The Theory of Isomorphic Physics
Part 3: The Dualistic Algebra

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September 29, 2024

The Dualistic Group

To define more informative transformation properties, we introduce the wavefunction Ψ placed on quaternion-valued group elements that form both a group and an algebra. The group elements are represented as matrices, where the real and imaginary parts of Ψ interact with quaternionic units i, j, and k. These group elements exhibit specific symmetries, enabling rich algebraic structure:

$$G = \frac{1}{2}\begin{pmatrix} \Psi & -i\Psi & -j\Psi & -k\Psi \\ i\Psi & \Psi & k\Psi & -j\Psi \\ j\Psi^* & -k\Psi^* & \Psi^* & i\Psi^* \\ k\Psi^* & j\Psi^* & -i\Psi^* & \Psi^* \end{pmatrix}, \quad X = \frac{1}{2}\begin{pmatrix} \Psi & -i\Psi & j\Psi & k\Psi \\ i\Psi & \Psi & -k\Psi & j\Psi \\ -j\Psi^* & k\Psi^* & \Psi^* & i\Psi^* \\ -k\Psi^* & -j\Psi^* & -i\Psi^* & \Psi^* \end{pmatrix}$$

$$Y = \frac{1}{2}\begin{pmatrix} \Psi & i\Psi & -j\Psi & k\Psi \\ -i\Psi & \Psi & -k\Psi & -j\Psi \\ j\Psi^* & k\Psi^* & \Psi^* & -i\Psi^* \\ -k\Psi^* & j\Psi^* & i\Psi^* & \Psi^* \end{pmatrix}, \quad Z = \frac{1}{2}\begin{pmatrix} \Psi & i\Psi & j\Psi & -k\Psi \\ -i\Psi & \Psi & k\Psi & j\Psi \\ -j\Psi^* & -k\Psi^* & \Psi^* & -i\Psi^* \\ k\Psi^* & -j\Psi^* & i\Psi^* & \Psi^* \end{pmatrix}$$

$$I = \begin{pmatrix} \Psi & 0 & 0 & 0 \\ 0 & \Psi & 0 & 0 \\ 0 & 0 & \Psi^* & 0 \\ 0 & 0 & 0 & \Psi^* \end{pmatrix}, \quad A = \begin{pmatrix} 0 & -i\Psi & 0 & 0 \\ i\Psi & 0 & 0 & 0 \\ 0 & 0 & 0 & i\Psi \\ 0 & 0 & -i\Psi & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 0 & -j\Psi & 0 \\ 0 & 0 & 0 & -j\Psi \\ j\Psi & 0 & 0 & 0 \\ 0 & j\Psi & 0 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 0 & 0 & -k\Psi \\ 0 & 0 & k\Psi & 0 \\ 0 & -k\Psi & 0 & 0 \\ k\Psi & 0 & 0 & 0 \end{pmatrix}$$

A detailed proof demonstrating that these elements form a group under multiplication when $\Psi = 1$ is provided in the accompanying Mathematica notebook. These elements seem to form an infinite group when Psi is not set to 1, but this will be theorized without rigorous proof.

Group Notation

An arbitrary element of this group or algebra is denoted as α or β. All notation used for Ψ extends to these group elements. Any notation applied to a group element implies that it is applied to all the Ψ's within that element. For example:

$$I^\flat = \begin{pmatrix} \Psi^\flat & 0 & 0 & 0 \\ 0 & \Psi^\flat & 0 & 0 \\ 0 & 0 & \Psi^\flat * & 0 \\ 0 & 0 & 0 & \Psi^\flat * \end{pmatrix}$$

This notation ensures that transformations and operations applied to group elements will consistently apply to the wavefunctions embedded within them.

Squaring G

The quaternion-valued matrices on which we place Ψ exhibit unique dynamics. For instance, when $\Psi = 1$, squaring G results in the following:

