Chapter 8

Random Bit Generation and Stream Ciphers

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Random Numbers

- A number of network security algorithms and protocols based on cryptography make use of random binary numbers:
  - Key distribution and reciprocal authentication schemes
  - Session key generation
  - Generation of keys for the RSA public-key encryption algorithm
  - Generation of a bit stream for symmetric stream encryption

There are two distinct requirements for a sequence of random numbers:

- Randomness
- Unpredictability

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Randomness

- The generation of a sequence of allegedly random numbers being random in some well-defined statistical sense has been a concern

Two criteria are used to validate that a sequence of numbers is random:

Uniform distribution
- The frequency of occurrence of ones and zeros should be approximately equal

Independence
- No one subsequence in the sequence can be inferred from the others

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Unpredictability

- The requirement is not just that the sequence of numbers be statistically random, but that the successive members of the sequence are unpredictable
- With "true" random sequences each number is statistically independent of other numbers in the sequence and therefore unpredictable
  - True random numbers have their limitations, such as inefficiency, so it is more common to implement algorithms that generate sequences of numbers that appear to be random
  - Care must be taken that an opponent not be able to predict future elements of the sequence on the basis of earlier elements

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Pseudorandom Numbers

- Cryptographic applications typically make use of algorithmic techniques for random number generation
- These algorithms are deterministic and therefore produce sequences of numbers that exhibit random properties
- If the algorithm is good, the resulting sequences will pass many tests of randomness and are referred to as *pseudorandom numbers*

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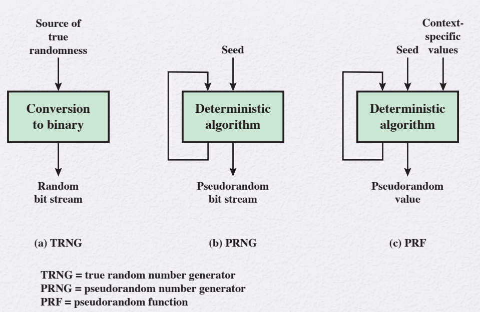

Figure 8.1 Random and Pseudorandom Number Generators

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True Random Number Generator (TRNG)

- Takes as input a source that is effectively random
- The source is referred to as an *entropy source* and is drawn from the physical environment of the computer
  - Includes things such as keystroke timing patterns, disk electrical activity, mouse movements, and instantaneous values of the system clock
  - The source, or combination of sources, serve as input to an algorithm that produces random binary output
- The TRNG may simply involve conversion of an analog source to a binary output
- The TRNG may involve additional processing to overcome any bias in the source

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Pseudorandom Number Generator (PRNG)

- Takes as input a fixed value, called the *seed*, and produces a sequence of output bits using a deterministic algorithm
  - Quite often the seed is generated by a TRNG
- The output bit stream is determined solely by the input value or values, so an adversary who knows the algorithm and the seed can reproduce the entire bit stream
- Other than the number of bits produced there is no difference between a PRNG and a PRF

Two different forms of PRNG

Pseudorandom number generator
- An algorithm that is used to produce an open-ended sequence of bits
- Input to a symmetric stream cipher is a common application for an open-ended sequence of bits

Pseudorandom function (PRF)
- Used to produce a pseudorandom string of bits of some fixed length
- Examples are symmetric encryption keys and nonces

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PRNG Requirements

- The basic requirement when a PRNG or PRF is used for a cryptographic application is that an adversary who does not know the seed is unable to determine the pseudorandom string
- The requirement for secrecy of the output of a PRNG or PRF leads to specific requirements in the areas of:
  - Randomness
  - Unpredictability
  - Characteristics of the seed

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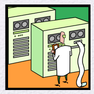
Randomness

- The generated bit stream needs to appear random even though it is deterministic
- There is no single test that can determine if a PRNG generates numbers that have the characteristic of randomness
  - If the PRNG exhibits randomness on the basis of multiple tests, then it can be assumed to satisfy the randomness requirement
- NIST SP 800-22 specifies that the tests should seek to establish three characteristics:
  - Uniformity
  - Scalability
  - Consistency



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Randomness Tests

- SP 800-22 lists 15 separate tests of randomness

Frequency test
- The most basic test and must be included in any test suite
- Purpose is to determine whether the number of ones and zeros in a sequence is approximately the same as would be expected for a truly random sequence

