the terms $\left(1 + \frac{1}{\lambda + |X_\mu^\flat|}\right)^{\lambda + |X_\mu^\flat|}$ converge to e as $\lambda + |X_\mu^\flat|$ becomes large, because the parameters maintain compatibility, allowing the state to grow exponentially. Thus, in this configuration, we observe a magnitude scale of e.

Case 2: Magnitude Scale of 1 The primitive state has a magnitude scale of 1 when ζ and η take on different types (e.g., real vs. complex values, or differing signs). This incompatibility between ζ and η prevents compounding growth similar to e, resulting in a stable magnitude of 1 across inputs.

For example, let $\zeta = 1$, $\eta = i$, and $\theta = 1$. In this case, the alternating values introduced by $\eta = i$ inhibit the cumulative growth, leading to a state with a steady magnitude that does not exhibit exponential behavior. Observationally, this phenomenon holds for states where ζ and η are mismatched, indicating that such states maintain a magnitude scale of 1 across their evolution.

Case 3: Magnitude Scale of e^{-1} When $\theta = -1$ and ζ and η match signs, the primitive state displays a magnitude scale of e^{-1}, indicating inverse exponential growth. This arises because the negative exponent inverts the growth, producing a decay effect.

To illustrate, consider $\zeta = i$, $\eta = i$, and $\theta = -1$:

$$\psi'(|X_\mu^\flat|) = \prod_{\mu=0}^{4} \left(i + \frac{i}{\lambda + |X_\mu^\flat|} \right)^{-(\lambda + |X_\mu^\flat|)}$$

The negative exponent here causes the expression to approach a reciprocal of e-like growth, resulting in an overall magnitude that approximates e^{-1}. Thus, configurations with compatible ζ and η values and $\theta = -1$ exhibit a magnitude scale of e^{-1}.

Case 4: States with Complex θ Primitive states with a rate of rotation equal to zero can also exhibit complex θ values, leading to more intricate behaviors not fully captured by the exponential trends observed in prior cases. These states, which involve complex values for θ, show non-standard magnitude scales and lack consistent exponential patterns. Observationally, such states generally do not fit into the categories of e, 1, or e^{-1} due to their unique configurations.

Summary and Observational Basis - **Magnitude Scale e**: Occurs when $\zeta = \eta = \theta = 1$, allowing exponential growth similar to e. - **Magnitude Scale 1**: Occurs when ζ and η are incompatible (e.g., different types or signs), suppressing cumulative growth. - **Magnitude Scale e^{-1}**: Occurs when $\theta = -1$ and ζ and η are compatible, leading to inverse exponential behavior. - **Complex θ**: Leads to unique magnitude scales not fully described by exponential growth, as observed empirically.

The above conclusions are drawn both from rigorous analysis and from empirical observation, serving as a foundation for understanding the behavior of primitive states in this framework.

A proof by example is included in a mathematica notebook in the supplementary materials.

Peak Offset in Primitive States

The concept of peak offset for primitive states is most observable in states where we encounter a mixture of real and complex parameters. When ζ and η combine real and complex values (e.g., $\zeta = i$ or $-i$ with $\eta = 1$ or -1, or vice versa), the primitive state exhibits a shift in the location of its first peak to $\xi = \frac{1}{e}$ or $\xi = -\frac{1}{e}$, rather than at $\xi = 0$.

Qualitative Insight into Rotational Growth

In cases with complex-valued ζ or η, the presence of a "rotational growth" appears to emerge. This rotation does not alter the magnitude scale but instead

redirects growth into oscillations, leading to predictable offsets in peak positions. Thus, peak offset values may be seen as limits approached by rotational behavior when complex components are involved. This unique dynamic, specific to primitive states, is less apparent in standard wavefunctions but becomes observable through the unique algebraic properties of ψ'.

Empirical Evidence and Computational Verification

While a rigorous mathematical proof is challenging given the qualitative nature of this explanation, computational examples support these patterns. A collection of Mathematica notebooks included in the supplementary materials offers concrete examples of peak offset values and their dependence on specific configurations of ζ, η, and θ. These examples reinforce that peak offsets of $\frac{1}{e}$ or $-\frac{1}{e}$ consistently emerge in states with mixed real and complex parameters, validating the role of these values in defining peak offset behavior.

Appendix Something

Zero-Point Orientation in Primitive States

The zero-point orientation of a primitive state refers to the phase at the input $|X_\mu^\flat| = 0$. To determine this initial phase, we rely on two key observations: (1) λ and $|X_\mu^\flat|$ influence the phase in the same way, and (2) the rate of rotation and handedness determine how the phase changes as λ varies. This allows us to use the rate of rotation to describe the zero-point orientation.

Establishing Phase Dependence on λ and $|X_\mu^\flat|$

To show that λ and $|X_\mu^\flat|$ influence the phase identically, we start with the expression for the primitive state:

$$\psi'(|X_\mu^\flat|) = \left(\zeta + \frac{\eta}{\lambda + |X_\mu^\flat|}\right)^{\theta(\lambda+|X_\mu^\flat|)}.$$

Observe that the term $(\lambda + |X_\mu^\flat|)$ functions as a single input variable, which scales the phase of ψ' proportionally. This implies that variations in λ shift the phase of ψ' in the same way as variations in $|X_\mu^\flat|$ would. Thus, we can apply the proven rate of rotation for $|X_\mu^\flat|$ to changes in λ and conclude that λ affects the phase identically.

Determining Zero-Point Orientation Using Rate of Rotation and Handedness

Since λ influences the phase in the same manner as $|X_\mu^\flat|$, we can use the rate of rotation and handedness (established for $|X_\mu^\flat|$) to describe the phase when

$|X_\mu^\flat| = 0$. Specifically: 1. The rate of rotation determines how rapidly the phase advances, completing one cycle every $1/r$ units. 2. Handedness (determined by the sign of the rotation rate) defines the direction of phase change, with left-handed states advancing in one direction and right-handed states in the opposite.

Thus, the zero-point orientation (the phase at $|X_\mu^\flat| = 0$) depends on the values of λ and handedness:

- For a rate of rotation r, the state completes one phase cycle every $\frac{1}{r}$ units.
- The handedness dictates the direction of phase progression.

This results in a consistent zero-point orientation for each primitive state, defined by λ and rate of rotation.

Zero-Point Orientation of the Standard Wavefunction

For the standard wavefunction, expressed as:

$$\psi_{\text{standard}}(x) = e^{ikx} = \cos(kx) + i\sin(kx),$$

the zero-point orientation represents the phase of the function at $x = 0$.

Evaluating at $x = 0$:

$$\psi_{\text{standard}}(0) = e^{i\cdot k\cdot 0} = \cos(0) + i\sin(0) = 1.$$

Thus, the zero-point orientation of the standard wavefunction is consistently 1. This property holds regardless of the wavefunction's frequency or wavelength, as the phase at $x = 0$ remains anchored at 1.

Conclusion: The standard wavefunction has a fixed zero-point orientation of 1. This aligns with the property in primitive states, where we define zero-point orientation as the initial phase reference for calculating phase evolution. This correspondence supports the isomorphism between the standard wavefunction and primitive states in terms of zero-point orientation.

Completeness of Primitive State Properties

To prove that the properties identified in the primitive states fully describe the states, we consider the two fundamental aspects of a complex function: phase and magnitude. We show that each aspect can be fully determined by the properties we have established, making these properties complete descriptors of the primitive state.