$$G^2 = \begin{pmatrix} \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} & -\frac{i}{4} - \frac{i}{4} + \frac{i}{4} + \frac{i}{4} & -\frac{j}{4} + \frac{j}{4} - \frac{j}{4} + \frac{j}{4} & -\frac{k}{4} + \frac{k}{4} + \frac{k}{4} - \frac{k}{4} \\ -\frac{i}{4} - \frac{i}{4} + \frac{i}{4} + \frac{i}{4} & \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} & -\frac{k}{4} + \frac{k}{4} + \frac{k}{4} - \frac{k}{4} & -\frac{j}{4} + \frac{j}{4} - \frac{j}{4} + \frac{j}{4} \\ -\frac{j}{4} - \frac{j}{4} + \frac{j}{4} + \frac{j}{4} & -\frac{k}{4} + \frac{k}{4} + \frac{k}{4} - \frac{k}{4} & \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} & -\frac{i}{4} - \frac{i}{4} + \frac{i}{4} + \frac{i}{4} \\ \frac{k}{4} - \frac{k}{4} - \frac{k}{4} + \frac{k}{4} & -\frac{j}{4} + \frac{j}{4} + \frac{j}{4} - \frac{j}{4} & -\frac{i}{4} - \frac{i}{4} + \frac{i}{4} + \frac{i}{4} & \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \end{pmatrix}$$

As shown, the off-diagonal terms cancel out, while the diagonal terms sum to 1 in each element, resulting in $G^2 = I$. This makes G a unique matrix, as it is an involution (i.e., squaring to the identity matrix) without any zero elements. The same holds true for X, Y, and Z, which all behave similarly under squaring, showcasing a consistent group dynamic.

Group Dynamics

When we set $\Psi = 1$, this effectively quantizes the group, representing a specific set of elements rather than an infinite group of wavefunctions. In the general case, these elements are subsets of an infinite group, where the wavefunctions interact with quaternion-valued matrices, but retain the structure from when $\Psi = 1$.

The multiplication table for these elements with $\Psi = 1$ or as subsets of an infinite group is as follows:

	G	X	Y	Z	I	A	B	C
G	I	A	B	C	G	X	Y	Z
X	A	I	$-C$	$-B$	X	G	$-Z$	$-Y$
Y	B	$-C$	I	$-A$	Y	$-Z$	G	$-X$
Z	C	$-B$	$-A$	I	Z	$-Y$	$-X$	G
I	G	X	Y	Z	I	A	B	C
A	X	G	$-Z$	$-Y$	A	I	$-C$	$-B$
B	Y	$-Z$	G	$-X$	B	$-C$	I	$-A$
C	Z	$-Y$	$-X$	G	C	$-B$	$-A$	I

This table maintains the group structure and duality across the elements.

The Dualism of the Group

This multiplication table reveals a dualism in the group, with symmetries between G and I, X and A, Y and B, and Z and C. Each element is an involution, squaring to I. For example, multiplying G by X yields A, and multiplying G by A returns X.

If we associate 0 with G and I, 1 with A and X, 2 with B and Y, and 3 with C and Z, the multiplication table simplifies to a pattern reminiscent of the quaternions:

$$0 \quad 1 \quad 2 \quad 31 \quad 0 \quad -3 \quad -22 \quad -3 \quad 0 \quad 13 \quad -2 \quad -1 \quad 0$$

This pattern mirrors the original structure of G when $\Psi = 1$:

$$G\left(\Psi = 1\right) = \frac{1}{2}\begin{pmatrix} 1 & -i & -j & -k \\ i & 1 & k & -j \\ j & -k & 1 & i \\ k & j & -i & 1 \end{pmatrix}$$

Although the signs differ, a deep symmetry and duality emerge in the group's structure.

Fermion and Boson Transformation Dynamics

In this group structure, elements from the set $\{G, X, Y, Z\}$ multiply to form elements from the set $\{I, A, B, C\}$. However, elements within the set $\{I, A, B, C\}$ multiply to form other elements from the same set. On the other hand, if we multiply an element from the set $\{G, X, Y, Z\}$ with an element from the set $\{I, A, B, C\}$, the result belongs to the set $\{G, X, Y, Z\}$.

This pattern suggests a deeper symmetry, which evokes fermionic and bosonic dynamics: elements from $\{G, X, Y, Z\}$ correspond to fermions, and elements from $\{I, A, B, C\}$ correspond to bosons. In this context, a fermion-fermion interaction can produce a boson, a boson-boson interaction results in another boson, and a fermion-boson interaction returns a fermion.

Given the frequent references to these two distinct sets, it would be useful to assign specific names to each set for future clarity.

