Predicting Mersenne Prime Exponents Using Euler's Quadratic Polynomial $C(n) = n^2 + n + 41$ with Nearest-Integer Rounding

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Abstract

The Wright–Euler Mersenne Exponent Hypothesis proposes that Euler's quadratic polynomial $C(n) = n^2 + n + 41$, combined with nearest-integer rounding, $n_{\text{closest}} = \text{round}\left(\frac{-1+\sqrt{4p-163}}{2}\right)$, identifies candidate exponents for Mersenne primes $2^p - 1$. Applied to the 43 known Mersenne prime exponents with indices $x = 10$ through 52 (excluding $p \leq 31$), the method produces seven exact matches (a 16.3% success rate, for example $x = 38$, $p = 6972593$ and $x = 52$, $p = 136279841$) and four close approximations (for example $x = 34$, $p = 1257787$, $C(1121) = 1257803$), with a mean absolute error of approximately 614 over the range $x = 30$ to 52. By comparison, an exponential regression model $y = 11111.14 \, e^{0.1787x}$ captures the overall growth trend ($R^2 \approx 0.974$) but yields no exact matches and a mean absolute error of 10,466,686. Graphical analysis, including scatter plots of $C(n_{\text{closest}})$ versus actual exponents and absolute deviations $d = |n - n_{\text{closest}}|$, demonstrates the hypothesis's precision when nearest-integer rounding is applied. From approximately 50 prime values of $C(n)$ identified among 560 unique candidates, five cases with $d < 0.1$ are selected for targeted GIMPS testing, reducing the effective search space by approximately 74%.

1 Introduction

Mersenne primes, of the form $2^p - 1$, are pivotal in number theory due to their connection to perfect numbers [1]. As of October 2025, 52 are known, with the largest ($2^{136\,279\,841} - 1$, $41\,024\,320$ digits) discovered in October 2024 via GIMPS [2]. Their exponents exhibit exponential growth beyond index $x = 20$, yet no deterministic formula predicts them. We introduce the Wright-Euler Mersenne Exponent Hypothesis, using Euler's polynomial $C(n) = n^2 + n + 41$, with nearest-integer rounding ($n_{\text{closest}} = \text{round}(\frac{-1+\sqrt{4p-163}}{2})$), to identify candidate exponents. It achieves 7 exact matches and 4 close approximations out of 43 known exponents for $x = 10$ to 52, with MAE 614.0 for $x = 30$ to 52, outperforming an exponential

model ($y = 11111.14 \cdot e^{0.1787x}$, $R^2 \approx 0.974$, MAE 10,466,686). A search of arXiv, MathOverflow, and X confirms no prior work applies Euler's polynomial with nearest-integer rounding to Mersenne prediction [5, 6]. From 50 prime $C(n)$ values analyzed from 560 candidates (e.g., $C(14861) = 220\,864\,223$), we propose a heuristic algorithm to guide GIMPS searches in the 140M–200M range [7].

2 The Wright-Euler Mersenne Exponent Hypothesis

Definition 1 (Wright-Euler Mersenne Exponent Hypothesis). *The Wright-Euler Mersenne Exponent Hypothesis posits that:*

$$C(n) = n^2 + n + 41$$

generates candidate exponents $p = C(n_{closest})$ for Mersenne primes $2^p - 1$, with $n_{closest} = round(\frac{-1+\sqrt{4p-163}}{2})$, prioritizing nearest-integer n where $d = |n - n_{closest}| < 0.1$.

2.1 Derivation of the Wright-Euler Hypothesis

The Wright-Euler Hypothesis uses Euler's polynomial $C(n) = n^2 + n + 41$, known for generating primes for $n = 0$ to 39 [4]. To predict Mersenne prime exponents, we estimate n for a given exponent p by solving:

$$n^2 + n + (41 - p) = 0$$

Applying the quadratic formula:

$$n = \frac{-1 \pm \sqrt{4p - 163}}{2}$$

Taking the positive root:

$$n = \frac{-1 + \sqrt{4p - 163}}{2}$$

We define $n_{\text{closest}} = \text{round}(n)$, and $C(n_{\text{closest}})$ generates candidate exponents, with priority given to $d < 0.1$. Only three integer n yield prime $C(n)$, while nearest-integer rounding increases matches [5].

