

Introduction to Homomorphic Encryption

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Privacy in the Cloud

- Many individuals and companies are outsourcing their storage and computing needs to the cloud
- This developments raise many privacy issues
 - Clients no longer have direct control of their data
- Privacy issues
 - Data privacy
 - Function privacy
 - Query privacy
 - Server privacy
- Current encryption schemes only guarantee data privacy
 - Data becomes unusable

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Homomorphism

- A homomorphism is a map (function) between two algebraic structures of the same type, that preserves the operation of the structures.

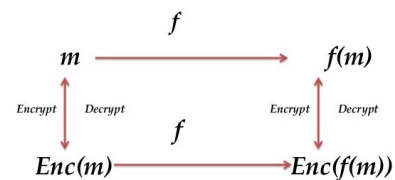
$$f : A \rightarrow B$$

$$f(x * y) = f(x) * f(y)$$

- The map f is a homomorphism or is said to preserve the operation $*$

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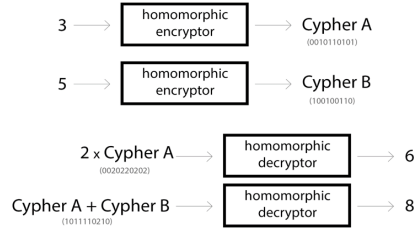
Homomorphic Encryption (HE)



- $ENC()$ – Homomorphic Encryption function
- $f()$ – Function that is preserved after the application of $ENC()$

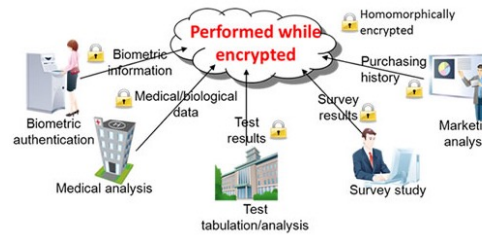
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Homomorphic Encryption



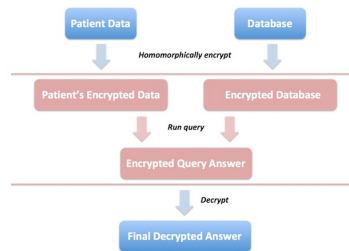
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Homomorphic Encryption



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Homomorphic Encryption – Use Case



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Types of Homomorphic Encryption Schemes

- Partially Homomorphic Encryption (PHE)
 - Supports only addition or multiplication operations on plaintext
- Fully Homomorphic Encryption (FHE)
 - Supports any arithmetic operation including addition and multiplication on plaintext
- Somewhat Homomorphic Encryption
 - Fully homomorphic encryption is only possible for fixed number of calculations after which the system becomes unstable

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Comparison of PHE Schemes

PHE Scheme	Supported Operations
RSA	Multiplication
ElGamal	Multiplication
Goldwasser-Micali	Addition
Paillier	Addition, Constant Multiplication

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FHE Schemes

- First FHE scheme was proposed in 2009 by **Craig Gentry**
 - PhD thesis at Stanford University
 - Scheme was based on Lattice-based cryptography
- In 2010 van Dijk et.al. proposed an improvement to Gentry's scheme
 - FHE over integers
 - Utilizes most of the ideas from Gentry's scheme but replaces Lattice-based algebraic structures with integers
- FHE schemes based on LWE (Learning with Errors)
 - Learning with errors is a problem in machine learning that is hard to solve

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FHE Schemes

- Not very efficient
- Only practical on small amounts of data
- In Dec 2020, IBM launched its homomorphic encryption service

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