Phase: Determined by Rate of Rotation, Zero-Point Orientation, and Peak Offset

The phase of a complex function at a given point can be described if we know: 1. **Rate of Rotation**: This property tells us how the phase evolves with respect to changes in the input, including any scalars (e.g., k or P) that affect the input. For a primitive state, the rate of rotation describes how quickly the phase completes a full cycle over the input range. Thus, if we know the rate of rotation, we know the scaling of phase evolution across inputs. 2. **Zero-Point Orientation**: This property establishes the phase of the state at $|X_\mu^\flat| = 0$, giving a starting reference for calculating the phase at any other point. 3. **Peak Offset**: The peak offset identifies the location of the first peak in the input range. Together with the zero-point orientation, this allows us to map the phase at other input values accurately.

Conclusion on Phase Completeness Knowing the rate of rotation, zero-point orientation, and peak offset enables a complete description of the phase for the primitive state. Given the zero-point phase, the first peak, and the phase evolution rate, we can calculate the phase at any input value, confirming that these properties are exhaustive for defining the phase.

Magnitude: Determined by Magnitude Scale and Curvature Type

For the magnitude, we identify two key components: 1. **Magnitude Scale**: This property represents the macro scale of the state's magnitude. For a flat state (as used in this proof), the magnitude scale alone suffices to describe the amplitude over the input range. Since physical states we consider are normalized to a magnitude scale of 1, this is directly comparable to the standard wavefunction's magnitude. 2. **Curvature Type**: While not relevant for flat states, the curvature type represents small-scale deviations from the magnitude scale due to curvature, adding detail in curved contexts. For the flat case, we can set aside the curvature type, making the magnitude scale alone the defining factor.

Conclusion on Magnitude Completeness The magnitude scale, in the absence of curvature, fully describes the state's magnitude. For physical states, the normalized magnitude scale of 1 matches that of the standard wavefunction. Thus, the magnitude is exhaustively described by the magnitude scale in the flat case.

Appendix LETTER

Completeness Proof for Flat States

In a flat configuration, the rate of rotation, zero-point orientation, peak offset, and magnitude scale completely describe the primitive state. These properties correspond to the core components of a complex function—phase and magnitude—and there are no additional independent properties needed to characterize the state. Therefore, we conclude that these properties provide a complete description of the primitive state in the flat case.

Position Magnitude in Primitive States

In this theory, the primitive position state $\underline{\psi}'(|X_\mu^\flat|)$ provides a natural position magnitude that grows linearly with the input $|X_\mu^\flat|$. Unlike the standard position eigenfunction in conventional quantum mechanics, where position is scaled by an operator, this model incorporates the scaling directly into the state itself.

Formulation of the Position Magnitude

The primitive position state is defined as:

$$\underline{\psi}'(|X_\mu^\flat|) = \Lambda^2 \left(\left(1 + \frac{1}{\Lambda + |X_\mu^\flat|}\right)^{(\Lambda + |X_\mu^\flat|)} - \left(1 + \frac{1}{\Lambda - |X_\mu^\flat|}\right)^{(\Lambda - |X_\mu^\flat|)} \right)$$

where Λ is a large parameter approaching infinity, acting as a "flattening" factor. The goal is to demonstrate that, as $\Lambda \to \infty$, the state behaves such that the output magnitude scales with $|X_\mu^\flat|$.

Analyzing the Magnitude Scaling

The difference between the terms in the expression above leads to a position magnitude that approximates $|X_\mu^\flat|$, specifically:

$$\lim_{\Lambda \to \infty} \left(\left(1 + \frac{1}{\Lambda + |X_\mu^\flat|}\right)^{(\Lambda + |X_\mu^\flat|)} - \left(1 + \frac{1}{\Lambda - |X_\mu^\flat|}\right)^{(\Lambda - |X_\mu^\flat|)} \right) = \frac{e|X_\mu^\flat|}{\Lambda^2}.$$

This relationship holds because: 1. As Λ grows, $\left(1 + \frac{1}{\Lambda + |X_\mu^\flat|}\right)^{(\Lambda + |X_\mu^\flat|)}$ becomes slightly larger, while $\left(1 + \frac{1}{\Lambda - |X_\mu^\flat|}\right)^{(\Lambda - |X_\mu^\flat|)}$ becomes slightly smaller. 2. The difference between these two terms is directly proportional to $|X_\mu^\flat|$ and scales with $\frac{1}{\Lambda^2}$.

Position Magnitude in the Flat Limit

In the limit as $\Lambda \to \infty$, the Λ^2 factor outside the difference restores the magnitude scale to match the original states. Thus, in the absence of curvature, this primitive state's structure results in an output magnitude that is identical to the input, effectively making:

$$\lim_{\Lambda \to \infty} \underline{\psi}'(|X_\mu^\flat|) = |X_\mu^\flat|,$$

which is the desired behavior for a position state.

In this way, the position magnitude is inherently encoded in the structure of the state, scaling linearly with the input $|X_\mu^\flat|$ in flat conditions.

Appendix LETTER

Proof of Isomorphism Between Primitive States and the Standard Wavefunction

We aim to prove that, under appropriate conditions and definitions, the primitive states defined in this theory are isomorphic to the standard quantum mechanical wavefunction

$$\psi_{\text{standard}}(x) = e^{i(kx - \omega t)}.$$

Definition of Isomorphism

In this context, we define two wavefunctions as isomorphic if:

- They exhibit the same magnitude behavior under scaling.
- Their phase evolution (rate of rotation) can be matched by adjusting scalar parameters.
- Variations in zero-point orientation and peak offset can be accounted for through phase factors without affecting the isomorphism.

Magnitude Scale

Both the primitive states and the standard wavefunction have a magnitude scale of 1 when defined in flat space, meaning they both yield constant magnitude over time and space in the absence of curvature. This aligns with the amplitude invariance in the standard wavefunction.

$$|\psi'| = 1 \quad \text{and} \quad |\psi_{\text{standard}}| = 1.$$

Rate of Rotation

The primitive states have rates of rotation (cycles per unscaled input) of ± 2 or ± 4. In comparison, the standard wavefunction has a rate of rotation of $\frac{1}{2\pi}$, meaning it completes one cycle every 2π increase in kx or ωt.

To reconcile these, we introduce a scaling factor for the wavenumber k or angular frequency ω, which effectively brings the rates of rotation into alignment. This scaling factor allows us to map the rate of rotation in primitive states to that in the standard wavefunction, ensuring a one-to-one correspondence.

Thus, the rate of rotation property does not inhibit the isomorphism, as it can be adjusted by scaling k or ω.

Phase Factors: Zero-Point Orientation and Peak Offset

The zero-point orientation (initial phase) and peak offset (location of the first phase extremum) vary between the primitive states and the standard wavefunction. However, these differences can be corrected by introducing a global phase factor, effectively aligning the two wavefunctions' initial phases.

Since isomorphism allows for phase adjustments, these phase differences do not impact the structural equivalence between the primitive states and the standard wavefunction.

Absence of Curvature

In this proof, we restrict our consideration to flat primitive states, which align with the curvature-free nature of the standard wavefunction. This ensures both wavefunctions are defined without deviations due to curvature, further supporting their structural equivalence.

Conclusion

Given the properties outlined:

- Both wavefunctions have identical magnitude behavior.
- Their rates of rotation can be aligned via a scaling factor.
- Variations in zero-point orientation and peak offset can be corrected with phase factors.

Therefore, by the definition of isomorphism, the primitive states are isomorphic to the standard wavefunction, with any differences accounted for through permissible transformations (scaling and phase adjustments).

Appendix LETTER

Proof of Isomorphism Between Primitive Position States and Position Eigenfunctions

We aim to prove that, under appropriate conditions, the primitive position states defined in this theory are isomorphic to the position eigenfunctions in standard quantum mechanics, specifically the form $x\psi(x)$, where the wavefunction is scaled by the spatial coordinate x.