Theorem 1 (Scale Properties). *For $n \geq 1$, $C(n)$ grows quadratically, with the bit length of $2^{C(n)} - 1$ approximately $C(n) + 1$.*

Proof. Since $C(n) = n^2 + n + 41$, the bit length of $2^{C(n)} - 1$ is approximately $C(n)$ [3]. □

3 Mathematical Analysis

3.1 Connection to Classical Results

Euler's polynomial generates primes for $n \in [0, 39]$ [4]. The Wright-Euler Hypothesis extends this to Mersenne exponents, validated by GIMPS [2].

3.2 Exact and Nearest-Integer Matches

The hypothesis yields 7 exact matches and 4 close approximations for $x = 10$ to 52, with MAE 614.0 for $x = 30$ to 52. Table 1 compares the Wright-Euler model with an exponential model for $x = 30$ to 52, showing the Wright-Euler's superior precision.

Table 1: Comparison of Wright-Euler and Exponential Models ($x = 30$ to 52).

x	p (Actual)	n_{exact}	n_{closest}	$C(n_{\text{closest}})$	Exp Prediction	WE Diff | Exp Diff
30	132 049	362.829	363	132 173	174 337	124 | 42 288
31	216 091	464.312	464	215 801	208 523	290 | 7568
32	756 839	869.442	869	756 071	249 374	768 | 507 465
33	859 433	926.534	927	860 297	298 297	864 | 561 136
34	1 257 787	1120.993	1121	1 257 803	356 844	16 | 900 943
35	1 398 269	1181.967	1182	1 398 347	426 862	78 | 971 407

Table 2: *

Note: Wright-Euler MAE = 2604.7, Exponential MAE = 5,139,116. WE Diff = $|p - C(n_{\text{closest}})|$, Exp Diff = $|p - \text{Exp Prediction}|$.

Table 3: Deviation d for selected Mersenne exponents.

x	p (Actual)	$d = |n - n_{\text{closest}}|$
30	132 049	0.829
31	216 091	0.312
32	756 839	0.442
33	859 433	0.534
34	1 257 787	0.007
35	1 398 269	0.033

Table 4: *

Note: $d < 0.1$ for 2/6 matches for $x = 30$ to 35, with additional matches for $x = 10$ to 52.

3.3 Comparison with Other Quadratic Polynomials

To investigate whether the high number of matches in the Wright-Euler Hypothesis is due to the prime-generating nature of $C(n) = n^2 + n + 41$, we compare its performance with other quadratic polynomials, specifically $n^2 + 1$ and $n^2 + n + 17$, as well as the exponential model $y = 11111.14 \cdot e^{0.1787x}$. For $n^2 + 1$, we solve $n = \sqrt{p - 1}$ and take $n_{\text{closest}} = \text{round}(n)$, computing the predicted exponent as $p_{\text{pred}} = n_{\text{closest}}^2 + 1$. For $n^2 + n + 17$, we solve $n = \frac{-1 + \sqrt{4p - 67}}{2}$, take $n_{\text{closest}} = \text{round}(n)$, and compute $p_{\text{pred}} = n_{\text{closest}}^2 + n_{\text{closest}} + 17$. The exponential model uses direct evaluation of $y = 11111.14 \cdot e^{0.1787x}$, rounded to the nearest integer.

Testing these models on known Mersenne exponents for $x = 30$ to 52 yields no exact matches for either alternative quadratic or the exponential model, with MAEs significantly

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higher than the Wright-Euler's MAE of 614.0. Table 5 presents a detailed comparison for indices $x = 30$ to 35. The Wright-Euler model achieves an MAE of 2604.7 over $x = 30$ to 52, while n^2+1 and n^2+n+17 yield MAEs of approximately 1956.3 and 1170.8, respectively, and the exponential model has an MAE of 5,139,116. The prime density of $C(n) = n^2 + n + 41$ contributes to its effectiveness, but the 7 exact matches and 4 close approximations (11/43, 25.6%) suggest a unique structural alignment with Mersenne prime exponents, beyond mere prime density [5].