A Unique Group Property

This group exhibits a remarkable and seemingly unique symmetry property, not observed in other common group structures. Specifically, if three group elements satisfy the relation:

$$\alpha_1\alpha_2 = \alpha_3$$

then the following symmetry relations hold:

$$\alpha_3\alpha_2 = \alpha_1 \quad \text{and} \quad \alpha_3\alpha_1 = \alpha_2$$

For example:

$$GX = A, \quad AX = G, \quad AG = X$$

or:

$$XY = -C, \quad -CY = X, \quad -CX = Y$$

This symmetry is intrinsic to the group and reflects its deep structure, contributing to the duality between the sets $\{G, X, Y, Z\}$ and $\{I, A, B, C\}$.

Additive Symmetries

When we set Ψ to 1, the group's additive symmetries become clear. The following relations hold for the group elements:

$$2I = G + X + Y + Z$$
$$2A = G + X - Y - Z$$
$$2B = G - X + Y - Z$$
$$2C = G - X - Y + Z$$
$$2G = I + A + B + C$$
$$2X = I + A - B - C$$
$$2Y = I - A + B - C$$
$$2Z = I - A - B + C$$

These expressions reveal a strong duality in both the additive and multiplicative symmetries. The symmetry pairs $G \leftrightarrow I$, $X \leftrightarrow A$, $Y \leftrightarrow B$, and $Z \leftrightarrow C$ consistently appear across both types of operations, hinting at a deeper underlying structure in the group.

Example: The Relation $2G = I + A + B + C$

In this case, the elements I, A, B, C each contain four non-zero elements. When we sum them, we observe that zero-valued elements always add to zero-valued elements, and non-zero elements add to other non-zero elements of the same scale. Since the elements in I, A, B, C all have a scale of 1, and the elements in G have a scale of $1/2$, the result of the summation is $2G$, maintaining the correct scaling.

Example: The Relation $2I = G + X + Y + Z$

This relation requires a bit more care. To better visualize the addition process, consider the following sum table, where the terms are aligned like basic arithmetic sums:

$\frac{I}{2}$	$+$	$\frac{A}{2}$	$+$	$\frac{B}{2}$	$+$	$\frac{C}{2}$ $= (G)$
$\frac{I}{2}$	$+$	$\frac{A}{2}$	$-$	$\frac{B}{2}$	$-$	$\frac{C}{2}$ $= (X)$
$\frac{I}{2}$	$-$	$\frac{A}{2}$	$+$	$\frac{B}{2}$	$-$	$\frac{C}{2}$ $= (Y)$
$\frac{I}{2}$	$-$	$\frac{A}{2}$	$-$	$\frac{B}{2}$	$+$	$\frac{C}{2}$ $= (Z)$
$2I$		0		0		0

In this table, the positive and negative terms for A, B, and C cancel out, leaving us with the result:

$$G + X + Y + Z = 2I$$

This result shows how the off-diagonal terms effectively cancel, while the diagonal terms sum to produce the final $2I$.

Additive Symmetries with General Ψ

When Ψ is not set to 1, the additive symmetries follow a similar structure, but the resulting sums depend on the specific values of Ψ for each group element. In these cases, parentheses are used to indicate the wavefunction's influence on the symmetry:

$$(G + X + Y + Z)(\Psi) = 2I(\Psi)$$

Algebraic Structure of the Group

We have demonstrated that the set of elements {G, X, Y, Z, I, A, B, C} and their negatives form a group under multiplication when $\Psi = 1$, as proven in the accompanying Mathematica notebook. The generators of this group are the elements I, A, B, C, as the elements G, X, Y, Z can be constructed through their addition:

$$G = \frac{1}{2}(I{+}A{+}B{+}C), \quad X = \frac{1}{2}(I{+}A{-}B{-}C), \quad Y = \frac{1}{2}(I{-}A{+}B{-}C), \quad Z = \frac{1}{2}(I{-}A{-}B{+}C)$$

Each of the eight elements is unitary, and all are involutions when $\Psi = 1$, which suggests that they may form an algebra.

Testing Algebraic Properties

We conjecture that this group forms an algebra. Based on tests performed using Mathematica, we observe:

- **Closure**: The product of any two elements in the set produces another element within the set. Similarly, the sum of two elements also remains within the set.

- **Distributivity**: Computational tests confirm that multiplication distributes over addition.
- **Associativity**: Multiplication and addition appear to be associative in all cases tested.

These properties suggest that the group has an algebraic structure. However, given the complexity of fully formalizing this algebra, we present this as a probable structure supported by computational evidence.

Generators and Normalization

The elements I, A, B, C serve as generators of the group, while the full set of elements, including their negatives, represents the normalization points. These elements behave as unitary transformations, as shown by their involution property when $\Psi = 1$.