Defining Position Magnitude

In the context of this theory, a primitive position state $\underline{\psi}'(|X_\mu^\flat|)$ possesses a natural position magnitude that grows linearly with the input $|X_\mu^\flat|$. Unlike the standard approach where position scaling is applied by an operator, here the scaling is inherently encoded in the state itself.

For a primitive position state, we define:

$$\underline{\psi}'(|X_\mu^\flat|) = \Lambda^2 \left(\left(1 + \frac{1}{\Lambda + |X_\mu^\flat|}\right)^{(\Lambda + |X_\mu^\flat|)} - \left(1 + \frac{1}{\Lambda - |X_\mu^\flat|}\right)^{(\Lambda - |X_\mu^\flat|)} \right)$$

where Λ is a large parameter approaching infinity, which acts as a "flattening" factor. As $\Lambda \to \infty$, this formulation yields:

$$\lim_{\Lambda \to \infty} \underline{\psi}'(|X_\mu^\flat|) = |X_\mu^\flat| \psi',$$

establishing that $\underline{\psi}'$ grows linearly with the input $|X_\mu^\flat|$.

Position Magnitude as a Distinct Property

The existence of a position magnitude property distinguishes primitive position states from other states that do not scale with $|X_\mu^\flat|$. For instance, a primitive state $\psi'(|X_\mu^\flat|)$ without position magnitude behaves as a standard wavefunction, whereas a primitive position state $\underline{\psi}'(|X_\mu^\flat|)$ behaves as a scaled wavefunction with the factor $|X_\mu^\flat|$. Thus, a primitive position state can be seen as structurally equivalent to $x\psi(x)$ in conventional quantum mechanics, where the spatial coordinate x serves as a scaling factor.

Unified Input and Relationship to x

In this theory, we define a unified input ξ that integrates space, time, and scalar components, allowing $X_\mu^\flat$ to represent position-related components t', x', S', etc., each scaled by E, p, m respectively. Through this scaling relationship:

$$x' = p \cdot x,$$

where x' represents a momentum-scaled coordinate that maintains equivalence to x in standard quantum mechanics. This scaling effectively bridges $|X_\mu^\flat|$ with the conventional spatial coordinate x and ensures that the position eigenfunction property holds in the mapping.

Isomorphism Properties

Based on the prior proofs of property, we observe: - **Magnitude Scale**: The primitive position state and the position eigenfunction $x\psi(x)$ share a similar scaling with x, enabling an isomorphic mapping. - **Phase Evolution**: The phase of the primitive states and the standard wavefunction evolve in isomorphic ways, allowing alignment under scalar adjustments. - **Phase Adjustments**: Differences in zero-point orientation and peak offset can be corrected using phase factors, aligning the two wavefunctions. - **Absence of Curvature**: We restrict consideration to flat primitive states to ensure both states are curvature-free, aligning with the standard wavefunction.

Conclusion

The primitive position states defined in this theory are isomorphic to the position eigenfunctions in standard quantum mechanics. This isomorphism is achieved because: 1. Both states exhibit position scaling through their respective input coordinates. 2. The phase properties align under scalar adjustments, and phase factors can account for any variations in orientation. 3. The flat nature of the primitive position state aligns with the structure of the position eigenfunction.

By these properties, we establish the isomorphism between primitive position states and position eigenfunctions in the absence of curvature.

Proof of Magnitude Properties in Epm States

In this proof, we aim to show that, given our definitions, the components of Epm states exhibit magnitudes corresponding to E, p, and m when evaluated in a flat space without curvature.

1. Definition of Position States with Magnitude of X'

By definition, primitive position states $\underline{\psi}'$ exhibit a position magnitude corresponding to the input scaled by the appropriate factor. Specifically:

$$\underline{\psi}'(|X_\mu^\flat|) \sim |X'_\mu|,$$

where X'_μ represents the scaled input in terms of E, p, or m. For instance:

- $t' = E \cdot t$,
- $x' = p_x \cdot x$,

- $S' = m \cdot S$.

Thus, in flat space without curvature, the magnitude of a position state component is X' for each respective input.

2. Definition of ϕ as a Scalar State

We define the scalar state ϕ as a primitive state with a unit magnitude:

$$\phi \in \{\psi \mid |P_\mu^\flat|_\mu = 1 \text{ for all } \mu\}.$$

In this configuration, ϕ effectively returns the input as the output magnitude, providing a reference state with no scaling effect from E, p, or m.

3. Obtaining Epm Magnitudes by Dividing Position States by ϕ

When we divide a position state $\underline{\psi}'(|X_\mu^\flat|)$ by the scalar state ϕ, we isolate the scaling factor associated with E, p, or m for each component:

$$\frac{\underline{\psi}'(|X_\mu^\flat|)}{\phi} = \frac{X'_\mu}{|X_\mu^\flat|}.$$

This yields:

- For $t' = E \cdot t$, we have $\frac{t'}{t} = E$.
- For $x' = p_x \cdot x$, we have $\frac{x'}{x} = p_x$.
- For $S' = m \cdot S$, we have $\frac{S'}{S} = m$.

Thus, each component of the state, when divided by ϕ, exhibits a magnitude equal to E, p_x, m, etc., depending on the associated factor in X'_μ.

Conclusion

We have shown that, by construction, the components of Epm states yield magnitudes directly proportional to E, p, and m when evaluated in flat space. This follows from the definitions of the position states and the scalar state ϕ, confirming that each component's magnitude is governed by the corresponding energy, momentum, or mass factor.

Appendix LETTER

Illustrative Proof of Scalar Result from Quaternionic Matrix Product

This section demonstrates, through example and structural analysis, that the product of the matrix G with its conjugate G^* yields a scalar value. Specifically,

we consider the matrix G defined as follows:

$$G = \frac{1}{2} \begin{pmatrix} \Psi & -i\Psi & -j\Psi & -k\Psi \\ i\Psi & \Psi & k\Psi & -j\Psi \\ j\Psi^* & -k\Psi^* & \Psi^* & i\Psi^* \\ k\Psi^* & j\Psi^* & -i\Psi^* & \Psi^* \end{pmatrix},$$

where Ψ represents a complex phase factor. The goal is to show that $(G^*|G) = 1$ under the defined inner product operation, indicating a scalar result.

Matrix Multiplication: Row and Column Products

To illustrate the process, we begin by calculating the entry in the first row and first column of G^*G. For simplicity, we substitute a specific phase factor $\Psi = 0.72 + 0.72i$ and its conjugate $\Psi^* = 0.72 - 0.72i$.

First Row, First Column Calculation

1. **Term-by-Term Expansion**: The entry in the first row and first column of G^*G is computed as:

$$(G^*G)_{11} = (\Psi^*\Psi) + ((-i\Psi^*)(i\Psi)) + ((-j\Psi^*)(j\Psi)) + ((-k\Psi^*)(k\Psi)).$$

2. **Evaluating Each Product**: - $\Psi^*\Psi = (0.72 - 0.72i)(0.72 + 0.72i) = 1$, - $(-i\Psi^*)(i\Psi) = 1$, - $(-j\Psi^*)(j\Psi) = 1$, - $(-k\Psi^*)(k\Psi) = 1$.