Runs test
- Focus of this test is the total number of runs in the sequence, where a run is an uninterrupted sequence of identical bits bounded before and after with a bit of the opposite value
- Purpose is to determine whether the number of runs of ones and zeros of various lengths is as expected for a random sequence

Maurer's universal statistical test
- Focus is the number of bits between matching patterns
- Purpose is to detect whether or not the sequence can be significantly compressed without loss of information. A significantly compressible sequence is considered to be non-random

Three tests

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Unpredictability

- A stream of pseudorandom numbers should exhibit two forms of unpredictability:
  - Forward unpredictability
    - If the seed is unknown, the next output bit in the sequence should be unpredictable in spite of any knowledge of previous bits in the sequence
  - Backward unpredictability
    - It should not be feasible to determine the seed from knowledge of any generated values
    - No correlation between a seed and any value generated from that seed should be evident
    - Each element of the sequence should appear to be the outcome of an independent random event whose probability is 1/2
- The same set of tests for randomness also provides a test of unpredictability
  - A random sequence will have no correlation with a fixed value (the seed)

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Seed Requirements

- The seed that serves as input to the PRNG must be secure and unpredictable
- The seed itself must be a random or pseudorandom number
- Typically, the seed is generated by TRNG

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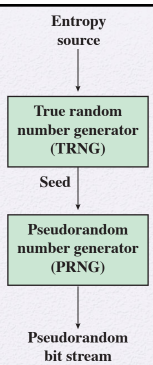

Figure 8.2 Generation of Seed Input to PRNG

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Algorithm Design

- Algorithms fall into two categories:
  - Purpose-built algorithms
    - Algorithms designed specifically and solely for the purpose of generating pseudorandom bit streams
  - Algorithms based on existing cryptographic algorithms
    - Have the effect of randomizing input data

Three broad categories of cryptographic algorithms are commonly used to create PRNGs:
• Symmetric block ciphers • Asymmetric ciphers • Hash functions and message authentication codes

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Linear Congruential Generator

- An algorithm first proposed by Lehmer that is parameterized with four numbers:

m	the modulus	$m > 0$
a	the multiplier	$0 < a < m$
c	the increment	$0 \leq c < m$
X_0	the starting value, or seed	$0 \leq X_0 < m$

- The sequence of random numbers $\{X_n\}$ is obtained via the following iterative equation:

$$X_{n+1} = (aX_n + c) \bmod m$$

- If m , a , c , and X_0 are integers, then this technique will produce a sequence of integers with each integer in the range $0 \leq X_n < m$
- The selection of values for a , c , and m is critical in developing a good random number generator

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Blum Blum Shub (BBS) Generator

- Has perhaps the strongest public proof of its cryptographic strength of any purpose-built algorithm
- Referred to as a *cryptographically secure pseudorandom bit generator* (CSPRBG)
  - A CSPRBG is defined as one that passes the *next-bit-test* if there is not a polynomial-time algorithm that, on input of the first k bits of an output sequence, can predict the $(k + 1)$st bit with probability significantly greater than 1/2
- The security of BBS is based on the difficulty of factoring n

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Blum Blum Shub (BBS) Generator

- Seed generation algorithm
  - Choose p and q, such that $p \equiv q \equiv 3 \pmod 4$
  - Find $n = p * q$
  - Choose random s, such that s is relatively prime to n i.e. neither p or q is is a factor of s
  - Use **s** as the seed of the algorithm
- We can choose several seed values for each value of n
- p and q are usually chosen to be large prime numbers
- Breaking the cipher requires factoring n into p and q (Integer Factorization Problem)

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Blum Blum Shub (BBS) Generator

- Random number generation algorithm
  - $x_0 = s^2 \bmod n$
  - For $i = 1$ to ∞
    - $x_i = (x_{i-1})^2 \bmod n$
    - $B_i = x_i \bmod 2$ (B_i is a bit value)

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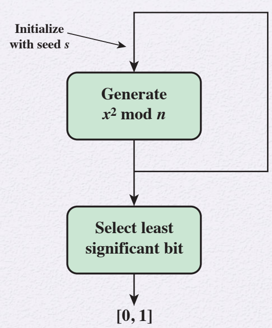