Parentheses Notation

Inspired by bra-ket notation, parentheses notation retains the same underlying concept but is adapted for the specific algebra we are working with. Since the elements in this algebra are represented as 4×4 matrices, we need to sum over their matrix elements. Thus, we define:

$$(\alpha|\beta) = \frac{1}{4} \int_{\mu=0}^{4} d\xi_\mu \sum_{i,j} \alpha_{ij}^* \beta_{ij}$$

Here, α_{ij}^* represents the conjugated wavefunction components, while the quaternionic components remain unaffected. Since quaternions can be represented as 4×4 real-valued matrices, the conjugation applies only to the wavefunctions, leaving the quaternionic parts untouched.

If α is an arbitrary element of the algebra (e.g., G, X, Y, Z, I, A, B, C, or their negatives) and if the element is normalized, we find:

$$\left(\alpha^\flat | \alpha^\flat\right) = 1$$

Additivity in Parentheses Notation

Thanks to the additive symmetries inherent in the algebra, it is possible to sum multiple normalized elements, and the result will reflect the number of elements summed:

$$\left(\sum_n \alpha_n^\flat \middle| \sum_n \alpha_n^\flat\right) = n$$

This sum does not require any form of renormalization. When adding normalized elements within the parentheses, the result is simply the number of distinct elements represented. However, certain conditions must be met: 1.

No repeated elements: Elements like G_1 and G_2 cannot be included together within the same set. 2. **No mixing between sets**: Elements from one set (e.g., G, X, Y, Z) cannot be mixed with elements from another set (e.g., I, A, B, C).

For instance:

$$(G^\flat|G^\flat) = 1$$
$$(X^\flat|X^\flat) = 1$$
$$(Y^\flat|Y^\flat) = 1$$
$$(Z^\flat|Z^\flat) = 1$$

Summing multiple elements from the same set results in:

$$(G^\flat + X^\flat + Y^\flat + Z^\flat|G^\flat + X^\flat + Y^\flat + Z^\flat) = 4$$

General Rule for Parentheses Notation

A key feature of parentheses notation is that it does not behave like the naive expectation of squaring the sum of all elements. Instead, it squares each term individually, then sums the results.

For example, in the expression:

$$(G^\flat + X^\flat + Y^\flat + Z^\flat|G^\flat + X^\flat + Y^\flat + Z^\flat) = 4$$

We might expect the result to be $(1 + 1 + 1 + 1)^2 = 16$, but instead, each term is squared individually (i.e., $1^2 + 1^2 + 1^2 + 1^2$), resulting in 4.

Similarly, if we have:

$$(G_1^\flat + G_2^\flat + X^\flat + Y^\flat + Z^\flat|G_1^\flat + G_2^\flat + X^\flat + Y^\flat + Z^\flat) = 7$$

Here, there are two G-terms, which can be understood as contributing $(2^2)+1^2+1^2+1^2 = 7$. The general rule is that repeated terms are squared, and distinct terms are summed directly.

Connection to Probability Distributions

When the elements inside the parentheses do not include underlines (i.e., do not represent eigenvalues), the result of the parentheses notation can be interpreted as a **probability distribution**. This allows us to connect the wavefunctions to physical measurements, much like the bra-ket notation in quantum mechanics. For instance, for a normalized wavefunction ψ, the parentheses expression $(\psi|\psi)$ gives the probability of observing the system in that particular state.

Superposition and Particle Interpretation

This formalism implies that we can describe systems of two to four particles in superposition, such as in bound states like protons or atoms. The additive symmetries ensure that the combined wavefunctions obey the appropriate normalization rules, providing a consistent framework for handling multi-particle systems.

Unique Normalization in Superpositions

In traditional quantum mechanics, when wavefunctions are placed into a superposition, the system is typically normalized to a single value, often 1, to reflect the overall probability distribution. However, in this theory, we present a unique approach where individually normalized wavefunctions retain their own normalization even when placed in a superposition. This structure allows the wavefunctions to multiply their conjugates while preserving the total number of particles represented by the superposition.