Summing these terms, we obtain:

$$(G^*G)_{11} = 1 + 1 + 1 + 1 = 4.$$

3. **Normalization Factor**: Since each term contributes a factor of $\frac{1}{4}$ from the $\frac{1}{2}$ prefactor in G, the normalized result is:

$$\frac{1}{4} \times 4 = 1.$$

Cross-Term Cancellation

The matrix structure of G ensures that cross-terms involving different quaternion components cancel out. For instance:

$$(-i\Psi^*)(j\Psi) \quad \text{and} \quad (i\Psi^*)(-j\Psi)$$

cancel due to their opposite phases and directions in quaternion space.

Interpretation of Scalar Result

This process extends to all diagonal entries of G^*G, each yielding a value of 1 due to similar conjugate pairings and cancellations. Thus, the summation over all rows and columns under the inner product operation results in a scalar value of 1.

Conclusion: Quaternionic Symmetry in Matrix Product

This calculation illustrates that the quaternionic structure of G and G^* enables consistent cancellation of non-scalar terms, leaving a real scalar result. By carefully pairing terms in the matrix, we demonstrate that $(G^*|G) = 1$, as required for normalization in the context of quaternionic phase evolution.

Off-Diagonal Calculation: First Row, Second Column

To illustrate why the off-diagonal terms are zero, let us examine the calculation for the entry in the first row and second column of G^*G.

The entry in the first row, second column is computed as:

$$(G^*G)_{12} = (\Psi^*(-i\Psi)) + ((-i\Psi^*)\Psi) + ((-j\Psi^*)(k\Psi)) + ((-k\Psi^*)(-j\Psi)) \,.$$

1. **Evaluate Each Product**: - $\Psi^*(-i\Psi) = (0.72 - 0.72i)(-i)(0.72 + 0.72i) = -i$, - $(-i\Psi^*)\Psi = (-i)(0.72 - 0.72i)(0.72 + 0.72i) = i$, - $(-j\Psi^*)(k\Psi) = (-j)(0.72-0.72i)(k)(0.72+0.72i) = -k$, - $(-k\Psi^*)(-j\Psi) = (-k)(0.72-0.72i)(-j)(0.72+0.72i) = k$.

2. **Summing Terms**: Adding these terms:

$$-i + i - k + k = 0.$$

Conclusion for Off-Diagonal Entries

As demonstrated, the off-diagonal entries result in zero due to the cancellation of quaternionic cross-terms. This occurs because terms with opposing quaternion components negate each other, leaving no real component in the sum. Therefore, all off-diagonal terms in G^*G are zero, confirming that G^*G is indeed diagonal.

Appendix LETTER

Proof of Scalar Result for $(G + X)^\dagger(G + X)$

In this section, we aim to prove that $(G + X)^\dagger(G + X)$ yields a scalar result of $2|\Psi|^2$ across the matrix, confirming that the structure of the matrices G and X leads to a consistent scalar outcome. A supplementary proof by example is provided in a Mathematica notebook in the supplementary materials.

Definition of Matrices G and X

The matrices G and X are defined as follows:

$$G = \frac{1}{2}\begin{pmatrix} \Psi & -i\Psi & -j\Psi & -k\Psi \\ i\Psi & \Psi & k\Psi & -j\Psi \\ j\Psi^* & -k\Psi^* & \Psi^* & i\Psi^* \\ k\Psi^* & j\Psi^* & -i\Psi^* & \Psi^* \end{pmatrix}, \quad X = \frac{1}{2}\begin{pmatrix} \Psi & -i\Psi & j\Psi & k\Psi \\ i\Psi & \Psi & -k\Psi & j\Psi \\ -j\Psi^* & k\Psi^* & \Psi^* & i\Psi^* \\ -k\Psi^* & -j\Psi^* & -i\Psi^* & \Psi^* \end{pmatrix}$$

where Ψ and Ψ^* represent complex-valued phase factors, and the quaternionic terms i, j, and k are used in the off-diagonal elements.

Calculation of Diagonal Terms in $(G + X)^\dagger(G + X)$

The matrix $(G + X)^\dagger(G + X)$ consists of products of the terms in G and X, with Ψ conjugated in $(G + X)^\dagger$ but not in $(G + X)$.

1. **Diagonal Consistency**: Each diagonal term in $(G+X)^\dagger(G+X)$ arises from the product of diagonal terms in $(G + X)^\dagger$ and $(G + X)$:

$$((G + X)^\dagger(G + X))_{ii} = |(G + X)_{ii}|^2 = 2|\Psi|^2.$$

This results in each diagonal term contributing $2|\Psi|^2$.

2. **Off-Diagonal Cancellation**: Off-diagonal terms involve products such as $i\Psi$ and $j\Psi$, which, due to the quaternionic structure, cancel out upon summation. For example: - In row 1, column 2, we have terms $(\Psi^*)(i\Psi) + (-i\Psi^*)(\Psi)$, which sum to zero because they are complex conjugates in quaternionic form. - This cancellation pattern extends across all off-diagonal entries due to the antisymmetric arrangement of i, j, k terms in G and X.

Summing and Normalization

With the diagonal terms contributing $2|\Psi|^2$ each, the total for the diagonal elements is:

$$\text{Sum of diagonal terms} = 4 \cdot 2|\Psi|^2 = 8|\Psi|^2.$$

Following the parentheses notation, we normalize by dividing by 4, yielding:

$$\frac{8|\Psi|^2}{4} = 2|\Psi|^2.$$

Conclusion

The symmetry and quaternionic cancellation within $(G + X)^\dagger(G + X)$ lead to a scalar result of $2|\Psi|^2$ across the entire matrix. This proof illustrates that the specific arrangement of terms in G and X guarantees this scalar result. A supplementary Mathematica notebook is included in the materials to provide a proof by example, demonstrating this result in concrete calculations.

Appendix LETTER

Demonstrating Independent Treatment of Scalars in $(G + X)^\dagger(G + X)$

To illustrate that scalar magnitudes associated with the matrices G and X are preserved independently in the computation of $(G + X)^\dagger(G + X)$, we calculate the first entry of the resulting matrix and show how the magnitudes of G and X combine directly in the outcome. This provides insight into the behavior of quaternionic terms with phase factors under conjugation.

Definitions of Ψ, Ψ^*, Φ, and Φ^*

We define:

$$\begin{aligned}
\Psi &= 3(0.72 + 0.72i) = 2.16 + 2.16i, \\
\Psi^* &= 3(0.72 - 0.72i) = 2.16 - 2.16i, \\
\Phi &= 5(0.6 + 0.8i) = 3 + 4i, \\
\Phi^* &= 5(0.6 - 0.8i) = 3 - 4i.
\end{aligned}$$

These values introduce phase factors and magnitudes of 3 and 5 for Ψ and Φ, respectively.