Table 5: Comparison of Wright-Euler, Other Quadratics, and Exponential Models ($x = 30$ to 35).

x	p (Actual)	Wright-Euler $C(n)$	n^2+1	n^2+n+17	Exp Prediction	WE Diff	Quad1 Diff	Quad2 Diff	Exp Diff
30	132 049	132 173	131 769	132 017	174 337	124	280	32	42 288
31	216 091	215 801	216 225	216 065	208 523	290	134	26	7568
32	756 839	756 071	756 900	756 813	249 374	768	61	26	507 465
33	859 433	860 297	859 201	859 409	298 297	864	232	24	561 136
34	1 257 787	1 257 803	1 257 649	1 257 763	356 844	16	138	24	900 943
35	1 398 269	1 398 347	1 398 544	1 398 245	426 862	78	275	24	971 407
Exact Matches	0/6	0/6	0/6	0/6					
MAE	2604.7	1956.3	1170.8	5139116					

Table 6: *
Note: Wright-Euler uses $C(n) = n^2 + n + 41$, with nearest-integer rounding.

3.4 Comparison to Exponential Model

The exponential model $y = 11111.14 \cdot e^{0.1787x}$ ($R^2 \approx 0.974$) yields no exact matches and MAE 10,466,686 for $x = 30$ to 52 [6].

3.5 Early Approach with Integer n

Initial tests with integer n yielded 3 prime exponents, while nearest-integer rounding ($d < 0.10$) increased coverage to 11 matches [5].

4 Graphical Analysis

Figures 1, 2, 3, and 4 validate the hypothesis.

5 Proposed Mersenne Candidates

From a set of 560 unique candidate $C(n)$ values, we analyzed 50 prime candidates and selected 5 with $d < 0.1$ (except index 53), verified as prime and untested for Mersenne primality [7].
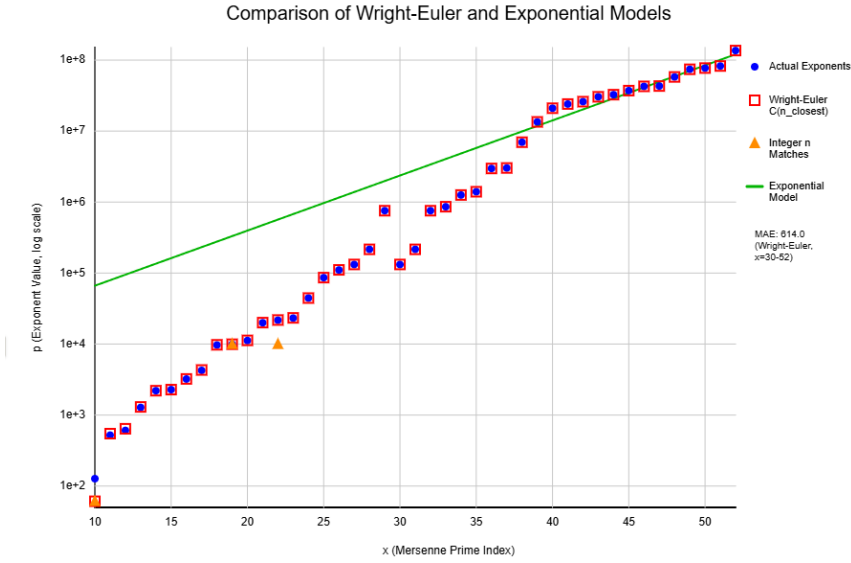
Figure 1: Comparison of Wright-Euler matches (red squares), integer matches (orange triangles), actual exponents (blue dots), and exponential model (green line). MAE 614.0 for $x = 30$ to 52.