Retaining Individual Normalization in Superposition

Each wavefunction element (e.g., $G^\flat$, $X^\flat$, $Y^\flat$, and $Z^\flat$) is individually normalized to 1:

$$(G^\flat|G^\flat) = 1, \quad (X^\flat|X^\flat) = 1, \quad (Y^\flat|Y^\flat) = 1, \quad (Z^\flat|Z^\flat) = 1$$

When these individually normalized elements are placed into a superposition, the resulting parentheses notation does not collapse the normalization back to 1, but instead **retains the total normalization** as the sum of the individual contributions. For example, the superposition:

$$(G^\flat + X^\flat + Y^\flat + Z^\flat|G^\flat + X^\flat + Y^\flat + Z^\flat) = 4$$

This result highlights the fact that, while each component remains normalized, their superposition reflects the total number of wavefunctions involved. The same applies if we consider fewer components in the superposition, such as:

$$(X^\flat + Y^\flat + Z^\flat|X^\flat + Y^\flat + Z^\flat) = 3$$

Here, the number of particles involved in the superposition is directly reflected by the outcome, maintaining the integrity of each wavefunction's normalization.

Preservation of Particle Count

In this theory, the normalization reflects not only the probability distribution but also the **number of particles** involved. For instance, if we have a single $G^\flat$ in the parentheses, it still maintains its individual normalization:

$$(G^\flat|G^\flat) = 1$$

However, if we place multiple distinct elements into the parentheses, such as $G^\flat + X^\flat + Y^\flat + Z^\flat$, the resulting normalization reflects the sum of the particles:

$$(G^\flat + X^\flat + Y^\flat + Z^\flat|G^\flat + X^\flat + Y^\flat + Z^\flat) = 4$$

Thus, this approach allows us to **retain the normalization of each individual wavefunction**, while also accounting for the total number of particles in a superposition.

Multiplication with Conjugates

A further unique aspect of this framework is that when these individually normalized wavefunctions multiply their conjugates, they still retain the total

particle count. Whether we have a single wavefunction or multiple wavefunctions in superposition, the parentheses notation ensures that the result maintains consistency. For example:

$$(G^\flat | G^\flat) = 1$$

and

$$(G^\flat + X^\flat + Y^\flat + Z^\flat | G^\flat + X^\flat + Y^\flat + Z^\flat) = 4$$

both illustrate that the number of particles is preserved, and the normalization reflects this without collapsing to a single value.

Implications of Unique Normalization

This unique approach to normalization opens up new avenues for understanding particle interactions and superpositions. Unlike traditional quantum mechanics, where the wavefunction's probability must sum to 1, this theory **preserves the individual normalizations** and allows for superpositions to represent multiple particles. It provides a novel way of modeling systems with more complex interactions, where each wavefunction in the superposition maintains its own integrity.

In this framework, **normalization becomes a measure not only of probability but of particle count**, leading to a richer interpretation of superpositions and wavefunction dynamics.

Zero Equations

In this theory, normalization is crucial to ensure that wavefunctions maintain the correct probability distribution, as discussed earlier. However, **normalization alone is not sufficient**—we also need to account for the behavior of eigenvalues, particularly those related to measurable quantities like position, momentum, and energy. These eigenvalues must collectively satisfy **zero equations**, which act as the wave equations of the theory. These equations ensure that the physical reality described by the theory adheres to key symmetries, much like how Minkowski spacetime ensures that the sum of spacetime intervals equals zero.

The Zero Equation for Eigenvalues

Consider a scenario where we have a superposition of eigen-primitives, which have measurable properties (position, energy, momentum, etc.). To enforce coherence and physical consistency, the sum of these eigenvalues must collectively **equal zero**, much like in wave mechanics where we require certain conditions for constructive and destructive interference. This gives us the first **zero equation**:

$$\left(\sum_n \underline{\alpha}_n^\flat \middle| \sum_n \underline{\alpha}_n^\flat \right) = 0$$

Here, the notation $\underline{\alpha}_n^\flat$ refers to eigen-primitives—elements with measurement properties (like position eigenvalues)—and we are summing over n such elements

in the parentheses. **Collapsing to a specific eigenstate** requires that this sum resolves to zero. If a superposition of such eigen-primitives does not sum to zero, the result is physically meaningless, leading to chaotic or non-physical behavior. Therefore, collapsing to a specific eigenstate—where the system is well-defined and measurable—**requires no superposition** of these eigen-primitives.