Constructing $G^\dagger$ and $X^\dagger$

Starting with matrices G and X, we obtain their conjugates, $G^\dagger$ and $X^\dagger$, by applying complex conjugation to the wavefunction values Ψ and Φ without altering the quaternion components.

$$G^\dagger = \frac{1}{2}\begin{pmatrix} \Psi^* & -i\Psi^* & -j\Psi^* & -k\Psi^* \\ i\Psi^* & \Psi^* & k\Psi^* & -j\Psi^* \\ j\Psi & -k\Psi & \Psi & i\Psi \\ k\Psi & j\Psi & -i\Psi & \Psi \end{pmatrix}$$

$$X^\dagger = \frac{1}{2}\begin{pmatrix} \Phi^* & -i\Phi^* & j\Phi^* & k\Phi^* \\ i\Phi^* & \Phi^* & -k\Phi^* & j\Phi^* \\ -j\Phi & k\Phi & \Phi & i\Phi \\ -k\Phi & -j\Phi & -i\Phi & \Phi \end{pmatrix}$$

Calculating the First Entry of $(G + X)^\dagger(G + X)$

To calculate the first entry of $(G+X)^\dagger(G+X)$, we add the first rows of $G^\dagger$ and $X^\dagger$ and the first columns of G and X:

$$\text{Row vector} = \frac{1}{2}\left(\Psi^* + \Phi^*, -i(\Psi^* + \Phi^*), (\Phi^* - \Psi^*)j, (\Phi^* - \Psi^*)k\right)$$

$$\text{Column vector} = \frac{1}{2}\begin{pmatrix} \Psi + \Phi \\ i(\Psi + \Phi) \\ (\Psi^* - \Phi^*)j \\ (\Psi^* - \Phi^*)k \end{pmatrix}$$

The first entry is computed by taking the dot product of the row and column vectors:

$$\begin{aligned}
\text{First entry} = &\left(\frac{\Psi^* + \Phi^*}{2}\right)\left(\frac{\Psi + \Phi}{2}\right) + \left(\frac{-i(\Psi^* + \Phi^*)}{2}\right)\left(\frac{i(\Psi + \Phi)}{2}\right) \\
&+ \left(\frac{(\Phi^* - \Psi^*)j}{2}\right)\left(\frac{(\Psi^* - \Phi^*)j}{2}\right) + \left(\frac{(\Phi^* - \Psi^*)k}{2}\right)\left(\frac{(\Psi^* - \Phi^*)k}{2}\right)
\end{aligned}$$

Applying the quaternion rules $i^2 = j^2 = k^2 = -1$, we simplify each term:
1. **First Term:**

$$\left(\frac{\Psi^* + \Phi^*}{2}\right)\left(\frac{\Psi + \Phi}{2}\right) = \frac{1}{4}|\Psi + \Phi|^2$$

2. **Second Term:**

$$\left(\frac{-i(\Psi^* + \Phi^*)}{2}\right)\left(\frac{i(\Psi + \Phi)}{2}\right) = \frac{1}{4}(\Psi^* + \Phi^*)(\Psi + \Phi)$$

3. **Third and Fourth Terms:** These yield the same expression:

$$\frac{1}{4}|\Psi^* - \Phi^*|^2(-1) = \frac{1}{4}|\Psi - \Phi|^2$$

Adding all terms gives:

$$\text{First entry} = \frac{1}{2}|\Psi + \Phi|^2 + \frac{1}{2}|\Psi - \Phi|^2 = |\Psi|^2 + |\Phi|^2.$$

With $|\Psi|^2 = 9$ and $|\Phi|^2 = 25$, we conclude:

$$\text{First entry} = 34$$

Conclusion

This calculation demonstrates that the final result incorporates the magnitudes of both G and X independently. If either G or X is scaled by a factor, this factor appears in the final sum as $G^2 + X^2$. Therefore, the calculation confirms that scalar magnitudes can be treated independently within $(G+X)^\dagger(G+X)$, a result supported by further examples provided in supplementary Mathematica notebooks.

Demonstrating that Entries (1,2) and (2,1) are Opposites in $(G + X)^\dagger(G + X)$

However, we also need to show that the off-diagonals cancel since they will be non-zero. This is shown rigorously in the proof by example, but illustrated anecdotally below.

Step 1: Define Ψ, Ψ^*, Φ, and Φ^*

Given:

$$\begin{aligned}
\Psi &= 3(0.72 + 0.72i) = 2.16 + 2.16i, \\
\Psi^* &= 3(0.72 - 0.72i) = 2.16 - 2.16i, \\
\Phi &= 5(0.6 + 0.8i) = 3 + 4i, \\
\Phi^* &= 5(0.6 - 0.8i) = 3 - 4i.
\end{aligned}$$

Step 2: Construct $G^\dagger$ and $X^\dagger$

For $G^\dagger$:

Replace Ψ with Ψ^* and Ψ^* with Ψ in G, without conjugating the quaternion units.

For $X^\dagger$:

Replace Φ with Φ^* and Φ^* with Φ in X, without conjugating the quaternion units.

Step 3: Form the Row and Column Vectors

Entry (1,2):

- **Row Vector ($\mathbf{Row}_1$):** First row of $(G+X)^\dagger$

$$\mathrm{Row}_1 = \frac{1}{2}\left(\Psi^* + \Phi^*,\ -i(\Psi^* + \Phi^*),\ (\Phi^* - \Psi^*)j,\ (\Phi^* - \Psi^*)k\right).$$

- **Column Vector ($\mathbf{Col}_2$):** Second column of $G+X$

$$\mathrm{Col}_2 = \frac{1}{2}\begin{pmatrix} -i(\Psi + \Phi) \\ \Psi + \Phi \\ -k(\Psi^* + \Phi^*) \\ -j(\Psi^* + \Phi^*) \end{pmatrix}.$$

Entry (2,1):

- **Row Vector ($\mathbf{Row}_2$):** Second row of $(G+X)^\dagger$

$$\mathrm{Row}_2 = \frac{1}{2}\left(i(\Psi^* + \Phi^*),\ \Psi^* + \Phi^*,\ (\Phi^* - \Psi^*)(-k),\ (\Phi^* - \Psi^*)j\right).$$

- **Column Vector ($\mathbf{Col}_1$):** First column of $G+X$

$$\mathrm{Col}_1 = \frac{1}{2}\begin{pmatrix} \Psi + \Phi \\ i(\Psi + \Phi) \\ (\Psi^* - \Phi^*)j \\ (\Psi^* - \Phi^*)k \end{pmatrix}.$$

Step 4: Compute the Entries

Compute the dot products:

$$\mathrm{Entry}_{1,2} = \mathrm{Row}_1 \cdot \mathrm{Col}_2, \quad \mathrm{Entry}_{2,1} = \mathrm{Row}_2 \cdot \mathrm{Col}_1.$$

Due to the properties of quaternion multiplication and the structure of the matrices, we find that:

$$\mathrm{Entry}_{1,2} = -\mathrm{Entry}_{2,1}.$$

Conclusion

The off-diagonal entries $(1,2)$ and $(2,1)$ of $(G+X)^{\dagger}(G+X)$ are opposites, illustrating the antisymmetric property in these positions.

Appendix LETTER

Proofs of Isomorphism and Morphisms for Position and Epm States

This section presents proofs establishing the isomorphism of position states to the Minkowski Spacetime interval (MST), Epm states to the energy-momentum relationship, and the morphism between MST and the energy-momentum relationship via ϕ.

Defining the Transformation ϕ

We define ϕ as a transformation that scales position states, converting them to Epm states. Formally, ϕ is defined by:

$$\phi \in \{\psi \mid |P_{\mu}^{\flat}|_{\mu} = 1 \text{ for all } \mu\}.$$

When $|P_{\mu}^{\flat}|$ is set to 1, ϕ yields quantized values of position, energy, momentum, or mass, depending on the state's rate of rotation. Position versions of ϕ yield a magnitude proportional to the input ξ, such that:

$$|\phi(\xi)| = \xi.$$

Position and Epm States

In this theory, position states ($\underline{\alpha_{\mu}}$) are related to Epm states ($\underline{\underline{\alpha_{\mu}}}$) by the transformation:

$$\underline{\underline{\alpha_{\mu}}} = \frac{\alpha_{\mu}}{\underline{\underline{\phi}}}.$$

Applying ϕ scales each component of the position state according to its associated quantity: time, space, or mass.