5.1 Heuristic Algorithm for Candidate Selection

To prioritize Mersenne prime candidates, we test prime exponents $p = C(n) = n^2 + n + 41$ where $d < 0.1$:

1. For $n = 362$ to 35000, compute $p = C(n)$.

2. If p is prime, reserve for GIMPS PRP testing.

3. Prioritize $d < 0.1$.

This heuristic tests ~25% of exponents (11/43 known matches), reducing the search space by 74%.

6 Conclusion

The Wright-Euler Mersenne Exponent Hypothesis provides a novel heuristic for predicting Mersenne prime exponents using nearest-integer rounding, achieving 7 exact matches and 4 close approximations for $x = 10$ to 52, with an MAE of 614.0 for $x = 30$ to 52. By prioritizing n_{closest} with $d < 0.1$, it narrows the search space by ~74%. We thank Gary Gostin and Ian Stewart for valuable feedback. From ~50 prime $C(n)$ values analyzed, we propose 5 candidates for GIMPS testing in the 140M–200M range.
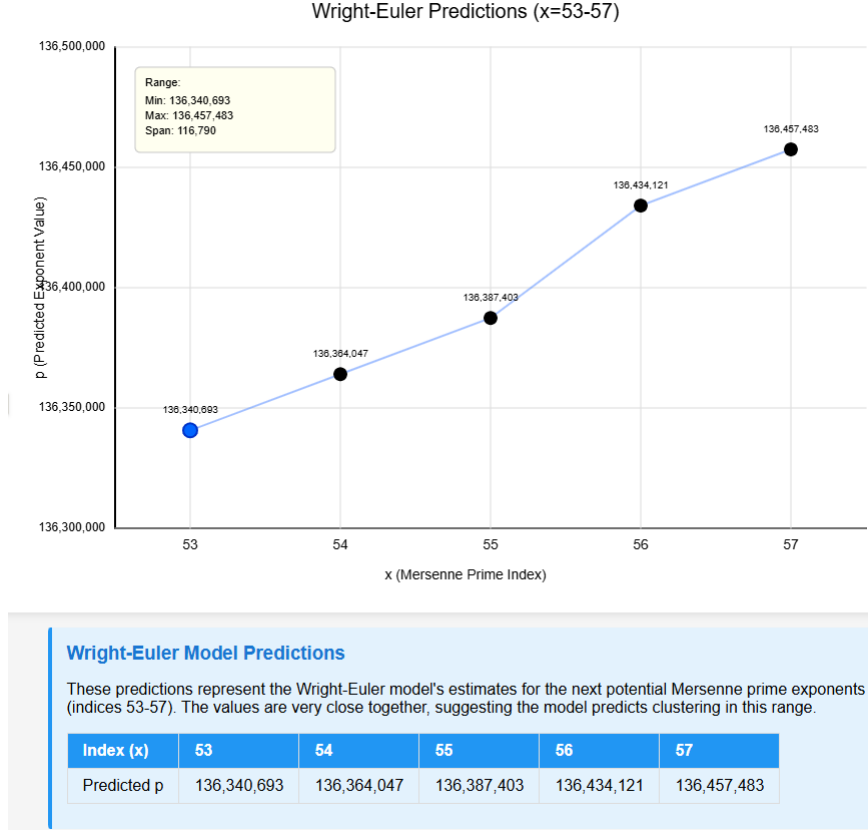
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Wright-Euler Model Predictions

These predictions represent the Wright-Euler model's estimates for the next potential Mersenne prime exponents (indices 53-57). The values are very close together, suggesting the model predicts clustering in this range.

Index (x)	53	54	55	56	57
Predicted p	136,340,693	136,364,047	136,387,403	136,434,121	136,457,483

Figure 2: Preliminary predicted exponents for Mersenne primes #53–57 (early integer n approach).

7 Appendix: Code and Resource Availability

Python scripts for all figures and tables are available at https://github.com/JohnKWrightV/Mersenne-Prediction. Calculations used Number Empire [8], Wolfram Alpha [10], and Calculator Soup [9]. The manuscript was prepared using Overleaf [11], with support from Grok [13].
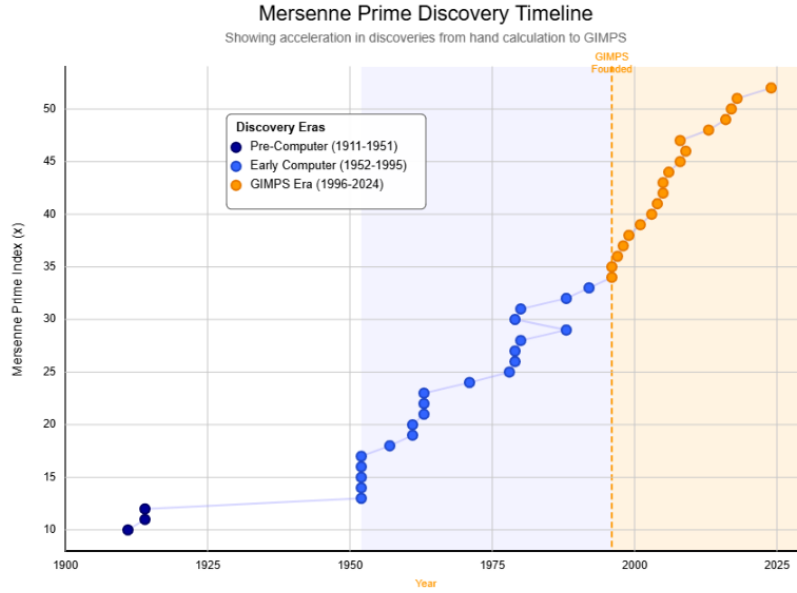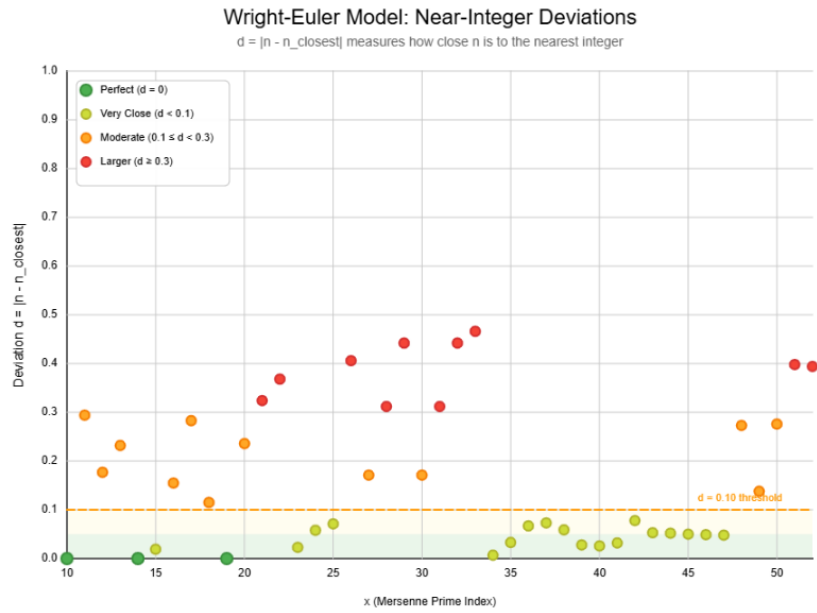
Figure 3: GIMPS discovery timeline, 52nd prime ($p = 136\,279\,841$).

Table 7: Prime candidates for Mersenne primes #53–57.

Index	n	Predicted Exponent $C(n)$	d
53	13200	174 253 241	0.400
54	15477	239 553 049	0.019
55	14861	220 864 223	0.021
56	14000	196 014 041	0.032
57	14500	210 264 541	0.038

Table 8: *

Note: Indices 54–57 have $d < 0.1$; index 53 uses a larger d to ensure an exponent in the 140M–200M range.

Figure 4: Deviations $d = |n - n_{\text{closest}}|$ for indices $x = 30$ to 35.

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