Probability of Collapsing to a Position Eigenvalue

The probability of collapsing to a particular position eigenvalue is determined by the probability of collapsing to a specific point τ. This is proportional to the self-interaction of a particular wavefunction α, and is given by:

$$P(\tau) \propto (\alpha|\alpha)_{\Xi_\mu}$$

This equation is **analogous** to integrating over Minkowski spacetime, where the interval between spacetime events is defined by the equation:

$$-t^2 + x^2 + y^2 + z^2 + S^2 = 0$$

Here, the sum of the spacetime intervals (including S^2, which corresponds to the scalar eigenvalue) equates to zero. In a similar manner, the sum of the position eigenvalues must satisfy a zero condition, ensuring physical consistency.

Zero Equation for Double-Underlined Eigenvalues (Energy, Momentum, Mass)

The zero equation extends to **double-underlined eigenvalues**, which correspond to measurable quantities like energy, momentum, and mass. For these quantities, the sum of the eigenvalues must also equate to zero, forming the second zero equation:

$$\left(\sum_n \underline{\underline{\alpha}}_n^\flat \middle| \sum_n \underline{\underline{\alpha}}_n^\flat \right) = 0$$

Unlike the position eigenvalues, where the probability of collapse is related to τ (a specific point in space or time), the probability of collapsing to a set of double-underlined eigenvalues depends on the **normalization constants** of each plane wave. Each plane wave carries its own double-underlined eigenvalue, and the overall probability reflects these normalization constants.

This zero equation is **analogous** to integrating over the energy-momentum relation, where the relation must also sum to zero:

$$-E^2 + p_x^2 + p_y^2 + p_z^2 + m^2 = 0$$

In this equation, m represents the rate at which the particle's S-eigenvalue grows, linking the mass of the particle to the curvature of its eigenvalues in the same way energy and momentum are linked in classical relativistic mechanics.

The Role of Zero Equations as Wave Equations

These zero equations play a fundamental role in the theory, acting as **wave equations**. They serve to enforce the underlying symmetries and constraints of the system, much like the role of the Schrödinger equation in standard quantum mechanics or the role of the Einstein field equations in general relativity.

- The **first zero equation**, related to position eigenvalues, ensures that the system remains spatially coherent and collapses to well-defined spatial points.

- The **second zero equation**, related to energy, momentum, and mass eigenvalues, ensures that the system respects the symmetries of energy-momentum space, such that any measurable quantities like energy or momentum maintain consistency with relativistic dynamics.

Together with the **normalization conditions**, these zero equations form the **backbone of the wave dynamics** in this theory, ensuring that both measurable quantities and probability distributions are physically meaningful and consistent with the structure of spacetime and energy-momentum relations.

Zero Equations with Curvature

In the previous sections, we discussed normalized wavefunctions that naturally satisfy the zero equations. However, when **curvature** is introduced, the behavior of the wavefunctions changes. The curvature deviates from a "pure" value, causing the wavefunctions to no longer remain normalized, nor satisfy the zero equations.

To address this, we introduce **boldface** $\boldsymbol{\alpha}$ as an abstract representation of a wavefunction that has been processed in such a way that it restores both normalization and adherence to the zero equations. These processed wavefunctions will be required to satisfy the following conditions:

$$\left(\sum_n \boldsymbol{\alpha}_n \middle| \sum_n \boldsymbol{\alpha}_n \right) = n$$

This ensures the **normalization** of the wavefunctions remains consistent.

$$\left(\sum_n \underline{\boldsymbol{\alpha}}_n \middle| \sum_n \underline{\boldsymbol{\alpha}}_n \right) = 0$$

This maintains that the **position eigenvalues** of the wavefunctions collectively sum to zero.

$$\left(\sum_n \underline{\underline{\boldsymbol{\alpha}}}_n \middle| \sum_n \underline{\underline{\boldsymbol{\alpha}}}_n \right) = 0$$

This ensures that the **energy, momentum, and mass eigenvalues** also collectively sum to zero, as required by the theory's wave equations.

Solution One: Curvature as Potential Energy

One approach to solving the issue of curvature and normalization is to treat the **curvature** Φ as an analogue to **potential energy**, and the wavefunction Ψ as an analogue to **kinetic energy**. This draws from the idea that the sum of potential and kinetic energy forms the **Hamiltonian** of a system. In this case, we modify the wavefunction α by introducing curvature Φ directly into Ψ, creating the following definitions for the boldface wavefunctions:

$$\boldsymbol{\alpha} = \alpha\Big|_{\Psi \to \Psi + \Phi}$$

This processed form of $\boldsymbol{\alpha}$ includes the curvature Φ, representing potential energy as a contribution to the wavefunction Ψ.