Figure 8.3 Blum Blum Shub Block Diagram

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Table 8.1 Example Operation of BBS Generator

i	X_i	B_i
0	20749	
1	143135	1
2	177671	1
3	97048	0
4	89992	0
5	174051	1
6	80649	1
7	45663	1
8	69442	0
9	186894	0
10	177046	0
11	137922	0
12	123175	1
13	8630	0
14	114386	0
15	14863	1
16	133015	1
17	106065	1
18	45870	0
19	137171	1
20	48060	0

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Table 8.5

	Pseudorandom Number Generators	True Random Number Generators
Efficiency	Very efficient	Generally inefficient
Determinism	Deterministic	Nondeterministic
Periodicity	Periodic	Aperiodic

Comparison of PRNGs and TRNGs

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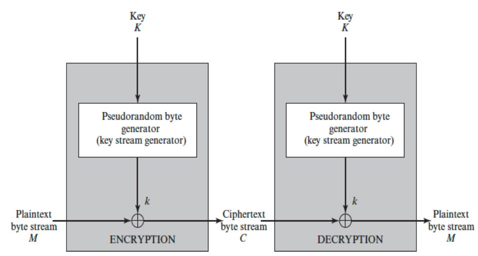

Figure 8.7 Stream Cipher Diagram

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Stream Cipher Design Considerations

The encryption sequence should have a large period
- A pseudorandom number generator uses a function that produces a deterministic stream of bits that eventually repeats; the longer the period of repeat the more difficult it will be to do cryptanalysis

The keystream should approximate the properties of a true random number stream as close as possible
- There should be an approximately equal number of 1s and 0s
- If the keystream is treated as a stream of bytes, then all of the 256 possible byte values should appear approximately equally often

A key length of at least 128 bits is desirable
- The output of the pseudorandom number generator is conditioned on the value of the input key
- The same considerations that apply to block ciphers are valid

With a properly designed pseudorandom number generator a stream cipher can be as secure as a block cipher of comparable key length
- A potential advantage is that stream ciphers that do not use block ciphers as a building block are typically faster and use far less code than block ciphers

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RC4

- Designed in 1987 by Ron Rivest for RSA Security
- Variable key size stream cipher with byte-oriented operations
- Based on the use of a random permutation
- Eight to sixteen machine operations are required per output byte and the cipher can be expected to run very quickly in software
- RC4 is used in the WiFi Protected Access (WPA) protocol that are part of the IEEE 802.11 wireless LAN standard
- It is optional for use in Secure Shell (SSH) and Kerberos
- RC4 was kept as a trade secret by RSA Security until September 1994 when the RC4 algorithm was anonymously posted on the Internet on the Cypherpunks anonymous remailers list

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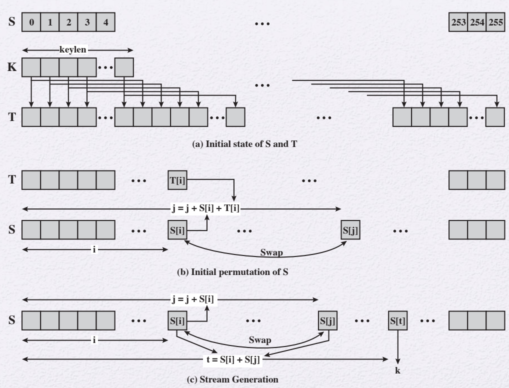

Figure 8.7 RC4

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Strength of RC4

- A fundamental vulnerability was revealed in the RC4 key scheduling algorithm that reduces the amount of effort to discover the key
- Recent cryptanalysis results exploit biases in the RC4 keystream to recover repeatedly encrypted plaintexts
- As a result of the discovered weaknesses the IETF issued RFC 7465 prohibiting the use of RC4 in TLS
- In its latest TLS guidelines, NIST also prohibited the use of RC4 for government use

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Summary

- Explain the concepts of randomness and unpredictability with respect to random numbers
- Present an overview of requirements for pseudorandom number generators
- Present an overview of stream ciphers and RC4

- Understand the differences among true random number generators, pseudorandom number generators, and pseudorandom functions
- Explain how a block cipher can be used to construct a pseudorandom number generator

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