TMA1201 Discrete Structures & Probability

Sets

$|A|$ cardinality: # of element

$\subset$ proper subset: $\subseteq$ & $\neq$

$P(A)$ power set: set of all subsets

$|P(A)| = 2^{|A|}$

$A \Delta B = (A-B) \cup (B-A) = (A \cup B) - (A \cap B)$

$A = (A-B) \cup (A \cap B)$

Relations

$R \circ T = \{(x \text{ of } T, y \text{ of } R)\}$

	T	F
Reflexive	$\forall x \in A.\ xRx.$	$\exists x \in A.\ x \not R x.$
Symmetric	$\forall x, y \in A.$ If xRy, then yRx.	$\exists x, y \in A.\ xRy$ but $y \not R x$.
Antisymmetric	$\forall x, y \in A.$ If xRy & yRx, then $x = y$.	$\exists x, y \in A, x \neq y.$ xRy & yRx.
Transitive	$\forall x, y, z \in A.$ If xRy & yRz, then xRz.	$\exists x, y, z \in A.\ xRy$ & yRx but $x \not R z$.

Equivalence Relation: reflexive, symmetric, transitive.

Partial order: reflexive, antisymmetric, transitive.

Functions

Function: ① Every $\in$ in domain is related to an $\in$ in codomain.

② No $\in$ in domain is related to >1 $\in$ in codomain.

If a function is one-to-one and onto, then its inverse function exists.

If range of $g \subseteq$ domain of f, then $f \circ g$ exists.

Logic

$p \to q$: False when p is T and q is F.

- converse $q \to p$
- inverse $\neg p \to \neg q$
- contrapositive $\neg q \to \neg p$

contingency: neither tautology nor contradiction

Negation	Equivalent Statement
$\neg \exists x P(x)$	$\forall x \neg P(x)$
$\neg \forall x P(x)$	$\exists x \neg P(x)$

$\forall$ specify $\to$

$\exists$ specify $\wedge$

	T	F
$\forall x \forall y\, P(x,y)$	Every pair (x,y) is T.	A pair (x,y) is F.
$\exists x \forall y\, P(x,y)$	An x is T for every y.	Every x is F for a y.
$\forall x \exists y\, P(x,y)$	For all x there's a y.	An x is F for every y.
$\exists x \exists y\, P(x,y)$	There's a pair (x,y) T.	All pair (x,y) is F.

Induction

Induction: ① IB: Prove $P(1)$ is T.

② IH: Assume $P(k)$ is T.

③ IS: Prove $P(k+1)$ is T.

Strong Induction:

① IB: Prove $P(1)$ is T.

② IH: Assume $P(n)$ is T for $n = 1, 2, \ldots, k$.

③ IS: Prove $P(k+1)$ is T.

Algorithm Complexity

$f(n)$ is

- $O(g(n))$ if $|f(n)| \leq B|g(n)|$ for $n > b$
- $\Omega(g(n))$ if $|f(n)| \geq A|g(n)|$ for $n > a$
- $\Theta(g(n))$ if $|f(n)|$ is $\Omega(g(n))$ & $O(g(n))$ for $n > k$

Graph

Handshaking Lemma: # of $\deg(v) = 2 \times$ # of e

Trail: Walk, no repeated edge.

Path: Trail, no repeated vertex.

Closed walk: Walk, start/end at same vertex.

Circuit/Cycle: Closed walk, no repeated edge.

Simple circuit: Circuit, no repeated vertex except the first/last.

Euler trail: Walk, pass every edge exactly once, pass every vertex at least once.

Euler circuit: Euler trail, closed walk.

Euler's Theorem: Connected graph with

① ≥ 3 odd degree vertices has no Euler trail.

② an odd number degree vertex has no Euler Circuit.

Hamitonian path: Path, pass every vertex exactly once.

Hamitonian Circuit: Simple circuit, pass every vertex exactly once.

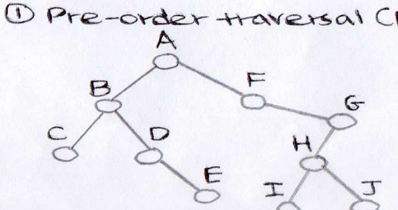
Dirac's Theorem: Simple graph with $n \geq 3$ vertices has HC if $\deg(v) \geq \frac{n}{2}$ for every vertex.

Ore's Theorem: Simple graph with $n \geq 3$ vertices has HC if $\deg(v) + \deg(w) \geq n$ for each pair of non-adjacent vertices v and w.

Tree

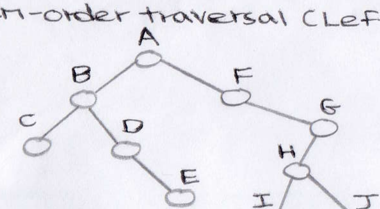
Tree traversal:

① Pre-order traversal (Root-Left-Right)



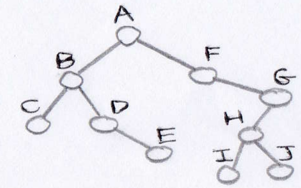
ABCDEFGHIJ

② In-order traversal (Left-Root-Right)

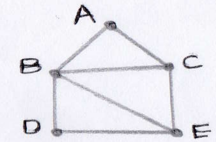


CBDEAFIHJG

③ Post-order traversal (Left-Right-Root)



CEDBIJHGFA



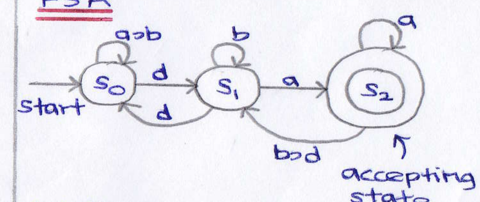
① Depth-first search (DFS)

Stack	Tree
A	—
AB	AB
ABC	AB, BC
ABCE	AB, BC, CE
ABCED	AB, BC, CE, ED
ABCE	AB, BC, CE, ED
ABC	AB, BC, CE, ED
AB	AB, BC, CE, ED
A	AB, BC, CE, ED
—	AB, BC, CE, ED

② Breadth-first search (BFS)

Queue	Tree
A	—
AB	AB
ABC	AB, AC
BC	AB, AC
BCD	AB, AC, BD
BCDE	AB, AC, BD, BE
CDE	AB, AC, BD, BE
DE	AB, AC, BD, BE
E	AB, AC, BD, BE
—	AB, AC, BD, BE

FSA



$L(M) =$