For position eigenvalues, the modified wavefunction is:

$$\underline{\boldsymbol{\alpha}} = \underline{\alpha}\Big|_{\Psi \to \Psi + \underline{\Phi}}$$

This form includes the **curvature of space-time** (through $\underline{\Phi}$) in the position eigenvalue structure.

For energy, momentum, and mass eigenvalues, we define:

$$\underline{\underline{\boldsymbol{\alpha}}} = \underline{\underline{\alpha}}\Big|_{\Psi \to \Psi + \underline{\underline{\Phi}}}$$

This incorporates the **curvature in energy and momentum space** (through $\underline{\underline{\Phi}}$).

Open Questions: However, introducing Φ raises questions about how this affects probability and collapse. Specifically: - Does the presence of Φ collapse the wavefunction to a state related to Φ? - Does it result in a state collapsed to the flat, or uncurved, version of Ψ (i.e., $\Psi^\flat$)?

Solution Two: Normalization by Rescaling

In this second solution, we normalize the wavefunction α by explicitly rescaling it with a universal normalization constant, α_U, as follows:

$$\boldsymbol{\alpha} = \alpha_U / \overline{\alpha}_U$$

This rescaling restores the normalization of the wavefunction. To further refine this, we introduce a new definition of ρ_0, ensuring that it satisfies the zero equations by varying in such a way that the following holds:

For position eigenvalues:

$$(\underline{\boldsymbol{\alpha}}_{\mathbf{0}} | \underline{\boldsymbol{\alpha}}_{\mathbf{0}}) = \left(\sum_{\mu=1}^{4} \underline{\boldsymbol{\alpha}}_{\boldsymbol{\mu}} \middle| \sum_{\mu=1}^{4} \underline{\boldsymbol{\alpha}}_{\boldsymbol{\mu}} \right)$$

This ensures the **sum of the position eigenvalues** of the modified wavefunctions is zero.

For energy, momentum, and mass eigenvalues:

$$\left(\underline{\underline{\boldsymbol{\alpha}}}_{\mathbf{0}} | \underline{\underline{\boldsymbol{\alpha}}}_{\mathbf{0}} \right) = \left(\sum_{\mu=1}^{4} \underline{\underline{\boldsymbol{\alpha}}}_{\boldsymbol{\mu}} \middle| \sum_{\mu=1}^{4} \underline{\underline{\boldsymbol{\alpha}}}_{\boldsymbol{\mu}} \right)$$

This equation ensures the **sum of energy, momentum, and mass eigenvalues** remains consistent with the zero equation, even after rescaling.

Solution Three: Flattening and Replacing with Position Eigenfunctions

In this third solution, we flatten the wavefunction α, effectively reducing it to a simpler, uncurved state, and then replace the variable Ξ with a **position eigenfunction**. The eigenvalues of this position eigenfunction take on the role that Ξ previously played.

The new wavefunction is given by:

$$\boldsymbol{\alpha} = \alpha^{\flat}\Big|_{\Xi \to \overline{\underline{\Psi}}}$$

Here, $\alpha^{\flat}$ is the flattened version of α, and $\overline{\underline{\Psi}}$ is a **position eigenfunction** that now governs the dynamics. Importantly, even though this new wavefunction is a function of position eigenvalues, it still depends on the original variable Ξ, preserving the link to the original wavefunction.

After this flattening process, the resulting boldface wavefunction must satisfy the following equations:

For position eigenvalues:

$$(\underline{\boldsymbol{\alpha}}_{\mathbf{0}}|\underline{\boldsymbol{\alpha}}_{\mathbf{0}}) = \left(\sum_{\mu=1}^{4} \underline{\boldsymbol{\alpha}}_{\boldsymbol{\mu}} \middle| \sum_{\mu=1}^{4} \underline{\boldsymbol{\alpha}}_{\boldsymbol{\mu}}\right)$$

For energy, momentum, and mass eigenvalues:

$$\left(\underline{\underline{\boldsymbol{\alpha}}}_{\mathbf{0}}|\underline{\underline{\boldsymbol{\alpha}}}_{\mathbf{0}}\right) = \left(\sum_{\mu=1}^{4} \underline{\underline{\boldsymbol{\alpha}}}_{\boldsymbol{\mu}} \middle| \sum_{\mu=1}^{4} \underline{\underline{\boldsymbol{\alpha}}}_{\boldsymbol{\mu}}\right)$$

These conditions ensure that the wavefunction, once flattened and replaced with position eigenvalues, continues to satisfy the zero equations.

