

① Number Systems and Codes

Number Systems

Decimal $\xrightarrow[\text{expand}]{\text{repeated } \div}$ Binary
Octal
Hexadecimal

Fraction - repeated multiplication $\uparrow \downarrow$

Binary: $+- \times \div$

Octal - Binary - Hexadecimal
 $+-$ $+-$

1 Octal digit = 3 Binary digit

1 Hex digit = 4 Binary digit

Codes

① Decimal - NBCD (8421)

15 = 0001 0101

② Decimal - 2421

15 = 0001 1011

③ Decimal - XS3

Decimal + 3(0011) → 8421

Decimal ← 8421 - 3(0011)

④ Binary - Gray code

B 0100 G 0010
↓
G 0110 B 0011
B 0110 G 1101
↓
G 0101 B 1001

Error Detection

Parity Protocol	1's	Parity Bit
Odd	Odd	0
	Even	1
Even	Odd	1
	Even	0

② Data Representation and Arithmetic

Sign-Magnitude

LSB	
0	+
1	-

+0 and -0.

Ones Complement

Complement to get -ve. +0 and -0.

Twos Complement

Complement and add 1 to get -ve.

Overflow Rule: two numbers of same sign added; result of opposite sign.

Subtraction Rule: $A - B = A + (-B)$

Floating Point:

1 bit	8 bits	23 bits
Sign Bit	Biased Exponent	Significand
	Actual Exponent + 127 → Binary	

$+- \times \div$ (make them have same exponent)

M = 0101, Q = 1010, -M = 1011
Booth Algorithm: $5 \times (-6) = -30$

M Register	A Register	Q Register	Q ₋₁
0101	0000	1010	0
	0000	0101	0 Shift
	+1011		
	1011	0101	0 A ← A-M
	1101	1010	1 Shift
	+0101		
	0010	1010	1 A ← A+M
	0001	0101	0 Shift
	+1011		
	1100	0101	0 A ← A-M
	1110	0010	1 Shift

Product

Finding Product

1110 0010

1's Complement

→ 0001 1101

2's Complement

→ 0001 1110 (30)

∴ Sign bit is 1

∴ -30

0010 1010

∴ Sign bit is 0

∴ 42

③ Introduction to Digital Logic and Boolean Algebra

Logic Gates

NOT

AND

OR

NAND

NOR

XOR

XNOR

\overline{AB} [00, 01, 10]

$\overline{A+B}$ [00]

$A \oplus B$ [10, 01]

$\overline{A \oplus B}$ [00, 11]

Boolean Algebra

$A+0=A$

$A \cdot A=A$

$A+1=1$

$A \cdot \overline{A}=0$

$A \cdot 0=0$

$\overline{\overline{A}}=A$

$A \cdot 1=A$

$A+AB=A$

$A+A=A$

$A+\overline{A}B=A+B$

$A+\overline{A}=1$

$(A+B)(A+C)=A+BC$

DeMorgan's Theorem

$\overline{X \cdot Y} = \overline{X} + \overline{Y}$

$\overline{X+Y} = \overline{X} \cdot \overline{Y}$

4 Design of Combinational Logic Circuits

Product of Sums & Sum of Products

		Output	A	\bar{A}	Term
SOP	AND-OR	1	1	0	Product/Min
POS	OR-AND	0	0	1	Sum/Max

Minimal to Standard Form:

$$F = (A+B+C')(CA'+C) \rightarrow X$$

$$= (A+B+C')(A'+C+B \cdot B')$$

$$= (A+B+C')(A'+C+B)$$

$\overline{\text{SOP}} \rightarrow \text{NAND-NAND}$

$\overline{\text{POS}} \rightarrow \text{NOR-NOR}$

Karnaugh Map

- ① Grouping 1's (SOP)/0's (POS) - 1, 2, 4, 8
- ② OR (SOP) / AND (POS) the terms.
- ③ Don't care: X could be 1/0.

A \ BC	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

Design Logic Circuit

- ① Truth Table.
- ② SOP (AND-OR) / POS (OR-AND).
- ③ Simplify - Boolean Algebra / K-Map.
- ④ Implement with logic gates.

5 Standard Combinational Logic Circuits

Adder / Subtractor

Half-Adder

A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$S = \bar{A}B + A\bar{B} = A \oplus B$$

$$C = AB$$

Full-Adder

A	B	C	S	C
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

A \ BC	00	01	11	10
0	0	1	0	1
1	1	0	1	0

$$S = A\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}C + \bar{A}BC$$

$$= A \oplus B \oplus C$$

A \ BC	00	01	11	10
0	0	0	1	0
1	0	1	0	1

$$C = AC + BC + AB$$

Half-Subtractor

A	B	D	B
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

$$D = \bar{A}B + A\bar{B} = A \oplus B$$

$$B = \bar{A}B$$

Full-Subtractor

A	B	C	D	B
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

A \ BC	00	01	11	10
0	0	1	0	1
1	1	0	1	0

$$D = A\bar{B}\bar{C} + \bar{A}\bar{B}C + ABC + \bar{A}B\bar{C}$$

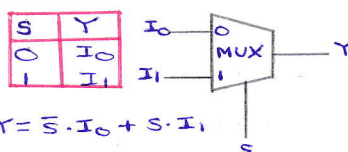
$$= A \oplus B \oplus C$$

A \ BC	00	01	11	10
0	0	1	1	1
1	0	0	1	0

$$B = \bar{A}C + BC + \bar{A}B$$

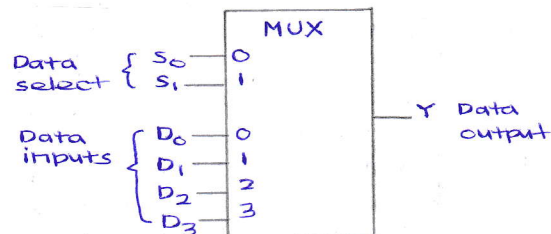
Multiplexer / Demultiplexer

2:1 MUX



4:1 MUX

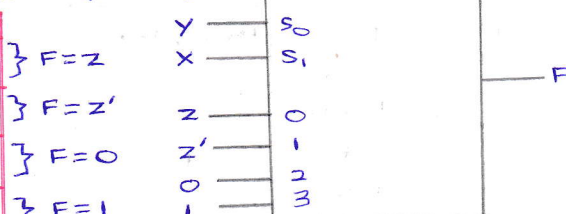
S_1	S_2	Y
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3



Implementation of Boolean Function using MUX

$$F = x'y'z + x'yz' + xyz' + xyz$$

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1



1:4 DEMUX

S_1	S_2	Y_0	Y_1	Y_2	Y_3
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

BCD Addition

- ① IF 4-bit sum > 9, add 6 (0110).
- ② IF a carry results after added 6, add carry to next 4-bit.

$$\begin{array}{r} 478 \\ + 137 \\ \hline 615 \end{array}$$

$$\begin{array}{r} 0100 \quad 0111 \quad 1000 \\ 0001 \quad 0011 \quad 0111 \\ \hline 0110 \quad 1011 \quad 1111 \\ 0000 \quad 0110 \quad 0110 \\ \hline 0110 \quad 0001 \quad 0101 \end{array}$$

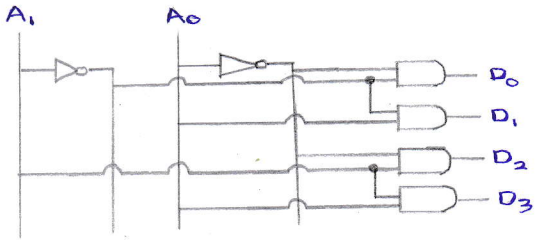
Decoder / Encoder

- ① Active-LOW output
 - only one low output
 - use NAND and NOT
- ② Active-HIGH output
 - only one high output
 - use AND and NOT

2x4 Decoder

- ① Active-HIGH output

A_1	A_0	D_0	D_1	D_2	D_3
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1



- ② Active-LOW output

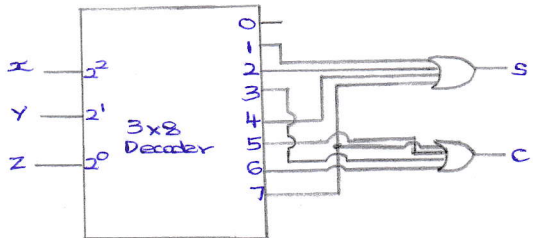
A ₁	A ₀	D ₀	D ₁	D ₂	D ₃
0	0	0	1	1	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	1	1	1	0

Decoder with Logic Gates

Full Address:

$$S(x, y, z) = \sum m(1, 2, 4, 7)$$

$$C(x, y, z) = \sum m(3, 5, 6, 7)$$



8x3 Encoder (Octal-to-Binary Encoder)

D_7	D_6	D_5	D_4	D_3	D_2	D_1	D_0	A_2	A_1	A_0
0	0	0	0	0	0	0	1		0	0
0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	1	0	0	0	1	0
0	0	0	0	1	0	0	0	0	1	1
0	0	0	1	0	0	0	0	1	0	0
0	0	1	0	0	0	0	0	1	0	1
0	1	0	0	0	0	0	0	1	1	0
1	0	0	0	0	0	0	0	1	1	1

4x2 Priority Encoder

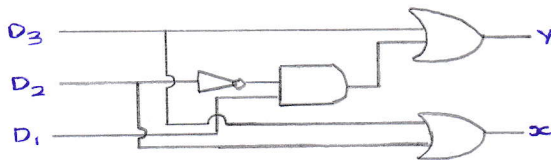
D_0	D_1	D_2	D_3	x	y
0	0	0	0	x	x
1	0	0	0	0	0
x	1	0	0	0	1
x	x	1	0	1	0
x	x	x	1	1	1

$D_0 D_1 \backslash D_2 D_3$	00	01	11	10
00	X	1	1	1
01		1	1	1
11			1	1
10			1	1

$$x = D_2 + D_3$$

$D_0 D_1 \backslash D_2 D_3$	00	01	11	10
00	X	1	1	1
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

$$y = D_3 + D_1 D_2'$$



D_0	D_1	D_2	D_3	x	y
0	0	0	0	x	x
1	0	0	0	0	0
x	1	0	0	0	1
x	x	1	0	1	0
x	x	x	1	1	1