The components of $\underline{\alpha_{\mu}}$ are thus associated with $(-t, x, y, z, -S)$, while the components of $\underline{\underline{\alpha_{\mu}}}$ are associated with (E, p_x, p_y, p_z, m). Importantly, S represents a scalar component, and the corresponding Epm component m is defined through growth in $-S$, ensuring that the term $-S^2$ in MST maps to $+m^2$ in the energy-momentum relationship. In both cases, these states must be evaluated at a given input ξ for discrete values that can satisfy MST and the energy-momentum relationship.

Proof A: Isomorphism of Position States to MST

To prove that position states are isomorphic to MST, we consider the position state $\underline{\alpha_\mu}$ evaluated at a discrete input ξ, where each component is scaled by its respective quantity:

$$\underline{\alpha_\mu}(\xi) = (-t, x, y, z, -S).$$

To demonstrate equivalence to the standard form of MST, we compute:

$$-t^2 + x^2 + y^2 + z^2 - S^2 = 0,$$

which aligns with the standard Minkowski Spacetime interval (MST) equation:

$$\eta^{\mu\nu} x_\mu x_\nu = 0.$$

Therefore, if we set:

$$(\underline{\alpha_\mu} \mid \underline{\alpha_\mu}) = 0,$$

this relationship satisfies MST for each discrete value of ξ, confirming that $\underline{\alpha_\mu}$ is isomorphic to MST.

Proof B: Isomorphism of Epm States to the Energy-Momentum Relationship

Now, consider the Epm state $\underline{\underline{\alpha_\mu}}$, defined as:

$$\underline{\underline{\alpha_\mu}}(\xi) = \left(\frac{-t}{\phi}, \frac{x}{\phi}, \frac{y}{\phi}, \frac{z}{\phi}, \frac{-S}{\phi} \right).$$

By construction, the components of $\underline{\underline{\alpha_\mu}}$ scale to yield magnitudes of E, p_x, p_y, p_z, m. Evaluating at a discrete value of ξ, we find:

$$-E^2 + p_x^2 + p_y^2 + p_z^2 + m^2 = 0,$$

which matches the standard energy-momentum relationship:

$$\eta^{\mu\nu} p_\mu p_\nu = 0.$$

Thus, we confirm:

$$(\underline{\underline{\alpha_\mu}} \mid \underline{\underline{\alpha_\mu}}) = 0,$$

which establishes that $\underline{\underline{\alpha_\mu}}$ is isomorphic to the energy-momentum relationship.

Proof C: Morphism Between MST and the Energy-Momentum Relationship via ϕ

We now show that if MST holds for a position state, then the energy-momentum relationship holds for its corresponding Epm state, and vice versa.

1. **Assume MST holds for the position state $\underline{\alpha_\mu}$:**

$$(\underline{\alpha_\mu} \mid \underline{\alpha_\mu}) = -t^2 + x^2 + y^2 + z^2 - S^2 = 0.$$

Applying the transformation ϕ to convert $\underline{\alpha_\mu}$ into $\underline{\underline{\alpha_\mu}}$ scales each component appropriately, yielding magnitudes of E, p_x, p_y, p_z, m such that:

$$(\underline{\underline{\alpha_\mu}} \mid \underline{\underline{\alpha_\mu}}) = -E^2 + p_x^2 + p_y^2 + p_z^2 + m^2 = 0.$$

This satisfies the energy-momentum relationship.

2. **Assume the energy-momentum relationship holds for the Epm state $\underline{\underline{\alpha_\mu}}$:**

$$(\underline{\underline{\alpha_\mu}} \mid \underline{\underline{\alpha_\mu}}) = -E^2 + p_x^2 + p_y^2 + p_z^2 + m^2 = 0.$$

Reversing the transformation (multiplying by ϕ) converts $\underline{\underline{\alpha_\mu}}$ back into $\underline{\alpha_\mu}$, with components scaled as $(-t, x, y, z, -S)$, satisfying:

$$(\underline{\alpha_\mu} \mid \underline{\alpha_\mu}) = -t^2 + x^2 + y^2 + z^2 - S^2 = 0.$$

Therefore, MST is satisfied.

Conclusion

The transformation ϕ establishes a morphism between position states satisfying MST and Epm states satisfying the energy-momentum relationship. By evaluating both states at discrete values of ξ, we ensure that each relationship holds in a way that is consistent across these foundational physical principles.

Appendix LETTER

Proof of the Hamiltonian in Terms of Position and Epm States

This proof establishes a framework for defining the Hamiltonian using linear terms for both time and space components and demonstrates that the time-dependent Schrödinger equation (TDSE) and time-independent Schrödinger equation (TISE) can be derived by distinguishing between H_t (time component) and H_s (space component).

1. Defining Linear Hamiltonian Terms Using Position and Epm States

The Hamiltonian, as a linear operator, can be defined with the position or Epm states in the theses alone, with a standard state in the paren. This setup yields linear terms for both time and space components:

$(\alpha \mid \underline{\alpha})$ defines linear position terms, and
$(\alpha \mid \underline{\underline{\alpha}})$ defines linear Epm terms.

2. Defining the Hamiltonian for the TDSE (Time-Dependent Schrödinger Equation)

In this context, we define the time component of the Hamiltonian as:

$$H_t = (\alpha \mid \underline{\underline{\alpha}}_t),$$

where $\underline{\underline{\alpha}}_t$ represents the time component of the Epm state.

To distinguish kinetic and potential energy contributions, we define:

$$T = (\alpha \mid \underline{\underline{\alpha}}_t^{\text{flat}}),$$
$$V = (\alpha \mid \underline{\underline{\alpha}}_t) - (\alpha \mid \underline{\underline{\alpha}}_t^{\text{flat}}),$$

where: - T corresponds to the expected (or "flat") rate, and - V represents deviation from the expected rate, contributing to changes over time.

Thus, we find that:

$$H_t = T + V.$$

This decomposition aligns with the Hamiltonian in the time-dependent Schrödinger equation, where T represents kinetic energy, V represents potential energy, and H_t represents the total energy, which can vary due to external forces.

3. Defining the Hamiltonian for the TISE (Time-Independent Schrödinger Equation)

Similarly, we define a spatial component Hamiltonian H_s for the time-independent Schrödinger equation, where:

$$H_s = (\alpha \mid \underline{\underline{\alpha}}_s),$$

with $\underline{\underline{\alpha}}_s$ representing the spatial Epm component. The components are then defined as:

$$T = (\alpha \mid \underline{\underline{\alpha}}_s^{\text{flat}}),$$
$$V = (\alpha \mid \underline{\underline{\alpha}}_s) - (\alpha \mid \underline{\underline{\alpha}}_s^{\text{flat}}).$$

This yields:

$$H_s = T + V.$$

The motivation here is that T represents the expected or flat value, while V introduces variations due to forces. This setup reflects the time-independent Hamiltonian operator in quantum mechanics, where the spatial components contribute to the total energy.

4. Relationship Between H_t and H_s

Importantly, H_t and H_s do not equate directly. Instead:

$$H_t^2 = H_s^2,$$

indicating that H_t represents a "hypotenuse" or total energy, while H_s is composed of components that correspond to the "legs" (spatial contributions) in an analogous geometric relationship.

By squaring H_t and H_s, we obtain a quadratic form that corresponds to the energy-momentum relationship:

$$(\underline{\underline{\alpha_\mu}} \mid \underline{\underline{\alpha_\mu}}) = -E^2 + p_x^2 + p_y^2 + p_z^2 + m^2 = 0.$$

In the absence of curvature, we find:

$$H_s^2 - H_t^2 = 0,$$

which aligns directly with the energy-momentum relationship, demonstrating the consistency of this Hamiltonian definition with foundational quantum mechanics.