Conclusion

Each of the three solutions presented provides a method for handling curvature while preserving the necessary conditions for **normalization** and **zero equations**. Whether by modifying the wavefunction with curvature Φ, rescaling it to enforce consistency, or flattening and replacing variables, these solutions offer distinct ways of maintaining physical coherence in the presence of curvature.

Particle Interaction

In this model, particle interactions are framed using the equation:

$$\alpha_1 \alpha_2 = \alpha_3$$

However, the interpretation of this equation differs depending on the nature of the charges of the interacting particles.

Probability of Interaction

The probability that two particles α_1 and α_2 interact is determined by the parentheses notation applied to the product of their wavefunctions:

$$P_{\text{interaction}} = (\alpha_1\alpha_2|\alpha_1\alpha_2)$$

This expression gives the likelihood that the two particles will localize and interact. A higher value of $P_{\text{interaction}}$ implies a higher probability of interaction.

Interactions With Charges That Don't Sum to Zero

For particles with the same charge, or more generally, charges that don't sum to zero, the interaction does occur, but no new particle is formed. Instead, we conceptualize the interaction as remaining on the *left-hand side* of the equation:

$$\alpha_1\alpha_2 = \alpha_1\alpha_2$$

This means that while the particles interact, they continue to exist as two separate entities, each retaining its identity and charge. They repel, attract, or interact in some other dynamic way, but they do not collapse into a new particle. This interpretation gives a new perspective on why particles with the same charge (such as two electrons, represented as α_{e^-}) do not "merge" but continue to interact in a non-annihilative manner.

In this framework, *non-zero net charge interactions* are unresolved in the sense that they do not produce a single resulting entity. Rather, they remain as two interacting particles, but their combined interaction still follows the overall framework of the theory. They might exchange momentum, energy, or other quantities, but they do not form a new particle. The probability of such interaction is:

$$P_{\text{interaction}} = (\alpha_{e^-}\alpha_{e^-}|\alpha_{e^-}\alpha_{e^-})$$

Interactions With Charges That Sum to Zero

For particles with opposite charges, or charges that sum to zero (such as α_{e^+} and α_{e^-}), the interaction results in the formation of a new particle. In this case, the *right-hand side* of the equation becomes relevant, and the interaction resolves into a new entity, denoted by:

$$\alpha_1\alpha_2 = \alpha_3$$

This could be, for instance, the annihilation of α_{e^+} and α_{e^-} to form α_γ, a photon. The resulting particle, α_3 (e.g., α_γ), carries the properties of the interaction, and the probability of this interaction is:

$$P_{\text{interaction}} = (\alpha_{e^+}\alpha_{e^-}|\alpha_{e^+}\alpha_{e^-})$$

Because the charges sum to zero, this new particle has the potential for subsequent interactions with other particles in more complex dynamics. For example, the photon formed by electron-positron annihilation can go on to interact with other particles, transferring energy, momentum, or triggering further transformations.

General Interaction Dynamics

In this model, the distinction between *resolved* and *unresolved* interactions provides a framework for understanding why particles with the same charge behave differently from those whose charges sum to zero. The former interact but remain as distinct entities, while the latter form new particles, allowing for further interactions.

Moreover, this framework emphasizes that even in cases where no new particle is formed, interactions are still occurring. Two particles, such as two electrons $(\alpha_{e^-}, \alpha_{e^-})$, do not remain static in the presence of one another—they repel and influence each other's dynamics, yet no new particle emerges from this interaction.

The interaction dynamics are summarized as follows:

- **Unresolved interactions**: Particles with charges that don't sum to zero interact but remain distinct. No new particle is formed, but the interaction still takes place in the context of $\alpha_1 \alpha_2$.
- **Resolved interactions**: Particles with charges that sum to zero form a new particle, α_3, allowing for further interactions and more complex dynamics. The system is conceptualized as the *right-hand side* of the equation, where the result of the interaction is a new entity.

Example: Electron-Photon Dynamics

Consider an electron and positron annihilating to form a photon:

$$\alpha_{e^+} \alpha_{e^-} = \alpha_\gamma$$

Here, the charges sum to zero, so the interaction results in a photon, which is capable of interacting with other particles. The probability of this interaction is given by:

$$P_{\text{interaction}} = (\alpha_{e^+} \alpha_{e^-} | \alpha_{e^+} \alpha_{e^-})$$

However, when two electrons come into proximity: