

TMAT101 Calculus

$$\lim_{n \rightarrow \infty} a_n$$

1. exists \Rightarrow sequence converges
2. DNE \Rightarrow sequence diverges

Geometric Series: $\sum_{n=1}^{\infty} ar^{n-1}$

1. $|r| < 1$: converges. $S_{\infty} = \frac{a}{1-r}$
2. $|r| \geq 1$: diverges.

p-Series: $\sum_{n=1}^{\infty} \frac{1}{n^p}$

1. $p > 1$: converges.
2. $p \leq 1$: diverges.

Comparison Test

1. Bigger series converges \Rightarrow smaller series converges.
2. Smaller series diverges \Rightarrow bigger series diverges.

Ratio Test: $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L$

1. $L < 1$: converges.
2. $L > 1$: diverges.
3. $L = 1$: inconclusive.

Integral Test: $\int_1^{\infty} f(x) dx = \lim_{N \rightarrow \infty} \int_1^N f(x) dx$

1. converges $\Rightarrow \sum_{n=1}^{\infty} a_n$ converges.
2. diverges $\Rightarrow \sum_{n=1}^{\infty} a_n$ diverges.

Test for Divergence: $\lim_{n \rightarrow \infty} a_n$

1. DNE or $\neq 0 \Rightarrow \sum_{n=1}^{\infty} a_n$ diverges.
2. $= 0 \xrightarrow{\text{maybe}} \sum_{n=1}^{\infty} a_n$ converges.

Root Test: $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = L$

1. $L < 1$: converges.
2. $L > 1$: diverges.
3. $L = 1$: inconclusive.