Conclusion

The above framework shows that: 1. Linear terms for the Hamiltonian (H_t and H_s) yield consistent interpretations with the time-dependent and time-independent Schrödinger equations. 2. Squaring the Hamiltonian terms leads to a quadratic form consistent with the energy-momentum relationship. 3. In the absence of curvature, $H_s^2 = H_t^2$, affirming the equivalence of these forms when evaluated under flat conditions.

1 Full Tables of Primitive States

This appendix provides the complete version of all tables referenced throughout the main body of the text. Below is the full table of the properties for primitive wavefunctions, which shows all 40 unique combinations of the parameters ζ, η, and θ, along with their associated physical properties.

2 Full Tables of Curvature Types

This appendix provides the complete version of all tables referenced throughout the main body of the text. Below is the full table of curvature types for different values of ν.

Notation	ζ	η	θ	Rate of Rotation	Curvature Type	Magnitude Scale	Zero-Point Orientation	Peak Offset
$\overleftarrow{\psi}'_{(-A)\odot}$	i	i	1	-4	$-A$	e	1	0
$\overrightarrow{\psi}'_{(A)\odot}$	i	$-i$	-1	4	A	e	1	0
$\overleftarrow{\psi}'_{(A)\odot}$	$-i$	i	-1	-4	A	e	1	0
$\overrightarrow{\psi}'_{(-A)\odot}$	$-i$	$-i$	1	4	$-A$	e	1	0
$\overrightarrow{\psi}'_{(A)\odot}$	-1	1	-1	2	A	e	1	0
$\overleftarrow{\psi}'_{(-A)\odot}$	-1	-1	1	-2	$-A$	e	1	0
$\overleftarrow{\psi}'_{(B)\vee}$	i	1	1	-4	B	1	1	$-1/e$
$\overrightarrow{\psi}'_{(-B)\wedge}$	i	1	-1	4	$-B$	1	1	$1/e$
$\overleftarrow{\psi}'_{(B)\wedge}$	i	-1	1	-4	B	1	1	$1/e$
$\overrightarrow{\psi}'_{(C)\vee}$	i	-1	-1	4	C	1	1	$-1/e$
$\overleftarrow{\psi}'_{(C)\vee}$	$-i$	1	-1	-4	C	1	1	$-1/e$
$\overrightarrow{\psi}'_{(C)\wedge}$	$-i$	1	1	4	C	1	1	$1/e$
$\overrightarrow{\psi}'_{(B)\vee}$	$-i$	-1	1	4	B	1	1	$-1/e$
$\overleftarrow{\psi}'_{(-B)\wedge}$	$-i$	-1	-1	-4	$-B$	1	1	$1/e$
$\overleftarrow{\psi}'_{(B)\vee}$	-1	i	1	-2	B	1	1	$-1/e$
$\overrightarrow{\psi}'_{(-B)\wedge}$	-1	i	-1	2	$-B$	1	1	$1/e$
$\overrightarrow{\psi}'_{(B)\wedge}$	-1	$-i$	1	2	B	1	1	$1/e$
$\overleftarrow{\psi}'_{(-B)\vee}$	-1	$-i$	-1	-2	$-B$	1	1	$-1/e$
$\overrightarrow{\psi}'_{(C)\oslash}$	i	i	-1	4	C	$1/e$	1	0
$\overleftarrow{\psi}'_{(-C)\oslash}$	i	$-i$	1	-4	$-C$	$1/e$	1	0
$\overrightarrow{\psi}'_{(-C)\oslash}$	$-i$	i	1	4	$-C$	$1/e$	1	0
$\overleftarrow{\psi}'_{(-A)\oslash}$	$-i$	$-i$	-1	-4	$-A$	$1/e$	1	0
$\overleftarrow{\psi}'_{(-C)\oslash}$	-1	1	1	-2	$-C$	$1/e$	1	0
$\overrightarrow{\psi}'_{(C)\oslash}$	-1	-1	-1	2	C	$1/e$	1	0
$\overrightarrow{\overline{\psi}}'_{(-A)\odot}$	1	1	1	0	$-A$	e	1	0
$\overrightarrow{\overline{\psi}}'_{(-D)\odot}$	1	i	$-i$	0	$-D$	e	1	0
$\overrightarrow{\overline{\psi}}'_{(A)\odot}$	1	-1	-1	0	A	e	1	0
$\overrightarrow{\overline{\psi}}'_{(-D)\odot}$	1	$-i$	i	0	$-D$	e	1	0
$\overline{\psi}'_{(F)\wedge}$	1	1	i	0	F	1	1	$1/e$
$\overline{\overline{\psi}}'_{(F)\vee}$	1	1	$-i$	0	F	1	1	$-1/e$
$\overline{\overline{\psi}}'_{(B)\wedge}$	1	i	1	0	B	1	1	$1/e$
$\overrightarrow{\overline{\psi}}'_{(-B)\vee}$	1	i	-1	0	$-B$	1	1	$-1/e$
$\overrightarrow{\overline{\psi}}'_{(B)\vee}$	1	$-i$	1	0	B	1	1	$-1/e$
$\overrightarrow{\overline{\psi}}'_{(-B)\wedge}$	1	$-i$	-1	0	$-B$	1	1	$1/e$
$\overline{\overline{\psi}}'_{(F)\vee}$	1	-1	i	0	F	1	1	$-1/e$
$\overline{\overline{\psi}}'_{(F)\wedge}$	1	-1	$-i$	0	F	1	1	$1/e$
$\overline{\overline{\psi}}'_{(C)\oslash}$	1	1	-1	0	C	$1/e$	1	0
$\overline{\overline{\psi}}'_{\oslash}$	1	i	i	0	0	$1/e$	1	0
$\overrightarrow{\overline{\psi}}'_{(-C)\oslash}$	1	-1	1	0	$-C$	$1/e$	1	0
$\overline{\overline{\psi}}'_{\oslash}$	1	$-i$	$-i$	0	0	$1/e$	1	0

Table 1: This table gives the resulting properties of a given set of Z, H, and Θ.

3 Full Tables of Primitive State Properties

This appendix provides the full version of all tables referenced in the main body. Below is the complete table showing the properties of wavefunctions for various combinations of parameters Z, H, and Θ.

4 Comprehensive definitions of various subsets of ψ using set theory.

5 wavefunction-symbols and Terms

Notation	Z	H	Θ	Rate of Rotation (κ)	Curvature Type	Magnitude Scale	Zero-Point Orientation	Peak Offset
$\underleftarrow{\underrightarrow{\psi}}_{(-a)\odot}$	i	i	1	-4	$-a$	e	1	0
$\underrightarrow{\underleftarrow{\psi}}_{(a)-1\odot}$	i	$-i$	-1	4	a	e	-1	0
$\underleftarrow{\underleftarrow{\psi}}_{(a)-1\odot}$	$-i$	i	-1	-4	a	e	-1	0
$\underrightarrow{\underrightarrow{\psi}}_{(-a)\odot}$	$-i$	$-i$	1	4	$-a$	e	1	0
$\underleftarrow{\underrightarrow{\psi}}_{(b)-1\vee}$	i	1	1	-4	b	1	-1	$-1/e$
$\underrightarrow{\underleftarrow{\psi}}_{(-b)\wedge}$	i	1	-1	4	$-b$	1	1	$1/e$
$\underleftarrow{\underleftarrow{\psi}}_{(b)-1\wedge}$	i	-1	1	-4	b	1	-1	$1/e$
$\underrightarrow{\underrightarrow{\psi}}_{(-b)\vee}$	i	-1	-1	4	$-b$	1	1	$-1/e$
$\underleftarrow{\underrightarrow{\psi}}_{(b)-1\wedge}$	$-i$	1	1	4	b	1	-1	$1/e$
$\underrightarrow{\underrightarrow{\psi}}_{(-b)\vee}$	$-i$	1	-1	4	$-b$	1	1	$-1/e$
$\underleftarrow{\underleftarrow{\psi}}_{(b)-1\vee}$	$-i$	-1	1	4	b	1	-1	$-1/e$
$\underrightarrow{\underleftarrow{\psi}}_{(-b)\wedge}$	$-i$	-1	-1	-4	$-b$	1	1	$1/e$
$\underleftarrow{\underrightarrow{\psi}}_{(-c)-1\oslash}$	i	i	-1	4	$-c$	$1/e$	-1	0
$\underrightarrow{\underrightarrow{\psi}}_{(c)\oslash}$	i	$-i$	1	4	c	$1/e$	1	0
$\underleftarrow{\psi}_{(c)\oslash}$	$-i$	i	1	4	c	$1/e$	1	0
$\underleftarrow{\underleftarrow{\psi}}_{(-c)-1\oslash}$	$-i$	$-i$	-1	-4	$-c$	$1/e$	-1	0
$\underrightarrow{\underrightarrow{\psi}}_{(-a)1\odot}$	1	1	1	0	$-a$	e	1	0
$\underleftarrow{\underrightarrow{\psi}}_{(-d)-i\odot}$	1	i	$-i$	0	$-d$	e	$-i$	0
$\underrightarrow{\underleftarrow{\psi}}_{(a)-1\odot}$	1	-1	-1	0	a	e	-1	0
$\underrightarrow{\underrightarrow{\psi}}_{(-d)i\odot}$	1	$-i$	i	0	$-d$	e	i	0
$\underleftarrow{\underrightarrow{\psi}}_{(-g)-i\wedge}$	1	1	i	0	$-g$	1	$-i$	$1/e$
$\underleftarrow{\underrightarrow{\psi}}_{(-g)i\vee}$	1	1	$-i$	0	$-g$	1	i	$-1/e$
$\underleftarrow{\underleftarrow{\psi}}_{(b)\wedge}$	1	i	1	0	b	1	-1	$1/e$
$\underleftarrow{\underleftarrow{\psi}}_{(-b)-1\vee}$	1	i	-1	0	$-b$	1	1	$-1/e$
$\underrightarrow{\underleftarrow{\psi}}_{(g)-1\vee}$	1	-1	i	0	g	1	$-i$	$-1/e$
$\underrightarrow{\underleftarrow{\psi}}_{(g)\wedge}$	1	-1	$-i$	0	g	1	i	$1/e$
$\underleftarrow{\underrightarrow{\psi}}_{(b)-1\vee}$	1	$-i$	1	0	b	1	-1	$-1/e$
$\underrightarrow{\underleftarrow{\psi}}_{(-b)\wedge}$	1	$-i$	-1	0	$-b$	1	1	$1/e$
$\underleftarrow{\underrightarrow{\psi}}_{(-c)-1\oslash}$	1	1	-1	0	$-c$	$1/e$	-1	0
$\underrightarrow{\underrightarrow{\psi}}_{(f)i\oslash}$	1	i	i	0	f	$1/e$	i	0
$\underrightarrow{\underrightarrow{\psi}}_{(c)\oslash}$	1	-1	1	0	c	$1/e$	1	0
$\underleftarrow{\underrightarrow{\psi}}_{(f)-i\oslash}$	1	$-i$	$-i$	0	f	$1/e$	$-i$	0

Table 2: Full table of wavefunction properties for various combinations of parameters Z, H, and Θ.

Symbol	Set-Theoretic Definition
ψ	Formed by multiplying/dividing Ψ' and $\underline{\Psi}'$, and potentially adding the results.
$\tilde{\psi}$	$\tilde{\psi} \in \{\psi \mid \text{rate of rotation} \in \{0, \pm\frac{1}{2}, \pm 1\}\}$
$\overline{\psi}$	$\overline{\psi} \in \{\psi \mid \text{rate of rotation} = 0\}$
$\overset{\rightharpoonup}{\psi}$	$\overset{\rightharpoonup}{\psi} \in \{\psi \mid \text{rate of rotation} = +\frac{1}{2}\}$
$\overset{\leftharpoondown}{\psi}$	$\overset{\leftharpoondown}{\psi} \in \{\psi \mid \text{rate of rotation} = -\frac{1}{2}\}$
$\overset{\rightarrow}{\psi}$	$\overset{\rightarrow}{\psi} \in \{\psi \mid \text{rate of rotation} = +1\}$
$\overset{\leftarrow}{\psi}$	$\overset{\leftarrow}{\psi} \in \{\psi \mid \text{rate of rotation} = -1\}$
$\overset{\leftrightarrow}{\psi}$	$\overset{\leftrightarrow}{\psi} \in \{\psi \mid \text{rate of rotation} = \pm 1\}$
$\overset{\leftharpoonup\rightharpoondown}{\psi}$	$\overset{\leftharpoonup\rightharpoondown}{\psi} \in \{\psi \mid \text{rate of rotation} = \pm\frac{1}{2}\}$
$\psi^\flat$	$\psi^\flat \in \{\psi \mid \text{has an exceedingly high value of } \lambda \text{ or } \Lambda \text{ such that curvature approaches } 0\}$
Ψ_p	$\Psi_p \in \{\psi \mid \text{represents the wavefunction of particle } p \text{ with unknown parameters}\}$
ϕ	$\phi \in \{\psi \mid P_\mu = 1 \text{ for all } \mu\}$
$\underline{\psi}$	$\underline{\psi} \in \{\psi \mid \text{has a non-multiplicative position magnitude}\}$
$\underline{\underline{\psi}}$	$\underline{\underline{\psi}} \in \{\psi \mid \underline{\psi}/\overline{\phi}\}$
$\psi_{(U_1,U_2,\dots)}$	$\psi_{(U_1,U_2,\dots)} \in \{\psi \mid \text{has curvature properties denoted by } U_1, U_2, \dots\}$

Table 3: Comprehensive definitions of various subsets of ψ using set theory.

Symbol	Set-Theoretic Definition
ψ	The composite wavefunction.
$\tilde{\psi}$	Composite wavefunction with any arbitrary rate of rotation between $+1$ and -1.
$\overline{\psi}$	Composite scalarfunction (with a rate of rotation of 0).
$\overset{\rightharpoonup}{\psi}$	Composite wavefunction with a rate of rotation of $+\frac{1}{2}$.
$\overset{\leftharpoondown}{\psi}$	Composite wavefunction with a rate of rotation of $-\frac{1}{2}$.
$\overset{\rightarrow}{\psi}$	Composite wavefunction with a rate of rotation of $+1$.
$\overset{\leftarrow}{\psi}$	Composite wavefunction with a rate of rotation of -1.
$\overset{\leftrightarrow}{\psi}$	Composite wavefunction with a rate of rotation of ± 1.
$\overset{\leftharpoonup\rightharpoondown}{\psi}$	Composite wavefunction with a rate of rotation of $\pm\frac{1}{2}$.
$\psi^\flat$	Composite wavefunction with no curvature.
Ψ_p	Particle wavefunction.
ϕ	Scalar wavefunction ϕ.
$\underline{\psi}$	Composite position wavefunction.
$\underline{\underline{\psi}}$	Composite Epm wavefunction.
$\psi_{(U_1,U_2,\dots)}$	Composite wavefunction with curvatures $U_1, U_2, \dots$.

Table 4: Comprehensive definitions of wavefunction symbols.